

Monte Carlo Studies of the Emergent Spacetime in the Polarized Type IIB Matrix Model

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The IKKT model (or the type IIB matrix model) has been investigated as a promising nonperturbative formulation of superstring theory. One of the recent developments concerning this model is the discovery of the dual supergravity solution corresponding to the model obtained after supersymmetry-preserving mass deformation, which is dubbed the polarized IKKT model. Here we perform Monte Carlo simulations of this model in the case of matrix size $N = 2$ for a wide range of the deformation parameter Ω . While we reproduce precisely the known result for the partition function obtained by the localization method developed for supersymmetric theories, we also calculate the observables, which were not accessible by the previous work, in order to probe the spacetime structure emergent from the dominant matrix configurations. In particular, we find that the saddle point corresponding to the original IKKT model is smoothly connected to the saddle represented by the fuzzy sphere dominant at large Ω , whereas the dominant configurations become diverging commuting matrices at small Ω .

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Introduction—The IKKT model (or the type IIB matrix model) [1,2] was originally proposed as a matrix regularization of type IIB superstring theory, which is conjectured to provide a constructive definition of second-quantized string theory in the large- N limit. It takes the form of the large- N reduced model of 10-dimensional $\mathcal{N} = 1$ super Yang-Mills theory [3–6], which can also be viewed as the effective theory of D instantons [7,8]. As a nonperturbative string-theoretic model without a predefined spacetime background, it offers a promising approach to studying the dynamical emergence of spacetime [9,10]. A key question, for example, is whether our $(3 + 1)$ -dimensional spacetime emerges as a dominant eigenvalue distribution of the matrices [11–21] from the underlying $(9 + 1)$ -dimensional superstring theory.

One of the issues that has not been explored until recently in this model is the gauge-gravity duality [22,23], which can be derived from superstring theory by considering coinciding N D p branes as a background. The low-energy effective theory of the D p branes is given by $(p + 1)$ -dimensional $U(N)$ super Yang-Mills theory, which can be thought of as a holographic description of the

supergravity solution in the large- N and strong coupling limits. The IKKT model corresponds to the extreme case $p = -1$, where the super Yang-Mills theory reduces to a matrix integral without a time coordinate unlike the BFSS model [24], which corresponds to $p = 0$. An important consequence of this is that there is no time derivative in the action, which implies that the Yang-Mills coupling constant can be absorbed by rescaling the matrices properly. The corresponding supergravity solution has only been discussed based on the D-instanton charge [25–27].

Recently it has been realized that these difficulties in the gauge-gravity duality for $p = -1$ can be overcome [28,29] by considering the *polarized IKKT matrix model*, which is obtained by applying a SUSY-preserving deformation [30] to the original model. The deformation introduces a mass scale Ω , which explicitly breaks the $SO(10)$ symmetry down to $SO(3) \times SO(7)$ and turns classical solutions into $\mathfrak{su}(2)$ representations. Following a similar analysis [31,32] in the SUSY-deformed BFSS (or the BMN) matrix model [33], a family of supergravity solutions preserving 16 supercharges has been identified as the holographic dual of the classical solutions in the polarized IKKT model [28,29].

For $\Omega \gg 1$, the path integral is dominated by the classical solution that corresponds to the N -dimensional irreducible representation of $\mathfrak{su}(2)$, which may be viewed as the maximal fuzzy sphere [34,35]. From the string-theoretic point of view, this can be understood as the Myers effect [36], where D instantons are polarized into a D1 brane with an S^2 worldvolume in the presence of a three-form flux [37], hence the name of the model.

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Another important aspect of the polarized IKKT matrix model is that the partition function can be calculated exactly [38] by the SUSY localization method [39] analogously to the BMN model [40]. In particular, it was found that the partition function diverges as $Z \sim \Omega^{-2(N-1)}$ in the $\Omega \rightarrow 0$ limit and does not converge to that of the original IKKT model, which is known to be finite [41–44]. Also it was found that there is a phase transition at $\Omega \sim \mathcal{O}(N^{-1/2})$ by investigating the model obtained by the localization method numerically [45].

In this paper, we perform Monte Carlo simulations of the polarized IKKT model in the case of matrix size $N = 2$ for a wide region of Ω . In particular, we capture the competing saddle points reliably by using parallel tempering, which was not done in the previous preliminary studies [46,47]. This plays a crucial role in reproducing the partition function obtained by the localization method precisely. Furthermore, we probe directly the spacetime structure emergent from the dominant matrix configurations, which is not accessible by the localization method. Thus, our results provide a complete understanding of the nature of the transition at intermediate Ω as well as the singularity in the $\Omega \rightarrow 0$ limit.

The polarized IKKT matrix model—The action of the Euclidean IKKT model is given by

$$S_{\text{IKKT}} = \text{tr} \left\{ -\frac{1}{4} [A_\mu, A_\nu]^2 - \frac{i}{2} \Psi_\alpha (\mathcal{C} \Gamma^\mu)_{\alpha\beta} [A_\mu, \Psi_\beta] \right\}, \quad (1)$$

where A_μ and Ψ_α are 10 bosonic and 16 fermionic $N \times N$ traceless hermitian matrices, which transform as a vector and a Majorana-Weyl spinor in 10D. Γ^μ are the 10-dimensional Euclidean gamma matrices after Weyl projection, and $\mathcal{C}$ is the charge conjugation matrix. The polarized IKKT model is obtained by adding the terms [30]

$$S_\Omega = \text{tr} \left\{ \frac{\Omega^2}{4^3} (3A_a^2 + A_I^2) + i\Omega [A_1, A_2] A_3 - \frac{\Omega}{8} \Psi_\alpha (\mathcal{C} \Gamma^{123})_{\alpha\beta} \Psi_\beta \right\} \quad (2)$$

corresponding to the SUSY deformation, where $a = 1, 2, 3$ and $I = 4, \dots, 10$ denote the polarized and unpolarized directions, respectively, and $\Gamma^{123} \equiv \Gamma^1 \Gamma^2 \Gamma^3$. The sign of Ω is irrelevant since it can be absorbed by $A_\mu \rightarrow -A_\mu$ and $\Psi_\alpha \rightarrow i\Psi_\alpha$.

At $\Omega \gg 1$, the path integral is dominated by the classical solution,

$$A_a = \frac{3}{8} \Omega J_a, \quad A_I = 0, \quad \Psi_\alpha = 0, \quad (3)$$

where J_a is the N -dimensional irreducible representation of $\mathfrak{su}(2)$, which represents a single fuzzy sphere in the polarized directions.

In the $\Omega \rightarrow 0$ limit, the polarized IKKT model has a diverging partition function, and it does not reduce to the original IKKT model as already mentioned. It was recently pointed out [38] that this singularity is due to the commuting matrix configurations.

In the original IKKT model, the classical solutions are indeed given by commuting matrices, which can be represented by diagonal matrices,

$$A_\mu = \text{diag}(x_\mu^{(1)}, \dots, x_\mu^{(N)}), \quad \text{where } \sum_{i=1}^N x_\mu^{(i)} = 0, \quad (4)$$

using $\text{SU}(N)$ symmetry. By integrating out the off-diagonal components at the one-loop level, one may attempt to obtain a low-energy effective theory [2], which is valid when all the diagonal components are separated from each other. Because of supersymmetry, the one-loop contributions from the bosonic and fermionic off-diagonal components cancel each other. However, the fermionic diagonal components do not have quadratic terms, which makes the integration over them nontrivial.

The situation simplifies for $\Omega \neq 0$ since the $\mathcal{O}(\Omega)$ fermionic mass term in (2) induces the quadratic terms of the fermionic diagonal components and one can integrate them out trivially. Thus, the one-loop effective theory becomes [38]

$$Z_{1\text{-loop}} = \Omega^{8(N-1)} \int dx \exp \left\{ -\frac{\Omega^2}{2^6} \left(3(x_a^{(i)})^2 + (x_I^{(i)})^2 \right) \right\}. \quad (5)$$

Saddle-point equation—In order to discuss the matrix configurations that dominate the path integral in the polarized IKKT model, let us first integrate out the fermionic matrices to obtain the bosonic integral,

$$Z(\Omega) = \int dA \text{Pf}[M(A)] e^{-S_b(A)} = \int dA e^{-S_{\text{eff}}(A)}, \quad (6)$$

where S_b is the bosonic part of the action and $\text{Pf}(M)$ denotes the Pfaffian of the antisymmetric matrix $M(A)$ that appears as the kernel in the fermionic part of the action. The effective action $S_{\text{eff}}(A)$ in Eq. (6) is given by

$$S_{\text{eff}}(A) = S_b(A) - \log \text{Pf}[M(A)], \quad (7)$$

and the saddle-point equation reads

$$0 = \frac{dS_{\text{eff}}}{dA} = \frac{dS_b}{dA} - \frac{1}{2} \text{Tr} \left(M^{-1} \frac{dM}{dA} \right). \quad (8)$$

In what follows, we discuss the solutions to Eq. (8) in the $N = 2$ case, for which the Pfaffian is real and positive semidefinite. This makes the saddle points real and enables

us to perform Monte Carlo simulations of the model (6) without suffering from the sign problem, which is not the case for larger N .

Fuzzy sphere saddle—Let us first consider the original IKKT model ($\Omega = 0$). For the present $N = 2$ case, one can use the $\text{SO}(10) \times \text{SU}(2)$ symmetry to bring an arbitrary matrix configuration into the form

$$A_a = x_a \frac{\sigma_a}{2}, \quad A_I = 0, \quad (9)$$

where σ_a ($a = 1, 2, 3$) are the Pauli matrices. Plugging this into Eq. (7) for $\Omega = 0$, we get

$$S = \frac{1}{4}B - 8 \log(2C), \quad (10)$$

where $B = (x_1 x_2)^2 + (x_2 x_3)^2 + (x_3 x_1)^2$ and $C = x_1 x_2 x_3$. The saddle point is obtained as

$$x_1 = x_2 = x_3 = 2^{3/4}, \quad (11)$$

up to the sign flip. This is different from the classical solution (4) due to the Pfaffian, which takes into account the full quantum effects of the fermionic matrices.

For $\Omega \neq 0$, we take Eq. (9) with $x_1 = x_2 = x_3 \equiv x$ as an ansatz in view of Eq. (11). The effective action Eq. (7) becomes

$$S_{\text{eff}} = \frac{3}{4}x^4 + \frac{9\Omega^2}{2^7}x^2 - \frac{\Omega}{2}x^3 - \log \left(2x^3 + \frac{3\Omega}{4}x^2 - \frac{\Omega^3}{64} \right)^8. \quad (12)$$

The saddle-point equation admits a closed-form solution, which shall be given elsewhere [48]. Here we present the asymptotic behaviors of the relevant saddle point identified in our simulation:

$$x = \begin{cases} 2^{3/4} + \frac{3}{32}\Omega + \mathcal{O}(\Omega^2) & \text{for } \Omega \ll 1, \\ \frac{3}{8}\Omega + \mathcal{O}(\Omega^{-3}) & \text{for } \Omega \gg 1. \end{cases} \quad (13)$$

This shows that, in the present case of $N = 2$, the unique saddle point in Eq. (11) of the original model is smoothly connected to the dominant solution (3) at $\Omega \gg 1$.

Commuting saddle—For $\Omega \ll 1$, the dominant saddle point is expected to be commuting matrices from the discussion below Eq. (4). Using the $\text{SO}(3) \times \text{SO}(7) \times \text{SU}(2)$ symmetry, the general form of the commuting matrices can be put into the form

$$A_3 = x \frac{\sigma_3}{2}, \quad A_{10} = y \frac{\sigma_3}{2}, \quad A_\mu = 0 (\text{otherwise}). \quad (14)$$

Plugging this into Eq. (7), we get

$$S = \frac{\Omega^2}{2^7} (3x^2 + y^2) - \log \left(\frac{\Omega^8}{2^{16}} E \right), \quad (15)$$

where $E = \{(x^2 + y^2)^2 + \Omega^2(-x^2 + y^2)/2^3 + \Omega^4/2^8\}^4$.

The corresponding saddle points are given by

$$\begin{aligned} \text{(i)} & x = \frac{1}{4\Omega} \sqrt{\frac{2^{14}}{3} + \Omega^4}, \quad y = 0, \\ \text{(ii)} & x = 0, \quad y = \frac{1}{4\Omega} \sqrt{2^{14} - \Omega^4}, \end{aligned} \quad (16)$$

up to the sign flip, both of which diverge as $\mathcal{O}(1/\Omega)$ for $\Omega \rightarrow 0$. In fact, the effective action Eq. (15) becomes small on the ellipse in the x - y plane including these saddle points in Eq. (16), along which the maximum and minimum are given by (i) and (ii), respectively. On the other hand, the fluctuations away from this ellipse are not suppressed at all. A better description in this regime is given by the one-loop effective theory in Eq. (5) obtained by treating bosonic and fermionic variables on equal footing.

Monte Carlo result—Let us first present our results for the partition function. More precisely, we present the derivative $d \log Z(\Omega)/d\Omega = -\langle \partial S / \partial \Omega \rangle$, which is directly accessible by Monte Carlo simulations as an expectation value. In Fig. 1, we plot our results for $0.3 \leq \Omega \leq 12$, which are in complete agreement with the results obtained by the SUSY localization method [38].

As Ω increases, we find that the partition function approaches the prediction from the fuzzy sphere saddle, whereas for $\Omega \rightarrow 0$, it approaches the result obtained from the one-loop effective theory in Eq. (5). In particular, we

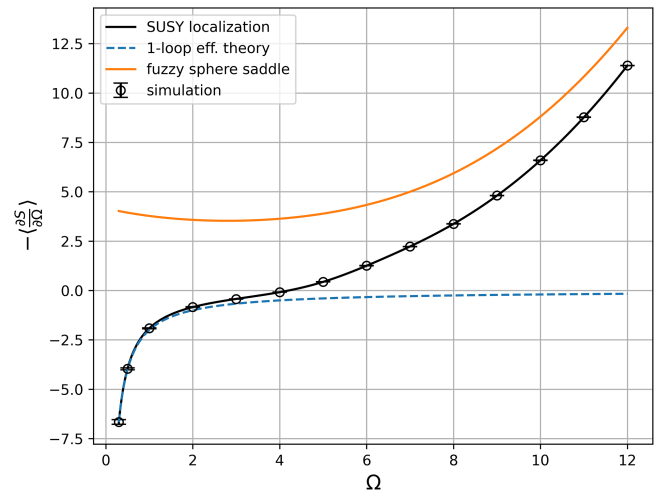


FIG. 1. The derivative of the partition function $d \log Z(\Omega)/d\Omega = -\langle \partial S / \partial \Omega \rangle$ obtained by our simulations is plotted against Ω (black circles). The black line represents the result obtained by the SUSY localization method. The orange line represents the result obtained for the fuzzy sphere saddle, whereas the blue dashed line represents the result obtained by the one-loop effective theory.

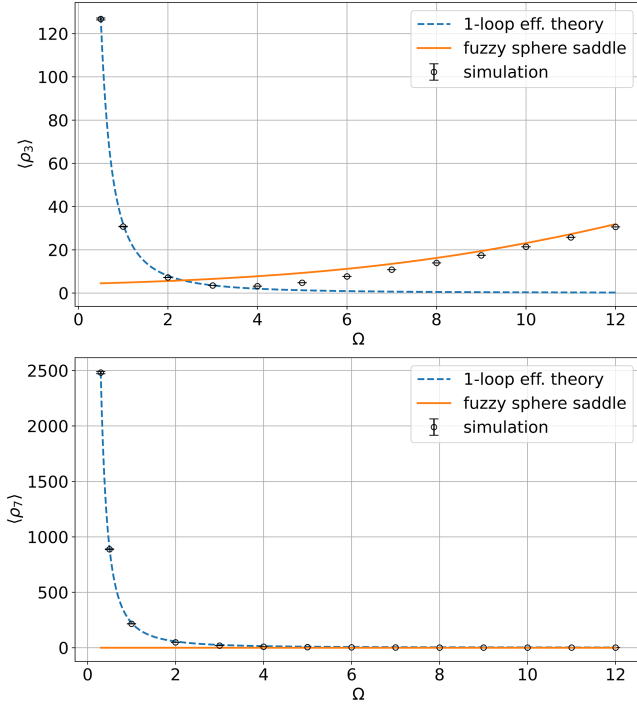


FIG. 2. The extent of spacetime ρ_3 (top) and ρ_7 (bottom) in the polarized and unpolarized directions obtained by simulations is plotted against Ω (black circles). The orange line represents the result obtained for the fuzzy sphere saddle, whereas the blue dashed line represents the result obtained by the one-loop effective theory.

note that $d \log Z(\Omega)/d\Omega \sim -2/\Omega$ as $\Omega \rightarrow 0$, which shows that $Z(\Omega) \sim \Omega^{-2}$ as expected for $N = 2$. Note also that the derivative $d \log Z(\Omega)/d\Omega$ changes its sign at $\Omega \sim 4$, where the dominant configurations are expected to switch from commuting matrices to the fuzzy sphere saddle.

In order to probe the spacetime structure emergent in the IKKT matrix model, the “extent of spacetime” $R^2 = \langle \text{tr}(A_\mu)^2 \rangle$ [10] has been commonly calculated. In the polarized IKKT model, it is useful to define

$$\rho_3 = \text{tr}(A_a)^2, \quad \rho_7 = \text{tr}(A_I)^2 \quad (17)$$

separately for the polarized $a = 1, 2, 3$ and unpolarized $I = 4, \dots, 10$ directions. From the viewpoint of the effective theory of D instantons, Eq. (17) shows how their distribution is affected by the background three-form flux.

In Fig. 2, we plot $\langle \rho_3 \rangle$ and $\langle \rho_7 \rangle$ against Ω . At large Ω , we find that $\langle \rho_3 \rangle$ grows quadratically with Ω and $\langle \rho_7 \rangle$ tends to vanish, as expected from the fuzzy sphere saddle (13). As $\Omega \rightarrow 0$, both $\langle \rho_3 \rangle$ and $\langle \rho_7 \rangle$ exhibit rapid growth, which agrees precisely with the predictions from the one-loop effective theory in Eq. (5). This further confirms that the partition function is dominated by commuting matrix configurations in this regime. (In fact, a particular linear combination $\langle \rho_3 \rangle/3 - \langle \rho_7 \rangle/7$ can be predicted by the

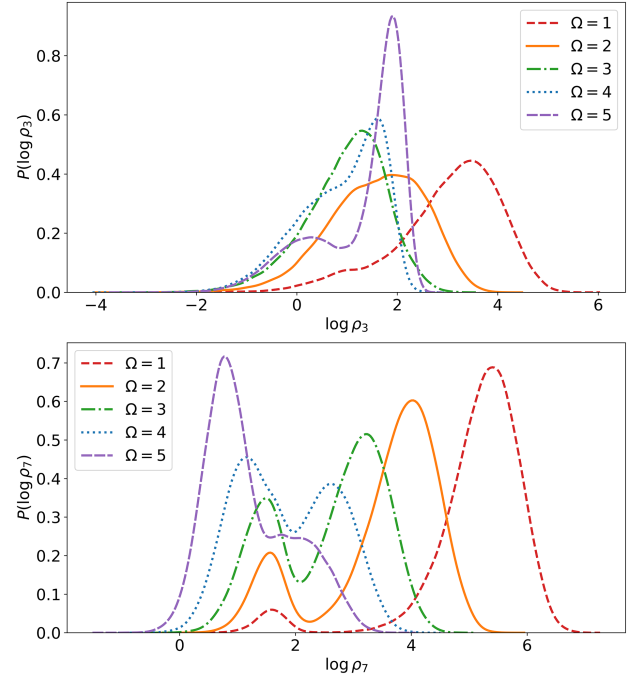


FIG. 3. Histograms of the quantities $\log \rho_3$ (top) and $\log \rho_7$ (bottom), which represent the extent of spacetime in the polarized and unpolarized directions, are shown for $\Omega = 1, 2, \dots, 5$.

localization method [38], and the prediction agrees with our data to the same precision as in Fig. 1.)

Finally, let us see more in detail what happens in the intermediate regime at $\Omega \sim 4$, where a change in the dominant configurations is anticipated. In Fig. 3, we plot the histogram of $\log \rho_3$ and $\log \rho_7$, where we see clear double-peak structures showing up at some values of Ω . Even at $\Omega = 1$, the ρ_7 distribution has two peaks, the right one corresponding to the commuting matrices and the left one corresponding to the fuzzy sphere saddle, which becomes the unique saddle of the original model as $\Omega \rightarrow 0$. We find that the right peak is by far dominating, which explains clearly why one cannot retrieve the original IKKT model in the $\Omega \rightarrow 0$ limit. As Ω increases, the right peak becomes smaller and comes closer to the left peak. In particular, at $\Omega \sim 4$, the two peaks become comparable as expected from the behavior of the partition function. At $\Omega \sim 5$, the two peaks merge since ρ_7 becomes small for both contributions.

On the other hand, the ρ_3 distribution starts to have two peaks at $\Omega = 5$, the right one corresponding to the fuzzy sphere saddle and the left one corresponding to the commuting matrices. As Ω increases further, the right peak shifts to the right and becomes more dominant.

While our results demonstrate a clear transition of the dominant configurations at $\Omega \sim 4$, the observables are found to be continuous as we have seen in Figs. 1 and 2 unlike the results of the localization method obtained for $N = 40$ [45]. We consider that this is simply due to the

chosen matrix size $N = 2$. At larger N , the dominance of one of the peaks occurs more rapidly as one crosses the critical Ω since the associated free energy is $\mathcal{O}(N^2)$. In fact, the calculation for small N is more tricky since one has to sample both peaks with the correct weight for a wide region of Ω , where the two peaks are actually very separated. This is made possible by using a sophisticated parallel tempering HMC algorithm as we discuss in a separate paper [48]. The agreement with the result of the localization method in Fig. 1 is achieved only with such calculations.

Discussion—In this Letter, we have performed Monte Carlo simulations of the polarized IKKT matrix model [30], which has attracted a lot of attention recently in the context of gauge-gravity duality [28,29,38]. In particular, by focusing on the simplest $N = 2$ case, we were able to identify all the saddle points that contribute to the path integral. While the validity of our simulations is confirmed by the precise agreement with the result of the localization method, the observables such as ρ_3 and ρ_7 , which are not accessible by the localization method, tell us clearly the spacetime structure of the dominant configurations depending on Ω . In particular, this clarified the geometric nature of the transition at intermediate Ω .

The fact that one cannot retrieve the original IKKT model in the $\Omega \rightarrow 0$ limit looks surprising at first sight. Here we point out that this effect is actually quite generic, and there is no need for SUSY or even fermions. For instance, let us consider a simple one-variable integral,

$$Z = \int_{-\infty}^{\infty} dx e^{-V(x)}, \quad (18)$$

with a polynomial “potential,”

$$V(x) = x^2 - \Omega^2 x^4 + a\Omega^4 x^6, \quad (19)$$

where $a > 0$ and $\Omega \in \mathbb{R}$. While this is an innocent-looking deformation of the Gaussian integral ($\Omega = 0$), the integral actually diverges in the $\Omega \rightarrow 0$ limit for $a < 1/4$.

To see that, we rewrite the integral (18) in terms of the rescaled variable $y \equiv \Omega x$ as

$$Z = \frac{1}{\Omega} \int_{-\infty}^{\infty} dy e^{-\tilde{V}(y)/\Omega^2}, \quad \tilde{V}(y) = y^2(1 - y^2 + ay^4). \quad (20)$$

For $a < 1/3$, one obtains a new minimum of $\tilde{V}(y)$ at $y = y_0$ [with $(y_0)^2 = (1 + \sqrt{1 - 3a})/3a$] in addition to the trivial one at $y = 0$. And for $a < 1/4$, the new minimum gives $\tilde{V}(y_0) < 0$, hence the divergence as $\Omega \rightarrow 0$. In terms of the original integral, this happens because the new minimum at $x = x_0 \equiv y_0/\Omega$ becomes infinitely deep $V(x_0) = -c/\Omega^2$ with $c = |\tilde{V}(y_0)| > 0$. Note also that the new minimum at $x = y_0/\Omega$ is pushed away to infinity as $\Omega \rightarrow 0$, which ensures the consistency with the fact that there is no such

minimum at $\Omega = 0$. In the polarized IKKT model, the role of the new minimum is played by the commuting matrices as we have seen in this Letter.

This new insight into the singularity in $\Omega \rightarrow 0$ may have broad implications in physics. For instance, if one applies it to quantum field theory (or to some quantum system), it implies that an infinitesimal deformation of the theory may lead to the emergence of a totally different vacuum (or a totally different ground state). In the case of superstring theory, it is known that there are tremendously many vacua, which are perturbatively stable, and each vacuum corresponds to a spacetime with different dimensionality accommodating quantum fields with different gauge symmetry. This is the situation that is commonly referred to as the string landscape. The IKKT matrix model is supposed to describe each vacuum in the string landscape as different saddle points. However, a small deformation may lead to a new saddle point, which is actually dominant.

In this regard, let us recall that the polarized IKKT model is a deformation of the *Euclidean* IKKT model [49–51] that can be obtained by applying a Wick rotation $A_0 = iA_{10}$ to the *Lorentzian* IKKT model. In fact, in the Lorentzian model, by adding a Lorentz invariant mass term $S_m = -\gamma \text{tr}(A_\mu A^\mu)$ to the action, one can obtain new saddle points, which represent expanding spacetime [12–18,21]. It would, therefore, be interesting to investigate the SUSY mass deformation of the Lorentzian model analogous to the polarized IKKT model.

Recently it was recognized that the Lorentzian model has a diverging partition function due to the noncompact Lorentz symmetry [52]. This led to a proposal of a well-defined model obtained by fixing the Lorentz symmetry by the Faddeev-Popov procedure [52]. Monte Carlo simulations of the Lorentzian model defined in this way were performed with the Lorentz invariant mass term omitting the fermionic matrices [53]. We are currently trying to extend this work to the SUSY deformed model including the fermionic matrices.

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Data availability—The data that support the findings of this Letter are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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