

Next-to-Leading-Order QCD Predictions for the Nucleon Form Factors

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We accomplish for the first time the next-to-leading-order QCD computations of the leading-twist contributions to the Dirac nucleon form factors by applying the hard-collinear factorization theorem rigorously. Our predictions for these baryon form factors indicate that the one-loop perturbative corrections to the hard-gluon-exchange contributions are numerically substantial for a wide range of momentum transfers. Including further the soft nonfactorizable contributions, we then perform the state-of-the-art analysis of the Dirac nucleon form factors, allowing for the most robust determinations of the nucleon distribution amplitudes from the comparison with the experimental data.

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Introduction—It is generally accepted that the nucleon electromagnetic form factors are among the simplest, most fundamental, and best-known measurable quantities for exploring diverse facets of the nonperturbative QCD dynamics that govern the complex nature of nucleon structure properties and for advancing our understanding toward the perturbative factorization formalism of hard exclusive processes. The pioneering investigation of nucleon tomography dates back to the elastic electron-nucleon scattering experiment in the early 1950s, long before the advent of QCD, conducted by Hofstadter and collaborators [1–4]. From then on, probing the internal structure of the composite nucleon state with unprecedented precision has become the central focus of the world-wide hadron-physics programs, reflected by the increasing number of electron accelerators and laboratories over the past decades [5–16]. Extensive experimental efforts in measuring the nucleon electromagnetic form factors with the Rosenbluth cross section separation method and with the polarization transfer technique have been well documented in several review articles (see Refs. [17–21] for an incomplete list).

Needless to say, the everlasting and tremendous experimental investment in exploiting the emergent nucleon structure naturally triggers a considerable amount of theoretical developments to achieve the field-theoretic

description of hard exclusive reactions (see Refs. [21–26] for an overview). It has been demonstrated that the underlying QCD dynamics of the nucleon electromagnetic form factors is dictated by the interplay of two competing mechanisms (namely, the soft Feynman mechanism [27] versus the hard-scattering mechanism). A systematic and successful approach to address the soft nonfactorizable contributions to the nucleon transition form factors has been formulated in the seminal works [28–30] with the inclusion of the leading and subleading contributions at $\mathcal{O}(\alpha_s^0)$ up to the twist-six accuracy [31], by adopting the light-cone sum rules (LCSR) technique [32,33], which receives numerous applications in the QCD-based calculations of B -meson decay matrix elements [34–40] and of heavy-baryon decay form factors [41–47]. Subsequently, the next-to-leading-order (NLO) QCD corrections to the LCSR predictions for the nucleon form factors have been computed at the leading-twist (i.e., twist-three) accuracy [48] with the covariant trace formalism and at the twist-four accuracy [49] with the Krankl-Manashov (KM) renormalization scheme [50]. In particular, the hard-gluon-exchange contributions to these nucleon form factors can be computed with the same LCSR method by performing two-loop computations of the considered correlation functions, at the price of slight model dependence of the nucleon disentanglement from the higher-mass background [51,52].

Instead, an elegant and model-independent description of the hard-scattering contributions to the nucleon form factors can be constructed with the perturbative factorization formalism, based upon the diagrammatic analysis [53–55] and the effective field theory approach [56,57]. As a matter of fact, the textbook tree-level computations of the leading-twist contributions to the nucleon form factors have been accomplished with the hard-collinear factorization prescription more than 40 years ago [58,59] (see Refs. [60–65] for subsequent recomputations and [66–71] for discussions on the applicability of perturbative QCD at

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accessible energies). Apparently, extending the leading-order (LO) QCD calculation of the twist-three hard-scattering contributions to the NLO accuracy will be in high demand for validating the collinear factorization of baryon transition matrix elements truly at the quantum level and for unraveling the intriguing pattern of perturbative expansion for multiparton scattering amplitudes. In this Letter, we will employ the modern QCD factorization formalism to extract the NLO hard coefficient function analytically, by performing the ultraviolet (UV) renormalization and infrared subtractions in a rigorous and factorization-compatible fashion. An emphasis will be then placed on phenomenological impacts of the thus determined radiative corrections to the hard-gluon-exchange contributions in the nucleon form factors.

Electromagnetic nucleon form factors—Adopting the customary definitions of the electromagnetic nucleon form factors enables us to write down [72]

$$\begin{aligned} \langle N(P') | j_\mu^{\text{em}}(0) | N(P) \rangle \\ = \bar{N}(P') \left[\gamma_\mu F_1(Q^2) - i \frac{\sigma_{\mu\nu} q^\nu}{m_N} F_2(Q^2) \right] N(P), \end{aligned} \quad (1)$$

where P (P') stands for the four-momentum carried by the initial (final) nucleon, $q = P - P'$ refers to the transfer momentum, $Q^2 = -q^2$, $N(P)$ [$N(P')$] corresponds to the nucleon spinor, and the electromagnetic current is given by $j_\mu^{\text{em}}(x) = Q_u \bar{u}(x) \gamma_\mu u(x) + Q_d \bar{d}(x) \gamma_\mu d(x)$. Introducing two light-cone vectors n_μ and $\bar{n}_\mu$ with the constraints $n^2 = \bar{n}^2 = 0$ and $n \cdot \bar{n} = 2$ leads to $P_\mu \approx (n \cdot P/2) \bar{n}_\mu$ and $P'_\mu \approx (\bar{n} \cdot P'/2) n_\mu$ at leading power in $\Lambda_{\text{QCD}}^2/Q^2$ [73]. The hard-scattering contribution to the Dirac form factor $F_1(Q^2)$ exhibits the scaling behaviour of $(\Lambda_{\text{QCD}}/Q)^4$ [74–76]. By contrast, the asymptotic prediction of the Pauli form factor $F_2(Q^2)$ turns out to be suppressed by an extra power of $\Lambda_{\text{QCD}}^2/Q^2$. We therefore concentrate on the NLO

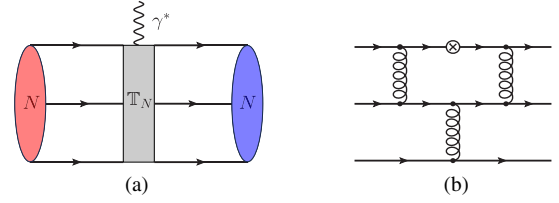


FIG. 1. (a) Schematic structure of the nucleon form factor. (b) One sample NLO diagram, where $\otimes$ marks an insertion of the electromagnetic current.

computation of $F_1(Q^2)$, whose factorization structure is sketched in Fig. 1.

According to the hard-collinear factorization prescription [53,55], the leading-power contribution to the Dirac form factor can be cast in the form of (see Refs. [56,57,77–81] for potential complications beyond the NLO accuracy)

$$\begin{aligned} F_1(Q^2) = \frac{(4\pi\alpha_s)^2}{Q^4} \int [\mathcal{D}x] \int [\mathcal{D}y] \mathbb{T}_N(x_i, y_i, Q^2, \mu_F) \\ \times [f_N(\mu_F)]^2 \varphi_N(x_i, \mu_F) \varphi_N(y_i, \mu_F), \end{aligned} \quad (2)$$

where the integration measure is defined as

$$\int [\mathcal{D}z] = \int_0^1 dz_1 dz_2 dz_3 \delta(z_1 + z_2 + z_3 - 1), \quad (3)$$

and μ_F refers to the factorization scale. The short-distance coefficient function $\mathbb{T}_N$ can be expanded perturbatively (similarly for any other QCD quantity),

$$\mathbb{T}_N = \mathbb{T}_N^{(0)} + \left(\frac{\alpha_s}{4\pi} \right) \mathbb{T}_N^{(1)} + \mathcal{O}(\alpha_s^2). \quad (4)$$

The twist-three nucleon distribution amplitude φ_N can be defined by the renormalized matrix element of the leading three-body light-ray operator [55,82]

$$\begin{aligned} \langle 0 | \epsilon_{ijk} \left[u_i^\uparrow(\tau_1 n) [\tau_1 n, \tau_0 n]_{i'i} C \not{n} u_j^\downarrow(\tau_2 n) [\tau_2 n, \tau_0 n]_{j'j} \right] \not{n} d_k^\uparrow(\tau_3 n) [\tau_3 n, \tau_0 n]_{k'k} | N^\uparrow(P) \rangle \\ = -\frac{f_N(\mu_F)}{2} (n \times P) \not{n} N^\uparrow(P) \int [\mathcal{D}x] \exp \left[-in \times P \sum_{i=1}^3 x_i \tau_i \right] \varphi_N(x_i, \mu_F), \end{aligned} \quad (5)$$

where the Wilson line $[\tau_i n, \tau_0 n]$ is defined as $[\tau_i n, \tau_0 n] = P \exp[i g_s \int_{\tau_0}^{\tau_i} dt n_\mu A^\mu(tn)]$ [83] and $q^{\uparrow(\downarrow)} = \frac{1}{2}(1 \pm \gamma_5)q$. The renormalization-group equation of the light-cone operator on the left-hand side of (5) motivates the following conformal expansion of φ_N [84]:

$$\varphi_N(x_i, \mu_F) = 120 x_1 x_2 x_3 \sum_{n=0}^{\infty} \sum_{k=0}^n \varphi_{nk}(\mu_F) \mathcal{P}_{nk}(x_i), \quad (6)$$

with the explicit expressions of $\mathcal{P}_{nk}$ displayed in [85].

The short-distance coefficient function $\mathbb{T}_N$ can be determined by inspecting the seven-point QCD matrix element

$$\Pi_\mu = \langle u(P'_1) u(P'_2) d(P'_3) | j_\mu^{\text{em}}(0) | u(P_1) u(P_2) d(P_3) \rangle, \quad (7)$$

where the external parton momenta are restricted to their leading components $P_i = x_i P$ and $P'_i = y_i P'$ at the leading power accuracy. We will employ dimensional regularization with $D = 4 - 2\epsilon$ in the entire calculation.

NLO QCD computation—In this section we will describe briefly the one-loop QCD computation of the bare amplitude Π_μ . We generate the NLO Feynman diagrams with FEYNARTS [86] and simultaneously by means of an in-house *Mathematica* routine. Taking into account the observation that certain Feynman diagrams cannot contribute due to the QCD equations of motion, the vanishing color (and/or electric charge) factors, as well as the Furry theorem, we obtain 1371 nonvanishing diagrams at one loop. The sample Feynman diagrams are shown in Fig. 1.

The Passarino-Veltman reduction [87] to the tensor structure of the amplitude is implemented first, and then the Dirac and color algebras are performed with in-house routines. The yielding one-loop scalar integrals are further reduced to a small set of master integrals with the packages

APART [88] and FIRE [89]. Exploiting further the fact that $P_i||P$ and $P'_i||P'$, we arrive at *only* two linearly independent master integrals at one loop.

QCD factorization formula—We will continue to derive the master formula for $\mathbb{T}_N$ by matching Π^μ onto the collinear matrix elements

$$\Pi^\mu = \frac{(4\pi\alpha_s)^2}{Q^6} \sum_k T_k \otimes \langle \mathcal{O}_k^\mu \rangle, \quad \mathcal{O}_k^\mu \in \{\mathcal{O}_N^\mu, \mathcal{O}_T^\mu, E_1^\mu, E_2^\mu, E_3^\mu, F_1^\mu, F_2^\mu, F_3^\mu, F_4^\mu\}, \quad (8)$$

where we have to extend the effective operator basis with the inclusion of the evanescent operators

$$\begin{aligned} \mathcal{O}_N^\mu &= [\bar{\chi}_u^\dagger \not{p} C^{-1} \bar{\chi}_u^\dagger] [\bar{\chi}_d^\dagger \not{p} \gamma_\perp^\mu \not{p} \xi_d^\dagger] [\xi_u^\dagger C \not{p} \xi_u^\dagger], & \mathcal{O}_T^\mu &= [\bar{\chi}_u^\dagger \not{p} \gamma_{\perp\beta} C^{-1} \bar{\chi}_u^\dagger] [\bar{\chi}_d^\dagger \not{p} \gamma_\perp^\beta \gamma_\perp^\mu \gamma_\perp^\alpha \not{p} \xi_d^\dagger] [\xi_u^\dagger C \gamma_{\perp\alpha} \not{p} \xi_u^\dagger], \\ E_1^\mu &= [\bar{\chi}_u \gamma_\perp^\mu \xi_u] [\bar{\chi}_u \gamma_\perp^\alpha \xi_u] [\bar{\chi}_d \gamma_{\perp\alpha} \xi_d] - \frac{1}{8} \mathcal{O}_N^\mu - \frac{1}{2} \mathcal{O}_T^\mu, & E_2^\mu &= [\bar{\chi}_u \gamma_\perp^\alpha \gamma_\perp^\beta \gamma_\perp^\mu \xi_u] [\bar{\chi}_u \gamma_{\perp\beta} \xi_u] [\bar{\chi}_d \gamma_{\perp\alpha} \xi_d] - \mathcal{O}_T^\mu, \\ E_3^\mu &= [\bar{\chi}_u \gamma_{\perp\alpha} \xi_u] [\bar{\chi}_u \gamma_{\perp\beta} \xi_u] [\bar{\chi}_d \gamma_\perp^\beta \gamma_\perp^\alpha \gamma_\perp^\mu \xi_d] - \frac{1}{4} \mathcal{O}_N^\mu, \\ F_1^\mu &= \underbrace{[\bar{\chi}_u^\dagger \not{p} C^{-1} \bar{\chi}_u^\dagger] [\bar{\chi}_d^\dagger \not{p} \gamma_\perp^\alpha \gamma_\perp^\beta \not{p} \xi_d^\dagger]}_{\mathcal{Q}_N} [\xi_u^\dagger C \gamma_{\perp\beta} \gamma_{\perp\alpha} \not{p} \xi_u^\dagger], & F_2^\mu &= \underbrace{[\bar{\chi}_u^\dagger \not{p} \gamma_{\perp\alpha} \gamma_{\perp\beta} C^{-1} \bar{\chi}_u^\dagger] [\bar{\chi}_d^\dagger \not{p} \gamma_\perp^\beta \gamma_\perp^\alpha \gamma_\perp^\mu \not{p} \xi_d^\dagger]}_{\mathcal{U}_N} \underbrace{[\xi_u^\dagger C \not{p} \xi_u^\dagger]}_{\mathcal{Q}_N}, \\ F_3^\mu &= \underbrace{[\bar{\chi}_u^\dagger \not{p} \gamma_{\perp\rho} C^{-1} \bar{\chi}_u^\dagger] [\bar{\chi}_d^\dagger \not{p} \gamma_\perp^\rho \gamma_\perp^\tau \gamma_\perp^\alpha \gamma_\perp^\beta \not{p} \xi_d^\dagger]}_{\mathcal{Q}_T} [\xi_u^\dagger C \gamma_{\perp\tau} \gamma_{\perp\beta} \gamma_{\perp\alpha} \not{p} \xi_u^\dagger], \\ F_4^\mu &= \underbrace{[\bar{\chi}_u^\dagger \not{p} \gamma_{\perp\alpha} \gamma_{\perp\beta} \gamma_{\perp\tau} C^{-1} \bar{\chi}_u^\dagger] [\bar{\chi}_d^\dagger \not{p} \gamma_\perp^\beta \gamma_\perp^\alpha \gamma_\perp^\tau \gamma_\perp^\mu \not{p} \xi_d^\dagger]}_{\mathcal{U}_T} \underbrace{[\xi_u^\dagger C \gamma_{\perp\tau} \not{p} \xi_u^\dagger]}_{\mathcal{Q}_T}. \end{aligned} \quad (9)$$

To reduce our notation to the essentials, we strip off all the color indices, position arguments and Wilson lines from $\mathcal{O}_k^\mu$, and represent them only by their flavor, chiral, and Dirac structures. The (anti)collinear quarks moving in the direction of $\bar{n}$ (n) are denoted by ξ (χ). The evanescent operators E_i^μ and F_j^μ vanish at $D = 4$. Moreover, F_j^μ can be further expressed in terms of the building blocks of the three-quark operators $\mathcal{P}_k \in \{\mathcal{Q}_N, \mathcal{U}_N, \mathcal{Q}_T, \mathcal{U}_T\}$ (with $\bar{\mathcal{P}}_k \equiv \mathcal{P}_k^\dagger \gamma_0$), where $\mathcal{U}_N$ and $\mathcal{U}_T$ are apparently evanescent.

We now write the perturbative expansion of Π^μ according to the tree-level matrix elements of $\mathcal{O}_k^\mu$,

$$\Pi^\mu = \frac{(4\pi)^4}{Q^6} \sum_k \sum_{\ell=0,1} \left(\frac{Z_\alpha \alpha_s}{4\pi} \right)^{\ell+2} A_k^{(\ell)} \otimes \langle \mathcal{O}_k^\mu \rangle^{(0)}, \quad (10)$$

where Z_α is the renormalization factor of the strong coupling [90–92] and $A_k^{(\ell)}$ denote the bare ℓ -loop QCD amplitudes. On the other hand, the effective matrix elements of the collinear operators take a similar form to (10),

$$\langle \mathcal{O}_k^\mu \rangle = \sum_m \sum_\ell \left(\frac{\alpha_s}{4\pi} \right)^\ell Z_{km}^{(\ell)} \otimes \langle \mathcal{O}_m^\mu \rangle^{(0)}, \quad (11)$$

where $Z_{km}^{(\ell)}$ stands for the matrix kernel of UV renormalization factors at the ℓ -loop order. Expanding further T_k in (8) according to $T_k = \sum_{\ell=0}^\infty (\alpha_s/4\pi)^\ell T_k^{(\ell)}$, we are then able to deduce the master formulas for the hard coefficients

$$\begin{aligned} T_{N(T)}^{(0)} &= A_{N(T)}^{(0)}, \\ T_{N(T)}^{(1)} &= A_{N(T)}^{(1)} + 2Z_\alpha^{(1)} A_{N(T)}^{(0)} - \sum_k Z_{kN(T)}^{(1)} \otimes T_k^{(0)}, \end{aligned} \quad (12)$$

where $Z_{NN}^{(1)}$ and $Z_{TT}^{(1)}$ have been worked out in [83]. The evanescent-to-physical operator mixing generates the non-vanishing *finite* renormalization constants $Z_{E_1N}^{(1)}$ and $Z_{E_3N}^{(1)}$, which are indispensable for establishing the perturbative factorization formula (see also [93–101]).

Employing the definitions of the matrix elements for $\mathcal{O}_N^\mu$ displayed in (5) and for $\mathcal{O}_T^\mu$ shown in [102], we can express

the desired coefficient function $\mathbb{T}_N$ as follows:

$$\begin{aligned} \mathbb{T}_N(x_1, x_2, x_3, y_1, y_2, y_3, Q^2, \mu_F) \\ = T_N(x_1, x_2, x_3, y_1, y_2, y_3, Q^2, \mu_F) + \left(\frac{1}{4}\right) \\ \times [T_T(x_1, x_3, x_2, y_1, y_3, y_2, Q^2, \mu_F) + (x_1 \leftrightarrow x_3) \\ + (y_1 \leftrightarrow y_3) + (x_1 \leftrightarrow x_3, y_1 \leftrightarrow y_3)]. \end{aligned} \quad (13)$$

Substituting the determined one-loop hard function into the factorization formula (2) and evaluating the convolution integral with the asymptotic nucleon distribution amplitude gives rise to [for the SU(3) color group]

$$\begin{aligned} F_1^{\text{Asy}}(Q^2) = \frac{(4\pi\alpha_s)^2}{Q^4} \frac{100}{3} f_N^2 \left\{ Q_u \left[1 + \frac{\alpha_s}{4\pi} \left(18 \ln \frac{\nu^2}{Q^2} \right. \right. \right. \\ \left. \left. + \frac{4}{3} \ln \frac{\mu_F^2}{Q^2} - \frac{1907}{15} + \frac{7438}{15} \zeta_3 - 368 \zeta_5 \right) \right] \\ \left. + 2Q_d \left[1 + \frac{\alpha_s}{4\pi} \left(18 \ln \frac{\nu^2}{Q^2} + \frac{4}{3} \ln \frac{\mu_F^2}{Q^2} \right. \right. \right. \right. \\ \left. \left. \left. + \frac{1721}{30} + \frac{209}{30} \zeta_3 + 6 \zeta_5 \right) \right] \right\}, \end{aligned} \quad (14)$$

where ν represents the renormalization scale and the *Mathematica* package POLYLOGTOOLS [103] turns out to be beneficial for handling the encountered fourfold integrals analytically. Including the subleading terms in the conformal expansion of φ_N results in the lengthy expression for the nonasymptotic correction, which are collected in Supplemental Material [104]. We further demonstrate that our NLO QCD computation of $F_1(Q^2)$ is indeed independent of the factorization scale at $\mathcal{O}(\alpha_s^3)$, by employing the one-loop evolution equation of φ_N [83].

Numerical analysis—We are now in a position to explore the phenomenological implication of the NLO QCD computation to the Dirac nucleon form factor. To achieve this goal, we first specify the twist-three nucleon distribution amplitude which has been extensively investigated with the method of QCD sum rules (QCDSR) [59,109–116], with the LCSR technique [29,30,49,117] and with the lattice simulation [102,118–120]. Three sample models of the initial condition $\varphi_N(x_i, \mu_0)$ at $\mu_0^2 = 2.0 \text{ GeV}^2$, labeled as ABO1 [49], LAT19 [120] (see Ref. [121] for an update), and COZ [109], will be employed in the numerical exploration for illustration purposes. Furthermore, we take the renormalization and factorization scales as $\nu^2 = \mu_F^2 = \langle z \rangle Q^2$ with $1/6 \leq \langle z \rangle \leq 1/2$, which corresponds to the expectation value of the maximum quark-gluon virtuality in the Feynman graphs (see Refs. [122–130] for further discussions).

Taking the ABO1 model as our default choice, we display in Fig. 2 the obtained predictions for the soft contributions from the LCSR approach at $\mathcal{O}(\alpha_s)$ [49] and

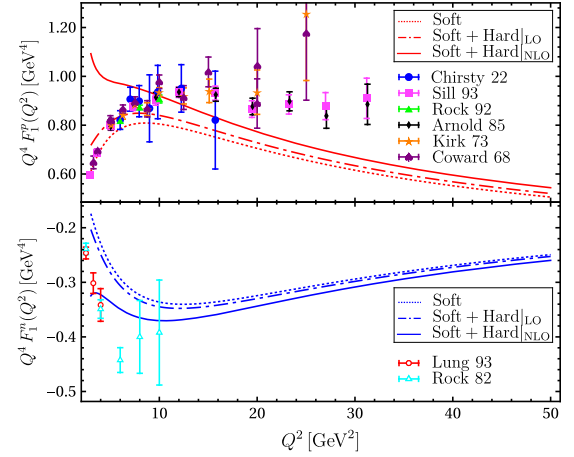


FIG. 2. Theory predictions for the Dirac nucleon form factors from the Feynman mechanisms (dotted curves) and from the combinations of the soft contributions with the hard-scattering mechanisms at LO (dash-dotted curves) and at NLO (solid curves). The experimental data points of the Dirac proton form factor (Christy *et al.* [131]; Sill *et al.* [132]; Rock *et al.* [133]; Arnold *et al.* [134]; Kirk *et al.* [135]; Coward *et al.* [136]) and of the Dirac neutron form factor (Lung *et al.* [137]; Rock *et al.* [138]) are further displayed.

for the hard-gluon-exchange contributions at LO and at NLO. The available experimental measurements of these two transition form factors [131–138] are further displayed for an exploratory comparison. It is evident that the hard-scattering contributions can shift the NLO LCSR predictions of the Dirac proton (neutron) form factor by an amount of $\{29\%, 11\%, 9\%, 8\%\}$ ($\{25\%, 7\%, 5\%, 4\%\}$) for $Q^2 = \{5.0, 15.0, 30.0, 50.0\} \text{ GeV}^2$, respectively [139]. In particular, the radiative corrections to the hard-scattering kernels in (2) can bring about enormous impacts on the corresponding tree-level predictions for a wide range of the kinematic domain. We further verify that this enlightening pattern of the perturbative α_s expansion remains unchanged for both the LAT19 and COZ models.

We proceed to present in Fig. 3 our predictions for the Dirac nucleon form factors with three sample models ABO1 [49], LAT19 [120], and COZ [109] of the nucleon distribution amplitudes. Inspecting the numerical features of these state-of-the-art results implies that the yielding theory predictions with the LCSR-inspired ABO1 model can lead to a reasonably good description of the experimental data for both the proton and neutron's Dirac form factors. On the contrary, the predicted electromagnetic form factors $F_1^p(Q^2)$ and $F_1^n(Q^2)$ with the QCDSR-based COZ model of the nucleon distribution amplitudes turn out to be significantly above their experimental values. This observation can be attributed to the fact that the QCDSR method tends to overestimate the hadronic matrix elements of higher conformal operators considerably [28–30,117,140,141]. The distinctive snapshot of the well-separated error bands for the two parameter sets

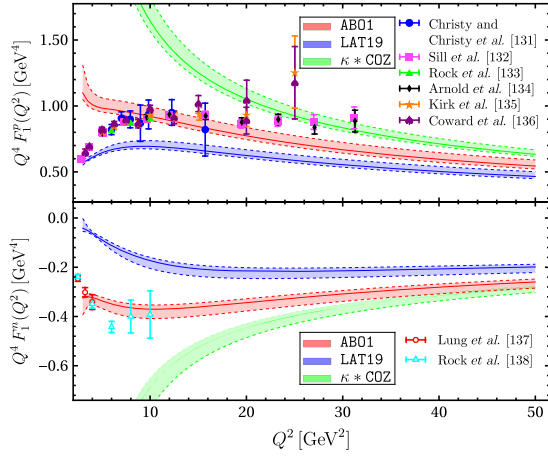


FIG. 3. Theory predictions for the Dirac nucleon form factors from three sample models (ABO1 [49], LAT19 [120], and COZ [109]) of $\varphi_N(x_i, \mu_0)$, obtained by combining the soft contributions from [49] with the NLO predictions of the hard-scattering contributions. Perturbative uncertainties from varying the factorization scale μ_F are indicated by the color bands. Note that the achieved results from the COZ model are rescaled with a factor of $\kappa = 0.5$. The references to the experimental data are summarized in the caption of Fig. 2.

ABO1 and LAT19 can be largely traced back to the absence of the lattice results for η_{10} and η_{11} in the twist-four distribution amplitudes Φ_4 and Ψ_4 , whose effects dominate the soft nonfactorizable contributions to the nucleon form factors [28,29,49].

We turn to address the phenomenological impact of extracting the shape parameters for the nucleon distribution amplitude φ_N with the NLO QCD computations of the hard-gluon-exchange contributions to the Dirac nucleon form factors. Inspecting the intrinsic dependence of the $\gamma^*N \rightarrow N$ transition form factors on the currently poorly constrained parameter $\varphi_{21}(\mu_0)$, we indeed observe that the inclusion of the higher-order QCD correction to the short-distance coefficient function can profoundly enhance the sensitivity of the nucleon form factors with regard to the local moment $\varphi_{21}(\mu_0)$ of the twist-three nucleon distribution amplitude.

Conclusions—In conclusion, we have carried out, for the first time, the complete NLO QCD computation of the Dirac nucleon form factor by applying the hard-collinear factorization theorem rigorously. Adopting three sample models for the nucleon distribution amplitudes, we demonstrated explicitly that the NLO corrections to the hard-scattering contributions can yield the substantial phenomenological impacts on the counterpart LO predictions in the kinematic region $5.0 \leq Q^2 \leq 50.0 \text{ GeV}^2$. Moreover, including the higher-order QCD correction turned out to be highly beneficial for an improved extraction of the shape parameters of the nucleon distribution amplitudes. Extending our factorization formalism to hadron form factors describing the electroproductions of

the Δ resonance and the negative-parity $N^*(1535)$ can be naturally expected for the sake of exploring the many facets of QCD dynamics dictating the baryon transition matrix elements.

Note added—Recently, the work [142] appeared, in which the NLO hard function was extracted [143]. The authors of [142] further corrected an error in the infrared subtractions, and their updated hard coefficients can now be constructed from our results obtained with the inclusion of evanescent operators [49,50,93–97].

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