Comment on "Role of Matter Interactions in Superradiant Phenomena"

Introduction—Recently, Mendonça *et al.* [1] investigated the Dicke-XXZ model and the Dicke-Ising model (DIM) using a variational unitary transformation accompanied by density matrix renormalization group calculations. For the case of the DIM [1]

$$\mathcal{H} = \omega a^\dagger a + \varepsilon \sum_{i=1}^{N} s_i^z + \frac{2g}{\sqrt{N}} \sum_{i=1}^{N} s_i^x (a + a^\dagger) - 4J \sum_{\langle i,j \rangle} s_i^z s_j^z \tag{1}$$

with spin-1/2 operators s_i^α, some of their findings contradict other studies [2–5]. Some of which were argued to be limited by their methods [2,3], while others were not mentioned [4,5]. We discuss two missing key features, namely, (1) the existence of an intermediate antiferromagnetic superradiant phase (AFM-S) and (2) the change in order of phase transitions for ferromagnetic Ising interactions.

Intermediate antiferromagnetic superradiant phase—For antiferromagnetic (AFM) interactions, mean-field calculations observe an intermediate AFM-S phase [2]. The existence of this phase is quantitatively verified by quantum Monte Carlo (QMC) simulations in Langheld *et al.* [4]. In contrast to this, Ref. [1] did not find this intermediate phase. As indicated by the orange and blue crosses in Fig. 1, the intermediate phase is quite limited in 1d [4]. This makes it difficult to find this phase, especially considering the chosen resolution and relatively small system size of presumably up to $N = 100$ used for the DIM in Ref. [1]. The quantum phase transition from the normal to the intermediate phase is of second order, as motivated by the exact solution of the low-energy spectrum in the antiferromagnetic normal phase by Ref. [6] and verified by QMC [4].

Order of phase transitions—For ferromagnetic interactions, Ref. [1] claims to find a second-order phase transition to the superradiant phase questioning the findings by Ref. [3] for vanishing and small longitudinal fields ε. Going to the limit of $\varepsilon = 0$ (or equivalently $J \to \infty$ in Fig. 1), the system gains an additional spin-flip symmetry [3]. This forces the quantum phase transition to be of first order by Landau theory, which is determined numerically with arbitrary precision using an exact self-consistent approach [4] and has been confirmed by exact analytical calculations [5]. While the spin-flip symmetry is explicitly broken for any finite ε making a second-order phase transition possible, the phase transition remains first order for small longitudinal fields [4]. At the multicritical point $J \approx \varepsilon/2 = 0.5$ [4] (see Fig. 1), the order of the quantum phase transition changes. Thus, the first-order phase transition line lies outside of the parameter regime discussed in Ref. [1], which potentially could be tracked when going to larger Ising interactions.

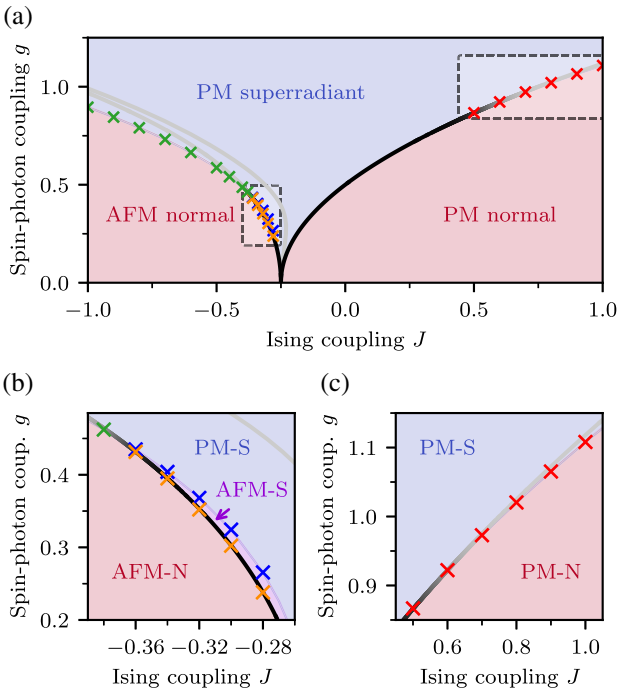


FIG. 1. Phase diagram of the DIM choosing $\omega = \varepsilon = 1$ as in Fig. 3(c) from Ref. [1]. Crosses and background colors show the QMC results using the algorithm from Ref. [4], black (grey) lines depict the mean-field results [2,6] consistent with (deviating from) the QMC analysis. Fading of the black mean-field lines visualizes the onset of deviation from the QMC data. (a) Full phase diagram containing antiferromagnetic normal (AFM-N), antiferromagnetic superradiant (AFM-S), paramagnetic normal (PM-N), and paramagnetic superradiant (PM-S) phases. The label PM-N deviates from Ref. [1], namely FM normal, as the phase does not break the $\mathbb{Z}_2$ symmetry spontaneously. (b) Close-up view into intermediate AFM-S phase, which was not found by Ref. [1]. (c) Close-up into emerging deviation from mean-field results [2,6] for ferromagnetic Ising interactions not included in Ref. [1].

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