

Cosmological Selection of a Small Weak Scale from Large Vacuum Energy: A Minimal Approach

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We present a minimal cosmological solution to the hierarchy problem. Our model consists of a light pseudoscalar and an extra Higgs doublet in addition to the standard model field content. We consider a landscape of vacua with varying values of the electroweak vacuum expectation value (VEV). The vacuum energy in our model peaks in a region of the landscape where the electroweak VEV is nonzero and much smaller than the cutoff. During inflation, due to exponential expansion, such regions of the landscape with maximal vacuum energy dominate the universe in volume, thus explaining the observed smallness of the electroweak scale with respect to the cutoff. The pseudoscalar potential in our model is that of a completely generic pseudogoldstone boson—not requiring the clockwork mechanism—and its field value never exceeds its decay constant or the Planck scale. Our mechanism is robust to the variation of other model parameters in the landscape as the electroweak VEV is varied. It also predicts a precise and falsifiable relationship between the masses and couplings of the different Higgs boson mass eigenstates. Moreover, the pseudoscalar in our model can account for the observed dark matter relic density.

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Introduction—The hierarchy problem has long been a central theme of particle physics research. Most solutions to the hierarchy problem embed the standard model into a larger framework where a new symmetry emerges as the Higgs mass squared, $\mu^2 \rightarrow 0$. This connects the Higgs mass to the symmetry-breaking scale, thus predicting new TeV-scale physics. There is thus a growing tension between these solutions and the absence of new physics at the Large Hadron Collider (LHC).

This has led to an alternative approach where $\mu^2 = 0$ is special not due to symmetry but because it separates the electroweak breaking and preserving phases. Models using this approach [1–18] propose mechanisms that allow the Higgs sector to spontaneously evolve to a near-critical state, thus resembling systems with self-organized criticality [19,20]. They use the electroweak vacuum expectation value (VEV) to trigger some cosmological dynamics that selects a small, negative Higgs mass squared from a landscape of values, $-\Lambda^2 \lesssim \mu^2 \lesssim \Lambda^2$, Λ being the cutoff. Unlike the anthropic approach [21,22], these models provide testable experimental signatures.

Our central observation is that there is already a physical quantity in the standard model—the Higgs vacuum energy contribution—that behaves in a special way at $\mu^2 = 0$. In the unbroken phase, this contribution is constant and independent of the Higgs potential parameters. In the broken phase, however, it drops monotonically with the electroweak VEV. We propose a model where the standard model (SM) Higgs sector is promoted to a two Higgs doublet model (2HDM) with an additional light pseudo-scalar, ϕ . The vacuum energy of the 2HDM- ϕ system decreases for large electroweak VEVs—a feature inherited from the SM. Unlike the SM, however, the vacuum energy contribution peaks at a non-zero value of the electroweak VEV, much smaller than the cutoff.

We assume a landscape of values for the parameters of the 2HDM- ϕ -potential, realized in an eternally inflating ‘multiverse’. We then utilize the observation made by [5] (and further developed in [6,8,11]) that maximal vacuum energy regions will expand at an exponentially faster rate compared to other regions and will eventually dominate the multiverse in volume. We show that with an appropriate choice of the measure for the multiverse, our mechanism can thus resolve the hierarchy problem. A schematic description of our mechanism has been presented in Fig. 1.

Our model has several attractive features: (1) we have a generic pseudo-Nambu goldstone boson (PNGB) potential that does not require a clockwork-like mechanism; (2) the field values of ϕ never exceed its ‘decay constant’, f , let alone the Planck scale; (3) unlike the anthropic

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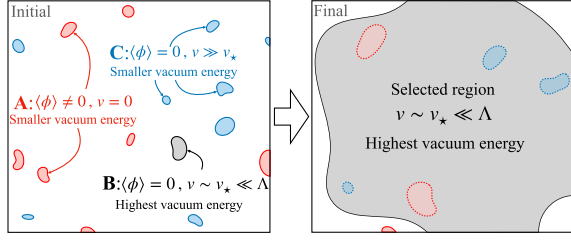


FIG. 1. A schematic description: gray patches (B) with maximal vacuum energy expand exponentially faster than others. The three cases (A, B, C) correspond to the three panels in Fig. 2.

mechanism [22], our model does not restrict the variation of other model parameters as the electroweak VEV is varied; (4) the number of e-folds in the slow-roll phase of inflation is not large; (5) there is a falsifiable prediction relating different masses and couplings of the 2HDM sector; and finally, (6) the pseudoscalar, ϕ , can account for dark matter.

Model—We consider a model with two Higgs doublets, H_1 and H_2 , and a pseudoscalar, ϕ . The potential is a sum of three terms,

$$V_H = V_\phi(\phi) + V_T(\phi, H_1, H_2) + V_{2HDM}(H_1, H_2), \quad (1)$$

where

$$V_\phi(\phi) = \mu_\phi^2 f^2 \left(-\frac{1}{2} \left(\frac{\phi}{f} \right)^2 + \lambda_\phi \left(\frac{\phi}{f} \right)^4 + \dots \right). \quad (2)$$

Here $\mu_\phi^2 \ll f^2$ and $\lambda_\phi \sim \mathcal{O}(1)$ are both positive. The ϕ value in our model never exceeds f so that we obtain a consistent effective field theory expansion. Such a potential can arise if ϕ is the PNGB of a global symmetry broken at a scale f , with, $M \sim \sqrt{\mu_\phi f}$, being the scale at which the symmetry is broken explicitly or via an anomaly.

The 2HDM potential is given by

$$\begin{aligned} V_{2HDM}(H_1, H_2) = & \mu_1^2 H_1^\dagger H_1 + \mu_2^2 H_2^\dagger H_2 + \lambda_1 (H_1^\dagger H_1)^2 \\ & + \lambda_2 (H_2^\dagger H_2)^2 + \lambda_3 (H_1^\dagger H_1)(H_2^\dagger H_2) \\ & + \lambda_4 (H_2^\dagger H_1)(H_1^\dagger H_2) \\ & + \frac{1}{2} (\lambda_5 (H_1^\dagger H_2)^2 + \lambda_5^* (H_2^\dagger H_1)^2), \end{aligned} \quad (3)$$

where we have assumed a $\mathbb{Z}_2$ symmetry, $H_1 \rightarrow -H_1$.

Finally, we consider a “trigger term” [10] that breaks the shift symmetry of the ϕ and the $\mathbb{Z}_2$ symmetry of the 2HDM sector,

$$V_T(\phi, H_1, H_2) = \frac{\mu_\phi^2 f^2}{2} \left(\kappa \left(\frac{H_1^\dagger H_2}{\mu_\phi f} \right) + \text{H.c.} \right) \left(\frac{\phi}{f} \right)^2. \quad (4)$$

All couplings in V_H , apart from κ and λ_5 , are real by hermiticity. Furthermore, we will assume a real and positive

κ —as any accompanying phase can always be removed by rotating H_2 —and rewrite $\lambda_5 = \hat{\lambda}_5 e^{i\alpha}$ with $\hat{\lambda}_5 < 0$. Note that perturbativity requires, $\kappa^2 \lesssim 16\pi^2 \lambda_\phi$. Due to this term, the electroweak VEV triggers a change in the ϕ potential, that makes the net vacuum energy contribution due to E peak at a small electroweak VEV.

The simplest possibility for such a trigger term, $V_T \sim |H|^2 \phi^2$, is problematic as we can close the Higgs loop to obtain $H^2 \phi^2 \rightarrow (\Lambda^2/16\pi^2) \phi^2$. This would mean that the change in the $V(\phi)$ that we wish to trigger by the electroweak VEV can already be caused by closing the Higgs loop unless $\Lambda \lesssim 4\pi v$ —which kills any hope of solving the hierarchy problem up to high scales. This is the rationale for considering the 2HDM in Eq. (3) and the trigger term in Eq. (4) which makes it impossible to close the Higgs loop [10]. The $\mathbb{Z}_2$ symmetry imposed on V_{2HDM} prevents terms like $(H_1^\dagger H_1)(H_1^\dagger H_2)$, that can generate such dangerous contributions at the two-loop level. The $\mathbb{Z}_2$ symmetry is, however, approximate as Eq. (4) breaks it. While this reintroduces a contribution to the ϕ -mass term at the two-loop level, the term generated is small enough not to spoil the effectiveness of the trigger term. We show this in detail in Supplemental Material [23].

We assume the Higgs sector is fully decoupled from the inflaton sector, and the total vacuum energy in a given landscape minimum is

$$\begin{aligned} \mathcal{V}\mathcal{E}^{(ijk)} = & \mathcal{V}\mathcal{E}_H[\{\alpha_H^{(i)}\}, P(\phi, H_1, H_2)] \\ & + \mathcal{V}\mathcal{E}_\chi[\{\beta_\chi^{(j)}\}, P(\chi)] + (\Lambda_{cc}^{(k)})^4, \end{aligned} \quad (5)$$

where χ is the inflaton field. The first term denotes the contribution from V_H in Eq. (1), the second term denotes the contribution from the inflaton potential and the third term is the cosmological constant contribution. Here $\{\alpha_H^{(i)}\}$ denotes a particular choice of Higgs parameters, namely $\{\mu_1^2, \mu_2^2, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \kappa, |\mu_\phi^2|, \lambda_\phi\}$; $\{\beta_\chi^{(j)}\}$ denotes a choice of parameters for the unspecified inflaton potential; and, $\Lambda_{cc}^{(k)}$, is a particular choice for the cosmological constant. The relevant parameters in our theory vary in the range, $-\Lambda^2 < \mu_i^2 < \Lambda^2$ and $-\Lambda^4 < \Lambda_{cc}^4 < \Lambda^4$, across the landscape. The scale up to which the hierarchy problem is solved is thus, Λ . The Mexican hat nature of V_ϕ is crucial to our mechanism and the sign of μ_ϕ^2 in Eq. (2) is thus assumed to be positive across the landscape. The landscape can arise either from string theory [24–26] or from a multiscalar potential (see Refs. [10,12]). We give a detailed description of the origin of a landscape of the latter kind in Supplemental Material [23].

In Eq. (5), $P(\chi)$ and $P(\phi, H_1, H_2)$, denote the volume-weighted distribution functions for the scalar field values during inflation. For the Higgs sector fields, $P(\phi, H_1, H_2)$ is the solution to the volume-weighted Fokker Planck equation (FPV). We will assume that $P(\phi, H_1, H_2)$ is

sharply peaked at the classical minima of Eq. (1), which requires (see Supplemental Material [23])

$$H_I^4/v_\star^4 \ll 1, \quad H_I^4/\mu_\phi^2 f^2 \ll 1, \quad f^2/M_{pl}^2 \ll 1, \quad (6)$$

H_I being the inflationary Hubble scale and $v_\star = 246$ GeV the observed electroweak VEV.

We will see that inflationary dynamics results in the multiverse being dominated by the vacuum state where each of the terms in Eq. (5)—in particular the Higgs contribution, $\mathcal{V}_\mathcal{H}$ —is maximized. This maximal vacuum energy would be of the order, $H_I^2 M_{pl}^2 \sim \Lambda^4$, as the maximal value of, Λ_{cc} , is of the order of the cutoff, Λ . The first equation of Eq. (6) thus implies an upper bound on the cutoff,

$$\Lambda \sim 10^{10} \text{ GeV} \sqrt{H_I/v_\star}. \quad (7)$$

To obtain the maximal value of $\mathcal{V}_\mathcal{H}$ in the landscape, first we vary only μ_1^2 and μ_2^2 and show that it is indeed maximized in regions with a hierarchically small VEV provided the quartics satisfy some requirements. We then carry out a variation of $\mathcal{V}_\mathcal{H}$ with respect to the quartics and show that—in regions where it is maximized—these requirements are automatically met.

Step 1: Variation of μ_1^2 and μ_2^2 —First, we assume that V_H is bounded from below, which requires

$$\lambda_3 + \lambda_4 + \hat{\lambda}_5 + 2\sqrt{\lambda_1 \lambda_2} \geq 0, \quad (8)$$

for $\lambda_4 + \hat{\lambda}_5 < 0$ and $\lambda_3 \geq -2\sqrt{\lambda_1 \lambda_2}$ for $\lambda_4 + \hat{\lambda}_5 > 0$. We then discuss the evaluation of $\mathcal{V}_\mathcal{H}$ in different classes of local minima.

Class I: $\langle H_1 \rangle = 0, \langle H_2 \rangle = 0, \langle \phi \rangle \neq 0$ In these regions the trigger term is not effective and ϕ has a Mexican hat potential that is minimized at, $\pm f/\sqrt{4\lambda_\phi}$. $\mathcal{V}_\mathcal{H}$ in this region is thus given by,

$$\mathcal{V}_\mathcal{H}^I = -\mu_\phi^2 f^2 / (16\lambda_\phi). \quad (9)$$

These solutions correspond to minima only if both μ_1^2 and μ_2^2 are positive.

Class II: Electroweak symmetry broken and $\langle \phi \rangle \neq 0$ —As we show in Supplemental Material [23], class II minima are disallowed if

$$\lambda_3 + \lambda_4 - \frac{\kappa^2}{8\lambda_\phi} - \left| \lambda_5 - \frac{\kappa^2}{8\lambda_\phi} \right| \leq -2\sqrt{\lambda_1 \lambda_2} \quad (10)$$

and $\lambda_4 + \hat{\lambda}_5 < 0$. We find that the vacuum energy in this case $\mathcal{V}_\mathcal{H}^I$ is always smaller than $\mathcal{V}_\mathcal{H}^I$ in Eq. (9).

Class III: Electroweak symmetry broken and $\langle \phi \rangle = 0$ —In landscape regions where $\langle H_1^\dagger H_2 \rangle \neq 0$, the trigger term gives a new contribution to the ϕ^2 term. This changes the

shape of the ϕ potential from a Mexican hat to one with a minimum at $\phi = 0$, if

$$-\mu_\phi^2 + (\kappa\mu_\phi/f)\langle (H_1^\dagger H_2 + \text{H.c.}) \rangle \geq 0. \quad (11)$$

The trigger term then becomes ineffective (as $\phi = 0$) giving no additional contribution to the 2HDM potential given by Eq. (3). Minima of this category exist only if

$$\lambda_4 + \hat{\lambda}_5 < 0 \quad (12)$$

The Higgs doublet VEV then takes the form $\langle H_1^T \rangle = (0, v_1/\sqrt{2})$ and $\langle H_2^T \rangle = (0, v_2 e^{i\xi}/\sqrt{2})$ with $v_1, v_2 > 0$ [27]. Minimizing the potential with respect to ξ and the CP even directions, we obtain

$$\mathcal{V}_\mathcal{H}^{III} = -\frac{1}{4}(\lambda_1 c_\beta^4 + \lambda_2 s_\beta^4 + \lambda_{345} s_\beta^2 c_\beta^2) v^4, \quad (13)$$

where $v = \sqrt{v_1^2 + v_2^2}$, $\tan \beta = v_2/v_1$, $\lambda_{345} = \lambda_3 + \lambda_4 + \hat{\lambda}_5$, $s_\beta = \sin \beta$, and $c_\beta = \cos \beta$. The minimization conditions imply that as μ_1^2 and μ_2^2 vary in the landscape, v and $\tan \beta$ also vary. The maximal value of $\mathcal{V}_\mathcal{H}^{III}$ with respect to the variation of v and $\tan \beta$ is given by

$$\mathcal{V}_\mathcal{H}^{III, \max} = -\frac{\mu_\phi^2 f^2}{4\kappa^2} (\lambda_{345} + 2\sqrt{\lambda_1 \lambda_2}), \quad (14)$$

which is realized in the following region of the landscape,

$$v_\star^2 = \mu_\phi f / (\kappa s_{\beta_\star} c_{\beta_\star}), \quad \tan^2 \beta_\star = \sqrt{\lambda_1 / \lambda_2}. \quad (15)$$

Notice that the electroweak VEV in Eq. (15) can be much smaller than the cutoff $\Lambda \sim \sqrt{H_I M_{pl}}$, as $M \sim \sqrt{\mu_\phi f}$ —the global symmetry breaking scale associated with the PNGB ϕ —can be naturally small. In fact, if we allow $|\mu_\phi^2|$ and f to vary in the landscape while keeping the sign of μ_ϕ^2 fixed, the vacua with the smallest $\mu_\phi f$ will be selected as they will have the highest value of $\mathcal{V}_\mathcal{H}^{III, \max}$. No other small parameters are required in our model and the quartics can all have $\mathcal{O}(1)$ values.

The landscape region defined by Eq. (15) has the highest $\mathcal{V}_\mathcal{H}$, not only among class III minima but in the whole landscape, provided the quartics satisfy some conditions. The first condition arises from requiring that $\mathcal{V}_\mathcal{H}^{III, \max}$ exceeds the vacuum energy for class I minima, $\mathcal{V}_\mathcal{H}^I$ which gives

$$\kappa^2 > 4\lambda_\phi (\lambda_{345} + 2\sqrt{\lambda_1 \lambda_2}). \quad (16)$$

We demand another condition on the quartics to eliminate the coexistence of multiple local minima for a given, $\{\alpha_H^i\}$. This is important because for multiple minima, the FPV equation might not prefer the highest one. For a given,

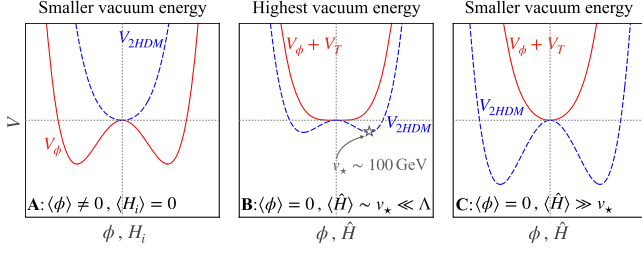


FIG. 2. The potential of ϕ and the Higgs doublets for varying values of the electroweak VEV, v . Assuming Eq. (10) is satisfied, only class I and III minima exist. H_I in this case can be any of the 8 components of the 2 Higgs doublets and $\hat{H} = \sqrt{\phi_3^2 + \phi_7^2}$, where ϕ_3 and ϕ_7 are the two CP even components. In the left panel, we show a minimum with $v = 0$ and a Mexican hat potential for ϕ . In the middle panel, we take, $v = v_*$, the threshold value required to change the ϕ potential to one with its minimum at the origin. In the right panel, we show that for, $v \gg v_*$, while the ϕ potential is still minimized at the origin, the total vacuum energy is smaller because of the 2HDM contribution. The middle panel clearly corresponds to vacua with maximal vacuum energy.

$\{\alpha_H^i\}$, class III minima cannot coexist (see Supplemental Material [23]) with class I minima but class II and III minima may coexist. If Eq. (10) is imposed, however, class II minima are disallowed and multiple minima cannot coexist. We illustrate the basic idea of our mechanism in Fig. 2 where we show the scalar potentials as the electroweak VEV is varied. For a more detailed derivation of the above results, see the supplementary material.

Step 2: Variation of the quartics—We have shown that the landscape regions defined by Eq. (15) have the maximal vacuum energy in the whole landscape provided the quartics satisfy the following: the potential is bounded [Eq. (8)], has class-III minima [Eq. (12)] and the two additional conditions discussed in the last paragraph [Eqs. (10) and (16)] are obeyed. We show in Supplemental Material [23] that as the quartics are varied, the regions satisfying these conditions are automatically selected as they have maximal vacuum energy.

Under well-motivated assumptions the multiverse eventually reaches a stationary state where it is dominated by the maximal energy vacuum [28–30]. This is the vacuum for which all the terms in Eq. (5), including the Higgs-sector contribution, $\mathcal{V}_{\mathcal{H}}$, are independently maximized. In this vacuum, the electroweak VEV, given by Eq. (15), is much smaller than the cutoff. Our assumption of decoupling between the Higgs and inflaton sectors implies that the transition from this vacuum to ones allowing slow-roll and reheating would not change the electroweak VEV. In Supplemental Material [23], we show that the volume of universes with an electroweak VEV not given by Eq. (15) is suppressed by a double exponential factor. We obtain this conclusion in the stationary measure which is free from issues like the youngness paradox.

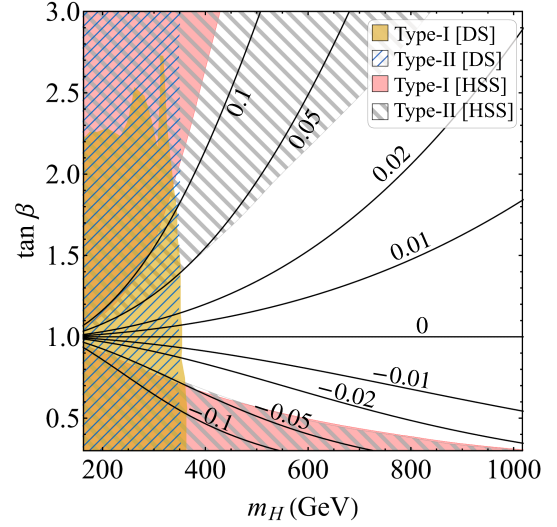


FIG. 3. The 2HDM parameter space for our model in the $\tan\beta$ - m_H plane. The black contours show predictions for $\cos(\beta - \alpha)$ as a function of m_H and $\tan\beta$. We also show bounds from Higgs signal strength (HSS) measurements [32,33] and direct searches (DS) [31] for both type I and II models.

Phenomenology—Eq. (15) provides the most important prediction of our model, $\tan^4\beta = (\lambda_1/\lambda_2)$. This can be rewritten using standard expressions (see e.g., [31]) as

$$\cos\alpha = \sqrt{\frac{m_h^2 - m_H^2 \tan^2\beta}{(m_h^2 - m_H^2)(1 + \tan^2\beta)}}. \quad (17)$$

While m_H , the mass of the heavier CP-even Higgs, can be measured by directly producing it at LHC, the measurement of the couplings of the charged Higgs and pseudo-scalar can determine $\tan\beta$. For a given value of $\tan\beta$, a measurement of the couplings of the SM-like Higgs, h , can then determine α , the CP-even mixing angle. In Fig. 3, we show the contours of $\cos(\beta - \alpha)$ predicted by Eq. (17) on the $\tan\beta$ - m_H plane. We also show bounds from Higgs signal strength measurements [32,33] as well as direct searches [31] in Fig. 3.

As far as ϕ is concerned, it's a weakly coupled state with a mass,

$$m_\phi^2 = (-\mu_\phi^2 + \kappa\mu_\phi v_*^2 s_{\beta_*} c_{\beta_*}/f), \quad (18)$$

that vanishes given Eq. (15), if we assume that the Higgs sector parameters are continuously scanned in the landscape. The leading term of the ϕ potential is then quartic. The ϕ energy density then falls as $\sim 1/a^4$ with the scale factor and is subdominant relative to the energy of the SM sector for a large enough reheating temperature or a small enough H_I (see Supplemental Material [23]).

If the parameters are scanned discretely, the cancellation in Eq. (18) is imperfect, resulting in a quadratic potential.

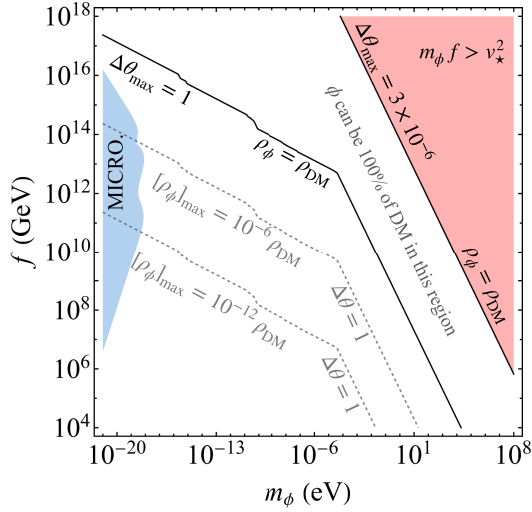


FIG. 4. The region in the m_ϕ - f parameter space where the pseudoscalar, ϕ , can account for the observed dark matter of the Universe (see main text for details).

The misalignment due to inflationary quantum spreading can then explain the observed dark matter (DM) abundance as shown in Fig. 4. The ϕ configuration also leads to an apparent violation of the equivalence principle [34,35]. We show, in blue, the resulting bounds from the MICROSCOPE experiment [36] (see Supplemental Material [23] for details). Our model also predicts the time variation of fundamental constants but not at a level that current experiments are sensitive to (see e.g., [37]).

Discussion—The key feature of our mechanism is the exponentially higher rate of inflationary expansion of patches with a small electroweak VEV, similar to some earlier works [5,6,8,11]. There is, however, a crucial difference between our model compared to previous ones: the light field, ϕ , does not scan the electroweak VEV and is involved only in the selection mechanism. Instead, as in [9,10,12,13], we assume a landscape having different values of the electroweak VEV. It is in this sense that our approach is minimal. This decoupling of scanning and selection, in fact, gives our model a unique simplicity and several advantages. In particular, our mechanism neither requires a clockwork sector [38–40] nor does it lead to ϕ values exceeding the decay constant or the Planck scale, thus avoiding the issues raised in [41].

To summarize, we have proposed a simple, nonanthropic selection mechanism that explains the smallness of the weak scale, provides a DM candidate, and a precise prediction in 2HDM phenomenology.

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- [23] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/x3fg-c5p2> for discussion on quantum effects, details on the 2HDM sector, origin of the landscape, dynamics of the multiverse and the phenomenology of the pseudoscalar.
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