

Measurement-Induced Crossover of Quantum Jump Statistics in Postselection-Free Many-Body Dynamics

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We reveal a nontrivial crossover of subsystem fluctuations of quantum jumps in continuously monitored many-body systems, which have a trivial maximally mixed state as a steady-state density matrix. While the fluctuations exhibit the standard volume law $\propto L$ following Poissonian statistics for sufficiently weak measurement strength, anomalous yet universal scaling law $\propto L^\alpha$ ($\alpha \sim 2.7$ up to $L = 20$) indicating super-Poissonian statistics appears for strong measurement strength. This drastically affects the precision of estimating the rate of quantum jumps: for strong (weak) measurement, the estimation uncertainty is enhanced (suppressed) as the system size increases. We demonstrate that the anomalous scaling of the subsystem fluctuation originates from an integrated many-body autocorrelation function and that the transient dynamics contributes to the scaling law rather than the Liouvillian gap. The measurement-induced crossover is accessed only from the postselection-free information obtained from the time and the position of quantum jumps and can be tested in ultracold atom experiments.

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Introduction—Dissipative quantum many-body phenomena induced by the coupling to environments offer a rich possibility for investigating physics unique to non-equilibrium situations [1–3]. One of the key ingredients is the measurement, which has been actively investigated in the context of open quantum systems [4,5]. The readout of measurement brings about nonequilibrium phenomena that cannot be seen in unconditional open quantum systems, such as measurement-induced entanglement transitions [6–33], unconventional non-Hermitian criticality [34–37], and measurement-altered universality and boundary transitions [38–42]. While these works demonstrate the significance to study an interplay between measurements and many-body effects, the so-called postselection problem makes it difficult to experimentally observe measurement-induced quantum many-body phenomena, despite efforts to evade the problem [29,43–56]. Thus, it is of

importance to investigate different type of measurement-induced physics that does not rely on postselections.

Notably, in quantum simulations of many-body trajectory dynamics under continuous monitoring, we can access times and positions of quantum jumps without postselection [3]. The statistics of such stochastic trajectories has been widely investigated both in classical [57,58] and open quantum [59–62] systems, where various properties have been demonstrated such as dynamical phase transitions [57,63] and thermodynamic uncertainty relations [58,64–66]. For instance, full counting statistics of quantum jumps in open systems are useful for capturing many-body physics [67–71] as well as few-body physics [72]. In particular, the variance of the number of jumps contains information that is not obtained from an average of physical quantities [72]; it is related with autocorrelations and relaxation rates [73–76]. For example, in driven dissipative many-body systems that break the detailed-balance conditions, diverging current fluctuations at the dissipative phase transition are studied [77,78]. Recently, it has been demonstrated that space-time correlations of measurement outcomes can reveal dynamical heterogeneity before thermalization in kinetically constrained many-body models [79].

However, it is highly nontrivial whether the quantum jump statistics exhibit universal many-body phenomena in the steady state characterized by the (unconditional) density

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matrix that heats up to a thermal state, which naturally appears in quantum many-body systems under particle-number measurement [20,29]. In particular, it is intriguing to investigate the impact of the measurement strength on fluctuations of quantum jumps even when the steady-state density matrix is independent of the measurement strength. This may lead to a novel measurement-induced phenomena distinct from, e.g., entanglement transitions.

In this Letter, we introduce the subsystem fluctuation of quantum jumps (SFQJ) as a new indicator to investigate measurement effects of continuously monitored many-body systems and discover its crossover in the steady state. In particular, we demonstrate that SFQJ obeys the Poissonian-type scaling $\propto L$ (L is the system size) for sufficiently weak measurement, whereas anomalous yet universal super-Poissonian-type scaling $\propto L^\alpha$ ($\alpha \sim 2.7$ up to $L = 20$) appears for strong measurement, although the unconditional steady state is always the trivial infinite-temperature state. This crossover results in two distinct regimes in estimating the rate of quantum jumps: for strong (weak) measurement, the uncertainty is enhanced (suppressed) when the system size increases. Strikingly, the crossover is unique to SFQJ and absent for fluctuations in the whole system, where the Poissonian-type scaling always appears. We elucidate that the anomalous scaling stems from the integrated many-body autocorrelation functions for the unconditional dynamics and that the universal exponents are not determined by the Liouvillian gap. Measurement-induced many-body phenomena presented in our Letter can be accessed in ultracold atom experiments without postselections. See Fig. 1 for the summary of our Letter.

Continuous monitoring and SFQJ—We consider interacting hard-core boson chains of length L ($= 2\mathbb{N}$) with the periodic boundary condition,

$$H = \sum_{j=1}^L \frac{J_{xy}}{2} (b_{j+1}^\dagger b_j + b_{j+1} b_j^\dagger) + \sum_{j=1}^L J_z n_{j+1} n_j, \quad (1)$$

where b_j is the bosonic annihilation operator satisfying the hard-core constraint $b_j^2 = 0$ and $n_j = b_j^\dagger b_j$ is the particle number operator. We call this model the XXZ spin chain as the Hamiltonian (1) is exactly mapped to the XXZ model with the anisotropy J_z/J_{xy} . Then, we study the dynamics under continuous monitoring of a local particle number by employing the quantum trajectory method [20,29]. The stochastic Schrödinger equation that governs the dynamics reads as [4]

$$d|\psi(t)\rangle = -iH|\psi(t)\rangle dt + \sum_{j=1}^L \left(\frac{n_j |\psi(t)\rangle}{\sqrt{\langle n_j \rangle}} - |\psi(t)\rangle \right) dN_j, \quad (2)$$

where $\langle \cdot \rangle$ denotes a quantum expectation value for the state $|\psi(t)\rangle$. Here, a discrete random variable $dN_j = 0, 1$ that

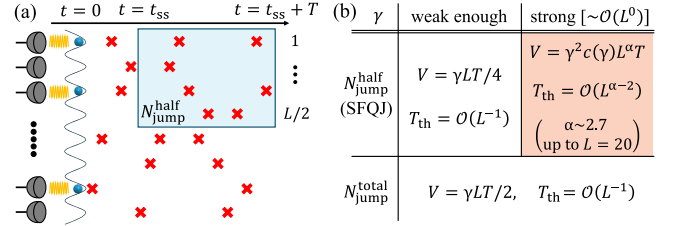


FIG. 1. (a) Schematic figure of our setup. We count the number of jumps (red cross marks) in a half chain $N_{\text{jump}}^{\text{half}}$ and take its variance along trajectory realizations. (b) Table of the main results for the variance V and estimation uncertainty T_{th} concerning the number of jumps. Subsystem fluctuation of quantum jumps (SFQJ) for strong measurement exhibits anomalous yet universal super-Poissonian statistics, and the estimation uncertainty is enhanced as the system size is increased, in stark contrast to the case for the weak measurement or the total fluctuation of quantum jumps.

counts the increment of a jump at site j is chosen according to $dN_j dN_k = \delta_{jk} dN_j$ and $E[dN_j] = \gamma \langle n_j \rangle dt$, where $E[\cdot]$ represents an ensemble average over the stochastic process. In the following, we assume that the initial state is prepared in the Néel state $|1010\cdots\rangle$. Importantly, the state $\rho(t)$ averaged over the measurement outcomes becomes a maximally mixed state $\rho_{\text{ss}} = I/D_0$ after long times, irrespective of measurement strengths $\gamma(>0)$. Here, I is the identity matrix, and D_0 is the dimension of the Hilbert space in the half-filling sector.

As a primary quantity of interest, we introduce the net number of quantum jumps for the half-chain subsystem [see Fig. 1(a)],

$$N_{\text{jump}}^{\text{half}}(T) = \sum_{j=1}^{L/2} \int_{t_{\text{ss}}}^{t_{\text{ss}}+T} dN_j, \quad (3)$$

where t_{ss} is sufficiently large such that $\rho(t_{\text{ss}}) \simeq \rho_{\text{ss}}$. We especially focus on the variance of $N_{\text{jump}}^{\text{half}}(T)$, or SFQJ, which is defined by

$$V[N_{\text{jump}}^{\text{half}}(T)] = E[N_{\text{jump}}^{\text{half}}(T)^2] - E[N_{\text{jump}}^{\text{half}}(T)]^2. \quad (4)$$

For comparison, we also define the number of quantum jumps for the whole system, $N_{\text{jump}}^{\text{total}}(T) = \sum_{j=1}^L \int_{t_{\text{ss}}}^{t_{\text{ss}}+T} dN_j$, and consider its variance. Note that the average jump number for $N_{\text{jump}}^{\text{total}}(T)$ is given by $E[N_{\text{jump}}^{\text{total}}(T)] = \gamma \langle \sum_{j=1}^L n_j \rangle T = \gamma LT/2$, which also leads to $E[N_{\text{jump}}^{\text{half}}(T)] = \gamma LT/4$ because of the symmetry. We remark that SFQJ is different from the so-called bipartite fluctuations $\langle n_{\text{half}}^2 \rangle - \langle n_{\text{half}} \rangle^2$ for $n_{\text{half}} = \sum_{i=1}^{L/2} n_i$ [48].

Measurement-induced crossover—We first demonstrate the measurement-induced crossover of SFQJ by numerically studying the Heisenberg model ($J_z = 1$). As shown in

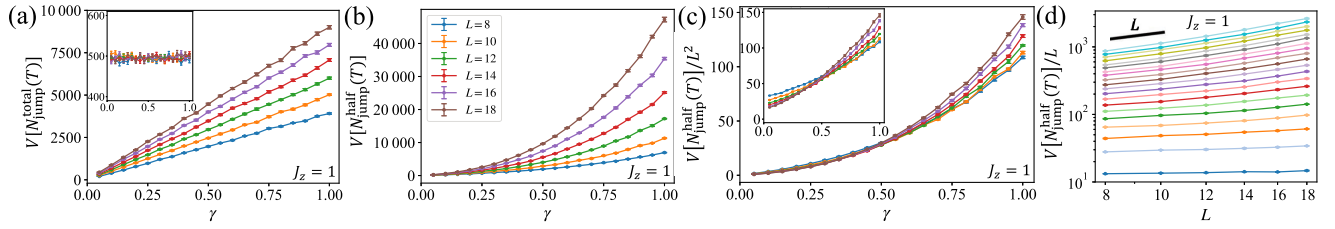


FIG. 2. Numerical results for the variance of quantum jumps for the Heisenberg model ($J_z = 1$) for 832 trajectories and $T = 990$. (a) Variance of net quantum jumps in the whole system $\{ \text{Inset: } V[N_{\text{jump}}^{\text{total}}(T)]/(\gamma L) \}$, (b) SFQJ, (c) $V[N_{\text{jump}}^{\text{half}}(T)]/L^2$ $\{ \text{Inset: } V[N_{\text{jump}}^{\text{half}}(T)]/(\gamma L^2) \}$, and (d) $V[N_{\text{jump}}^{\text{half}}(T)]/L$. The SFQJ exhibits a measurement-induced crossover of the system-size scaling. Data are plotted against γ for system sizes $L = 8, 10, \dots, 18$ in (a)–(c) and against L for measurement strengths $\gamma = 0.05, 0.1, \dots, 1$ from bottom to top in (d). We take the average over 20 time intervals for $t \in [200, 1190], [1190, 2180], \dots, [19010, 2 \times 10^4]$ [84].

Fig. 2(a), we see that $V[N_{\text{jump}}^{\text{total}}(T)]$ exhibits a trivial scaling proportional to γL , irrespective of γ . On the other hand, SFQJ shown in Fig. 2(b) does not behave linearly against γ or L , and we find a crossing in $V[N_{\text{jump}}^{\text{half}}(T)]/L^2$ around $\gamma_c \sim 0.5$ as seen in Fig. 2(c). This indicates a measurement-induced crossover of SFQJ in the system-size scaling, and in fact in Fig. 2(d), we see that the scaling is estimated to show a crossover from $\propto L^{1.13}$ for $\gamma = 0.05$ to $\propto L^{2.45}$ for $\gamma = 1$. However, we note that universal exponents are different from these values due to the finite-size effect. This is because SFQJ is given as a sum of two terms whose L dependence is different, as detailed later, which leads to the deviations of exponents from the universal ones for finite-system sizes. We can conduct the same numerical simulation for XX ($J_z = 0$) and XXZ ($J_z = 0.5, 1.5$) models as shown in Supplemental Material [80]. All of these models exhibit the measurement-induced crossover, signifying an anomalous enhancement of SFQJ in many-body quantum systems under continuous monitoring. We also mention that such an anomalous scaling does not exist in the Ising model ($J_{xy} = 0, J_z = 1$) due to particle-number conservation at a single site [80].

Relation to unconditional dynamics—We examine the measurement-induced crossover with the help of the unconditional dynamics of Eq. (2) and clarify that SFQJ is composed of two terms: dynamical activity with the volume law that governs SFQJ for sufficiently weak measurement strength and an integrated autocorrelation function that causes the anomalous enhancement of SFQJ. First, we should beware that SFQJ (4) is unique to quantum trajectories because measurement outcomes are averaged over all possible sequences in the unconditional dynamics. However, the counting variable is still evaluated by using the Liouvillian, and we can write down SFQJ in terms of the ensemble-averaged dynamics [72]. To start with, we calculate the noise $D_{\text{half}}(t) = dV[N_{\text{jump}}^{\text{half}}(t)]/dt$ for a subsystem in the steady state (see Appendix A in End Matter for the detailed derivation) as

$$D_{\text{half}}(T) = K_{\text{half}} + 2 \int_0^T d\tau \{ \text{Tr}[\mathcal{J}_{\text{half}} e^{\mathcal{L}\tau} \mathcal{J}_{\text{half}} \rho_{\text{ss}}] - J_{\text{half}}^2 \}, \quad (5)$$

where $\mathcal{L}(\rho) \equiv -i[H, \rho] + \gamma \sum_i (-\frac{1}{2}\{n_i, \rho\} + n_i \rho n_i)$ is the Liouvillian superoperator for the ensemble-averaged dynamics of Eq. (2), the subsystem superoperator $\mathcal{J}_{\text{half}}$ is defined as $\mathcal{J}_{\text{half}} \rho(t) \equiv \gamma \sum_{i=1}^{L/2} n_i \rho(t) n_i$, and $K_{\text{half}} \equiv \gamma \sum_{i=1}^{L/2} \text{Tr}[n_i \rho_{\text{ss}}]$ and $J_{\text{half}} \equiv \text{Tr}[\mathcal{J}_{\text{half}} \rho_{\text{ss}}]$ both reduce to $K_{\text{half}} = J_{\text{half}} = \gamma L/4$, which is nothing but (half of) the dynamical activity in the steady state, $E[dN_{\text{jump}}^{\text{total}}(t)]/dt = 2E[dN_{\text{jump}}^{\text{half}}(t)]/dt = \gamma L/2$. By numerically calculating the long-time dynamics in Eq. (5), we find that the noise becomes constant for large T and does not involve a contribution of $\mathcal{O}(T)$ [72]. Then, we obtain SFQJ as $V[N_{\text{jump}}^{\text{half}}(T)] = D_{\text{half}}T$, which leads to

$$V[N_{\text{jump}}^{\text{half}}(T)] = V_{\text{act}}(T) + V_{\text{anom}}(T), \\ = \frac{\gamma L T}{4} + 2\gamma^2 T \int_0^T d\tau \langle n'_{\text{half}}(\tau) n'_{\text{half}} \rangle_{\infty}, \quad (6)$$

where $n'_{\text{half}}(\tau) = n_{\text{half}}(\tau) - \langle n_{\text{half}} \rangle_{\infty}$, the Heisenberg picture for the adjoint Liouvillian $\tilde{\mathcal{L}}(A) \equiv i[H, A] + \gamma \sum_i (-\frac{1}{2}\{n_i, A\} + n_i A n_i)$ is represented as $n_{\text{half}}(\tau) = e^{\tilde{\mathcal{L}}\tau} n_{\text{half}}$, and $\langle \cdot \rangle_{\infty}$ stands for the expectation value with respect to ρ_{ss} . This demonstrates that the super-Poissonian-type anomalous enhancement of SFQJ in the measurement-induced crossover originates from the integrated autocorrelation function

$$C_{\text{auto}}^{\text{half}} \equiv \int_0^T d\tau C_{\text{auto}}^{\text{half}}(\tau) \equiv \int_0^T d\tau \langle n'_{\text{half}}(\tau) n'_{\text{half}} \rangle_{\infty}. \quad (7)$$

Here, the positive (zero) correlation $V[N_{\text{jump}}^{\text{half}}(T)] > E[N_{\text{jump}}^{\text{half}}(T)]$ ($V[N_{\text{jump}}^{\text{half}}(T)] = E[N_{\text{jump}}^{\text{half}}(T)]$) is referred to as the super-Poissonian (Poissonian) statistics. We remark

that, as the particle number in the whole chain is a conserved quantity, the anomalous term is absent for the whole system, and the variance $V[N_{\text{jump}}^{\text{total}}(T)]$ reduces to the dynamical activity, which shows a Poissonian-type volume law irrespective of the measurement strength.

Origin of the measurement-induced crossover—Using the quantum trajectory method, we numerically simulate the Heisenberg chain and obtain the contribution of V_{anom} and V_{act} to SFQJ. The two terms V_{anom} and V_{act} are quantitatively evaluated with the help of the analytical expression (6) as shown in Fig. 3(a), where they cross at the critical measurement strength γ_c , and the leading contribution to SFQJ changes from V_{act} (Poissonian) to V_{anom} (super-Poissonian) as γ is increased. This causes a measurement-induced crossover of SFQJ, illustrated in Fig. 2, which offers the novel platform to investigate many-body phenomena under measurement absent in the entire system. We note that the anomalous scaling is characterized by a mechanism distinct from, e.g., measurement-induced entanglement transitions, whose presence depends on interactions in one dimension [85].

In Fig. 3(b) and in other parameters of J_z [80], we find by performing the finite-size scaling analysis for $L = 8, 10, \dots, 18$ that

$$V_{\text{anom}} \propto L^\alpha, \quad (8)$$

where $\alpha = 2.64 \pm 0.02, 2.66 \pm 0.02, 2.67 \pm 0.02, 2.67 \pm 0.02$ for $J_z = 0, 0.5, 1, 1.5$ with $\gamma = 1$, respectively. Here, the statistical uncertainty of the scaling exponent arises from averaging over different time intervals. Remarkably, this indicates that the anomalous scaling of SFQJ is characterized by the universal exponent $\alpha \sim 2.7$. Since V_{anom} is also proportional to $\gamma^2 c(\gamma)$, where $c(\gamma)$ is an increasing polynomial function of γ with $c(0) \geq 0$ [80], SFQJ is dominated by $V_{\text{anom}} \propto L^\alpha$ for strong $\gamma = \mathcal{O}(L^0)$ instead of $V_{\text{act}} \propto L$ for weak γ [which should be $\mathcal{O}(L^0)$], the fact of which is the direct origin of the crossover of SFQJ [see Fig. 1(b)]. Notably, the anomalous yet universal

super-Poissonian statistics of SFQJ even persists in the thermodynamic limit, further highlighting the significance of the measurement-induced universality (see Appendix B for further evaluation of the rigidity of α). Physically, such an enhancement of SFQJ is regarded as a kind of Zeno effect [86], which indicates that, if a quantum jump occurs in a subsystem, the next jump is also likely to occur in the same subsystem.

We argue that the global U(1) symmetry without internal particle number conservations is the essential physical factor for the measurement-induced universality. In Appendix B in End Matter, we indeed demonstrate that the anomalous scaling with a quantitatively similar universal exponent at quarter filling and even in nonintegrable models with U(1) symmetry. Note that, if the total particle conservation is broken, the super-Poissonian statistics of the fluctuation would emerge in the whole system as well as the subsystem owing to the Zeno effect.

Impact on the precision—One interesting consequence of the above anomalous scaling is the drastically enhanced uncertainty in estimating the rate of quantum jumps occurring in the subsystem. Let us estimate the rate of quantum jumps in the subsystem by $\bar{I}_{\text{half}}(T) = N_{\text{jump}}^{\text{half}}(T)/T$ [72]. We then obtain

$$E[\bar{I}_{\text{half}}(T)] = \frac{E[N_{\text{jump}}^{\text{half}}(T)]}{T} = J_{\text{half}} \propto \gamma L, \quad (9)$$

$$V[\bar{I}_{\text{half}}(T)] = \frac{V[N_{\text{jump}}^{\text{half}}(T)]}{T^2} = \frac{D_{\text{half}}}{T} \propto \frac{\gamma L + \gamma^2 c(\gamma) L^\alpha}{T}. \quad (10)$$

To estimate the true rate $E[\bar{I}_{\text{half}}(T)]$ accurately, we shall evaluate the estimation uncertainty defined by the (squared) coefficient of variation,

$$\frac{V[\bar{I}_{\text{half}}(T)]}{\{E[\bar{I}_{\text{half}}(T)]\}^2} = \frac{D_{\text{half}}}{J_{\text{half}}^2 T} \equiv \frac{T_{\text{th}}}{T}, \quad (11)$$

where $T_{\text{th}} \equiv D_{\text{half}}/J_{\text{half}}^2$ is frequently discussed in thermodynamic and kinetic uncertainty relations [72]; here, we rather focus on the L dependence in the large- T dynamics. For sufficiently weak γ , we find

$$T_{\text{th}} \propto L^{-1}. \quad (12)$$

This means that the error for evaluating $E[\bar{I}_{\text{half}}(T)]$ diminishes fast enough and $E[\bar{I}_{\text{half}}(T)]$ is accurately determined for larger system sizes. On the other hand, for strong measurement satisfying $\gamma \sim \mathcal{O}(L^0)$, we obtain

$$T_{\text{th}} \propto L^{\alpha-2}, \quad (13)$$

with $\alpha \sim 2.7$ (up to $L = 20$), which means that the uncertainty in estimating $E[\bar{I}_{\text{half}}(T)]$ is enhanced as the

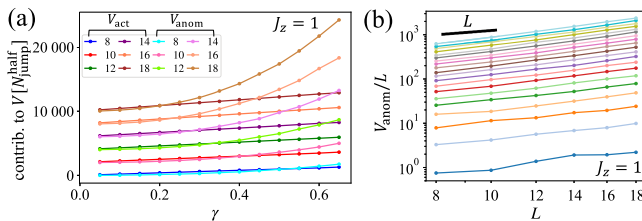


FIG. 3. Numerical results for V_{anom} and V_{act} based on the quantum trajectory method for the Heisenberg model. (a) Leading contribution of V_{anom} and V_{act} changes at a critical measurement strength. (b) System-size scaling of V_{anom}/L . Data are shifted by 2000 as L is increased as $L = 8, 10, \dots, 18$ in (a) to improve visualization, and measurement strengths are $\gamma = 0.05, 0.1, \dots, 1$ from bottom to top in (b). The parameters and methods are the same as in Fig. 2.

system size is increased [see Fig. 1(b)]. We remark that, as the estimation uncertainty in Eq. (13) is proportional to $L^{\alpha-2}$, a particular super-Poissonian-type variance satisfying $\alpha > 2$ enhances T_{th} with increasing L . Moreover, the estimation uncertainty (11) is related to the ergodicity of a quantum trajectory for the jump statistics [87], i.e., the concept that the long-time average of a single realization yields the ensemble average in almost all trajectories. Equation (13) means that the ergodicity of a quantum trajectory tends to be disturbed for larger system sizes in XXZ spin chains under continuous monitoring. We note that the fluctuation for a whole system always follows the trivial Poissonian statistics, whose estimation uncertainty reduces to Eq. (12) for arbitrary γ .

Autocorrelation function and Liouvillian gap—Finally, it is illustrative to what extent the scaling (8) is affected by the Liouvillian gap Δ , the smallest nonzero value among the real parts of the eigenvalues of $-\mathcal{L}$ (see Appendix C in End Matter). This is because the asymptotic decay of the single-site autocorrelation function is dominated by Δ as $|C_{\text{auto}}^{\text{single}}(\tau)| \equiv |\langle n'_1(\tau)n'_1 \rangle_{\infty}| \sim e^{-\Delta\tau}$ ($\tau \rightarrow \infty$) [88,89]. We especially compare the exponent of $C_{\text{auto}}^{\text{single}} \equiv \int_0^T d\tau C_{\text{auto}}^{\text{single}}(\tau)$ with that solely extracted from the Liouvillian gap, $C_{\text{auto}}^{\text{single}} \stackrel{?}{\sim} \int_0^T e^{-\Delta\tau} \sim 1/\Delta$. In Fig. 4, we depict the exponent of the inverse Liouvillian gap obtained from the exact diagonalization and that of $C_{\text{auto}}^{\text{single}}$ calculated from the quantum trajectory method for the Heisenberg model. We find that the Liouvillian gap estimated from the finite-size scaling analysis reads as $\Delta \propto 1/L^{1.95}$ for $\gamma = 1$, which is close to the analytical result $\Delta \propto 1/L^2$ for $\gamma \gg 1/L$ [90,91]. On the other hand, we obtain $C_{\text{auto}}^{\text{single}} \propto L^{0.74 \pm 0.04}$ for $\gamma = 1$, which indicates that the scaling for the integrated autocorrelation function is largely governed by the transient dynamics [89] rather than the Liouvillian gap. We emphasize that, as the dynamical activity is proportional to L^0 for a single site, the exponent for $C_{\text{auto}}^{\text{single}}$ is also anomalous. As for $C_{\text{auto}}^{\text{half}}$ shown in Fig. 4, which reads as $C_{\text{auto}}^{\text{half}} \propto L^{2.62 \pm 0.03}$ for $\gamma = 1$ [92], we find that not only the diagonal elements corresponding to $C_{\text{auto}}^{\text{single}}$

but also the off-diagonal components $C_{\text{auto}}^{ij} \equiv \int_0^T d\tau \langle n'_i(\tau)n'_j \rangle_{\infty}$ ($i \neq j$) therein contribute to the scaling of $C_{\text{auto}}^{\text{half}}$ because $C_{\text{auto}}^{\text{half}} \gg L/2 \cdot C_{\text{auto}}^{\text{single}}$ holds. Thus, we cannot estimate the anomalous scaling exponent for $C_{\text{auto}}^{\text{half}}$ given in Eq. (8) by the Liouvillian gap.

Experimental proposal and conclusions—Our main discoveries, i.e., the measurement-induced crossover of SFQJ and its anomalous scaling, are accessible in experiments without postselection because the only information that we need is the times and positions of quantum jumps. For example, in ultracold atoms, hard-core bosons are naturally realized by employing the Tonks-Girardeau gas [93,94], and SFQJ is obtained by recording the photon scattering with a probe light [95,96] and quantum-gas microscopes [97,98]. We then realize the measurement-induced crossover by tuning the light intensity, while this may be affected by heating effects as the detection fidelity of scattered photons would be limited. We also suggest changing the system size instead by fixing the probe intensity; SFQJ shows the volume law for sufficiently small system sizes, whereas it exhibits anomalous universal scaling $\propto L^{\alpha}$ ($\alpha \sim 2.7$ up to $L = 20$) for large system sizes.

We emphasize that the measurement-induced crossover, which is the gradual change of SFQJ at the finite system size [99], is significant irrespective of its persistence in the thermodynamic limit. This is because crossover phenomenon away from thermodynamic limit (say, order of 10 or 10^2), which is relevant for experiments of trapped ions and ultracold atoms in optical lattices, has served as a variety of significant research topics. In open quantum systems, the qualitative difference between weak dissipation of $\gamma = o(L^0)$ and strong dissipation of $\gamma = \mathcal{O}(L^0)$ was studied for, e.g., the instantaneous decay rate of autocorrelation functions [89] and the computational complexity under noise in random circuit sampling [100].

In this Letter, we have discovered hitherto unnoticed crossover of measurement-induced many-body properties free from postselection, even though the unconditional steady state trivially becomes a maximally mixed state insensitive to measurement strength. To highlight this, we have shown that SFQJ serves as a new indicator for quantum jumps in many-body systems under measurement, in stark contrast to the fluctuation in the whole system. Our results give a new twist for investigating measurement-induced many-body universality in open quantum systems. The next step would be the evaluation of the universality class and investigation of the impact of dissipative phase transitions [101] on the anomalous scaling in prototypical many-body models. In addition, it is worth studying the relation between our results and well-known universal scalings in many-body problems, e.g., long-time tails in hydrodynamics [102]. Finally, analytically evaluating the universal exponent obtained in our Letter is an intriguing future problem.

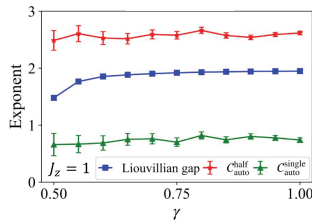


FIG. 4. Exponents a , b , and c of the Liouvillian gap $\Delta \propto 1/L^a$ (blue), $C_{\text{auto}}^{\text{single}} \propto L^b$ (green), and $C_{\text{auto}}^{\text{half}} \propto L^c$ (red) obtained from the finite-size scaling analysis with $L = 8, 10, 12, 14$ for the Heisenberg model. The anomalous scaling of SFQJ cannot be captured by the Liouvillian gap.

The numerical calculations were partly carried out with the help of QuSpin [103].

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Data availability—The data that support the findings of this Letter are openly available [104], embargo periods may apply.

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End Matter

Appendix A: Derivation of Eq. (5)—The noise given in Eq. (5) is calculated with the help of the dynamics described by Eq. (2), which is the stochastic Schrödinger equation that is mainly used for numerical simulations [72]. To prove it, we first calculate the two-point correlation function,

$$F(t, t') = E[I_{\text{half}}(t)I_{\text{half}}(t')] - E[I_{\text{half}}(t)]E[I_{\text{half}}(t')], \quad (\text{A1})$$

where $I_{\text{half}}(t) = dN_{\text{jump}}^{\text{half}}(t)/dt$ and $F(t, t + \tau) = F(t + \tau, t)$ is satisfied for $\tau > 0$. To evaluate the first term in $F(t, t + \tau)$, we need to calculate the joint probability $P[dN_i(t) = 1, dN_j(t + \tau) = 1]$, which represents that we have quantum jumps at times t and $t + \tau$ without any other restrictions in between. This is because the following is satisfied:

$$\begin{aligned} E[I_{\text{half}}(t)I_{\text{half}}(t + \tau)] &= \frac{1}{dt^2} \sum_{i,j}^{L/2} P[dN_i(t) = 1, dN_j(t + \tau) = 1] \\ &= \frac{1}{dt^2} \sum_{i,j}^{L/2} E\{P[dN_j(t + \tau) = 1 | dN_i(t) = 1, \rho_c(t)] p_i^c(t)\}, \end{aligned} \quad (\text{A2})$$

where we introduced the conditional probability $P[\cdot | \cdot]$ by using $p_i^c(t) \equiv \text{Tr}[\mathcal{L}_i \rho_c(t)] dt$ with the conditional density

matrix $\rho_c(t)$ on a particular sequence of quantum jumps that satisfies $E[\rho_c(t)] = \rho(t)$. Here, $\mathcal{L}_i$ is the super-operator defined as $\mathcal{L}_i(\rho) \equiv \gamma n_i \rho n_i$. Note that $p_i^c(t)$ stands for the probability that a quantum jump occurs at time t for $\rho_c(t)$. Equation (2) indicates that, if a quantum jump occurs at site i , the conditional density matrix of the system $\rho_c(t)$ is updated as

$$\rho_c'(t) \equiv \frac{\mathcal{L}_i \rho_c(t)}{\text{Tr}[\mathcal{L}_i \rho_c(t)]} = \frac{\mathcal{L}_i \rho_c(t) dt}{p_i^c(t)}. \quad (\text{A3})$$

By using the ensemble-averaged dynamics of Eq. (2), we obtain the probability that a quantum jump occurs at time $t + \tau$ with respect to the time-evolved state $e^{\mathcal{L}\tau} \rho_c'(t)$ as

$$\begin{aligned} P[dN_j(t + \tau) = 1 | dN_i(t) = 1, \rho_c(t)] &= \text{Tr}[\mathcal{L}_j e^{\mathcal{L}\tau} \rho_c'(t)] dt = \frac{(dt)^2}{p_i^c(t)} \text{Tr}[\mathcal{L}_j e^{\mathcal{L}\tau} \mathcal{L}_i \rho_c(t)]. \end{aligned} \quad (\text{A4})$$

Then, Eq. (A2) is calculated with the help of Eq. (A4) as

$$E[I_{\text{half}}(t)I_{\text{half}}(t + \tau)] = \text{Tr}[\mathcal{J}_{\text{half}} e^{\mathcal{L}\tau} \mathcal{J}_{\text{half}} \rho(t)]. \quad (\text{A5})$$

The case $\tau = 0$ is treated separately, leading to

$$E[I_{\text{half}}(t)^2] = \frac{1}{(dt)^2} \sum_{i=1}^{L/2} E[dN_i(t)] = \frac{1}{dt} K_{\text{half}}(t), \quad (\text{A6})$$

which results in

$$F(t, t + \tau) = \delta(\tau) K_{\text{half}}(t) + \text{Tr}[\mathcal{J}_{\text{half}} e^{\mathcal{L}\tau} \mathcal{J}_{\text{half}} \rho(t)] - J_{\text{half}}(t) J_{\text{half}}(t + \tau). \quad (\text{A7})$$

Here, we introduce $K_{\text{half}} \equiv \gamma \sum_{i=1}^{L/2} \text{Tr}[n_i \rho(t)]$. As SFQJ (4) is rewritten by using Eq. (A1) as

$$V[N_{\text{jump}}^{\text{half}}(T)] = \int_{t_{\text{ss}}}^{T+t_{\text{ss}}} dt \int_{t_{\text{ss}}}^{T+t_{\text{ss}}} dt' F(t, t'), \quad (\text{A8})$$

we find that the noise $D_{\text{half}}(t) = dV[N_{\text{jump}}^{\text{half}}(t)]/dt$ is given by

$$\begin{aligned} D_{\text{half}}(T) &= 2 \int_{t_{\text{ss}}}^{T+t_{\text{ss}}} dt' F(T + t_{\text{ss}}, t'), \\ &= 2 \int_0^T d\tau F(T + t_{\text{ss}} - \tau, T + t_{\text{ss}}). \end{aligned} \quad (\text{A9})$$

Finally, by substituting Eq. (A7) into (A9), we obtain

$$\begin{aligned} D_{\text{half}}(T) &= 2 \int_0^T d\tau \delta(\tau) K_{\text{half}}(T + t_{\text{ss}} - \tau) \\ &\quad + 2 \int_0^T d\tau \left\{ \text{Tr}[\mathcal{J}_{\text{half}} e^{\mathcal{L}\tau} \mathcal{J}_{\text{half}} \rho(T + t_{\text{ss}} - \tau)] \right. \\ &\quad \left. - J_{\text{half}}(T + t_{\text{ss}} - \tau) J_{\text{half}}(T + t_{\text{ss}}) \right\}, \end{aligned} \quad (\text{A10})$$

which reduces to Eq. (5) by replacing $K_{\text{half}}(T + t_{\text{ss}} - \tau)$, $J_{\text{half}}(T + t_{\text{ss}} - \tau)$, $J_{\text{half}}(T + t_{\text{ss}})$, and $\rho(T + t_{\text{ss}} - \tau)$ with their steady-state values.

Appendix B: Numerical results for large system sizes and different initial states—To further confirm the universality of the super-Poissonian statistics of SFQJ, we perform numerical simulations under continuous monitoring for large system sizes and change the initial condition to be at quarter filling $|\psi(0)\rangle = |1001010\dots\rangle$. First, we calculate SFQJ for $L = 20$ at half filling as shown in Figs. 5(a) and 5(b). By performing the finite-size scaling analysis for $L = 8, 10, \dots, 20$, we obtain the scaling exponents for $V_{\text{anom}} \propto L^\alpha$ as $\alpha = 2.65 \pm 0.02, 2.69 \pm 0.02, 2.68 \pm 0.02, 2.66 \pm 0.02$ for $J_z = 0, 0.5, 1, 1.5$ with $\gamma = 1$, respectively. These values are almost the same as the results calculated from the finite-size scaling up to $L = 18$ obtained in the main text and support the universality of the exponent.

Then, we perform the numerical simulation for the quarter-filling case. We remark that, if the system is at quarter filling, we have to change the rate of quantum

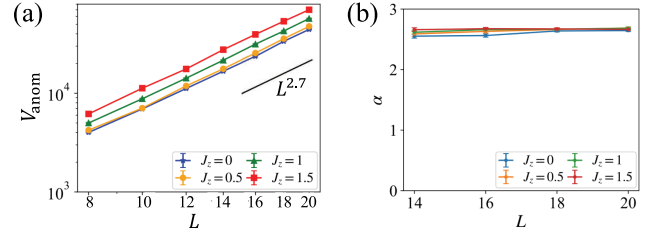


FIG. 5. (a) System-size scaling of V_{anom} and (b) the exponent α , up to $L = 20$ for $\gamma = 1$ at half filling. We find the universal scaling $\propto L^\alpha$ ($\alpha \sim 2.7$) irrespective of the values of J_z .

jumps properly during numerical calculations. Accordingly, ρ_{ss} should be described in the quarter-filling sector with the use of its Hilbert space dimension D'_0 , and the first term in Eqs. (5) and (6) is modified since quarter-filling condition gives $E[N_{\text{jump}}^{\text{half}}(T)] = \gamma LT/8$ instead of $\gamma LT/4$. As shown in Fig. 6(a), we find the measurement-induced crossover characterized by the crossing of V_{anom} and V_{act} , where the super-Poissonian statistics of SFQJ emerges for strong measurement.

Moreover, by performing the finite-size scaling analysis for $L = 8, 12, 16, 20$ as illustrated in Fig. 6(b), we obtain the anomalous scaling of SFQJ as $V_{\text{anom}} \propto L^\alpha$, where $\alpha = 2.69 \pm 0.02, 2.70 \pm 0.02, 2.69 \pm 0.02, 2.72 \pm 0.02$ for $J_z = 0, 0.5, 1, 1.5$ with $\gamma = 1$, respectively. Remarkably, these values are quantitatively similar to the exponents obtained for the half-filling condition and suggest that the anomalous scaling of SFQJ exhibits the universal behavior that does not depend on initial conditions.

Furthermore, we tested the robustness of the universality under a global U(1) symmetry for nonintegrable models, by incorporating next nearest neighbor interactions $H_{\text{NN}} = \sum_{j=1}^L J'_z n_{j+2} n_j$ to the Hamiltonian (1). Notably, we obtain $\alpha = 2.70 \pm 0.02, 2.70 \pm 0.02, 2.71 \pm 0.02, 2.66 \pm 0.02$ ($J'_z = 0.5$) and $\alpha = 2.69 \pm 0.01, 2.72 \pm 0.02, 2.71 \pm 0.02, 2.68 \pm 0.02$ ($J'_z = 1$), up to $L = 20$ with $\gamma = 1$ for

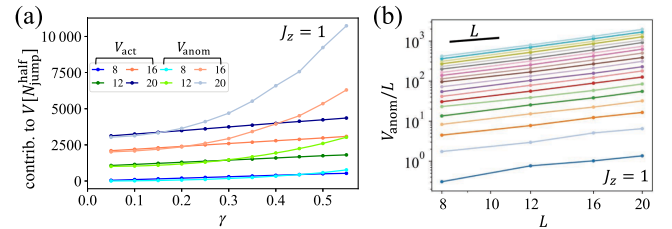


FIG. 6. Numerical results for V_{anom} and V_{act} based on the quantum trajectory method for the Heisenberg model at quarter filling. (a) Leading contribution of V_{anom} and V_{act} changes at a critical measurement strength similar to the half-filling case. (b) System-size scaling of V_{anom}/L . Data are shifted by 1000 as L is increased as $L = 8, 12, 16, 20$ in (a) to improve visualization, and measurement strengths are $\gamma = 0.05, 0.1, \dots, 1$ from bottom to top in (b). The other parameters and methods are the same as in Fig. 2.

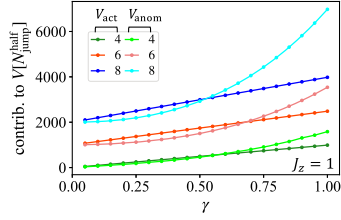


FIG. 7. V_{anom} and V_{act} for the Heisenberg model for system sizes $L = 4, 6, 8$ obtained with the use of Eq. (C1). The leading contribution of V_{anom} and V_{act} changes at a critical measurement strength consistent with Fig. 3(a). We set $T = 990$ and have numerically checked that $C_{\text{auto}}^{\text{half}}$ becomes almost constant for large T . Data are shifted by 1000 as L is increased.

$J_z = 0, 0.5, 1, 1.5$, respectively. This result demonstrates that the universality is robust even for nonintegrable Hamiltonians under measurement, going beyond the integrable XX and XXZ Hamiltonians.

Finally, we tried to extrapolate the exponent to the thermodynamic limit, where we obtained $\alpha = 2.90 \pm 0.09, 2.91 \pm 0.10, 2.82 \pm 0.08, 2.66 \pm 0.08$ ($J'_z = 0$), $\alpha = 2.85 \pm 0.10, 2.94 \pm 0.10, 2.83 \pm 0.09, 2.79 \pm 0.09$ ($J'_z = 0.5$), and $\alpha = 2.81 \pm 0.08, 2.86 \pm 0.09, 3.02 \pm 0.09, 2.81 \pm 0.10$ ($J'_z = 1$), for $J_z = 0, 0.5, 1, 1.5$, respectively. Then, we see that almost all parameter sets (10 out of 12) take the universal value $\alpha = 2.85$ within the error bar in the thermodynamic limit. We note that the statistical error is rather large due to the limitation of samples, and this causes a slight deviation for two sets of parameters from $\alpha = 2.85$. Thus, though the exact value may slightly deviate to a larger value, this fact still supports the universality of the exponent.

Appendix C: Spectral decomposition of the Liouvillian—We explain the spectral decomposition of the Liouvillian for the unconditional dynamics of Eq. (2) for calculating the Liouvillian gap as well as V_{anom} and V_{act} , assuming the diagonalizability of $\mathcal{L}$. By employing the basis transformation to the doubled Hilbert space for the density matrix $\rho = \sum_{i,j} \rho_{ij} |i\rangle\langle j|$ as $|i\rangle\langle j| \mapsto |ij\rangle := |i\rangle \otimes |j\rangle \in \mathcal{H} \otimes \mathcal{H}$ [72,105–107], Eq. (6) is rewritten as

$$V[N_{\text{jump}}^{\text{half}}(T)] = \frac{\gamma LT}{4} + \frac{2\gamma^2 T}{D_0} \sum_{j \neq 0} \frac{e^{\lambda_j T} - 1}{\lambda_j} \times (n'_{\text{half}} | \rho_j^R \rangle \langle \rho_j^L | n'_{\text{half}}), \quad (\text{C1})$$

where the spectral decomposition of the Liouvillian is given by $\mathcal{L} = \sum_{j \neq 0} \lambda_j | \rho_j^R \rangle \langle \rho_j^L |$, the inner product is defined as $(A|B) = \text{Tr}[A^\dagger B]$, and we restricted both the bra and ket spaces to a half-filling sector. By sorting the eigenvalues of $\mathcal{L}$ to satisfy $0 = \lambda_0 > \text{Re}[\lambda_1] \geq \text{Re}[\lambda_2] \geq \dots \geq \text{Re}[\lambda_{D_0^2-1}]$, where we confirm that the unconditional steady state is unique, the Liouvillian gap Δ that dominates the asymptotic decay rate [108,109] is extracted from the largest real part of the eigenvalue except for λ_0 as $\Delta = -\text{Re}\lambda_1$. As the Lindblad equation is invariant under the transformation by a real constant in the particle-number jump operator [110], we refer to the Liouvillian gap in the Heisenberg model with dephasing [90,91]. In addition, as shown in Fig. 7, we obtain V_{anom} and V_{act} with the use of Eq. (C1). Clearly, we find a crossing of V_{anom} and V_{act} and that the critical γ_c quantitatively agrees well with that in Fig. 3(a) for $L = 8$.