

Magic Barrier before Thermalization

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We investigate the time dependence of antiflatness in the entanglement spectrum, a measure for nonstabilizerness and lower bound for nonlocal quantum magic resource, on a subsystem of a linear SU(2) plaquette chain during thermalization. Tracing the time evolution of a large number of initial states, we find that the antiflatness exhibits a barrierlike maximum during the time period when the entanglement entropy of the subsystem grows rapidly from the initial value to the microcanonical entropy. The location of the peak is strongly correlated with the time when the entanglement exhibits the strongest growth. This behavior is found for generic highly excited initial computational basis states and persists for coupling constants across the ergodic regime, revealing a universal structure of the entanglement spectrum during thermalization. We conclude that quantitative simulations of thermalization for non-Abelian gauge theories require quantum computing. We speculate that this property generalizes to other quantum chaotic systems, a conjecture supported by analogous behavior observed in real-time simulations of the mixed-field Ising model.

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Introduction—Quantum computers are expected to eventually outperform classical supercomputers for a number of important applications by harvesting the resources opened up by quantum entanglement of qubits or qudits. However, identifying such applications is not as straightforward as one might imagine. While it is easy to prepare highly entangled quantum states, in particular by the Clifford set of gates comprising the Hadamard gate, the phase gate and the two-qubit CNOT gate, this entanglement alone does not enable dramatic gains in computational power. In fact, the Gottesman-Knill (GK) theorem [1] states that any quantum circuit built solely from Clifford gates (also known as the stabilizer circuits) can be simulated in polynomial time on a classical digital computer.

Not all unitary quantum circuits are stabilizer circuits. A universal gate set requires at least one additional gate, e.g., the $\pi/8$ (T) gate. The GK theorem implies that only

quantum circuits requiring such non-Clifford gates can realize true quantum advantage.

In practice, it would be most helpful to have a quantitative measure of the potential advantage of quantum computation over classical computation for simulating a given quantum system. One such measure has become known as nonstabilizerness or “magic” [2–5], but its experimental or numerical determination is challenging. Recently, the Stabilizer Rényi entropy (SRE) was introduced as a computable measure of nonstabilizerness [6] and has been studied in many systems including matrix product states [7], random quantum circuits [8] and states [9], lattice models and gauge theories [10–14], nuclei [15], neutrinos [16], and Bell inequalities [17]. In high-energy proton-proton reaction at the Large Hadron Collider (LHC), identifying quantum systems with nonstabilizerness involves studying certain processes such as the top quark-antiquark pair ($t\bar{t}$) production [18–22]. In part, the investigation of entanglement at the LHC arises from their promise to provide a novel avenue in search for possible signals of physics beyond the Standard Model. For instance, the authors of [23] considered a general form of nonstabilizerness for mixed quantum states, which is defined based on the second SRE [6,21]. They found the result of their analysis for the $t\bar{t}$ data to agree with the Standard Model expectations.

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A quantity closely related to nonstabilizerness is the antiflatness of the entanglement spectrum of a quantum system [24]. The reduced density matrix ρ_A of a subsystem A is defined by

$$\rho_A = \text{Tr}_{A^c}(\rho), \quad (1)$$

where A^c denotes the complement of the subsystem. The entanglement entropy

$$S_A = -\text{Tr}(\rho_A \log \rho_A), \quad (2)$$

provides a measure of the entanglement in the full system wave function. Li and Haldane [25] showed that the spectrum of the so-called entanglement Hamiltonian $H_A = -\log \rho_A$ contains additional information beyond just the total amount of entanglement, which has been used to study thermalization in Z_2 lattice gauge theory [26]. If this spectrum is flat, the so-called antiflatness

$$\mathcal{F}_A = \text{Tr}(\rho_A^3) - [\text{Tr}(\rho_A^2)]^2, \quad (3)$$

vanishes. Refer to [27] for another related measure.

It has been shown that nonstabilizerness can be decomposed into local and nonlocal contributions [28,29] and $\mathcal{F}_A$ provides a lower bound on nonlocal nonstabilizerness [28]. While local nonstabilizerness characterizes non-Clifford features that can be attributed to individual subsystems and thus removed by local unitaries, nonlocal contribution captures intrinsically multipartite, nonstabilizer correlations that cannot be eliminated locally and require entanglement to exist [28–31]. The total amount of nonstabilizerness can be viewed as the sum of these two components, with the nonlocal one quantifying genuinely nonclassical correlations beyond both stabilizer structure and entanglement. Hence, antiflatness bounds the hardness of classical simulations from below.

Here we pursue the question whether the description of thermalization of an isolated quantum system requires quantum computing, using SU(2) lattice gauge theory on a linear plaquette chain as an example. This system can be viewed as an extreme simplification of the more complex process of thermalization in a highly excited system of quarks and gluons as it is created in relativistic heavy ion collisions [32]. We will show that in this model system the time dependence of antiflatness of the entanglement spectrum and the growth of the entanglement entropy are closely connected. While not constituting a general proof, our results support the notion that an accurate description of gauge field thermalization requires quantum computing.

Method—We investigate pure SU(2) lattice gauge theory (LGT) in $2 + 1$ dimensions as a representative framework for exploring generic non-Abelian gauge dynamics. To access real-time phenomena, we employ the Hamiltonian formulation of SU(2) LGT, which provides a natural setting for studying out-of-equilibrium processes beyond the limitations of Euclidean lattice and perturbative

approaches. The discretized Kogut-Susskind (KS) Hamiltonian for unit lattice spacing can be written as [33]

$$H = \frac{g^2}{2} \sum_{\text{links}} (E_i^a)^2 - \frac{2}{g^2} \sum_{\mathbf{n}} \square(\mathbf{n}), \quad (4)$$

where g is the coupling constant, E_i^a is the electric field operator along the direction $i = \hat{x}$ or $\hat{y}$ with the SU(2) index a (both of which are implicitly summed over), and $\square(\mathbf{n})$ denotes the plaquette operator at $\mathbf{n} = (n_x, n_y)$, i.e., the trace of the product of four link variables (Wilson lines) around an elementary plaquette. The first and second part of the Hamiltonian represent the electric and magnetic energy of the gauge field, respectively.

The discretized KS Hamiltonian can be represented in the electric basis, which labels the state on each link by the SU(2) quantum numbers $|j m_L m_R\rangle$. In this basis the electric energy $(E_i^a)^2$ is diagonal with eigenvalues $j(j+1)$. The matrix elements of the plaquette operator may be expressed as a combination of Wigner $6j$ symbols [34]. Truncation of the local Hilbert space of each link to representations with $0 \leq j \leq j_{\max}$ renders the total Hilbert space finite-dimensional and allows for exact numerical diagonalization of the Hamiltonian.

In this Letter we study an aperiodic seven-plaquette chain with the electric field representation truncated at $j_{\max} = 1$, but our investigation can also be performed for other electric field truncations and boundary conditions. Figure 1 schematically illustrates the corresponding link configuration of the system. In the specific case where vertices contain at most three adjoining links, as here, the physical states of the theory are unambiguously represented by the irreducible representation j of each link variable [34,35]. Asymmetric boundary conditions are enforced by setting the external links to $\{j_{\text{ext}}\} = \{0, 0, 0, 1\}$, such that the Hamiltonian has no spatial symmetries [36]. The Hilbert space dimension of the seven-plaquette SU(2) chain with $j_{\max} = 1$ and the above boundary conditions is 36334. Similar systems were studied in [37–40], where it was demonstrated that these quantum systems exhibit both ergodic and nonergodic coupling regimes. In particular, for sufficiently large g^2 , the magnetic contribution to the Hamiltonian becomes too small to generate chaotic energy-level statistics on a small lattice. The KS Hamiltonian has two integrable limits at a given lattice size: $g^2 \rightarrow \infty$ and $g^2 \rightarrow 0$, but for intermediate (ergodic) coupling (here,

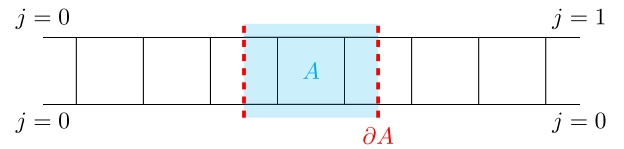


FIG. 1. Seven-plaquette chain with asymmetric boundary conditions. Subsystem A is defined as the middle plaquette and adjacent links.

$0.4 \lesssim g^2 \lesssim 1.5$) [41], the eigenvalue distribution of the seven-plaquette system follows closely the prediction for a Gaussian orthogonal ensemble (GOE) and the system exhibits quantum chaos. In this Letter, we focus specifically on the ergodic coupling regime of the SU(2) LGT on seven plaquettes, motivated by the fact that the classical SU(2) theory is chaotic [43] and, upon quantization, is expected to obey the eigenstate thermalization hypothesis [37,38,44].

To compute entanglement entropy and antiflatness for subsystems of the plaquette chain we use the technique described in [39]. Each electric basis state can be written as a tensor product of the j -quantum numbers of individual links $|\{j\}\rangle$, subject to the Gauss law constraint for physical states. The boundary of a subsystem A cuts through its adjacent horizontal links such that the reduced density matrix remains invariant under time-independent gauge transformations at each vertex (see Fig. 1). The cut links result in a direct sum structure in the reduced density matrix ρ_A , which can be diagonalized to obtain the full entanglement spectrum, the entanglement entropy S_A and the antiflatness $\mathcal{F}_A$. The subsystem A considered in the following consists of the middle plaquette with adjacent cut links as depicted in Fig. 1.

To analyze the thermalization dynamics, we initialize the system in pure states that are not eigenstates of the Hamiltonian but whose mean energy falls into a given energy window. To achieve this, we employ computational basis states (electric energy eigenstates) within the desired energy window. Numerical calculations were performed using our custom code available at [45].

Time dependence of entanglement and antiflatness—In order to understand the dynamics of quantum complexity during thermalization, we track the time dependence of entanglement entropy $S_A(t)$ and antiflatness $\mathcal{F}_A(t)$ of the subsystem A for the chosen globally evolving pure states. For S_A this evolution is well understood. Starting from an

initial pure product state, we have $S_A(0) = 0$. As the unitary dynamics proceeds, entanglement spreads and the entanglement entropy rises steadily until it saturates at a value consistent with the thermal entropy of the subsystem [40]. This behavior, shown by the solid blue line in Fig. 2(a) for an arbitrarily chosen highly excited initial electric basis state, reflects the effective thermalization of the subregion: while the global state remains pure, the local reduced state approaches a mixed quasi-thermal ensemble, as expected for a system obeying the eigenstate thermalization hypothesis [46–48].

In contrast, the antiflatness $\mathcal{F}_A(t)$, being a witness of nonstabilizerness, displays a more intricate profile, as shown by the solid red line in Fig. 2(a). At early time, the subsystem has negligible nonlocal nonstabilizerness since it is close to a classical configuration. As the dynamics scrambles information across the full system, the reduced state ρ_A becomes highly nonclassical, resulting in a sharp rise of the antiflatness. As $S_A(t)$ approaches saturation, $\mathcal{F}_A(t)$ falls again steeply as ρ_A approaches a nearly maximally mixed form characteristic of a high-energy system in (microcanonical) thermal equilibrium, which can be efficiently studied on classical digital computers. The peak of the antiflatness appears at time t_{MB} , which approximately equals the time t_{EE} when $S_A(t)$ is growing most rapidly.

The dynamics of the entanglement spectrum can also be visualized directly as in Fig. 2(b), where we show the eigenvalues of the entanglement Hamiltonian at various times during the thermalization process. The initially flat spectrum develops a visible nonflat structure during the time when $\mathcal{F}_A(t)$ is nonvanishing. At late times, in thermal equilibrium, the spectrum returns to an almost flat (uniform) structure where all eigenvalues approximately approach $\log[\dim(\mathcal{H}_A)]$, where $\mathcal{H}_A$ denotes the Hilbert space on the subsystem A . The appearance of the

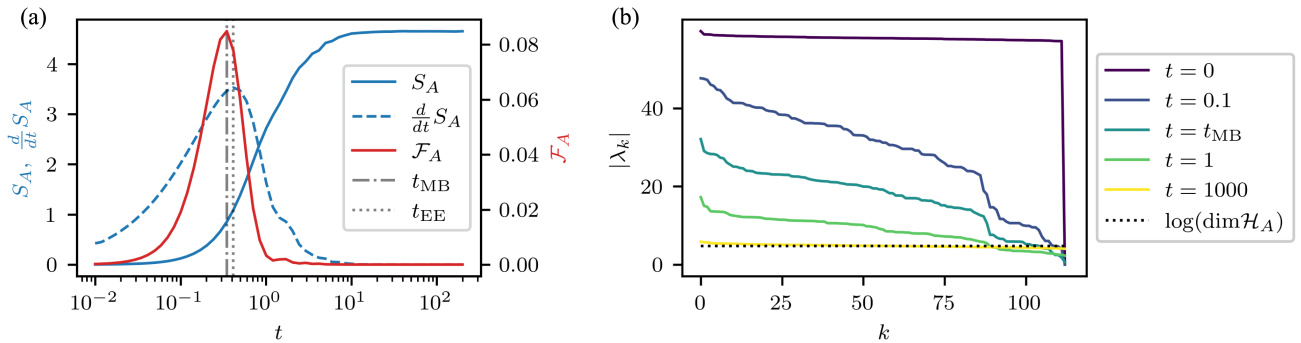


FIG. 2. (a) Real-time evolution of entanglement entropy S_A (blue solid line, left scale) and antiflatness $\mathcal{F}_A$ (red solid line, right scale) of a small one-plaquette subsystem with cut links in the middle of an aperiodic seven-plaquette chain with $j_{\text{max}} = 1$, ergodic coupling $g^2 = 1$ and asymmetric boundary conditions $\{j_{\text{ext}}\} = \{0, 0, 0, 1\}$ for a randomly chosen, highly excited initial electric basis state with energy $E - E_0 \approx 19.17$, where E_0 is the ground state energy. The blue dashed line shows the entanglement entropy growth rate. t_{MB} and t_{EE} , indicated by gray lines, correspond to the time of the magic barrier and maximum entanglement growth rate, respectively. (b) Entanglement spectra of the above state at different times, including $t_{\text{MB}} \approx 0.348$, during the thermalization process. λ_k denotes the k th eigenvalue of the entanglement Hamiltonian $H_A = -\log \rho_A$. The flat-spectrum limit is given by $|\lambda_k| = \log[\dim(\mathcal{H}_A)] \approx 4.727$.

antiflatness peak signals that the subsystem acquires strong “quantumness” during the thermalization process. However, the quantum features detected by the antiflatness are transient: they dominate the intermediate time regime of thermalization, but are washed out again as the subsystem relaxes toward thermal equilibrium. Similar barrierlike behavior was also found for nonstabilizerness in state preparation algorithms such as adiabatic annealing [49] and for operator entanglement dynamics in conformal field theory [50].

Together, entanglement entropy and antiflatness offer complementary insights: the former captures the irreversible buildup of correlations between the subsystem and its complement, while the latter identifies the transient temporal regime when the dynamics is the most “quantum” and the most difficult to simulate classically.

Universal magic barrier and entropy growth—We now study whether the appearance of the magic barrier is state-independent and quantify the correlation between the time of the antiflatness peak and that of the maximum growth rate of the entanglement entropy. We compute the time evolution of entanglement entropy and antiflatness of a small subsystem with one plaquette and dangling links in the middle of an aperiodic seven-plaquette chain with $j_{\max} = 1$ and asymmetric boundary conditions $\{j_{\text{ext}}\} = \{0, 0, 0, 1\}$ for all 2728 electric basis states inside the highly excited energy window $E - E_0 \in [19.71, 20.21]$, where E_0 is the ground state energy.

The result is shown in Fig. 3(a). It can be seen that at early time, $\log(t) < -1$, the entanglement entropies of different initial states grow at different rates. This is analogous to the behavior of classical chaotic systems with a broad Lyapunov spectrum, where the coarse-grained entropies for different initial states are found to grow at different rates at early time, depending on their degree of

overlap with the modes characterized by large Lyapunov exponents. However, in all cases the antiflatness exhibits a broad, nevertheless very prominent, peak during the period when the state’s entanglement grows. It is important to emphasize that we analyze all electric basis states in the energy window of interest and observe no outliers, i.e., all states exhibit the same qualitative behavior.

In order to better analyze the profile of the magic barrier with respect to the thermalization process, we make use of the universal behavior of entanglement entropy growth for subsystems found in [40]. We fit the corresponding time evolution for each state to the function $S_A(t) = S_A(0) + \{[S_A(\infty) - S_A(0)]/[1 + (t/t_0)^{-2\kappa}]\}$, where $S_A(0) = 0$ and $S_A(\infty)$ approaches a quasithermal value that only depends on the state’s energy. The fit parameters κ and t_0 correspond to a thermalization speed and time scale, respectively. By using $\kappa \log(t/t_0)$ instead of t as a “thermalization clock” we effectively synchronize the thermalization process of each state. As a result, as seen in Fig. 3(b), the entanglement entropy and antiflatness as functions of this “thermalization clock” show smaller variations and more manifest functional profiles.

Qualitatively, the functional profiles suggest that the magic barrier emerges around the time when the entanglement entropy grows the fastest. To quantify this observation, we compare the magic barrier time t_{MB} to the time t_{EE} of maximal entanglement entropy growth, for all physical electric basis states—defined as those with energies below the mean of the spectrum. This physical ensemble contains 18 389 noneigenstates, for which a 2D histogram of $\log(t_{\text{MB}})$ and $\log(t_{\text{EE}})$ are depicted in Fig. 3(c). We find a strong positive correlation between the two quantities. The Pearson correlation coefficient between $\log(t_{\text{MB}})$ and $\log(t_{\text{EE}})$ is $r = 0.927$, indicating a highly significant relationship. Furthermore, as indicated by the color

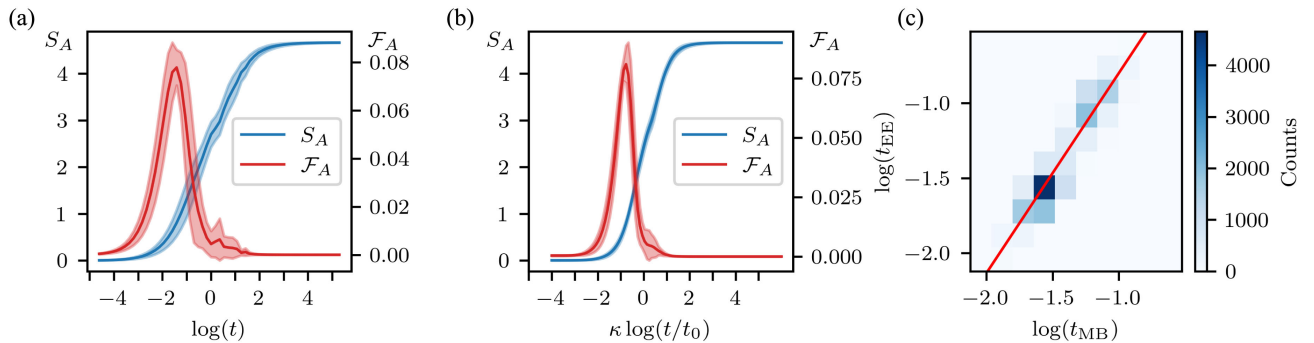


FIG. 3. (a) Real-time evolution of S_A and $\mathcal{F}_A$ for a small one-plaquette subsystem with cut links in the middle of an aperiodic seven-plaquette chain with $j_{\max} = 1$ and asymmetric boundary conditions $\{j_{\text{ext}}\} = \{0, 0, 0, 1\}$ at ergodic coupling $g^2 = 1$ for all electric basis states within the highly excited energy window $E - E_0 \in [19.71, 20.21]$. The energy window contains 2728 states. (b) Rescaled time evolution of the same system and state ensemble. The real time t is replaced by $\kappa \log(t/t_0)$ with state-dependent fit parameters κ and t_0 , such that the thermalization of different states is synchronized. The solid lines and bands in (a) and (b) are the ensemble means and 1σ bands. The left and right scales correspond to S_A and $\mathcal{F}_A$, respectively. (c) 2D histogram showing the joint distribution of magic barrier time t_{MB} and time of maximum entanglement entropy growth t_{EE} on logarithmic scales for all 18 389 physical electric basis states of the above system with energies below the spectrum mean. Color intensity corresponds to bin counts. The red line represents a linear fit with slope 1.331 and intercept 0.530.

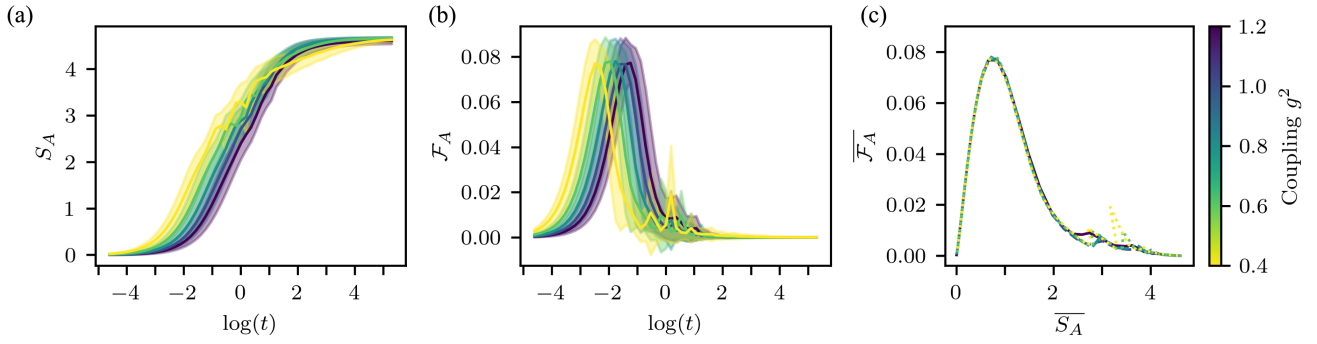


FIG. 4. Coupling dependence of entanglement dynamics. Real-time evolution of (a) S_A and (b) $\mathcal{F}_A$ for a small one-plaquette subsystem with cut links in the middle of an aperiodic seven-plaquette chain with $j_{\max} = 1$ and asymmetric boundary conditions $\{j_{\text{ext}}\} = \{0, 0, 0, 1\}$ at different ergodic couplings $g^2 \in \{1.2, 1.0, 0.8, 0.6, 0.4\}$. The ensemble of electric basis states is chosen such that at $g^2 = 0.6$ they lie in the highly excited energy window $E - E_0 \in [26.48, 26.98]$. This ensemble contains 2995 states. The solid lines and bands in (a) and (b) are the ensemble means and 1σ -bands. (c) Relations between the ensemble means $\bar{S}_A$ and $\bar{\mathcal{F}}_A$ for different ergodic couplings, which exhibit very mild coupling dependence.

distribution of the histogram, the time scales t_{MB} and t_{EE} roughly coincide for the majority of investigated states.

Coupling constant dependence—In order to make predictions about the emergent magic barrier for the physical limit of $\text{SU}(2)$ LGT, it is necessary to study its coupling dependence. With respect to the continuum limit, we are especially interested in the weak coupling limit, where g^2 is small but the theory is still ergodic.

We perform similar analyses as above for four other coupling values $g^2 = 0.4, 0.6, 0.8$, and 1.2 , all of which exhibit quantum chaotic energy level statistics. The ensemble of initial electric basis states for all five couplings is chosen such that for $g^2 = 0.6$ they lie in the highly excited, narrow energy window $E - E_0 \in [26.48, 26.98]$. In Fig. 4(a), we compare the entanglement entropy growth of this ensemble for the different couplings and find that the growth is faster at smaller coupling. In particular, we find a maximum of average growth rate $(d\bar{S}_A/dt)_{\max} = \{3.631, 4.443, 5.665, 7.563, 11.106\}$ for $g^2 = \{1.2, 1.0, 0.8, 0.6, 0.4\}$, consistent with a $(d\bar{S}_A/dt)_{\max} \propto g^{-2}$ scaling, which reflects the g^{-2} factor in the magnetic energy (plaquette operator) of the KS Hamiltonian (4) that controls the mixing rate between electric basis states. As a result, the computational basis states, which are eigenstates of the electric energy, get scrambled faster under unitary time evolution when the strength of the coupling is reduced.

The time t_{EE} at which the entanglement entropy growth reaches its maximum rate lies within a linear growth regime, consistent with the findings in [51]. The quantity $(dS_A/dt)_{\max}$ corresponds to the slope in this linear regime and can be interpreted as the entanglement velocity, i.e., the characteristic speed of the wave-front carrying entanglement from A^c into A [52].

Furthermore, these findings are in accordance with the time evolution of antiflatness. In Fig. 4(b), the magic barrier is seen to emerge earlier for decreasing coupling, while maintaining its height and shape. Instead of tracing

entanglement entropy and antiflatness as functions of time individually, we can also calculate $\mathcal{F}_A$ as a function S_A and thus obtain a thermalization profile of the magic barrier instead of a temporal profile. This profile is found to be almost identical for all tested ergodic couplings, as shown in Fig. 4(c), where both quantities are ensemble averaged. These results show that the entanglement spectrum has a coupling-independent structure and the magic barrier emerges during the equilibration process, as long as the system is ergodic and the subsystem thermalizes. In [53], we show the coupling-independent structure of the entanglement spectrum exemplarily for three randomly chosen states at $t = t_{\text{MB}}$.

Other quantum chaotic models—In order to investigate whether the magic barrier phenomenon is special to non-Abelian gauge theories or may also be observed in other quantum ergodic models, we repeat the main analysis of this Letter for the mixed-field Ising model. The results shown in [53] indicate that the antiflatness of a small subsystem exhibits a similar barrierlike maximum during the time when its entanglement entropy grows rapidly from the initial value to the microcanonical entropy. Further, for the most ergodic field strengths, we find a coupling-independent relation between the antiflatness and entanglement entropy during local equilibration, similar to the one found for the $\text{SU}(2)$ gauge theory.

Conclusion—We have studied the equilibration dynamics of subsystem entanglement entropy and antiflatness under unitary evolution in a $(2+1)$ -dimensional $\text{SU}(2)$ lattice gauge theory constrained on a linear plaquette chain with $j_{\max} = 1$. We find that the antiflatness, a lower bound on nonlocal nonstabilizerness, exhibits a universal barrierlike peak during the period of fast entanglement growth. This behavior occurs for all highly excited electric basis states and persists across ergodic couplings, revealing a coupling-independent structure of the entanglement spectrum during thermalization. The timing of the barrier is strongly

correlated with that of the maximal entanglement growth, demonstrating that the thermalization dynamics in the $SU(2)$ gauge theory involves simultaneous generation of high nonstabilizerness and high entanglement. Supported by similar behavior found for the mixed-field Ising model, we speculate that this generic barrier phenomenon may persist across a broad class of quantum chaotic theories, implying that quantitative simulations of thermalization in these theories inherently require quantum computation.

Promising directions for future work include testing this conjecture in other chaotic systems—most notably $SU(3)$ gauge theory, examining the evolution of antiflatness during specific physical processes such as string breaking in fermionic models and glueball formation [57–59], and computing nonstabilizerness directly via measures for qudit systems rather than through its antiflatness bound. We plan to pursue these investigations in future work.

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Data availability—The data that support the findings of this article are openly available [45].

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