Dimensional Reduction Is Supersymmetric at Three Loops

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We resolve the long-standing claim that regularization by dimensional reduction (DR) fails to preserve supersymmetry in supersymmetric Yang–Mills (SYM) theories at three loops. Earlier results reported a mismatch between the Yukawa and ghost-gluon β functions in $\mathcal{N}=2$ SYM, suggesting a breakdown of supersymmetry. We show that this discrepancy does not originate from DR itself but from subtleties in the treatment of the Clifford algebra. A corrected three-loop calculation restores full supersymmetric behavior, and we demonstrate that the same issue would first affect $\mathcal{N}=4$ SYM only at five loops, consistent with existing four-loop results. Our findings confirm that DR preserves supersymmetry for $\mathcal{N}=1$, 2, and 4 SYM through the loop orders examined.

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Introduction—Supersymmetry (SUSY) is a proposed symmetry relating bosons and fermions, pairing particles of differing spin into common multiplets. In supersymmetric gauge theories, this symmetry imposes strong constraints on quantum corrections, often leading to improved ultraviolet behavior and, in some cases, exact nonrenormalization theorems, see, e.g., Ref. [1]. A central quantity for assessing these quantum properties is the β function,

$$\mu\frac{d}{d\mu}a(\mu)=\beta(a). \tag{1}$$

It describes how a coupling $a(\mu)$ evolves with the renormalization scale μ, and SUSY typically requires that all couplings related by SUSY transformations run identically. Any mismatch in their β functions is therefore a direct and unambiguous signal of SUSY breaking at the quantum level, either a physical breaking or an artifact of the regularization scheme.

Supersymmetric Yang-Mills (SYM) theories are based on the supersymmetric vector multiplet and are symmetric under a local non-Abelian gauge group $\mathrm{SU}(N)$. They come in several versions, depending on the number of supercharges $\mathcal{N}$ in the SUSY algebra. Correspondingly, we have $\mathcal{N}=1$, 2, and 4 vector supermultiplets. The $\mathcal{N}=1$ SYM multiplet contains one gauge boson and one Weyl fermion; the $\mathcal{N}=2$ multiplet contains one gauge boson, two Weyl fermions, and two real scalars; and the $\mathcal{N}=4$ multiplet contains one gauge boson, four Weyl fermions, and six real scalars. In this Letter, we use the term "fermion" instead of "gaugino" to refer to the superpartners, as such a suffix "-ino" is not readily applicable to the scalar superpartners in the extended $\mathcal{N}=2$, 4 vector multiplets. The $\mathcal{N}=8$ vector multiplet contains gravitons and thus does not define a SYM theory. All component fields transform in the adjoint representation of the gauge group.

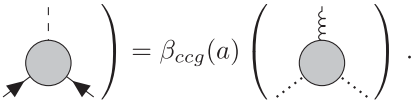
These increasing field contents impose increasingly rigid symmetry structures: $\mathcal{N}=1$ allows for nontrivial renormalization, $\mathcal{N}=2$ already exhibits strong constraints such as the one-loop exactness of the gauge β function [2], and $\mathcal{N}=4$ SYM is ultraviolet finite, with vanishing β functions to all loop orders [3,4]. Because of these properties, SYM theories have long served as testing grounds for regularization schemes and precision checks of SUSY at the quantum level.

To compute the β function and the other renormalization group functions explicitly for SYM, we require a regularization procedure that maintains manifestly both the gauge symmetry and SUSY. Dimensional regularization [5,6] in $d=4-2\varepsilon$ is a successful procedure for gauge theories, and a supersymmetric variant, dimensional reduction (DR), was proposed in 1979 [7], where the number of scalars n_s is analytically continued to $n_s+2\varepsilon$. Shortly thereafter, Siegel published a follow-up paper highlighting mathematical inconsistencies in the definition of DR [8]. The first explicit three-loop calculation implementing DR for the $\mathcal{N}=1$, 2, and 4 SYM theories appeared in 1982 [9,10]. In that work, it was observed that the β function for the Yukawa coupling β_{ffs} did not match the one for the ghost-gluon coupling β_{ccg} in any of the theories, despite SUSY-imposed constraints that these couplings should run identically,

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$$\beta(a) = \beta_{ffs}(a) \left(\right. \left. \right) = \beta_{ccg}(a) \left(\right. \left. \right) . \tag{2}$$



A mismatch between β_{ffs} and β_{ccg}, i.e., a violation of Eq. (2), signaled a breakdown of SUSY.

A few decades later, in 2009, the three-loop calculation was repeated in [11] (see also Ref. [12]). The updated results showed that the three-loop Yukawa and ghost-gluon vertices behave supersymmetrically in $\mathcal{N} = 1$, 4 SYM, but not in $\mathcal{N} = 2$ SYM, while consistent SUSY behavior was confirmed for all other three-point vertices, with the sole exception of the $\mathcal{N} = 2$ Yukawa coupling as summarized in Table I. In 2011, the calculations in [13] further established that the four-loop β functions for all vertices in $\mathcal{N} = 4$ SYM vanish, confirming fully consistent supersymmetric behavior.

The violation of SUSY in the $\mathcal{N} = 2$ Yukawa β function was attributed to an inconsistency of DR. However, this raised an unresolved question: why did DR fail only for $\mathcal{N} = 2$ SYM, but not for $\mathcal{N} = 1$, 4? The situation was paradoxical: the minimal $\mathcal{N} = 1$ theory and the maximally extended $\mathcal{N} = 4$ theory were protected from these inconsistencies, while the intermediate $\mathcal{N} = 2$ case was not. Moreover, no explanation was given for why all other vertices and propagators remained supersymmetrically consistent, or why the mismatch appeared only at three loops.

In the following, we provide answers to these questions. We show that the issue originates from an inconsistent handling of the Clifford algebra in the calculation procedure. This inconsistency leaves all three-loop vertices and propagators unaffected except for the three-loop $\mathcal{N} = 2$ Yukawa coupling, and, once corrected, the $\mathcal{N} = 2$ Yukawa β function becomes fully supersymmetric. We further demonstrate that in $\mathcal{N} = 4$ SYM the same inconsistency would first appear only at five loops, explaining why the four-loop result of [13] remained supersymmetric.

The generalized Lagrangian—For our work, we will be taking an approach based on the formalism developed in [15], where SUSY theories emerge within a generalized theory at certain values of some free parameters. We start by writing the following generalized Lagrangian for SYM theories:

$$\begin{aligned}\mathcal{L}^{\text{gen}}_{\text{SYM}} = \frac{1}{N}\text{Tr}\Big[&-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + i\bar{\lambda}_A\bar{\sigma}^\mu D_\mu\lambda^A + \frac{1}{2\xi}(\partial^\mu A_\mu)^2 \\ &+ \partial^\mu\bar{c}D_\mu c + \frac{1}{2}D_\mu\phi^I D^\mu\phi_I - \frac{1}{4}g^2[\phi^I,\phi^J][\phi_I,\phi_J] \\ &+ g\phi_I Z^I_{AB}\lambda^A\lambda^B + g\phi_I(\bar{Z}^I)^{AB}\bar{\lambda}_A\bar{\lambda}_B\Big],\end{aligned} \tag{3}$$

where $F_{\mu\nu}$ is the gauge field strength tensor, $\lambda_A, \bar{\lambda}^A$ are Weyl spinors, and ϕ^I are real scalars. Every field $\Phi = \Phi^b F^b$, with F^b being the generators in the adjoint representation of SU(N), under which the fields transform. The $\{I, J\}$ and $\{A, B\}$ are flavor indices for the fermions and scalars, respectively, with $n_s = \delta^I_I$ and $n_f = \delta^A_A$. We express the perturbative expansion in terms of the coupling $a = (g/4\pi)^2$. A similar such generalized Lagrangian for the Gross-Neveu-Yukawa model was used in [15], where the number of scalars and fermions (n_s, n_f) could be varied.

As there are no evanescent interaction terms in the SYM theories, there should be no such terms in $\mathcal{L}^{\text{gen}}_{\text{SYM}}$ either. It was shown in [15] that the flavor elements $Z^I, \bar{Z}^J$ then must satisfy the following relation:

$$Z^I\bar{Z}^J + Z^J\bar{Z}^I = 2\delta^{IJ}\mathbb{I}_Z, \tag{4}$$

where the identity $\mathbb{I}_Z$ is the same size as the Z^I's. This relation completely fixes the flavor factors of the diagrams in terms of n_s, n_f, as the Feynman diagrams only depend on the trace of alternating even products of the $Z^I, \bar{Z}^J$.

Much like the emergence of SUSY observed in [15], the $\mathcal{N} = 1$, 2, 4 SYM theories emerge from the generalized Lagrangian. Indeed, by substituting the representations of Eq. (4) for each of the values $(n_s, n_f) = \{(0, 1), (2, 2), (6, 4)\}$ into the classical $\mathcal{L}^{\text{gen}}_{\text{SYM}}$, we get the classical Lagrangians for $\mathcal{N} = 1$, 2, 4 SYM theories, respectively.

TABLE I. Results for the gauge and Yukawa vertices of [11]. Here, ccg refers to the ghost (c) gauge (g) vertex and ffs refers to the fermion (f) scalar (s) Yukawa vertex; see Eq. (2). C_A and d_{44} denote the adjoint quadratic and quartic Casimir invariants of the SU(N) gauge group [14]. The $\mathcal{N} = 2$ gauge β_{ccg} and Yukawa β_{ffs} functions are unequal. However, β_{ccg} is one-loop exact as expected by holomorphy; see Ref. [2]. Thus only the $\mathcal{N} = 2$ Yukawa β_{ffs} function violates SUSY.

$\mathcal{N} =$	$\beta_{ccg}(a)$	$\beta_{ffs}(a)$
1	$-(6a^2C_A + 12a^3C_A^2 + 42a^4C_A^3)$	$-(6a^2C_A + 12a^3C_A^2 + 42a^4C_A^3)$
2	$-4a^2C_A$	$-4a^2C_A - 8a^4(C_A^3(4 - 3\zeta_3) + d_{44}(12 + 72\zeta_3))$
4	0	0

Here, we briefly summarize the trace identities (see Ref. [15] for details) for positive integer values of n_s, before we discuss the extension to the regularized case. For even $n_s > 0$, we can construct

$$\Gamma^I = \begin{pmatrix} 0 & Z^I \\ \bar{Z}^I & 0 \end{pmatrix}. \tag{5}$$

The Γ^I's then satisfy the Clifford algebra,

$$\Gamma^I\Gamma^J + \Gamma^J\Gamma^I = 2\delta^{IJ}\mathbb{I}_\Gamma, \tag{6}$$

and we can define the analog of γ_5 for even $n_s > 0$ as

$$\Gamma_{(n_s+1)} = \frac{i^{\frac{n_s}{2}}}{n_s!}\epsilon_{I_1\ldots I_{n_s}}\Gamma^{I_1}\ldots\Gamma^{I_{n_s}}, \tag{7}$$

where the overall phase arises from the Euclidean metric in the Clifford algebra in Eq. (6). The even alternating traces of the $Z^I, \bar{Z}^J$ can then be obtained as

$$\begin{aligned} \mathrm{Tr}[Z^{I_1}\ldots\bar{Z}^{I_{2n}}] &= \frac{1}{2}\mathrm{Tr}[\Gamma^{I_1}\ldots\Gamma^{I_{2n}}(1+\Gamma_{(n_s+1)})], \\ \mathrm{Tr}[\bar{Z}^{I_1}\ldots Z^{I_{2n}}] &= \frac{1}{2}\mathrm{Tr}[\Gamma^{I_1}\ldots\Gamma^{I_{2n}}(1-\Gamma_{(n_s+1)})]. \end{aligned} \tag{8}$$

The part of the trace dependent on $\Gamma_{(n_s+1)}$ denotes the *chiral* part, and the remaining is the *achiral* one. Traces with odd numbers of flavor elements are not relevant for the theory as there are no diagrams that generate them (see, again, Ref. [15] for details). We list here a few specific cases relevant for the three-loop calculation,

$$\mathrm{Tr}[Z^I\bar{Z}^J] = \begin{cases} \mathrm{Tr}[\mathbb{I}_Z](\delta^{IJ} + i\epsilon^{IJ}) & \text{for } n_s = 2, \\ \mathrm{Tr}[\mathbb{I}_Z]\delta^{IJ} & \text{for } n_s > 2, \end{cases} \tag{9}$$

where the chiral contribution does not "unlock" for higher values of n_s because there are too few Γ matrices in the trace of Eq. (8) to produce a nonvanishing Levi-Civita tensor. Similarly, in $\mathcal{N} = 4$ SYM with $n_s = 6$, there will be no flavor Levi-Civita tensors unless we have a trace of six Z^I's, which does not occur before five loops.

The case of $\mathcal{N} = 1$ SYM ($n_s = 0$, $n_f = 1$) is different: the representation being one-dimensional over $\mathbb{R}$ implies that it is a real representation, with $Z^I = \bar{Z}^I$. Hence, Eq. (4) reduces to the usual Clifford algebra in Eq. (6) for $n_s = 0$. Thus, there is no chiral part in the flavor traces of $\mathcal{N} = 1$ SYM.

Regularization: In conventional DR, used in the results of [9,11,16], the Dirac matrices and gauge bosons are kept in integer dimensions $D = 4$, 6, or 10 and then reduced to $d = 4 - 2\varepsilon$. By contrast, our implementation of DR using the formalism of [15], the Dirac matrices, and gauge bosons are directly treated in d dimensions.

To obtain emergent SUSY, we compute the β function for the vertices from $\mathcal{L}^{\mathrm{gen}}_{\mathrm{SYM}}$, and at every loop order, we solve for the values of (n_s, n_f) that satisfy the SUSY relation (2). This yields the solutions $(2\varepsilon, 1)$, $(2+2\varepsilon, 2)$, and $(6+2\varepsilon, 4)$, corresponding to $\mathcal{N} = 1, 2$, and 4 SYM. As we have nonintegral n_s at the SUSY points, the flavor traces need to be regularized. The achiral part of the traces extends straightforwardly to the regularized case. It is in the chiral part that we encounter the γ_5 problem.

Fortunately, at three loops, contributions from the Levi-Civita tensor are limited to simple poles; accordingly, a simplified regularization can be employed [17],

$$\Gamma_{(n_s+1)} = \frac{i^{\frac{n_s}{2}}}{n_s!}\epsilon_{I'_1\ldots I'_{n_s}}\Gamma^{I'_1}\ldots\Gamma^{I'_{n_s}} + \mathcal{O}(\varepsilon), \text{ even } n_s \geq 2. \tag{10}$$

Here, the indices I' denote quantities with exact integer dimensions. Consequently, the primary challenge in supersymmetric regularization here reduces to addressing the γ_5 problem. At higher loop orders, it might be useful to employ the systematic framework of Weyl consistency conditions [18,19], which has been effectively applied to the computation of the standard model gauge β function at four loops [17].

For $\mathcal{N} = 1$ SYM, since there are no chiral components in the traces, we do not expect any fundamental obstacle in regularizing the flavor factors at any loop order. Since we are using Weyl fermions, the traces from fermion loops involve Pauli matrices instead of Dirac matrices. In this case, the Levi-Civita tensor appears in traces involving four or more matrices,

$$\begin{aligned} \mathrm{Tr}[\sigma^\mu\bar{\sigma}^\nu\sigma^\rho\bar{\sigma}^\lambda] &= 2[(\delta^{\mu\nu}\delta^{\rho\lambda} - \ldots) - \epsilon^{\mu\nu\rho\lambda}] \\ \mathrm{Tr}[\bar{\sigma}^\mu\sigma^\nu\bar{\sigma}^\rho\sigma^\lambda] &= 2[(\delta^{\mu\nu}\delta^{\rho\lambda} - \ldots) + \epsilon^{\mu\nu\rho\lambda}]. \end{aligned} \tag{11}$$

Note that the Pauli matrices above are Wick-rotated, and the simplified regularization for the ϵ tensor is used.

Restoring SUSY—Since the computational steps in [11,16] are not laid out in detail, it is not possible to conclusively identify the calculational subtleties there. Nonetheless, we succeeded in reproducing the results shown in Table I by excluding the Levi-Civita tensor in either the flavor traces, the Pauli traces, or both. In this section, we will demonstrate that, despite these subtleties with the Levi-Civita tensor, both $\mathcal{N} = 1$, 4 SYM theories and all vertices in $\mathcal{N} = 2$ SYM, except for the Yukawa vertex, yield results consistent with SUSY. Subsequently, we will present the correction to the $\mathcal{N} = 2$ Yukawa vertex.

The Levi-Civita tensor can contribute a nonzero value to a Feynman diagram only if it is fully contracted with another Levi-Civita tensor within the diagram. Consequently, it affects only diagrams containing an even number of (open or closed) fermion chains. Up to three loops, the maximum number of such chains is 2. Therefore, in this section, we will focus solely on these diagrams.

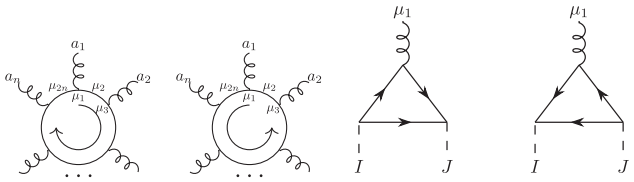


FIG. 1. Left: fermion loop connected to gluons only (arrows indicate fermion flow). Right: fermion loop with scalars.

Consider first the case of closed fermion chains (i.e., loops) that are connected solely to gluons (Fig. 1, left). In this class of diagrams, the fermion flow proceeds in both clockwise and anticlockwise directions; however, these contributions are multiplied by the same integrand. Therefore, the sum of these diagrams is proportional to

$$\begin{aligned}
&\mathrm{Tr}[F^{a_1}\ldots F^{a_n}]\mathrm{Tr}[\sigma_{\mu_1}\ldots\bar{\sigma}_{\mu_{2n}}] \\
&\quad+(-1)^n\mathrm{Tr}[F^{a_n}\ldots F^{a_1}]\mathrm{Tr}[\bar{\sigma}_{\mu_{2n}}\ldots\sigma_{\mu_1}] \\
&=\ldots+(-1)^n\mathrm{Tr}[(F^{a_1})^T\ldots(F^{a_n})^T]\mathrm{Tr}[\sigma^T_{\mu_1}\ldots\bar{\sigma}^T_{\mu_{2n}}] \\
&=\ldots+(-1)^{4n}\mathrm{Tr}[F^{a_1}\ldots F^{a_n}]\mathrm{Tr}[\bar{\sigma}_{\mu_1}\ldots\sigma_{\mu_{2n}}] \\
&=(\text{colour term})(\mathrm{Tr}[\sigma_{\mu_1}\ldots\bar{\sigma}_{\mu_{2n}}]+\mathrm{Tr}[\bar{\sigma}_{\mu_1}\ldots\sigma_{\mu_{2n}}]),
\end{aligned} \tag{12}$$

where F^a is the color matrix in the adjoint representation. We employed the invariance of traces under transpose, along with the identities $(F^{a_i})^T=-F^{a_i}$ and $\sigma^T_\mu=(-i\sigma_2)\bar{\sigma}_\mu(-i\sigma_2)$. Substituting Eq. (8) into the last line, it becomes evident that the Levi-Civita contribution cancels out. Therefore, in the case of a fermion loop connected solely to gluons, subtleties associated with the chiral part of the traces will have no effect.

The situation differs when scalars are involved. For instance, consider the triangle loop depicted on the right of Fig. 1 in $\mathcal{N}=2$ SYM. Upon summing over both orientations of the fermions in the loop, the Levi-Civita contributions do not cancel,

$$\begin{aligned}
&\mathrm{Tr}[\sigma_{\mu_1}\ldots\bar{\sigma}_{\mu_4}]\mathrm{Tr}[Z^I\bar{Z}^J]+\mathrm{Tr}[\bar{\sigma}_{\mu_1}\ldots\sigma_{\mu_4}]\mathrm{Tr}[\bar{Z}^IZ^J] \\
&\propto(\ldots-\epsilon_{\mu_1\ldots\mu_4})(\ldots+i\epsilon^{IJ})+(\ldots+\epsilon_{\mu_1\ldots\mu_4})(\ldots-i\epsilon^{IJ}).
\end{aligned} \tag{13}$$

This illustrates a general principle: for Levi-Civita terms to survive cancellation, chiral contributions must be present in both the flavor and spinor traces of the fermion loop. This is because the double negatives multiply to produce a positive, preventing the chiral terms from canceling. It is important to note that in $\mathcal{N}=1$ SYM, there are no flavor chiral terms, while in $\mathcal{N}=4$ SYM, the flavor chiral contributions only become relevant at five loops. This accounts for why, even with an inconsistent implementation of the Levi-Civita tensor, the three-loop results in these theories remain unaffected.

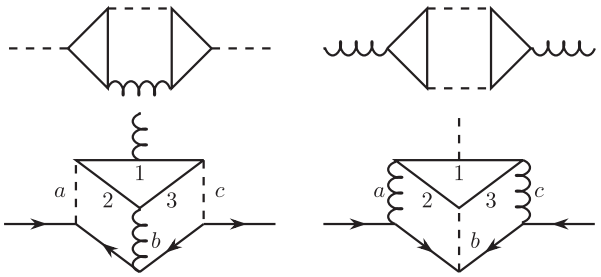


FIG. 2. Sample topologies of diagrams that yield a nonvanishing Levi-Civita contribution in the integrand with gluons, fermions (solid lines, arrows indicate fermion flow), and scalars (dashed lines).

In the majority of diagrams containing fermion loops, as in Fig. 1 on the right, the Levi-Civita tensors still vanish in the integrand up to three loops. Using the infrared rearrangement procedure [20,21] for renormalizing vertices, the momentum of the external bosonic leg is set to zero. This effectively equalizes the two fermion momenta at that vertex, enabling the use of the relation $p^\mu p^\nu\sigma^\mu\bar{\sigma}^\nu=p^2$, which reduces two Pauli matrices within the trace. When this reduction occurs in a trace of four σ^μ matrices, the Levi-Civita contribution cancels out.

This leaves us with the four types of diagrams shown in Fig. 2, in which the Levi-Civita tensor remains present at least within the integrand. For each of the diagrams in the bottom row of Fig. 2, there are a total of $2\times3\times3=18$ diagrams, corresponding to the three choices of placing the external vertex on lines 1, 2, or 3, the three permutations of the bosonic lines a, b, c, and the two possible directions of fermion flow.

After integration, the two propagator corrections and the pole part of the gluon vertex correction also vanish due to the symmetries of the integral. Only the Yukawa vertex diagrams (bottom right) yield a nonvanishing Levi-Civita pole contribution. The values of the corresponding integrals are identical under permutations of a, b, c once the external vertex is fixed. Let $I^{(n)}$ denote the value of the diagram with the external vertex on line n. Then, with the adjoint quadratic and quartic Casimir invariants C_A and d_{44},

$$\begin{aligned}
I^{(1)}&=i\left(8d_{44}+\frac{2}{3}C_A^3\right)\frac{1}{\varepsilon}+\mathcal{O}(\varepsilon^0), \\
I^{(2)}=I^{(3)}&=i\left(8d_{44}-\frac{1}{3}C_A^3\right)\left(-\frac{1}{3}+\zeta_3\right)\frac{1}{\varepsilon}+\mathcal{O}(\varepsilon^0).
\end{aligned} \tag{14}$$

Summing over the three permutations of a, b, c gives

$$\begin{aligned}
&3(I^{(1)}+I^{(2)}+I^{(3)}) \\
&\quad=i\frac{2}{3}[C_A^3(4-3\zeta_3)+d_{44}(12+72\zeta_3)]\frac{1}{\varepsilon}.
\end{aligned} \tag{15}$$

The above contribution is multiplied by a factor of 12 while calculating $\beta_{ffs}(a)$, which cancels exactly the error term for $\mathcal{N}=2$ SYM in Table I at $\mathcal{O}(a^4)$, thereby restoring SUSY, as now Eq. (2) holds, and the β_{ffs} is also one-loop exact, as expected from holomorphy [2].

Implementation: For the full computation of the theories, we generate the diagrams up to three loops using Qgraf [22]. The symbolic manipulations are performed with the computer algebra system FORM [23–25], which we use to implement the Feynman rules, identify topologies, and carry out the color, flavor, and spinor traces, this time including the correct Levi-Civita tensors. The loop integrations are computed with FORCER [26], which is capable of evaluating two-point functions up to four loops. The renormalization of the bare quantities in the modified minimal subtraction scheme proceeds in the standard way. As a check of the entire setup, we have also performed the renormalization of the $\mathcal{N}=0$ SUSY case, namely QCD, in standard dimensional regularization up to four loops [27,28] (for QCD β function computations in DR, see Refs. [29,30]). Throughout the calculation a general covariant gauge was used, and we verified explicitly that all final results of the β function are independent of the gauge parameter. Finally, substituting the appropriate values of (n_s, n_f) yields SUSY-consistent results for all $\mathcal{N}=1$, 2, 4 SYM theories.

Our reasoning predicts that $\mathcal{N}=1$ SYM, which lacks any flavor Levi-Civita tensors, should be free of the γ_5 problem at all loop orders; see also Refs. [31,32]. In contrast, $\mathcal{N}=4$ SYM could in principle be affected when six flavor matrices appear, which can happen only in the five-loop topologies such as the diagram shown in Fig. 3. To verify this, we have evaluated the four-loop β functions for $\mathcal{N}=1$, 4 SYM, dropping the Levi-Civita terms. We find

$$\beta_{\mathcal{N}=1} = -(6a^2C_A + 12a^3C_A^2 + 42a^4C_A^3 + 204a^5C_A^4),$$
$$\beta_{\mathcal{N}=4} = 0. \tag{16}$$

Here, both the Yukawa and gauge β functions are equal, i.e., Eq. (2) holds. The four-loop $\mathcal{N}=1$ gauge β_{ccg} was calculated in [30] and agrees with Eq. (16), whereas the four-loop calculation of the $\mathcal{N}=1$ Yukawa β_{ffs} function in DR performed by us is a novel result. The $\mathcal{N}=4$ case matches with [13]. The four-loop $\mathcal{N}=2$ case needs a nontrivial handling of the γ_5 problem and will be the topic of a future work.

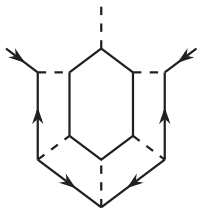


FIG. 3. Diagram where Levi-Civita contributions appear for the first time in $\mathcal{N}=4$ SYM, resembling the *Urdhwa Pundra* (upward lotus) symbol of Lord Vishnu in Hindu mythology.

Conclusions—We have shown that the alleged failure of DR in SYM theories was caused by subtleties in the treatment of Levi-Civita tensors, correcting which restored SUSY at three loops. By using the formalism of the generalized Lagrangian, we were able to quantify the source of the apparent breakdown of supersymmetric regularization. This helped us go beyond just fixing the three-loop result, and enabled us to make general higher loop predictions for the regularization of the $\mathcal{N}=1$, 2, 4 SYM theories. We then carried out four-loop calculations that verified our predictions, in addition to producing a novel result.

We have demonstrated here that, there is no inherent problem in supersymmetric regularization, the only problem is regularizing γ_5, which is not a SUSY specific problem. Thus, the generalized SUSY DR framework suggests intriguing possibilities for formulating supersymmetric theories directly in $d=4-2\varepsilon$ dimensions, with potential implications for studies in the planar ('t Hooft) limit, where simplifications of color structures often allow for exact or quasiexact results. It may also prove relevant for applying DR to other highly symmetric theories with extended scalar sectors and Yukawa interactions, such as half-maximal ($\mathcal{N}=4$) or maximal ($\mathcal{N}=8$) supergravity and questions related to their ultraviolet finiteness, which involve double copies of the $\mathcal{N}=4$ SYM theory.

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Data availability—There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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