

# Next-to-Next-to-Next-to-Leading-Order QCD Corrections to Photon-Pair Production

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The production of two isolated photons at the Large Hadron Collider has applications in Higgs-boson phenomenology and in new-physics searches. In fact, measurements would provide more insight if a percent-level theoretical prediction were available. We account for next-to-next-to-next-to-leading-order strong-interaction effects in the cross section for the first time. This is not only necessary to demonstrate the convergence of perturbation theory, but also results in a reduction of the theoretical uncertainty by a factor of 3 down to 3%. With this, we pave the way to improved analyses.

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**Introduction**—Photons are among the cleanest objects at hadron colliders: they deposit their energy in the electromagnetic calorimeters, can be selected efficiently by dedicated photon triggers, and yield diphoton spectra with excellent invariant-mass,  $m_{\gamma\gamma}$ , resolution. In fact, the Higgs-boson decay channel into two photons was instrumental in establishing discovery. The same channel remains a source of highly accurate data on the Higgs-boson transverse-momentum and rapidity distributions [1,2], and, not the least, associated jet activity. Most importantly, it allows for measurements of the mass [3,4] of the fundamental scalar with competitive uncertainties. The same clean invariant-mass spectrum that made  $H \rightarrow \gamma\gamma$  powerful also makes diphoton production a prime channel for new-particle searches. For instance, it provides strong constraints on a multitude of fashionable models at high  $m_{\gamma\gamma}$  [5,6], whereas low- $m_{\gamma\gamma}$  measurements are being conducted in order to exclude or discover dark-matter candidates with masses below the electroweak scale [7,8]. The high-luminosity phase of the LHC (HL-LHC) promises more measurements with higher statistics and improved systematics due to the constant methodological progress on the experimental-analysis side.

How important is reliable theoretical modeling of prompt diphoton production—the background process to the

signals listed above? The complexity of Monte Carlo event generators and the numerous approximations they rely on, make it desirable to assess the quality of modeling in the context of purely perturbative predictions. Consider the  $H \rightarrow \gamma\gamma$  signal as an example. The signal excess over the continuum background is about 4% in the main  $m_{\gamma\gamma}$  window; see e.g., Ref. [3]. In order to perform the analysis, the background is actually fitted as a smooth curve from the region outside the signal window. Reference [1] quotes an uncertainty of about 4% on the inclusive  $pp \rightarrow H \rightarrow \gamma\gamma$  cross section from uncertainty on the shape of the fit function alone as one of the largest systematic errors. This is where an accurate theory prediction could help, especially if percent-level accuracy would be certified. Cross-section measurements are, in fact, performed in a multitude of specialized phase-space regions adapted to the goals of a given analysis. This fine graining will be even more pronounced at the HL-LHC. A simple fit function is rarely applicable in such cases and the shape of the background is obtained from Monte Carlo simulations with the normalization given from inclusive fits. Clearly, a percent-level perturbative prediction would be invaluable in validating the simulations. One should also remember that the concept of background diphoton production ignores quantum interference effects between signal and background amplitudes. These percent-level effects are not negligible [9]. However, their inclusion is only justified if the background process without interference is known to matching precision.

Perturbative predictions for diphoton production present a major challenge. Inclusion of strong-interaction effects at next-to-leading (NLO) and next-to-next-to-leading order (NNLO) [10,11] showed that while theory starts to agree with the data at NNLO [12], no convergence is observed when increasing the order of the approximation. Instead, both NLO and NNLO calculations yield cross-section values well outside the theory uncertainty predicted by

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the previous order. Furthermore, this theory uncertainty increases order by order and reaches about 8% at NNLO. The reason for the unsatisfactory behavior lies to a large extent in the fact that new parton-scattering channels appear at NLO (quark gluon) and NNLO (gluon gluon), while the leading order is generated by electromagnetic quark-antiquark annihilation. Hence, reliability of the perturbative expansion can only be assessed with results beyond NNLO.

In this Letter, we present the theoretical prediction for the fiducial cross section for diphoton production at next-to-NNLO order (N<sup>3</sup>LO) in QCD. We finally observe perturbative convergence together with a significant reduction of scale uncertainty to about 2%. There are two other sources of uncertainty that affect our result. One is due to Monte Carlo integration, while the other is an artifact of the computational method, and is referred to as power corrections. We estimate both uncertainties at the same level of 2%.

Our result is first in the class of genuine 2-to-2 scattering processes at hadron colliders. It could not have been obtained without methodological progress, despite substantial computational resources used. In particular, we have drastically improved the efficiency of the NNLO  $pp \rightarrow \gamma\gamma$  + jet calculation, and obtained analytic one-loop amplitudes for  $pp \rightarrow \gamma\gamma + 2$  jets using techniques that have only recently been proposed.

**Calculation**—N<sup>3</sup>LO predictions for simpler hadron-collider processes have been obtained with a multitude of methods: reverse unitarity [13,14] in Refs. [15–21], projection to Born [22] in Refs. [23–25],  $q_T$  slicing [26] in Refs. [27–35]. Since there is no subtraction scheme able to handle our problem [36], we apply the  $q_T$ -slicing method [26], which, at this order, is currently restricted to colorless final states. Approaches [39–44] to overcome this limitation have not exceeded NNLO so far [45]. A different slicing approach that could also be applied is  $N$ -jettiness slicing [47–52]. However, it was only completed [53,54] for our case when the present study was already under way.

In the framework of  $q_T$  slicing, the diphoton production cross section is decomposed as

$$\sigma_{\text{N}^3\text{LO}}^{pp \rightarrow \gamma\gamma} = \int_0^{q_T^{\text{cut}}} dq_T \frac{d\sigma_{\text{N}^3\text{LO}}^{pp \rightarrow \gamma\gamma}}{dq_T} + \int_{q_T^{\text{cut}}}^\infty dq_T \frac{d\sigma_{\text{NNLO}}^{pp \rightarrow \gamma\gamma j}}{dq_T}, \quad (1)$$

where  $q_T \equiv |\mathbf{q}_T|$  is the transverse momentum of the photon pair. A finite  $q_T^{\text{cut}} > 0$  regulates infrared divergences due to the jet radiation in the second term. On the other hand, if  $q_T^{\text{cut}}$  is chosen sufficiently small, we can approximate the first term by means of the factorization theorem [55–59]

$$\begin{aligned} \frac{d\sigma_{\text{N}^3\text{LO}}^{pp \rightarrow \gamma\gamma}}{dq_T^2 dm_{\gamma\gamma} dy} &= \sum_{a,b} H_{ab}(m_{\gamma\gamma}) \int \frac{d^2 \mathbf{b}_T}{(2\pi)^2} e^{iq_T \cdot \mathbf{b}_T} \\ &\times \tilde{f}_a(x_1, \mathbf{b}_T) \tilde{f}_b(x_2, \mathbf{b}_T) + \mathcal{O}(q_T/m_{\gamma\gamma}), \end{aligned} \quad (2)$$

where  $m_{\gamma\gamma}$  is the diphoton invariant mass and  $y$  the rapidity,  $x_{1,2} = m_{\gamma\gamma}/\sqrt{s} \exp(\pm y)$ .  $H_{ab}$  contains the Born cross section and virtual corrections and  $\tilde{f}_i$  is the transverse-momentum-dependent parton distribution function. It combines beam and soft functions,  $\tilde{f}_i(x, \mathbf{b}_T) \equiv B_i(x, \mathbf{b}_T) \sqrt{S(\mathbf{b}_T)}$ , and enters multiplicatively for the partons  $a$  and  $b$  when expressed through the impact parameter  $\mathbf{b}_T$ , the Fourier conjugate of  $\mathbf{q}_T$ . Furthermore, it is related to regular parton distribution functions (PDFs)  $f_i$  via convolution [55],

$$\tilde{f}_i(x, \mathbf{b}_T) = \sum_j \int_x^1 \frac{dz}{z} \tilde{\mathcal{I}}_{ij}^{\text{TMD}}(z, \mathbf{b}_T) f_j\left(\frac{x}{z}\right) + \mathcal{O}(\mathbf{b}_T \Lambda_{\text{QCD}}), \quad (3)$$

with the matching kernel  $\tilde{\mathcal{I}}_{ij}^{\text{TMD}}$  calculated through N<sup>3</sup>LO in Refs. [60,61]. Starting from the ancillary files of Ref. [61], we perform the  $\mathbf{b}_T$  and  $\mathbf{q}_T$  integrals analytically and take the virtual amplitudes through three-loop order that contributes to  $H_{ab}$  from Refs. [62,63].

The second term of Eq. (1) requires an NNLO calculation that can, in principle, be performed with existing subtraction schemes [38,64–74]. In particular, Refs. [38,67] have enabled precise predictions for  $2 \rightarrow 3$  processes [75–81] including  $pp \rightarrow \gamma\gamma j$  [82,83]. Here, we use STRIPPER, a C++ implementation of the NNLO sector-improved residue subtraction scheme [67,84,85] starting from the setup described in detail in Ref. [82]. Unfortunately, the requirement of a very small  $q_T^{\text{cut}}$  parameter introduces major technical obstacles. We therefore implement several improvements compared to Ref. [82]. First, we employ hybrid floating-point precision, double or quadruple, using the qd library [86]. Phase-space regions with particularly large cancellations are calculated in quadruple precision by default. Second, a significant speedup of the Monte Carlo integration was achieved by modifying, as described in Supplemental Material [87], the selector functions, which are integral parts of the STRIPPER methodology. Third, we use amplitudes of very high numerical stability. The seven-point tree-level amplitudes are computed with the avhlib library [99,100] and the five-point two-loop amplitude from Ref. [82] is upgraded to full color thanks to Ref. [101]. The previously used OpenLoops library [102,103] does not offer sufficient precision for the six-point one-loop amplitudes. While the library NJet [104] can calculate one-loop amplitudes as needed with octuple precision, the computation time makes a calculation with acceptable statistical precision unfeasible. Therefore, we implement stable and compact analytic expressions for the amplitudes; see next section.

Note that the cross section contains an independently gauge-invariant contribution from the loop-induced  $gg \rightarrow \gamma\gamma$  amplitude at NNLO. The  $gg \rightarrow \gamma\gamma g$  amplitude relevant at N<sup>3</sup>LO is obtained through OpenLoops. Since this loop-induced N<sup>3</sup>LO contribution is obtained with an effective

NLO calculation, which had already been performed in Ref. [11], the precision is sufficient in this case. With the three-loop  $gg \rightarrow \gamma\gamma$  [105] and two-loop  $gg \rightarrow \gamma\gamma g$  [106] amplitudes, all ingredients are available to include this contribution at N<sup>4</sup>LO. However, we choose not to do so in order to obtain a pure N<sup>3</sup>LO prediction.

We work in a theory with  $n_f = 5$  flavors without top-quark loops and a QED coupling of  $\alpha = 1/137$ . The strong coupling and PDF values are obtained through the LHAPDF interface [107] from the NNPDF3.0 [108] PDF set [109], which was also used for the theory predictions in Ref. [12]. We set the factorization and renormalization scales to  $\mu_F = \mu_R = m_{\gamma\gamma}$  and use standard seven-point scale variation to obtain uncertainty bands. As in Ref. [12], we perform the calculation for the LHC at 13 TeV with fiducial cuts  $p_{T,\gamma_{1(2)}} > 40(30)$  GeV,  $|\eta_\gamma| < 2.37$  excluding  $1.37 < |\eta_\gamma| < 1.52$ ,  $\Delta R_{\gamma\gamma} > 0.4$ , using a hybrid scheme for photon isolation [110,111] which combines the smooth-cone condition

$$E_T^{\max}(r) = 0.15 E_{T,\gamma} \frac{1 - \cos(r)}{1 - \cos(R_{\max}^s)}, \quad r < R_{\max}^s \quad (4)$$

with the hard-cone condition

$$E_T^{\max}(r) = 0.09 E_{T,\gamma}, \quad r < R_{\max}^h, \quad (5)$$

where  $E_T^{\max}(r)$  is the maximal allowed sum of transverse energies of all partons within angular distance  $r$  of a photon, and we use  $R_{\max}^s = 0.1$  and  $R_{\max}^h = 0.2$ .

Six-point one-loop amplitudes—As announced in the previous section, we obtain analytic expressions for the scattering amplitudes of the processes  $0 \rightarrow \gamma\gamma gg q\bar{q}$  and  $0 \rightarrow \gamma\gamma Q\bar{Q} q\bar{q}$  at one-loop order using finite-field-based reconstruction. While an analytic derivation of a six-point one-loop amplitude has already been performed using high-precision floating-point evaluations of the amplitude [112], the present calculation constitutes the first full calculation of such an amplitude using finite-field reconstruction [113,115].

The method of rational reconstruction over finite fields [117,118] consists of two steps. First, we develop a framework to calculate the amplitude numerically for arbitrary finite-field phase-space points, and second, we construct compact rational ansätze for the analytic expressions. The unknown coefficients in the latter can be obtained given a sufficient number of random finite-field probes.

Finite-field reconstruction is routinely used in the literature to calculate analytic expressions of two-loop five-point QCD amplitudes. In the five-point case, the ansätze can be built using Mandelstam variables and a single parity-odd quantity, which has been done to obtain massless [119–124] and massive [78,125,126] amplitudes. This is impractical for a generalization to six-point kinematics because the Gram determinant  $G(p_1, p_2, p_3, p_4, p_5, p_6)$  is

nonzero when expressed through Mandelstam variables but evaluates to zero in four dimensions. A solution is given by intrinsically four-dimensional momentum-twistor variables [127], which have been used for the finite-field reconstruction of massless [80,106,128–131] and massive [132–137] five-point two-loop amplitudes as well. Momentum-twistor variables, however, have the disadvantage that they do not have a simple connection to physical limits and are related to Mandelstam variables via rational—not polynomial—identities, introducing additional denominators. Indeed, Ref. [119] has found significantly simplified results when Mandelstam variables are used compared to twistor variables used in Ref. [131]. Alternatively, phase-space variables which are both four-dimensional and have an intuitive behavior in infrared limits are given by spinor products in the spinor-helicity formalism. Their disadvantage is that standard finite-field reconstruction algorithms require a set of independent variables, whereas spinor products are related via Schouten and momentum-conservation identities. This can be tackled with methods from algebraic geometry as proposed in Ref. [116], which have been applied to the calculation of a number of massless [138–140] and massive [141–146] five-point amplitudes. There, the finite-field probes were obtained either from previous analytic computations, through the unitarity method [147–149], or its implementation in the software package Caravel [150]. However, the latter currently only supports the calculation of finite-field probes of amplitudes for five-point kinematics.

The reconstructed expressions of the amplitude have been made publicly available [151]. Details of the computation and subsequent implementation in efficient C++ code can be found in Supplemental Material [87] to this Letter.

**Results**—Our main result is the behavior of the fiducial  $pp \rightarrow \gamma\gamma$  cross section at N<sup>3</sup>LO for the slicing variable  $r_{\text{cut}} \equiv q_T^{\text{cut}}/m_{\gamma\gamma} \ll 1$  shown in Fig. 1. The left plot demonstrates the drastic growth of various contributions to the NNLO  $pp \rightarrow \gamma\gamma j$  cross section for small  $r_{\text{cut}}$ , which results in large cancellations between real and virtual corrections as well as integrated subtraction terms. Not illustrated are cancellations within each contribution [152]. These various cancellations are responsible for the substantial computational resources used—in units of million CPUh: 2.7 for the real-virtual-finite (RVF), 2.3 for the double-real-finite (RRF), 2.4 for the double-real-single-unresolved (RRSU), 0.5 for the double-real-double-unresolved (RRDU), and 0.6 for the double-virtual-finite (VVF) contribution (other contributions required significantly less).

The middle plot of Fig. 1 presents the cancellation between the two terms in Eq. (1). We observe that the  $r_{\text{cut}}$  dependence of the sum is smaller than its statistical uncertainty over a large range. Furthermore, the separate divergence of the terms is clearly visible for  $r_{\text{cut}} < 10^{-3}$ .

Finally, the right plot of Fig. 1 shows the results for the fiducial N<sup>3</sup>LO cross section, including the statistical error

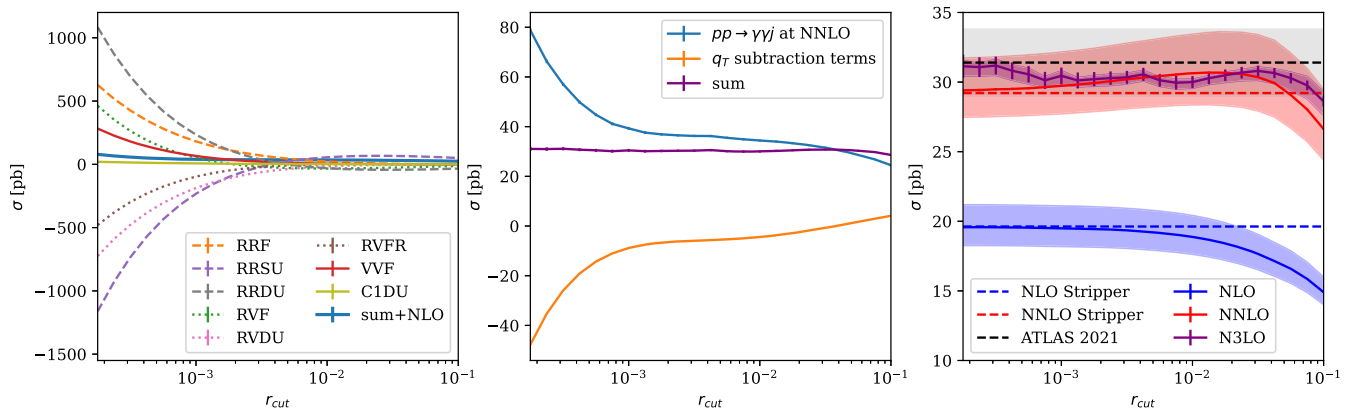


FIG. 1. The left plot shows  $p p \rightarrow \gamma \gamma j$  cross sections at NNLO for different  $r_{\text{cut}}$  values decomposed into multiple contributions (a precise definition can be found in Ref. [85]). The first two letters indicate contributions which are double virtual (VV), real virtual (RV), double real (RR), or a single convolution with splitting kernels (C1). The letters F, SU, DU, and FR correspond respectively to finite, single-unresolved, double-unresolved, and finite-remainder contributions. The NNLO cross section and the contribution of the  $q_T$  subtraction terms [Eq. (1)] is shown in the middle plot. Their sum converges to  $\sigma_{\text{N}^3\text{LO}}^{p p \rightarrow \gamma \gamma}$  as  $r_{\text{cut}} \rightarrow 0$ . The right plot displays the results of the  $q_T$ -slicing method additionally for NLO and NNLO, where the cross section can also be directly calculated with STRIPPER for comparison. It also contains a comparison to experimental data presented by ATLAS in Ref. [12]. The error bars correspond to the statistical uncertainty of the Monte Carlo integration, whereas the shaded bands indicate the uncertainty from seven-point scale variation for the theory predictions and the experimental uncertainty reported by ATLAS, respectively. The numerical values to reproduce the right plot can be found in Supplemental Material [87].

and the theoretical uncertainty from scale dependence, estimated with a seven-point variation. Although the scale dependence is affected by residual statistical uncertainty, it remains significantly smaller than the 8% at NNLO. In view of the weak  $r_{\text{cut}}$  dependence for small cutoff values, and the comparison of slicing and direct calculation at NNLO, any remaining dependence on the slicing parameter is expected to lie well within the statistical uncertainty. For our fiducial cross-section prediction, we choose  $r_{\text{cut}} = 3.2 \times 10^{-4}$  for which the power corrections at NNLO amount to 0.3 pb.

As long as the  $\text{N}^3\text{LO}$  power corrections are hidden in the statistical uncertainty band of  $\pm 0.6$  pb, they too should not exceed 0.6 pb. Estimating them at twice the power corrections of the NNLO result should be considered conservative. There have been attempts to remove at least the linear power corrections in  $r_{\text{cut}}$  due to the asymmetric cuts on the two-body phase space [153]. Unfortunately, Ref. [153] shows that precisely in the case of diphoton production, this leads to even larger differences between the exact NNLO cross section and the one obtained with  $q_T$  slicing. In Ref. [154], it has been pointed out that the problem is related to the very specific photon-isolation criteria. The remedy proposed there seems to improve the situation slightly, but is based on a special treatment of only some of the contributions at NNLO and does not seem to be extensible to  $\text{N}^3\text{LO}$  in a justifiable way. Hence, for now, the only way to deal with power corrections at this order is to take as small  $r_{\text{cut}}$  as possible.

Our result reads

$$\sigma_{\text{N}^3\text{LO}}^{pp \rightarrow \gamma\gamma} = 31.2(6)_{-0.7}^{+0.5} \text{ pb } (\pm 0.6 \text{ pb power corrections}), \quad (6)$$

where the statistical uncertainty is quoted in parentheses, the scale uncertainty is provided as lower and upper bounds of the variation range, and the power-corrections estimate is separated. The scale uncertainty is obviously affected by the statistical uncertainty. Thus, we recommend adding the

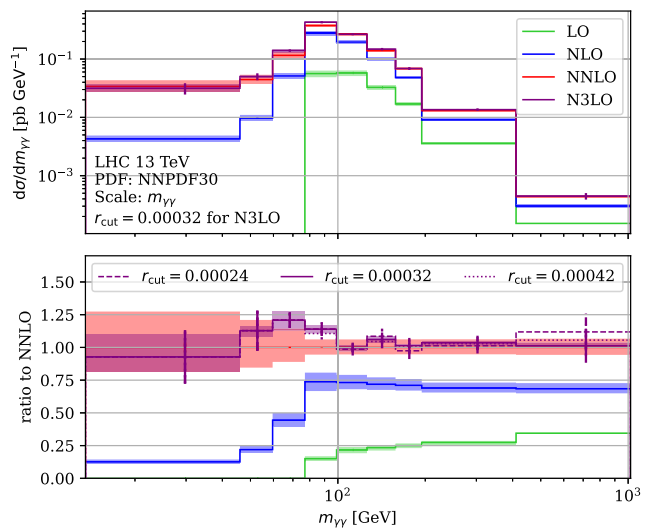


FIG. 2. The invariant-mass distribution of the photon pair through  $N^3LO$ . The top panel contains absolute predictions, whereas the bottom panel is normalized to the NNLO values. While the distributions through NNLO are calculated through direct computation with STRIPPER, the  $N^3LO$  values are obtained through  $q_T$  slicing for fixed  $r_{\text{cut}}$ . The remaining dependence on  $r_{\text{cut}}$  is included in the lower panel and lies well within the statistical uncertainty.

uncertainties in quadrature to obtain the total error estimate. In summary, our result reduces the uncertainty on the total cross section to 3%, while also being consistent with the NNLO value, indicating convergence of the perturbation series. Additionally, it is in agreement with the ATLAS measurement [12]

$$\sigma_{\text{ATLAS}}^{pp \rightarrow \gamma\gamma} = 31.4 \pm 0.1(\text{stat}) \pm 2.4(\text{syst}) \text{ pb} \quad (7)$$

also displayed in Fig. 1.

Lastly, since  $q_T$  slicing yields fully differential information, we can obtain distributions for arbitrary observables. Figure 2 shows the differential cross section  $d\sigma^{pp \rightarrow \gamma\gamma}/dm_{\gamma\gamma}$  through N<sup>3</sup>LO. As we have optimized the phase-space integration to minimize the statistical uncertainty of the total cross section, there remain large error bars for  $m_{\gamma\gamma}$  regions that contribute less. Overall, the statistical uncertainty is comparable with the scale uncertainty at NNLO. Nevertheless, the distribution shows perturbative convergence across different regions of the phase space, as well as stable behavior as the value of the slicing parameter  $r_{\text{cut}}$  is varied.

**Conclusions**—Theoretical predictions for  $pp \rightarrow \gamma\gamma$  are plagued by large higher-order corrections, but our N<sup>3</sup>LO result finally shows perturbative convergence. The N<sup>3</sup>LO corrections are small, in agreement with the NNLO QCD uncertainty estimates, and lead to a significant reduction of the scale sensitivity. They shift the theoretical prediction slightly upward, closer to the central value of the ATLAS measurement, although this is of little relevance given the large experimental systematic uncertainty. Since the calculation has been performed in a fully differential manner, extending to different phase spaces or observables based on the photons' kinematics is straightforward.

This is a first N<sup>3</sup>LO QCD calculation for a process with actual  $2 \rightarrow 2$  kinematics. It marks a milestone in the advancement and automation of higher-order predictions, specifically at NNLO. Besides technical improvements of the cross-section integration, it required substantial effort to increase the stability and evaluation speed of the one-loop matrix elements.

Despite all improvements and considerable computational resources used, the precision of the calculated N<sup>3</sup>LO QCD cross sections is limited by the statistical uncertainty arising from the Monte Carlo integration of the NNLO QCD cross section of the  $pp \rightarrow \gamma\gamma j$  process in the limit of vanishing photon-pair transverse momentum. The cancellation between positive and negative contributions in subtracted cross sections is a major limiting factor. Improving on this issue would make a substantial difference in the capabilities of existing NNLO QCD frameworks and their application to N<sup>3</sup>LO slicing computations. Turning this argument around, local N<sup>3</sup>LO subtraction schemes might offer improved efficiency by avoiding regions with extreme

numerical cancellations. Furthermore, they would remove the uncertainty due to power corrections.

Based on our results, one can claim that diphoton production at N<sup>3</sup>LO has a theoretical uncertainty of 2% as long as the computational methodology itself does not introduce additional errors. This should be sufficient to improve various measurements by constraining Monte Carlo simulations currently used.

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**Data availability**—The data that support the findings of this article are openly available.

- [1] G. Aad *et al.* (ATLAS Collaboration), Measurements of the Higgs boson inclusive and differential fiducial cross-sections in the diphoton decay channel with pp collisions at  $\sqrt{s} = 13$  TeV with the ATLAS detector, *J. High Energy Phys.* **08** (2022) 027.
- [2] A. Hayrapetyan *et al.* (CMS Collaboration), Measurements of inclusive and differential Higgs boson production cross sections at  $\sqrt{s} = 13.6$  TeV in the  $H \rightarrow \gamma\gamma$  decay channel, *J. High Energy Phys.* **09** (2025) 070.
- [3] G. Aad *et al.* (ATLAS Collaboration), Measurement of the Higgs boson mass with  $H \rightarrow \gamma\gamma$  decays in  $140 \text{ fb}^{-1}$  of  $s = 13$  TeV pp collisions with the ATLAS detector, *Phys. Lett. B* **847**, 138315 (2023).
- [4] A measurement of the Higgs boson mass in the diphoton decay channel with the CMS detector, Technical Report (CERN, Geneva, 2026).
- [5] G. Aad *et al.* (ATLAS Collaboration), Search for resonances decaying into photon pairs in  $139 \text{ fb}^{-1}$  of  $pp$  collisions at  $\sqrt{s} = 13$  TeV with the ATLAS detector, *Phys. Lett. B* **822**, 136651 (2021).

- [6] A. Hayrapetyan *et al.* (CMS Collaboration), Search for new physics in high-mass diphoton events from proton-proton collisions at  $\sqrt{s} = 13$  TeV, *J. High Energy Phys.* **08** (2024) 215.
- [7] G. Aad *et al.* (ATLAS Collaboration), Search for boosted diphoton resonances in the 10 to 70 GeV mass range using 138 fb<sup>-1</sup> of 13 TeV pp collisions with the ATLAS detector, *J. High Energy Phys.* **07** (2023) 155.
- [8] A. Hayrapetyan *et al.* (CMS Collaboration), Search for a narrow resonance with a mass between 10 and 70 GeV decaying to a pair of photons in proton-proton collisions at  $\sqrt{s} = 13$  TeV, [arXiv:2603.03250](#).
- [9] P. Bargiela, F. Buccioni, F. Caola, F. Devoto, A. von Manteuffel, and L. Tancredi, Signal-background interference effects in Higgs-mediated diphoton production beyond NLO, *Eur. Phys. J. C* **83**, 174 (2023).
- [10] S. Catani, L. Cieri, D. de Florian, G. Ferrera, and M. Grazzini, Diphoton production at hadron colliders: A fully-differential QCD calculation at NNLO, *Phys. Rev. Lett.* **108**, 072001 (2012); **117**, 089901(E) (2016).
- [11] J.M. Campbell, R.K. Ellis, Y. Li, and C. Williams, Predictions for diphoton production at the LHC through NNLO in QCD, *J. High Energy Phys.* **07** (2016) 148.
- [12] G. Aad *et al.* (ATLAS Collaboration), Measurement of the production cross section of pairs of isolated photons in *pp* collisions at 13 TeV with the ATLAS detector, *J. High Energy Phys.* **11** (2021) 169.
- [13] C. Anastasiou and K. Melnikov, Higgs boson production at hadron colliders in NNLO QCD, *Nucl. Phys.* **B646**, 220 (2002).
- [14] C. Anastasiou and K. Melnikov, Pseudoscalar Higgs boson production at hadron colliders in NNLO QCD, *Phys. Rev. D* **67**, 037501 (2003).
- [15] C. Anastasiou, C. Duhr, F. Dulat, F. Herzog, and B. Mistlberger, Higgs boson gluon-fusion production in QCD at three loops, *Phys. Rev. Lett.* **114**, 212001 (2015).
- [16] C. Anastasiou, C. Duhr, F. Dulat, E. Furlan, T. Gehrmann, F. Herzog, A. Lazopoulos, and B. Mistlberger, High precision determination of the gluon fusion Higgs boson cross-section at the LHC, *J. High Energy Phys.* **05** (2016) 058.
- [17] B. Mistlberger, Higgs boson production at hadron colliders at N<sup>3</sup>LO in QCD, *J. High Energy Phys.* **05** (2018) 028.
- [18] C. Duhr, F. Dulat, and B. Mistlberger, Higgs boson production in bottom-quark fusion to third order in the strong coupling, *Phys. Rev. Lett.* **125**, 051804 (2020).
- [19] C. Duhr, F. Dulat, and B. Mistlberger, Drell-Yan cross section to third order in the strong coupling constant, *Phys. Rev. Lett.* **125**, 172001 (2020).
- [20] C. Duhr, F. Dulat, and B. Mistlberger, Charged current Drell-Yan production at N<sup>3</sup>LO, *J. High Energy Phys.* **11** (2020) 143.
- [21] J. Baglio, C. Duhr, B. Mistlberger, and R. Szafron, Inclusive production cross sections at N<sup>3</sup>LO, *J. High Energy Phys.* **12** (2022) 066.
- [22] M. Cacciari, F. A. Dreyer, A. Karlberg, G. P. Salam, and G. Zanderighi, Fully differential vector-boson-fusion Higgs production at next-to-next-to-leading order, *Phys. Rev. Lett.* **115**, 082002 (2015); **120**, 139901(E) (2018).
- [23] X. Chen, T. Gehrmann, E. W. N. Glover, A. Huss, B. Mistlberger, and A. Pelloni, Fully differential Higgs boson production to third order in QCD, *Phys. Rev. Lett.* **127**, 072002 (2021).
- [24] F. A. Dreyer and A. Karlberg, Vector-Boson Fusion Higgs Production at Three Loops in QCD, *Phys. Rev. Lett.* **117**, 072001 (2016).
- [25] F. A. Dreyer and A. Karlberg, Vector-boson fusion Higgs pair production at N<sup>3</sup>LO, *Phys. Rev. D* **98**, 114016 (2018).
- [26] S. Catani and M. Grazzini, An NNLO subtraction formalism in hadron collisions and its application to Higgs boson production at the LHC, *Phys. Rev. Lett.* **98**, 222002 (2007).
- [27] L. Cieri, X. Chen, T. Gehrmann, E. W. N. Glover, and A. Huss, Higgs boson production at the LHC using the *q<sub>T</sub>* subtraction formalism at N<sup>3</sup>LO QCD, *J. High Energy Phys.* **02** (2019) 096.
- [28] G. Billis, B. Dehnadi, M. A. Ebert, J. K. L. Michel, and F. J. Tackmann, Higgs PT spectrum and total cross section with fiducial cuts at third resummed and fixed order in QCD, *Phys. Rev. Lett.* **127**, 072001 (2021).
- [29] S. Camarda, L. Cieri, and G. Ferrera, Drell-Yan lepton-pair production: *q<sub>T</sub>* resummation at N<sup>3</sup>LL accuracy and fiducial cross sections at N<sup>3</sup>LO, *Phys. Rev. D* **104**, L111503 (2021).
- [30] X. Chen, T. Gehrmann, N. Glover, A. Huss, T.-Z. Yang, and H. X. Zhu, Dilepton rapidity distribution in Drell-Yan production to third order in QCD, *Phys. Rev. Lett.* **128**, 052001 (2022).
- [31] S. Camarda, L. Cieri, and G. Ferrera, Fiducial perturbative power corrections within the *q<sub>T</sub>* subtraction formalism, *Eur. Phys. J. C* **82**, 575 (2022).
- [32] X. Chen, T. Gehrmann, E. W. N. Glover, A. Huss, P. F. Monni, E. Re, L. Rottoli, and P. Torrielli, Third-order fiducial predictions for Drell-Yan production at the LHC, *Phys. Rev. Lett.* **128**, 252001 (2022).
- [33] X. Chen, T. Gehrmann, N. Glover, A. Huss, T.-Z. Yang, and H. X. Zhu, Transverse mass distribution and charge asymmetry in W boson production to third order in QCD, *Phys. Lett. B* **840**, 137876 (2023).
- [34] T. Neumann and J. Campbell, Fiducial Drell-Yan production at the LHC improved by transverse-momentum resummation at N<sup>4</sup>LLp+N<sup>3</sup>LO, *Phys. Rev. D* **107**, L011506 (2023).
- [35] J. Campbell and T. Neumann, Third order QCD predictions for fiducial W-boson production, *J. High Energy Phys.* **11** (2023) 127.
- [36] See, however, Ref. [37] for recent progress in antenna subtraction [38] at N<sup>3</sup>LO for *e<sup>+</sup>e<sup>-</sup>* collisions.
- [37] X. Chen, P. Jakubčík, M. Marcoli, and G. Stagnitto, Jet production at electron-positron colliders at next-to-next-to-next-to-leading order in QCD, *Phys. Lett. B* **869**, 139804 (2025).
- [38] A. Gehrmann-De Ridder, T. Gehrmann, and E. W. N. Glover, Antenna subtraction at NNLO, *J. High Energy Phys.* **09** (2005) 056.
- [39] L. Buonocore, M. Grazzini, J. Haag, L. Rottoli, and C. Savoini, Effective transverse momentum in multiple jet production at hadron colliders, *Phys. Rev. D* **106**, 014008 (2022).

- [40] L. Buonocore, M. Grazzini, J. Haag, L. Rottoli, and C. Savoini, Exploring slicing variables for jet processes, *J. High Energy Phys.* **12** (2023) 193.
- [41] R.-J. Fu, R. Rahn, D. Y. Shao, W. J. Waalewijn, and B. Wu, qT slicing with multiple jets, *Phys. Rev. Lett.* **135**, 171903 (2025).
- [42] L. Buonocore, M. Grazzini, F. Guadagni, J. Haag, S. Kallweit, and L. Rottoli, Jet production at NNLO: Exploring a new scheme [arXiv:2512.03954](#).
- [43] J. Gao, H. T. Li, H. X. Zhu, and Y. J. Zhu, Transverse-momentum subtraction for semi-inclusive deep-inelastic scattering, [arXiv:2602.06364](#).
- [44] L. Dong, S. Fang, J. Gao, H. T. Li, D. Y. Shao, and Y. J. Zhu, NNLO QCD corrections to hadron production in DIS at finite transverse momentum, [arXiv:2602.22972](#).
- [45] Notice, however, the recent progress for lepton-hadron collisions in Ref. [46].
- [46] L. Dong, S. Fang, J. Gao, H. T. Li, D. Y. Shao, H. X. Zhu, and Y. J. Zhu, Two-dimensional transverse-momentum subtraction and semi-inclusive deep-inelastic scattering at N<sup>3</sup>LO in QCD, [arXiv:2603.29673](#).
- [47] R. Boughezal, C. Focke, X. Liu, and F. Petriello, W-boson production in association with a jet at next-to-next-to-leading order in perturbative QCD, *Phys. Rev. Lett.* **115**, 062002 (2015).
- [48] J. Gaunt, M. Stahlhofen, F. J. Tackmann, and J. R. Walsh, N-jettiness Subtractions for NNLO QCD Calculations, *J. High Energy Phys.* **09** (2015) 058.
- [49] M. A. Ebert, B. Mistlberger, and G. Vita, N-jettiness beam functions at N<sup>3</sup>LO, *J. High Energy Phys.* **09** (2020) 143.
- [50] D. Baranowski, A. Behring, K. Melnikov, L. Tancredi, and C. Wever, Beam functions for N-jettiness at N<sup>3</sup>LO in perturbative QCD, *J. High Energy Phys.* **02** (2023) 073.
- [51] R. Brüser, Z. L. Liu, and M. Stahlhofen, Three-loop quark jet function, *Phys. Rev. Lett.* **121**, 072003 (2018).
- [52] P. Banerjee, P. K. Dhani, and V. Ravindran, Gluon jet function at three loops in QCD, *Phys. Rev. D* **98**, 094016 (2018).
- [53] D. Baranowski, M. Delto, K. Melnikov, A. Pikelner, and C.-Y. Wang, Zero-jettiness soft function to third order in perturbative QCD, *Phys. Rev. Lett.* **134**, 191902 (2025).
- [54] D. Baranowski, M. Delto, K. Melnikov, A. Pikelner, and C.-Y. Wang, Triple real-emission contribution to the zero-jettiness soft function at N<sup>3</sup>LO in QCD, *J. High Energy Phys.* **04** (2024) 084.
- [55] J. C. Collins, D. E. Soper, and G. F. Sterman, Transverse momentum distribution in Drell-Yan pair and W and Z boson production, *Nucl. Phys.* **B250**, 199 (1985).
- [56] G. Bozzi, S. Catani, D. de Florian, and M. Grazzini, Transverse-momentum resummation and the spectrum of the Higgs boson at the LHC, *Nucl. Phys.* **B737**, 73 (2006).
- [57] T. Becher and M. Neubert, Drell-Yan production at small  $q_T$ , transverse parton distributions and the collinear anomaly, *Eur. Phys. J. C* **71**, 1665 (2011).
- [58] M. G. Echevarria, A. Idilbi, and I. Scimemi, Factorization theorem for Drell-Yan at low  $q_T$  and transverse momentum distributions on-the-light-cone, *J. High Energy Phys.* **07** (2012) 002.
- [59] J.-Y. Chiu, A. Jain, D. Neill, and I. Z. Rothstein, A formalism for the systematic treatment of rapidity logarithms in quantum field theory, *J. High Energy Phys.* **05** (2012) 084.
- [60] M. A. Ebert, B. Mistlberger, and G. Vita, Transverse momentum dependent PDFs at N<sup>3</sup>LO, *J. High Energy Phys.* **09** (2020) 146.
- [61] M.-x. Luo, T.-Z. Yang, H. X. Zhu, and Y. J. Zhu, Unpolarized quark and gluon TMD PDFs and FFs at N<sup>3</sup>LO, *J. High Energy Phys.* **06** (2021) 115.
- [62] E. W. N. Glover and M. E. Tejeda-Yeomans, Two loop QCD helicity amplitudes for massless quark massless gauge boson scattering, *J. High Energy Phys.* **06** (2003) 033.
- [63] F. Caola, A. Von Manteuffel, and L. Tancredi, Diphoton amplitudes in three-loop quantum chromodynamics, *Phys. Rev. Lett.* **126**, 112004 (2021).
- [64] A. Daleo, T. Gehrmann, and D. Maitre, Antenna subtraction with hadronic initial states, *J. High Energy Phys.* **04** (2007) 016.
- [65] J. Currie, E. W. N. Glover, and S. Wells, Infrared structure at NNLO using antenna subtraction, *J. High Energy Phys.* **04** (2013) 066.
- [66] A. Huss *et al.* (NNLOJET Collaboration), NNLOJET: A parton-level event generator for jet cross sections at NNLO QCD accuracy, [arXiv:2503.22804](#).
- [67] M. Czakon, A novel subtraction scheme for double-real radiation at NNLO, *Phys. Lett. B* **693**, 259 (2010).
- [68] G. Somogyi and Z. Trocsanyi, A New subtraction scheme for computing QCD jet cross sections at next-to-leading order accuracy, [arXiv:hep-ph/0609041](#).
- [69] V. Del Duca, C. Duhr, A. Kardos, G. Somogyi, Z. Szőr, Z. Trócsányi, and Z. Tulipánt, Jet production in the CoLoR-FulNNLO method: Event shapes in electron-positron collisions, *Phys. Rev. D* **94**, 074019 (2016).
- [70] V. Del Duca, C. Duhr, L. Fekeshazy, F. Guadagni, P. Mukherjee, G. Somogyi, F. Tramontano, and S. Van Thurenhout, NNLOCAL: Completely local subtractions for color-singlet production in hadron collisions, *J. High Energy Phys.* **05** (2025) 151.
- [71] F. Caola, K. Melnikov, and R. Rönsch, Nested soft-collinear subtractions in NNLO QCD computations, *Eur. Phys. J. C* **77**, 248 (2017).
- [72] F. Caola, K. Melnikov, and R. Rönsch, Analytic results for color-singlet production at NNLO QCD with the nested soft-collinear subtraction scheme, *Eur. Phys. J. C* **79**, 386 (2019).
- [73] S. Y. Klein and L. Simon, history: A tool for fully-differential cross sections at next-to-next-to-leading order, [arXiv:2603.13729](#).
- [74] L. Magnea, E. Maina, G. Pelliccioli, C. Signorile-Signorile, P. Torrielli, and S. Uccirati, Local analytic sector subtraction at NNLO, *J. High Energy Phys.* **12** (2018) 107; **06** (2019) 13.
- [75] H. A. Chawdhry, M. L. Czakon, A. Mitov, and R. Poncelet, NNLO QCD corrections to three-photon production at the LHC, *J. High Energy Phys.* **02** (2020) 057.
- [76] M. Czakon, A. Mitov, and R. Poncelet, Next-to-next-to-leading order study of three-jet production at the LHC, *Phys. Rev. Lett.* **127**, 152001 (2021); **129**, 119901(E) (2022).

- [77] X. Chen, T. Gehrmann, E. W. N. Glover, A. Huss, and M. Marcoli, Automation of antenna subtraction in colour space: Gluonic processes, *J. High Energy Phys.* **10** (2022) 099.
- [78] H. B. Hartanto, R. Poncelet, A. Popescu, and S. Zoia, Next-to-next-to-leading order QCD corrections to  $Wb\bar{b}$  production at the LHC, *Phys. Rev. D* **106**, 074016 (2022).
- [79] M. Alvarez, J. Cantero, M. Czakon, J. Llorente, A. Mitov, and R. Poncelet, NNLO QCD corrections to event shapes at the LHC, *J. High Energy Phys.* **03** (2023) 129.
- [80] S. Badger, M. Czakon, H. B. Hartanto, R. Moodie, T. Peraro, R. Poncelet, and S. Zoia, Isolated photon production in association with a jet pair through next-to-next-to-leading order in QCD, *J. High Energy Phys.* **10** (2023) 071.
- [81] S. Badger, C. Branchicchio, M. Becchetti, M. Czakon, H. B. Hartanto, R. Poncelet, and S. Zoia, Higher-order QCD corrections to top-quark pair production in association with a jet, [arXiv:2511.11431](https://arxiv.org/abs/2511.11431).
- [82] H. A. Chawdhry, M. Czakon, A. Mitov, and R. Poncelet, NNLO QCD corrections to diphoton production with an additional jet at the LHC, *J. High Energy Phys.* **09** (2021) 093.
- [83] F. Buccioni, X. Chen, W.-J. Feng, T. Gehrmann, A. Huss, and M. Marcoli, Precise predictions for event shapes in diphoton production at the LHC, *Phys. Rev. Lett.* **134**, 171901 (2025).
- [84] M. Czakon and D. Heymes, Four-dimensional formulation of the sector-improved residue subtraction scheme, *Nucl. Phys. B* **890**, 152 (2014).
- [85] M. Czakon, A. van Hameren, A. Mitov, and R. Poncelet, Single-jet inclusive rates with exact color at  $\mathcal{O}(\alpha_s^4)$ , *J. High Energy Phys.* **10** (2019) 262.
- [86] Y. Hida, X. S. Li, and D. H. Bailey, Algorithms for quad-double precision floating point arithmetic, in *Proceedings 15th IEEE Symposium on Computer Arithmetic. ARITH-15 2001* (2001), pp. 155–162, [10.1109/ARITH.2001.930115](https://doi.org/10.1109/ARITH.2001.930115).
- [87] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/wlcl-c61b> for describing technical aspects of the six-point one-loop computation and selector function in the STRIPPER framework, which includes Refs. [88–98].
- [88] J. A. M. Vermaseren, New features of FORM, [arXiv:math-ph/0010025](https://arxiv.org/abs/math-ph/0010025).
- [89] J. Klappert and F. Lange, Reconstructing rational functions with FireFly, *Comput. Phys. Commun.* **247**, 106951 (2020).
- [90] J. Klappert, S. Y. Klein, and F. Lange, Interpolation of dense and sparse rational functions and other improvements in FireFly, *Comput. Phys. Commun.* **264**, 107968 (2021).
- [91] L. Chen, A prescription for projectors to compute helicity amplitudes in D dimensions, *Eur. Phys. J. C* **81**, 417 (2021).
- [92] P. Maierhöfer, J. Usovitsch, and P. Uwer, Kira—A Feynman integral reduction program, *Comput. Phys. Commun.* **230**, 99 (2018).
- [93] J. Klappert, F. Lange, P. Maierhöfer, and J. Usovitsch, Integral reduction with Kira 2.0 and finite field methods, *Comput. Phys. Commun.* **266**, 108024 (2021).
- [94] J. M. Henn, A. Matijašić, and J. Miczajka, One-loop hexagon integral to higher orders in the dimensional regulator, *J. High Energy Phys.* **01** (2023) 096.
- [95] H. A. Chawdhry, p-adic reconstruction of rational functions in multiloop amplitudes, *Phys. Rev. D* **110**, 056028 (2024).
- [96] G. De Laurentis, Lips: p-adic and singular phase space, *J. Phys. Conf. Ser.* **3206**, 012118 (2026).
- [97] The FLINT team, FLINT: Fast Library for Number Theory (2025), version 3.2.1, <https://flintlib.org>.
- [98] F. Eschment, Advancements in the theory of QCD amplitudes and their application to photon-pair production at the LHC at next-to-next-to-next-to-leading order, Ph.D. thesis, RWTH Aachen University, 2026.
- [99] A. van Hameren, `avhlib`, <https://bitbucket.org/hameren/avhlib>.
- [100] M. Bury and A. van Hameren, Numerical evaluation of multi-gluon amplitudes for High Energy Factorization, *Comput. Phys. Commun.* **196**, 592 (2015).
- [101] B. Agarwal, F. Buccioni, A. von Manteuffel, and L. Tancredi, Two-loop helicity amplitudes for diphoton plus jet production in full color, *Phys. Rev. Lett.* **127**, 262001 (2021).
- [102] F. Cascioli, P. Maierhofer, and S. Pozzorini, Scattering amplitudes with open loops, *Phys. Rev. Lett.* **108**, 111601 (2012).
- [103] F. Buccioni, J.-N. Lang, J. M. Lindert, P. Maierhöfer, S. Pozzorini, H. Zhang, and M. F. Zoller, OpenLoops 2, *Eur. Phys. J. C* **79**, 866 (2019).
- [104] S. Badger, B. Biedermann, P. Uwer, and V. Yundin, Numerical evaluation of virtual corrections to multi-jet production in massless QCD, *Comput. Phys. Commun.* **184**, 1981 (2013).
- [105] P. Bargiela, F. Caola, A. von Manteuffel, and L. Tancredi, Three-loop helicity amplitudes for diphoton production in gluon fusion, *J. High Energy Phys.* **02** (2022) 153.
- [106] S. Badger, C. Brønnum-Hansen, D. Chicherin, T. Gehrmann, H. B. Hartanto, J. Henn, M. Marcoli, R. Moodie, T. Peraro, and S. Zoia, Virtual QCD corrections to gluon-initiated diphoton plus jet production at hadron colliders, *J. High Energy Phys.* **11** (2021) 083.
- [107] A. Buckley, J. Ferrando, S. Lloyd, K. Nordström, B. Page, M. Rüfenacht, M. Schönherr, and G. Watt, LHAPDF6: Parton density access in the LHC precision era, *Eur. Phys. J. C* **75**, 132 (2015).
- [108] Approximate  $N^3$ LO PDF sets have become available recently. Given the timescale of the present computation, and to compare to Ref. [12] we still use the NNPDF3.0 PDF set. As the main focus of this Letter is perturbative convergence and the technical challenges, we believe predictions based on recent PDFs will not change the message of our work.
- [109] R. D. Ball *et al.* (NNPDF Collaboration), Parton distributions for the LHC Run II, *J. High Energy Phys.* **04** (2015) 040.
- [110] F. Siegert, A practical guide to event generation for prompt photon production with Sherpa, *J. Phys. G* **44**, 044007 (2017).
- [111] X. Chen, T. Gehrmann, N. Glover, M. Höfer, and A. Huss, Isolated photon and photon + jet production at NNLO QCD accuracy, *J. High Energy Phys.* **04** (2020) 166.
- [112] G. Laurentis and D. Maître, Extracting analytical one-loop amplitudes from numerical evaluations, *J. High Energy Phys.* **07** (2019) 123.

- [113] Note that in Ref. [114], the MHV 6-gluon amplitude has been computed in terms of momentum-twistor variables using finite-field reconstruction. We find, however, that the expressions of more complicated helicity configurations of our desired amplitude are too complicated when expressed through momentum-twistor variables, making a reconstruction unfeasible.
- [114] F. Sarandrea, Analytic QCD amplitudes using finite field evaluation techniques, Ph.D. thesis, Durham University, 2023.
- [115] This is also the first independent application of the methods from Ref. [116], in contrast to the previous computations involving at least one of its authors.
- [116] G. De Laurentis and B. Page, Ansätze for scattering amplitudes from p-adic numbers and algebraic geometry, *J. High Energy Phys.* **12** (2022) 140.
- [117] A. von Manteuffel and R.M. Schabinger, A novel approach to integration by parts reduction, *Phys. Lett. B* **744**, 101 (2015).
- [118] T. Peraro, Scattering amplitudes over finite fields and multivariate functional reconstruction, *J. High Energy Phys.* **12** (2016) 030.
- [119] S. Abreu, J. Dormans, F. Febres Cordero, H. Ita, B. Page, and V. Sotnikov, Analytic form of the planar two-loop five-parton scattering Amplitudes in QCD, *J. High Energy Phys.* **05** (2019) 084.
- [120] S. Badger, D. Chicherin, T. Gehrmann, G. Heinrich, J.M. Henn, T. Peraro, P. Wasser, Y. Zhang, and S. Zoia, Analytic form of the full two-loop five-gluon all-plus helicity amplitude, *Phys. Rev. Lett.* **123**, 071601 (2019).
- [121] S. Abreu, B. Page, E. Pascual, and V. Sotnikov, Leading-color two-loop QCD corrections for three-photon production at hadron colliders, *J. High Energy Phys.* **01** (2021) 078.
- [122] H. A. Chawdhry, M. Czakon, A. Mitov, and R. Poncelet, Two-loop leading-color helicity amplitudes for three-photon production at the LHC, *J. High Energy Phys.* **06** (2021) 150.
- [123] S. Abreu, F. Febres Cordero, H. Ita, B. Page, and V. Sotnikov, Leading-color two-loop QCD corrections for three-jet production at hadron colliders, *J. High Energy Phys.* **07** (2021) 095.
- [124] H. A. Chawdhry, M. Czakon, A. Mitov, and R. Poncelet, Two-loop leading-colour QCD helicity amplitudes for two-photon plus jet production at the LHC, *J. High Energy Phys.* **07** (2021) 164.
- [125] S. Abreu, F. Febres Cordero, H. Ita, M. Klinkert, B. Page, and V. Sotnikov, Leading-color two-loop amplitudes for four partons and a W boson in QCD, *J. High Energy Phys.* **04** (2022) 042.
- [126] S. Badger, H. B. Hartanto, and S. Zoia, Two-loop QCD corrections to  $Wbb^-$  production at hadron colliders, *Phys. Rev. Lett.* **127**, 012001 (2021).
- [127] A. Hodges, Eliminating spurious poles from gauge-theoretic amplitudes, *J. High Energy Phys.* **05** (2009) 135.
- [128] S. Badger, C. Brønnum-Hansen, H. B. Hartanto, and T. Peraro, First look at two-loop five-gluon scattering in QCD, *Phys. Rev. Lett.* **120**, 092001 (2018).
- [129] S. Abreu, F. Febres Cordero, H. Ita, B. Page, and M. Zeng, Planar two-loop five-gluon amplitudes from numerical unitarity, *Phys. Rev. D* **97**, 116014 (2018).
- [130] S. Badger, C. Brønnum-Hansen, H. B. Hartanto, and T. Peraro, Analytic helicity amplitudes for two-loop five-gluon scattering: the single-minus case, *J. High Energy Phys.* **01** (2019) 186.
- [131] S. Abreu, J. Dormans, F. Febres Cordero, H. Ita, and B. Page, Analytic form of planar two-loop five-gluon scattering amplitudes in QCD, *Phys. Rev. Lett.* **122**, 082002 (2019).
- [132] S. Badger, H. B. Hartanto, J. Kryś, and S. Zoia, Two-loop leading-colour QCD helicity amplitudes for Higgs boson production in association with a bottom-quark pair at the LHC, *J. High Energy Phys.* **11** (2021) 012.
- [133] S. Badger, H. B. Hartanto, J. Kryś, and S. Zoia, Two-loop leading colour helicity amplitudes for  $W^\pm\gamma + j$  production at the LHC, *J. High Energy Phys.* **05** (2022) 035.
- [134] S. Badger, H. B. Hartanto, Z. Wu, Y. Zhang, and S. Zoia, Two-loop amplitudes for  $\mathcal{O}(\alpha_s^2)$  corrections to  $W\gamma\gamma$  production at the LHC, *J. High Energy Phys.* **12** (2024) 221.
- [135] S. Badger, H. B. Hartanto, R. Poncelet, Z. Wu, Y. Zhang, and S. Zoia, Full-colour double-virtual amplitudes for associated production of a Higgs boson with a bottom-quark pair at the LHC, *J. High Energy Phys.* **03** (2025) 066.
- [136] S. Badger, M. Becchetti, C. Brancaccio, H. B. Hartanto, and S. Zoia, Numerical evaluation of two-loop QCD helicity amplitudes for  $gg \rightarrow t\bar{t}g$  at leading colour, *J. High Energy Phys.* **03** (2025) 070.
- [137] S. Badger, M. Becchetti, C. Brancaccio, M. Czakon, H. B. Hartanto, R. Poncelet, and S. Zoia, Double virtual QCD corrections to  $t\bar{t} + \text{jet}$  production at the LHC, *J. High Energy Phys.* **05** (2026) 044.
- [138] S. Abreu, G. De Laurentis, H. Ita, M. Klinkert, B. Page, and V. Sotnikov, Two-loop QCD corrections for three-photon production at hadron colliders, *SciPost Phys.* **15**, 157 (2023).
- [139] G. De Laurentis, H. Ita, M. Klinkert, and V. Sotnikov, Double-virtual NNLO QCD corrections for five-parton scattering. I. The gluon channel, *Phys. Rev. D* **109**, 094023 (2024).
- [140] G. De Laurentis, H. Ita, and V. Sotnikov, Double-virtual NNLO QCD corrections for five-parton scattering. II. The quark channels, *Phys. Rev. D* **109**, 094024 (2024).
- [141] J.M. Campbell, G. De Laurentis, and R.K. Ellis, Vector boson pair production at one loop: analytic results for the process  $q\bar{q}\ell\bar{\ell}\ell'\bar{\ell}'g$ , *J. High Energy Phys.* **07** (2022) 096.
- [142] J.M. Campbell, G. De Laurentis, and R. K. Ellis, Analytic amplitudes for a pair of Higgs bosons in association with three partons, *J. High Energy Phys.* **10** (2024) 230.
- [143] G. De Laurentis, H. Ita, B. Page, and V. Sotnikov, Compact two-loop QCD corrections for  $Vjj$  production in proton collisions, *J. High Energy Phys.* **06** (2025) 093.
- [144] J.M. Campbell, G. De Laurentis, and R. K. Ellis, Analytic reconstruction with massive particles: One-loop amplitudes for  $0 \rightarrow q\bar{q}t\bar{t}H$ , *J. High Energy Phys.* **07** (2025) 147.

- [145] J.M. Campbell, G. De Laurentis, and R.K. Ellis, An analytic result for the  $0 \rightarrow ggHHH$  amplitude, *J. High Energy Phys.* **01** (2025) 157.
- [146] G. De Laurentis, H. Ita, V. Kuschke, M. Ruf, and V. Sotnikov, Two-loop leading-color QCD corrections for Higgs plus two-jet production in the heavy-top limit, [arXiv:2605.04009](https://arxiv.org/abs/2605.04009).
- [147] Z. Bern, L. J. Dixon, D. C. Dunbar, and D. A. Kosower, One loop n point gauge theory amplitudes, unitarity and collinear limits, *Nucl. Phys.* **B425**, 217 (1994).
- [148] R. K. Ellis, W. T. Giele, and Z. Kunszt, A numerical unitarity formalism for evaluating one-loop amplitudes, *J. High Energy Phys.* **03** (2008) 003.
- [149] W. T. Giele, Z. Kunszt, and K. Melnikov, Full one-loop amplitudes from tree amplitudes, *J. High Energy Phys.* **04** (2008) 049.
- [150] S. Abreu, J. Dormans, F. Febres Cordero, H. Ita, M. Kraus, B. Page, E. Pascual, M. S. Ruf, and V. Sotnikov, Caravel: A C++ framework for the computation of multi-loop amplitudes with numerical unitarity, *Comput. Phys. Commun.* **267**, 108069 (2021).
- [151] M. Czakon, T. Generet, F. Eschment, and R. Poncelet, Ancillary files for “next-to-next-to-next-to- leading order qcd corrections to photon-pair production” (2026), [10.5281/zenodo.19565380](https://zenodo.org/record/19565380).
- [152] T. Janßen, R. Poncelet, and S. Schumann, Sampling NNLO QCD phase space with normalizing flows, *J. High Energy Phys.* **09** (09) (2025) 194.
- [153] L. Buonocore, S. Kallweit, L. Rottoli, and M. Wiesemann, Linear power corrections for two-body kinematics in the qT subtraction formalism, *Phys. Lett. B* **829**, 137118 (2022).
- [154] J. Campbell, T. Neumann, and G. Vita, Projection-to-Born-improved subtractions at NNLO, *J. High Energy Phys.* **05** (2024) 172.
- [155] [www.dirac.ac.uk](http://www.dirac.ac.uk).