

Scalar Relics from the Hot Big Bang

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In this Letter, we motivate the fact that couplings between a scalar field and the standard model with strengths $10^{-6}(m_\phi/\text{eV})^{-1/4}$ relative to gravity yield the total measured cosmological dark matter abundance over a broad mass range of 10^{-12} to 10^{14} eV. Remarkably, this result holds with minimal sensitivity to whether the scalar couples to electrons, photons, hadrons, or other particles at laboratory energy scales, thereby linking fifth force experiments to the search for dark matter.

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The prototypical example of a fifth force is described by the addition of a massive singlet scalar to the standard model (SM), whose exchange among SM particles leads to the emergence of a Yukawa potential between them. Because a singlet scalar is not associated with any conserved charges, it can couple to any SM operator, generically mediating forces between all known particles. However, the energy scales involved in laboratory tests for fifth forces and those of the early Universe can differ by many orders of magnitude. In this companion Letter to Ref. [1], we argue that the SM generically generates a cosmological abundance of the scalar field, nearly independent of how the coupling manifests in the laboratory due to the running of gauge couplings, dimensional transmutation, and spontaneous symmetry breaking, implying a sharp target for fifth force experiments and dark matter haloscopes.

Consider a real scalar field ϕ with Lagrangian

$$\mathcal{L}_\phi = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m_\phi^2\phi^2 + \mathcal{L}_{\text{int}}, \quad (1)$$

where m_ϕ is the mass of ϕ and $\mathcal{L}_{\text{int}}$ contains interactions of ϕ with SM particles. The dimensionless scalar field $\varphi = \sqrt{4\pi G}\phi$, where G is Newton's constant, characterizes ϕ relative to gravitational potentials. Because ϕ has no internal symmetries, it can multiply any SM operator, so variation in its classical expectation value effectively generates space and time dependence of the SM parameters (i.e., the fundamental constants of the SM). The

field-dependence of a generic SM parameter ζ is then described by the scalar response function $d'_\zeta(\varphi) \equiv \zeta(\varphi)^{-1}[d\zeta(\varphi)/d\varphi]$ [2]. We consider only the leading interactions of ϕ , in which case $\zeta[\varphi] = \zeta[0](1 + d_\zeta^{(1)}\varphi)$, where $d_\zeta^{(1)} \approx 1$ corresponds to a gravitational strength interaction.

In the early Universe, the SM plasma generates an effective potential for the scalar $V_{\text{eff}}(\phi)$ whose minimum is displaced relative to ϕ 's vacuum expectation value [3–8]. In a homogeneous Universe with fixed parameters $\{\zeta\}$, the thermal path integral evaluated over the SM species at finite temperature T corresponds to the thermodynamic pressure of the SM sector. Thus, the effective potential for ϕ becomes the ϕ -dependent pressure of the system, where the fundamental constants acquire their local ϕ -dependent values [9,10]:

$$V_{\text{eff}}(\phi) = -P[\{\zeta(\varphi)\}]. \quad (2)$$

At high temperature, the leading contribution to the pressure is independent of the fundamental constants. Therefore, the force on ϕ is driven by the largest subleading terms, primarily controlled by the subset of dimensionless SM couplings $\{g_i\}$. These terms capture the interactions between various particles in the plasma, and we denote this pressure of interactions $P_{\text{int}} \sim \sum_i g_i^2(\varphi)T^4$ [11–13].

A central feature of the SM is the fact that the constants of nature flow and mix as a function of the energy scales of the interactions taking place. Hence the numerical values of fundamental constants observed in low-energy laboratory experiments, often taken to be $\{e, m_e, \Lambda_{\text{QCD}}, m_u, m_d\}$ [14–71] [the strength of electromagnetism, the mass of the electron, the confinement scale of quantum chromodynamics (QCD), and the masses of the up and down quarks, respectively], are different than those at higher energies reached in the early Universe. The associated

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couplings of the scalar to photons, electrons, gluons, and light quarks correspondingly obtain temperature-dependent values [72].

In particular, the dimensionless coupling g_i at one energy scale $\bar{\mu}$ is a function of all the other fundamental constants at some fixed reference scale $\bar{\mu}_0$:

$$g_i[\bar{\mu}, \varphi] = g_i(\bar{\mu}, \varphi; \{\zeta_j[\bar{\mu}_0, \varphi]\}), \quad (3)$$

where $\zeta_j[\bar{\mu}, \varphi]$ indicates the value of the SM parameter ζ_j at the energy scale $\bar{\mu}$ and the background scalar value φ . Differentiating the set of relations of Eq. (3) gives the scale dependence of the response functions, which we truncate at leading order in φ [2]:

$$d_{g_i}^{(1)}(\bar{\mu}) = \sum_{j=1} \frac{\partial \log g_i[\bar{\mu}, \varphi]}{\partial \log \zeta_j[\bar{\mu}_0, \varphi]} \Big|_{\varphi=0} d_{\zeta_j}^{(1)}(\bar{\mu}_0). \quad (4)$$

Setting $\bar{\mu} = T$ and $\bar{\mu}_0 = 0$ to match onto the values of the fundamental constants in the early and present Universe, respectively, this mixing implies that the leading order temperature dependence of the effective potential is

$$\frac{\partial V_{\text{eff}}(\varphi)}{\partial \varphi} = -T^4 \sum_i \frac{\partial P_{\text{int}}(\{g_j[T, \varphi]\})}{T^4 \partial \log \zeta_i(0, \varphi)} \Big|_{\varphi=0} d_{\zeta_i}^{(1)} + \mathcal{O}(T^2). \quad (5)$$

A substantial literature exists on precision computations of pressures in the SM [73–80]. As we are concerned with small variations of the fundamental parameters, we truncate these results at leading order in the SM parameters.

From these formulae, it is clear that the various T^4 contributions to $V_{\text{eff}}(\varphi)$ depend on all SM parameters as measured at some experimentally relevant reference scale and that the dependence of any mass scale or interaction strength on ϕ generates such a term. For simplicity, we assume that the Universe reheats instantaneously and that the SM bath achieves a maximum temperature T_{RH} [81]. The scalar field then acquires an irreducible nonthermal relic abundance from the changing effective potential:

$$\begin{aligned} \frac{\Omega_\varphi}{\Omega_{\text{DM}}} &\propto \left(\sum_i \frac{\partial P_{\text{int}}(\{g_j[T_{\text{RH}}, \varphi]\})}{T_{\text{RH}}^4 \partial \log \zeta_i(0, \varphi)} \Big|_{\varphi=0} d_{\zeta_i}^{(1)} \right)^2 \\ &\times \sqrt{\frac{m_\phi}{H_{\text{eq}}}} \times \begin{cases} \log^2 \left(\frac{T_{\text{RH}}^2}{M_{\text{pl}} m_\phi} \right), & m_\phi \lesssim \frac{T_{\text{RH}}^2}{M_{\text{pl}}}, \\ \left(\frac{T_{\text{RH}}^2}{M_{\text{pl}} m_\phi} \right)^{5/2}, & m_\phi \gtrsim \frac{T_{\text{RH}}^2}{M_{\text{pl}}}, \end{cases} \end{aligned} \quad (6)$$

where $\Omega_{\varphi, \text{DM}}$ are the fractions of the critical density in the scalar and dark matter, respectively. The contribution of other processes to the scalar relic abundance, including thermal scattering and inflationary fluctuations, is estimated in Ref. [1], and it is shown that they can be consistently neglected over much of the available parameter space. As the interaction pressure is determined only by

dimensionless couplings at leading order, the first line of Eq. (6) simplifies to the sum over products of $d_{\zeta_i}^{(1)}$ and the corresponding dimensionless couplings. In the case that the dimensionless couplings are $\mathcal{O}(1)$ and $m_\phi < H_{\text{RH}}$, Eq. (6) can be inverted:

$$d_{\zeta_i}^{(1)} \sim 10^{-6} (m_\phi / \text{eV})^{-1/4}. \quad (7)$$

The sense in which this sets a prior on parameter space can be clarified by considering the putative discovery of a laboratory fifth force with a coupling to matter exceeding Eq. (7). Such a large coupling, given the measured abundance of cosmological dark matter, implies that the relic abundance is suppressed by a cancellation between terms in the bracketed sum in Eq. (6). For T_{RH} greater than the electroweak scale, this can only happen if the scalar couples to the standard model exclusively through a superrenormalizable coupling to the Higgs, where φ modulates only the Higgs mass parameter above the electroweak scale [5,82,83]. Even in this special case, our conclusions are largely unchanged: one simply cuts off the thermal effective potential at the electroweak scale, where the cancellation occurs. Further, the ratios of the scalar's couplings at laboratory scales are fixed by renormalization group running from the electroweak scale: $d_{\Lambda_{\text{QCD}}}^{(1)} = -(6/27)d_{m_f}^{(1)}$ and $d_e^{(1)} = -(16\alpha_{\text{EM}}/3\pi)d_{m_f}^{(1)}$, where m_f are fermion masses.

More generally, when the pressure of interactions is suppressed by a precise cancellation, this typically happens only at a single temperature. Differential running of masses and couplings prevents such a cancellation from persisting across scales. As a result, couplings to heavy particles not accessible in laboratory tests will usually contribute additional pressure, increasing the relic abundance of ϕ , implying that the typical allowed couplings are smaller than Eq. (7).

We illustrate the coupling strengths which yield the correct dark matter relic abundance in Fig. 1, scanning over the allowable reheat temperatures and under the assumption that the scalar only modulates only one of the electron mass $[d_{m_e}^{(1)}]$, the electromagnetic coupling constant $[d_e^{(1)}]$, or the QCD scale $[d_{\Lambda_{\text{QCD}}}^{(1)}]$ ($\sim$ baryon masses) at laboratory scales—an assumption often [14–32,34–71] (though not always [33,84]) made when displaying the sensitivity of experimental searches [85]. The similarity of each panel in the figure shows that a scalar mediating a fifth force couples to the early Universe plasma in a way largely independent of its low-energy interactions. Scalars with mass smaller than the Hubble rate at reheating $T_{\text{RH}}^2/M_{\text{pl}}$ are insensitive to the reheat temperature, in contrast to scalars with larger mass; scalar couplings comparable to $d_{\zeta_i}^{(1)} \sim 10^{-6} (m_\phi / \text{eV})^{-1/4}$ yield a relic abundance that matches the dark matter density so long as $m_\phi \lesssim T_{\text{RH}}^2/M_{\text{pl}}$. A low reheat temperature can

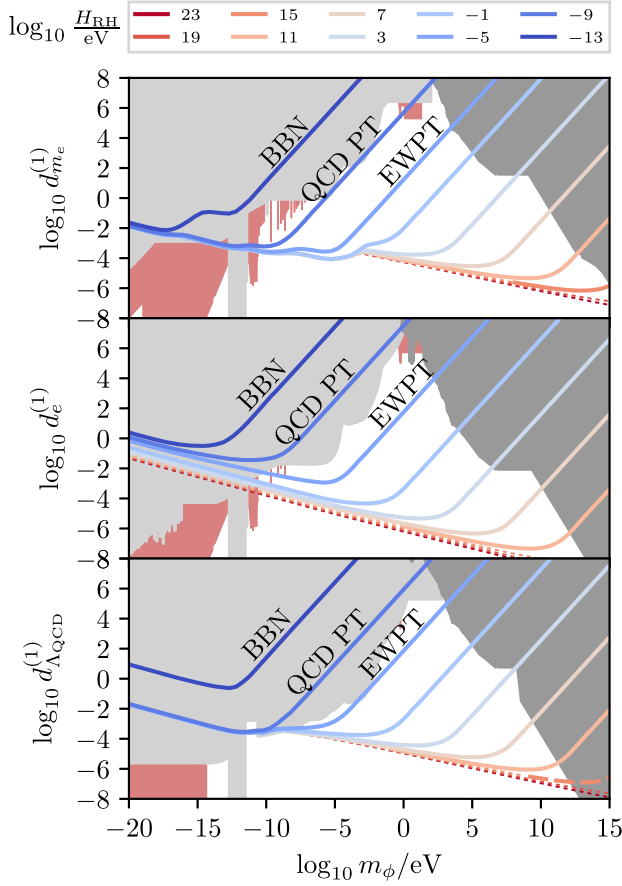


FIG. 1. Parameter space of scalar couplings that yields the correct relic abundance as a function of the Hubble rate at reheating $H_{\text{RH}} \approx T_{\text{RH}}^2/M_{\text{pl}}$. The labels big bang nucleosynthesis (BBN), QCD PT, and EWPT correspond to instantaneous reheating to temperatures just above the onset of BBN, the QCD phase transition, and the electroweak phase transition, respectively. Dashed lines indicate where the scalar cannot account for all of the dark matter due to isocurvature constraints, and dotted lines mark where misalignment from inflationary fluctuations is expected to dominate over thermal misalignment. For further subleading considerations, see Fig. 6 of Ref. [1]. Each panel corresponds to a scalar that couples *only* to one particular atomic constituent at laboratory energies: the electron mass, the electromagnetic coupling constant, and the QCD scale. The particular case of a coupling to the electron mass has a somewhat stronger dependence on the details of the scalar coupling (see the discussion in Ref. [1])—for this figure, we assume that the scalar couples to the electron mass through the Higgs potential, meaning that every other heavy fundamental fermion obtains the same coupling. Observational bounds consisting of equivalence principle tests [14–18] and inverse square law tests [19–30], atomic clocks and spectroscopy [31–40], resonant cavities and interferometry [41–45], resonant bar and mechanical and atomic oscillators [46,47], and astrophysical limits [48–50,86–88] are in light gray, and astrophysical and cosmological bounds on decaying scalar dark matter are in dark gray [51–58]. Prospective sensitivity curves for experiments are plotted in red for (top) [59–64], (middle) [59–62,64–68], and (bottom) [61,66].

yield deviations from the from the universal scaling [Eq. (7)]. The diversity of scalar behavior at a low reheating temperature can be understood by comparing the low H_{RH} (blue) curves among the three panels of Fig. 1. Planned experiments will probe these couplings [59,60], particularly around scalar masses of 10^{-12} eV where current astrophysical constraints are weak and sensitivity to the reheating temperature is minimal.

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- [1] D. Cyncynates and O. Simon, companion paper, Minimal targets for dilaton direct detection, *Phys. Rev. D* **112**, 055002 (2025).
- [2] T. Damour and J.F. Donoghue, Equivalence principle violations and couplings of a light dilaton, *Phys. Rev. D* **82**, 084033 (2010).
- [3] T. Damour and A.M. Polyakov, The string dilaton and a least coupling principle, *Nucl. Phys. B* **423**, 532 (1994).
- [4] B. Batell and A. Ghalsasi, Thermal misalignment of scalar dark matter, *Phys. Rev. D* **107**, L091701 (2023).
- [5] B. Batell, A. Ghalsasi, and M. Rai, Dynamics of dark matter misalignment through the Higgs portal, *J. High Energy Phys.* **01** (2024) 038.
- [6] X. Gan and D. Liu, Cosmologically varying kinetic mixing, *J. High Energy Phys.* **11** (2023) 031.
- [7] A. Alachkar, M. Fairbairn, and D.J.E. Marsh, Dilatonic couplings and the relic abundance of ultralight dark matter, *Phys. Rev. Lett.* **134**, 191003 (2025).
- [8] A conservative naturalness criterion would be that the UV cutoff of SM loops is above the reheating temperature and small enough that the mass is not strongly renormalized: $m_\phi^2 \gtrsim (d_{g_i}^{(1)} g_i)^2 \Lambda_{\text{UV}}^4 / M_{\text{pl}}^2 \gtrsim (d_{g_i}^{(1)} g_i)^2 H_{\text{RH}}^2$, where g_i is as defined later in the text.
- [9] L. Dolan and R. Jackiw, Symmetry behavior at finite temperature, *Phys. Rev. D* **9**, 3320 (1974).

- [10] S. Weinberg, Gauge and global symmetries at high temperature, *Phys. Rev. D* **9**, 3357 (1974).
- [11] W. Buchmuller, K. Hamaguchi, and M. Ratz, Gauge couplings at high temperature and the relic gravitino abundance, *Phys. Lett. B* **574**, 156 (2003).
- [12] W. Buchmuller, K. Hamaguchi, O. Lebedev, and M. Ratz, Dilaton destabilization at high temperature, *Nucl. Phys. B* **699**, 292 (2004).
- [13] B. Lillard, M. Ratz, T. Tait, M. P., and S. Trojanowski, The flavor of cosmology, *J. Cosmol. Astropart. Phys.* **07** (2018) 056.
- [14] A. Hees, O. Minazzoli, E. Savalle, Y. V. Stadnik, and P. Wolf, Violation of the equivalence principle from light scalar dark matter, *Phys. Rev. D* **98**, 064051 (2018).
- [15] E. Fischbach and C. Talmadge, Ten years of the fifth force, in *31st Rencontres de Moriond: Dark Matter and Cosmology, Quantum Measurements and Experimental Gravitation* (1996), pp. 443–451, [arXiv:hep-ph/9606249](https://arxiv.org/abs/hep-ph/9606249).
- [16] P. Touboul *et al.* (MICROSCOPE Collaboration), MICROSCOPE mission: Final results of the test of the equivalence principle, *Phys. Rev. Lett.* **129**, 121102 (2022).
- [17] G. L. Smith, C. D. Hoyle, J. H. Gundlach, E. G. Adelberger, B. R. Heckel, and H. E. Swanson, Short range tests of the equivalence principle, *Phys. Rev. D* **61**, 022001 (1999).
- [18] S. Schlamminger, K. Y. Choi, T. A. Wagner, J. H. Gundlach, and E. G. Adelberger, Test of the equivalence principle using a rotating torsion balance, *Phys. Rev. Lett.* **100**, 041101 (2008).
- [19] E. G. Adelberger, B. R. Heckel, and A. E. Nelson, Tests of the gravitational inverse square law, *Annu. Rev. Nucl. Part. Sci.* **53**, 77 (2003).
- [20] A. S. Konopliv, S. W. Asmar, W. M. Folkner, Ö. Karatekin, D. C. Nunes, S. E. Smrekar, C. F. Yoder, and M. T. Zuber, Mars high resolution gravity fields from MRO, Mars seasonal gravity, and other dynamical parameters, *Icarus* **211**, 401 (2011).
- [21] V. M. Mostepanenko and G. L. Klimchitskaya, The state of the art in constraining axion-to-nucleon coupling and non-Newtonian gravity from laboratory experiments, *Universe* **6**, 147 (2020).
- [22] D. J. Kapner, T. S. Cook, E. G. Adelberger, J. H. Gundlach, B. R. Heckel, C. D. Hoyle, and H. E. Swanson, Tests of the gravitational inverse-square law below the dark-energy length scale, *Phys. Rev. Lett.* **98**, 021101 (2007).
- [23] J. G. Lee, E. G. Adelberger, T. S. Cook, S. M. Fleischer, and B. R. Heckel, New test of the gravitational $1/r^2$ law at separations down to 52 μm , *Phys. Rev. Lett.* **124**, 101101 (2020).
- [24] S.-Q. Yang, B.-F. Zhan, Q.-L. Wang, C.-G. Shao, L.-C. Tu, W.-H. Tan, and J. Luo, Test of the gravitational inverse square law at millimeter ranges, *Phys. Rev. Lett.* **108**, 081101 (2012).
- [25] W.-H. Tan *et al.*, Improvement for testing the gravitational inverse-square law at the submillimeter range, *Phys. Rev. Lett.* **124**, 051301 (2020).
- [26] Y. J. Chen, W. K. Tham, D. E. Krause, D. Lopez, E. Fischbach, and R. S. Decca, Stronger limits on hypothetical Yukawa interactions in the 30–8000 nm range, *Phys. Rev. Lett.* **116**, 221102 (2016).
- [27] J. Ke, J. Luo, C.-G. Shao, Y.-J. Tan, W.-H. Tan, and S.-Q. Yang, Combined test of the gravitational inverse-square law at the centimeter range, *Phys. Rev. Lett.* **126**, 211101 (2021).
- [28] J. K. Hoskins, R. D. Newman, R. Spero, and J. Schultz, Experimental tests of the gravitational inverse square law for mass separations from 2-cm to 105-cm, *Phys. Rev. D* **32**, 3084 (1985).
- [29] A. A. Geraci, S. J. Smullin, D. M. Weld, J. Chiaverini, and A. Kapitulnik, Improved constraints on non-Newtonian forces at 10 microns, *Phys. Rev. D* **78**, 022002 (2008).
- [30] A. O. Sushkov, W. J. Kim, D. A. R. Dalvit, and S. K. Lamoreaux, New experimental limits on non-Newtonian forces in the micrometer range, *Phys. Rev. Lett.* **107**, 171101 (2011).
- [31] M. Filzinger, S. Dörscher, R. Lange, J. Klose, M. Steinle, E. Benkler, E. Peik, C. Lisdat, and N. Huntemann, Improved limits on the coupling of ultralight bosonic dark matter to photons from optical atomic clock comparisons, *Phys. Rev. Lett.* **130**, 253001 (2023).
- [32] O. Tretiak, X. Zhang, N. L. Figueroa, D. Antypas, A. Brogna, A. Banerjee, G. Perez, and D. Budker, Improved bounds on ultralight scalar dark matter in the radio-frequency range, *Phys. Rev. Lett.* **129**, 031301 (2022).
- [33] R. Oswald *et al.*, Search for dark-matter-induced oscillations of fundamental constants using molecular spectroscopy, *Phys. Rev. Lett.* **129**, 031302 (2022).
- [34] X. Zhang, A. Banerjee, M. Leyser, G. Perez, S. Schiller, D. Budker, and D. Antypas, Search for ultralight dark matter with spectroscopy of radio-frequency atomic transitions, *Phys. Rev. Lett.* **130**, 251002 (2023).
- [35] T. Kobayashi, A. Takamizawa, D. Akamatsu, A. Kawasaki, A. Nishiyama, K. Hosaka, Y. Hisai, M. Wada, H. Inaba, T. Tanabe, and M. Yasuda, Search for ultralight dark matter from long-term frequency comparisons of optical and microwave atomic clocks, *Phys. Rev. Lett.* **129**, 241301 (2022).
- [36] K. Beloy *et al.* (BACON Collaboration), Frequency ratio measurements at 18-digit accuracy using an optical clock network, *Nature (London)* **591**, 564 (2021).
- [37] K. Van Tilburg, N. Leefer, L. Bougas, and D. Budker, Search for ultralight scalar dark matter with atomic spectroscopy, *Phys. Rev. Lett.* **115**, 011802 (2015).
- [38] S. Aharony, N. Akerman, R. Ozeri, G. Perez, I. Savoray, and R. Shaniv, Constraining rapidly oscillating scalar dark matter using dynamic decoupling, *Phys. Rev. D* **103**, 075017 (2021).
- [39] A. Hees, J. Guéna, M. Abgrall, S. Bize, and P. Wolf, Searching for an oscillating massive scalar field as a dark matter candidate using atomic hyperfine frequency comparisons, *Phys. Rev. Lett.* **117**, 061301 (2016).
- [40] N. Sherrill *et al.*, Analysis of atomic-clock data to constrain variations of fundamental constants, *New J. Phys.* **25**, 093012 (2023).
- [41] E. Savalle, A. Hees, F. Frank, E. Cantin, P.-E. Pottie, B. M. Roberts, L. Cros, B. T. Mcallister, and P. Wolf, Searching for dark matter with an optical cavity and an unequal-delay interferometer, *Phys. Rev. Lett.* **126**, 051301 (2021).

- [42] S. M. Vermeulen *et al.*, Direct limits for scalar field dark matter from a gravitational-wave detector, *Nature (London)* **600**, 424 (2021).
- [43] L. Aiello, J. W. Richardson, S. M. Vermeulen, H. Grote, C. Hogan, O. Kwon, and C. Stoughton, Constraints on scalar field dark matter from colocated Michelson interferometers, *Phys. Rev. Lett.* **128**, 121101 (2022).
- [44] C. J. Kennedy, E. Oelker, J. M. Robinson, T. Bothwell, D. Kedar, W. R. Milner, G. E. Marti, A. Derevianko, and J. Ye, Precision metrology meets cosmology: Improved constraints on ultralight dark matter from atom-cavity frequency comparisons, *Phys. Rev. Lett.* **125**, 201302 (2020).
- [45] A. S. Göttel, A. Ejlli, K. Karan, S. M. Vermeulen, L. Aiello, V. Raymond, and H. Grote, Searching for scalar field dark matter with LIGO, *Phys. Rev. Lett.* **133**, 101001 (2024).
- [46] A. Branca *et al.*, Search for an ultralight scalar dark matter candidate with the AURIGA detector, *Phys. Rev. Lett.* **118**, 021302 (2017).
- [47] W. M. Campbell, B. T. McAllister, M. Goryachev, E. N. Ivanov, and M. E. Tobar, Searching for scalar dark matter via coupling to fundamental constants with photonic, atomic and mechanical oscillators, *Phys. Rev. Lett.* **126**, 071301 (2021).
- [48] E. Hardy and R. Lasenby, Stellar cooling bounds on new light particles: Plasma mixing effects, *J. High Energy Phys.* **02** (2017) 033.
- [49] S. Bottaro, A. Caputo, G. Raffelt, and E. Vitagliano, Stellar limits on scalars from electron-nucleus bremsstrahlung, *J. Cosmol. Astropart. Phys.* **07** (2023) 071.
- [50] A. Afzal *et al.* (NANOGrav Collaboration), The NANOGrav 15 yr data set: Search for signals from new physics, *Astrophys. J. Lett.* **951**, L11 (2023); **971**, L27(E) (2024).
- [51] R. Janish and E. Pinetti, Hunting dark matter lines in the infrared background with the James Webb Space Telescope, *Phys. Rev. Lett.* **134**, 071002 (2025).
- [52] D. Cadamuro and J. Redondo, Cosmological bounds on pseudo Nambu-Goldstone bosons, *J. Cosmol. Astropart. Phys.* **02** (2012) 032.
- [53] D. Wadekar and Z. Wang, Strong constraints on decay and annihilation of dark matter from heating of gas-rich dwarf galaxies, *Phys. Rev. D* **106**, 075007 (2022).
- [54] W. Yin *et al.*, First result for dark matter search by WINERED, *Phys. Rev. Lett.* **134**, 051004 (2025).
- [55] E. Todarello, M. Regis, J. Reynoso-Cordova, M. Taoso, D. Vaz, J. Brinchmann, M. Steinmetz, and S. L. Zoutendijke, Robust bounds on ALP dark matter from dwarf spheroidal galaxies in the optical MUSE-Faint survey, *J. Cosmol. Astropart. Phys.* **05** (2024) 043.
- [56] D. Grin, G. Covone, J.-P. Kneib, M. Kamionkowski, A. Blain, and E. Jullo, A telescope search for decaying relic axions, *Phys. Rev. D* **75**, 105018 (2007).
- [57] P. Carenza, G. Lucente, and E. Vitagliano, Probing the blue axion with cosmic optical background anisotropies, *Phys. Rev. D* **107**, 083032 (2023).
- [58] S. Porras-Bedmar, M. Meyer, and D. Horns, Novel bounds on decaying axion-like-particle dark matter from the cosmic background, *Phys. Rev. D* **110**, 103501 (2024).
- [59] L. Badurina, O. Buchmueller, J. Ellis, M. Lewicki, C. McCabe, and V. Vaskonen, Prospective sensitivities of atom interferometers to gravitational waves and ultralight dark matter, *Phil. Trans. R. Soc. A* **380**, 20210060 (2021).
- [60] A. Arvanitaki, S. Dimopoulos, and K. Van Tilburg, Sound of dark matter: Searching for light scalars with resonant-mass detectors, *Phys. Rev. Lett.* **116**, 031102 (2016).
- [61] A. Arvanitaki, S. Dimopoulos, and K. Van Tilburg, Resonant absorption of bosonic dark matter in molecules, *Phys. Rev. X* **8**, 041001 (2018).
- [62] M. Abe *et al.*, Matter-wave atomic gradiometer interferometric sensor (MAGIS-100), *Quantum Sci. Technol.* **6**, 044003 (2021).
- [63] D. Antypas *et al.*, New horizons: Scalar and vector ultralight dark matter, *arXiv:2203.14915*.
- [64] J. Manley, D. J. Wilson, R. Stump, D. Grin, and S. Singh, Searching for scalar dark matter with compact mechanical resonators, *Phys. Rev. Lett.* **124**, 151301 (2020).
- [65] M. Hirschel, V. Vadakkumbatt, N. P. Baker, F. M. Schweizer, J. C. Sankey, S. Singh, and J. P. Davis, Superfluid helium ultralight dark matter detector, *Phys. Rev. D* **109**, 095011 (2024).
- [66] A. Arvanitaki, J. Huang, and K. Van Tilburg, Searching for dilaton dark matter with atomic clocks, *Phys. Rev. D* **91**, 015015 (2015).
- [67] A. A. Geraci, C. Bradley, D. Gao, J. Weinstein, and A. Derevianko, Searching for ultralight dark matter with optical cavities, *Phys. Rev. Lett.* **123**, 031304 (2019).
- [68] I. Kozyryev, Z. Lasner, and J. M. Doyle, Enhanced sensitivity to ultralight bosonic dark matter in the spectra of the linear radical SrOH, *Phys. Rev. A* **103**, 043313 (2021).
- [69] C. A. J. O'Hare and E. Vitagliano, Cornering the axion with CP -violating interactions, *Phys. Rev. D* **102**, 115026 (2020).
- [70] S. Adachi *et al.* (POLARBEAR Collaboration), Constraints on axionlike polarization oscillations in the cosmic microwave background with POLARBEAR, *Phys. Rev. D* **108**, 043017 (2023).
- [71] G. Raffelt, Limits on a CP -violating scalar axion-nucleon interaction, *Phys. Rev. D* **86**, 015001 (2012).
- [72] In contrast to Ref. [2] and much of the literature after it, which uses the notation $d_e^{(1)}$ and $d_g^{(1)}$, these coupling functions are written in our notation as $d_{\text{aEM}}^{(1)}$ and $d_{\Lambda\text{QCD}}^{(1)}$, respectively.
- [73] P. Arnold and C. Zhai, The three loop free energy for high temperature QED and QCD with fermions, *Phys. Rev. D* **51**, 1906 (1995).
- [74] J. I. Kapusta and C. Gale, *Finite-temperature field theory: Principles and applications* (Cambridge University Press, Cambridge, England, 2007).
- [75] K. Kajantie, M. Laine, K. Rummukainen, and Y. Schroder, The pressure of hot QCD up to $g_6 \ln(1/g)$, *Phys. Rev. D* **67**, 105008 (2003).
- [76] A. Gynther, Electroweak phase diagram at finite lepton number density, *Phys. Rev. D* **68**, 016001 (2003).
- [77] A. Gynther and M. Vepsäläinen, Pressure of the standard model near the electroweak phase transition, *J. High Energy Phys.* **03** (2006) 011.

- [78] A. Gynther and M. Vepsäläinen, Pressure of the standard model at high temperatures, *J. High Energy Phys.* **01** (2006) 060.
- [79] A. Gynther, Thermodynamics of electroweak matter, Other thesis, 2006, [arXiv:hep-ph/0609226](https://arxiv.org/abs/hep-ph/0609226).
- [80] M. Laine and M. Meyer, Standard model thermodynamics across the electroweak crossover, *J. Cosmol. Astropart. Phys.* **07** (2015) 035.
- [81] Our results are insensitive to the reheating history provided $m_\phi < H_{\text{RH}}$; earlier dynamics are effectively instantaneous from the scalar's perspective. The study of heavier scalars with different reheat histories is left to future work.
- [82] Y. Shtanov, Light scalaron as dark matter, *Phys. Lett. B* **820**, 136469 (2021).
- [83] Y. Shtanov, Initial conditions for the scalaron dark matter, *J. Cosmol. Astropart. Phys.* **10** (2022) 079.
- [84] A. Banerjee, G. Perez, M. Safronova, I. Savoray, and A. Shalit, The phenomenology of quadratically coupled ultra light dark matter, *J. High Energy Phys.* **10** (2023) 042.
- [85] We treat the light quarks as massless to avoid the technical difficulties associated with their running across the QCD phase transition, which would detract from our main analysis while having minimal quantitative impact.
- [86] M. Baryakhtar, M. Galanis, R. Lasenby, and O. Simon, Black hole superradiance of self-interacting scalar fields, *Phys. Rev. D* **103**, 095019 (2021).
- [87] S. Hoof, D. J. E. Marsh, J. Sisk-Reynés, J. H. Matthews, and C. Reynolds, Getting more out of black hole superradiance: A statistically rigorous approach to ultralight boson constraints, [arXiv:2406.10337](https://arxiv.org/abs/2406.10337).
- [88] D. F. G. Fiorillo, A. Lella, C. A. J. O'Hare, and E. Vitagliano, Leading bounds on micro- to picometer fifth forces from neutron star cooling, [arXiv:2506.19906](https://arxiv.org/abs/2506.19906).
- [89] C. O'Hare, cajohare/axionlimits: Axionlimits, <https://cajohare.github.io/AxionLimits/> (2020).