

Ma and Cui Reply: Recently, Ancilotto argued that the shell-shaped BEC proposed in our Letter [1] was not the true ground state of the three-component mixture described in that work and instead there is a lower energy “dimer” configuration [2]. Ancilotto also explored state formation, finding that a naive trap-release protocol was unable to produce the core-shell structure. In this reply we confirm that our core-shell structure is an excited state for finite-size systems. Nonetheless, the core-shell structure is found to be locally stable under external perturbations, and if one pays careful attention to mode matching, a trap-release protocol can produce this metastable structure.

The equilibrium core-shell and dimer states can be obtained by choosing different initial states in the energy minimization process [3]. We find that the core-shell state indeed has a higher energy than the dimer one for finite-size systems, with energy cost mainly from the surface of the shell. As the shell becomes larger, however, the relative energy difference decreases due to less contribution from the surface compared to the bulk. In the thermodynamic limit with vanishing surface contribution, the two states become degenerate in energy per particle, which is solely determined by equilibrium densities of correlated (1, 2)–(2, 3) droplets as derived in [1].

The stability of the core-shell state can be inferred from its all-real excitation spectra [4], shown in Fig. 4 of [1]. To further confirm its stability, we apply a small (symmetry-breaking) field gradient $h_{sb}(\{\mathbf{r}_i\}) = B'(z_3 - z_1)$ [5], which particularly favors dimer configuration as the ground state. We consider the thin shell case in Fig. 2(a3) of [1], which was criticized in [2] as extremely unstable. However, Fig. 1 shows that the shell structure is only slightly deformed under a small B' , rather than ending up with dimer state. This directly manifests the intrinsic stability of the core-shell structure—under perturbation it will adjust its own densities such that the surface tension reaches a balance with external force and helps to resist further structural distortion. The common component-2 also plays a positive role here; its core-shell structure efficiently glues 1 and 3 as a similar shape. Still, it is possible that the thermal or high-order quantum fluctuations (beyond those captured by our extended GP ansatz) could destabilize the core-shell structure. Nonetheless our current calculations strongly suggest that it is observable.

In [1], we proposed to first prepare the core-shell structure in an isotropic harmonic trap (not favoring dimer state) and then release it from the trap. The system will quickly relax to the ground state core-shell under the optimized mode matching [6]. We have confirmed this in Fig. 2: after release from the trap, the system quickly

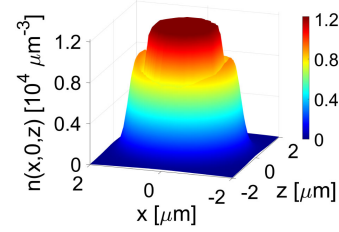


FIG. 1. Deformation of core-shell structure under a small field gradient $B' = 0.08E_0/l_0$ [$l_0 = 1 \mu\text{m}$ and $E_0 = \hbar^2/(m_K l_0^2)$]. Atom numbers are the same as Fig. 2(a3) of [1]. Here we just plot out n_1 (shell) and n_3 (core) at $y = 0$; n_2 (not shown here) exhibits a similar core-shell structure.

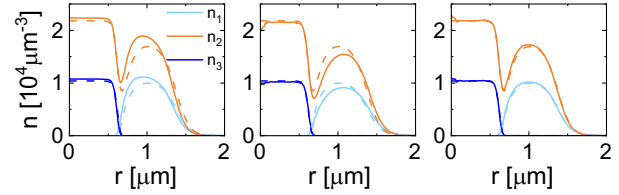


FIG. 2. Radial densities after the system is released from an isotropic harmonic trap: $t(\text{ms}) = 0$ (a), 0.15 (b), and 0.3 (c). Initial trap frequencies are $\omega_K = 5 \text{ KHz}$, $\omega_{\text{Na}} = \omega_K \sqrt{m_K/m_{\text{Na}}}$, and atom numbers are $(N_1, N_2, N_3)/10^5 = (1, 1.86, 0.1)$. Dashed lines show static densities in free space. Interaction parameters are the same as Fig. 2 of [1].

relaxes and ends up with breathing oscillations around free-space profiles. This dynamical core-shell structure is also found to be robust under small perturbations. Here we emphasize the importance of mode matching in this process, which was violated in [2]—the initial and final states therein have opposite core and shell components [7]. As a result, during the releasing dynamics (1, 2) and (2, 3) tend to flow oppositely, and the large releasing energy can also result in splash of the whole system [8]. Together with certain symmetry-breaking noise, splitting small droplets with strange geometry may be the result [2].

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Received 2 April 2025; accepted 17 October 2025;
published 29 October 2025

DOI: [10.1103/wcfn-lyb8](https://doi.org/10.1103/wcfn-lyb8)

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[1] Y. Ma and X. Cui, Shell-shaped quantum droplet in a three-component ultracold Bose gas, *Phys. Rev. Lett.* **134**, 043402 (2025).

- [2] F. Ancilotto, Comment on “Shell-shaped quantum droplet in a three-component ultracold Bose gas,” *Phys. Rev. Lett.* **135**, 159301 (2025).
- [3] The core-shell state is obtained from a rotationally invariant initial state. During the imaginary time evolution, it always preserves $SO(3)$ rotational symmetry due to the same symmetry of Hamiltonian. In comparison, the dimer state is from a symmetry-broken initial state where (1, 2) and (2, 3) droplets are arranged side by side.
- [4] Should the system be unstable against any density fluctuation, its associated mode will develop an imaginary part such that the fluctuation grows exponentially with time to destabilize the system. For all of the atom numbers considered in our Letter, we do not observe such instability.
- [5] Note that a field gradient on component-2 can be gauged away in the moving frame of this component, so only the field gradients on two of the components (here 1 and 3) are independent.
- [6] Z. Guo, F. Jia, L. Li, Y. Ma, J. M. Hutson, X. Cui, and D. Wang, Lee-Huang-Yang effects in the ultracold mixture of ^{23}Na and ^{87}Rb with attractive interspecies interactions, *Phys. Rev. Res.* **3**, 033247 (2021).
- [7] Mode matching requires that the densities of initial (in a trap) and final (without a trap) states match each other as much as possible. If this requirement is violated, there will be a huge amount of internal energy to release during the dynamics, leading to complex dynamical outcomes. In [2], the initial and final states are substantially different, as seen from the switched core and shell components therein. This may be due to the combined effect of tight trapping potential and large $N_{1,2}$ taken in [2], such that (1, 2) prefers to stay in the trap center (core) to avoid large potential energy and accordingly (2,3) becomes the shell.
- [8] Y. Ma and X. Cui, Quantum-fluctuation-driven dynamics of droplet splashing, recoiling, and deposition in ultracold binary Bose gases, *Phys. Rev. Res.* **5**, 013100 (2023).