

Defect Conformal Manifolds from Phantom Noninvertible Symmetries

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(Received 24 June 2025; accepted 23 October 2025; published 17 November 2025)

We explore a general mechanism that allows $(1+1)$ d CFTs to have interesting interface conformal manifolds even in the absence of any continuous internal symmetry or supersymmetry. This is made possible by the breaking of an enhanced continuous symmetry—which is generically noninvertible—arising in the folded theory. We provide several examples and showcase the power of the symmetry-based approach by computing the evolution of the reflection coefficient along the defect conformal manifold. We also discuss higher-dimensional generalizations and we comment on no-go theorems.

DOI: [10.1103/w43n-sybv](https://doi.org/10.1103/w43n-sybv)

Introduction—Conformal manifolds are a rare and intriguing phenomenon. In CFTs with spacetime dimensions greater than 2 their occurrence is typically restricted to free or supersymmetric theories. Marginal deformations usually acquire anomalous dimensions and become irrelevant or relevant. Exact marginality—and thus the existence of a conformal manifold—requires special mechanisms, often relying on supersymmetry.

The situation is markedly different for defects and boundaries in CFTs [1–3]. A continuous global symmetry G might be broken on a p -dimensional defect $\mathcal{D}$. The breaking shows up in the Ward identity as

$$\partial_\mu J^\mu = t(x)\delta_p(\Sigma_{\mathcal{D}}) \quad t(x) = t_a(x)T^a \quad T^a \in \mathfrak{g}. \quad (1)$$

The tilt operator on the r.h.s. of (1) has protected conformal dimension $\Delta_t = p$. Its exponential generates a defect conformal manifold [4]. Such manifolds are thus common in theories with continuous symmetries, their existence being protected by the symmetry-breaking pattern at the defect.

These spaces are homogeneous [3], and several quantities—such as the g -function [9] and reflection coefficient $\mathcal{R}$ [10]—remain constant along them.

In this Letter, we examine a distinct scenario. Even when the bulk lacks continuous symmetries, codimension 1 conformal interfaces can still host conformal manifolds. This seems paradoxical, as no mechanism appears to

protect the marginality of defect operators. We argue that these arise from the breaking of emergent symmetries after folding, which we term *phantom* symmetries.

The mechanism is as follows: Consider an interface $\mathcal{I}$ between CFT_L and CFT_R , with respective symmetry $\mathcal{C}_L$ and $\mathcal{C}_R$. The interface is conformal if it is invariant under the reduced conformal group $SO(d-1, 2)$. Using the residual conformal symmetry, we map the interface to a conformal boundary condition for the folded theory $\text{CFT}_L \times \overline{\text{CFT}}_R$. This is known as the folding trick.

In several cases the symmetry after folding enhances to $\mathcal{C}_{L \times \bar{R}} \supseteq \mathcal{C}_L \times \overline{\mathcal{C}}_R$ [11], which may include *continuous* symmetries, generically noninvertible. The spontaneous breaking of continuous noninvertible symmetries, recently analyzed in [13], leads to moduli spaces with orbifold geometry. Likewise, their breaking by boundary conditions yields orbifold boundary conformal manifolds. While this continuous symmetry is lost upon unfolding, such boundary conformal manifold becomes a nontrivial interface conformal manifold.

This symmetry-based approach yields properties of interfaces related by broken phantom symmetries, including their g functions and reflection coefficients. Unlike ordinary symmetry-breaking cases, where both are invariant under the symmetry action, here the g function remains constant but the reflection coefficient $\mathcal{R}$ varies.

We also expect phantom symmetries to constrain other key features of defects, such as the defect central charge c_{eff} [14–18], and fusion rules [19–21], potentially following [22]. We leave these directions to future work.

It is tempting to conjecture that breaking a (possibly phantom) continuous symmetry is the only way to generate nonsupersymmetric conformal manifolds of codimension 1 defects. [23]

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TABLE I. Conformal defects in the Ising CFT.

Ising	$(1, \theta)$ $\theta \in [0, \pi/2)$	(ϵ, θ) $\theta \in [\pi/2, \pi)$	(σ, θ') $\theta' \in [0, \pi/2]$
$R = \sqrt{2}$ orbifold		$ D^+, \theta\rangle$	$ N^+, \theta'\rangle$
Free fermion	$\chi^* = e^{-2i\theta}\bar{\chi}$		$\chi = e^{-2i\theta'}\bar{\chi}$
g function	1		$\sqrt{2}$
$\mathcal{R}$	$\cos^2(2\theta)$		$\cos^2(2\theta')$
Phantom sym.	$L_\theta^{(m)}$		$L_{\theta'}^{(w)}$

Our results highlight how symmetry and its breaking offer a framework to explore the rich, largely uncharted landscape of defect CFTs. This forms part of a broader program to leverage (generalized) symmetries to constrain dynamical defects and solitonic excitations [24–35].

Phantom symmetry in the (1+1)d Ising CFT—Conformal defects in the Ising CFT have been classified [36,37] and organize into continuous families, labeled by a primary field and a point on an interval (see Table I).

The topological Verlinde lines sit at [38,39]: $1 = (1, \pi/4)$, $\eta = (\epsilon, 3\pi/4)$, and $\mathcal{N} = (\sigma, \pi/4)$. The first two families are continuously connected, as $(\epsilon, \pi/2) = (1, \pi/2)$.

Nontopological defect lines arise by deforming Verlinde lines by the exactly marginal energy operator ϵ , thus generating a defect conformal manifold.

The existence of these families is surprising, as the Ising CFT possesses only discrete symmetries. Furthermore, the families in Table I are parametrized by an orbifold $S^1/\mathbb{Z}_2$, which is incompatible with the breaking of a continuous symmetry G . Both phenomena admit a natural explanation upon folding.

The folded theory is described by the $\mathbb{Z}_2$ orbifold of the $c = 1$ compact boson $\phi \sim \phi + 2\pi$ at radius $R = \sqrt{2}$ [40]. As observed in [41], this theory admits two continuous families of noninvertible topological defects $L_\theta^{(m)} = 2 \cos(-2\theta \int (\star d\phi/2\pi))$, $L_{\theta'}^{(w)} = 2 \cos(2i\theta' \int (d\phi/2\pi))$, with fusion rules [42]

$$L_{\theta_1}^{(m/w)} \times L_{\theta_2}^{(m/w)} = L_{\theta_1+\theta_2}^{(m/w)} + L_{\theta_1-\theta_2}^{(m/w)}. \quad (2)$$

This “cosine” symmetry has been extensively studied [13,41,43–45]. Dirichlet and Neumann boundary conditions of the free boson, upon orbifolding, break $L_\theta^{(m)}$ and $L_{\theta'}^{(w)}$, respectively, reproducing the Ising conformal defects. Notably, $L_\theta^{(m)}$ and $L_{\theta'}^{(w)}$ exhibit a mixed ’t Hooft anomaly, preventing the existence of a boundary state invariant under both [46,47].

An alternative perspective will facilitate the generalizations: Consider the spectrum of the Ising CFT in the $\mathbb{Z}_2$ -twisted Hilbert space $\mathcal{H}_\eta$. It contains three primary

fields, $\varphi_{1/2,0}$, $\bar{\varphi}_{0,1/2}$, $\mu_{1/16,1/16}$. The exactly marginal deformation ϵ appears in the OPE $\varphi \times \bar{\varphi} \sim \epsilon$. In the folded theory, the fermions give rise to two currents: $j = \varphi_L \varphi_R$, $\bar{j} = \bar{\varphi}_L \bar{\varphi}_R$ in the twisted sector of the diagonal $\mathbb{Z}_2$ symmetry $\eta_L \eta_R$ [48]. Vector and axial currents are $j_V = (j + \bar{j})/2$, $j_A = (j - \bar{j})/2$.

Although conserved, these currents are nonlocal: To build the symmetry generators $L_\theta^{(m)}$ and $L_{\theta'}^{(w)}$ we deform the defect $1 + \eta_L \eta_R$ as $L_\theta^{(m)} = (1 + \eta_L \eta_R) \exp(i\theta \int \star j_V)$, $L_{\theta'}^{(w)} = (1 + \eta_L \eta_R) \exp(i\theta' \int \star j_A)$. This defines a deformation of a symmetry-reflecting topological defect, recently formalized in [30] (see also Ref. [49]). Current conservation and the protected dimension $\Delta_j = \Delta_{\bar{j}} = 1$ assure topological invariance of the deformation.

These additional symmetries are also manifest in the fermionic description [10,36,37,50–52] in terms of a single Dirac fermion [53]: its $U(1)_V \times U(1)_A$ symmetries are absent in the Majorana CFT.

This picture elucidates the link between the tilt operator and the energy deformation: the tilt $j_V^\perp = \frac{1}{2} \bar{\psi} \gamma^1 \psi = \varphi_L \bar{\varphi}_R + \bar{\varphi}_L \varphi_R$ on the D_θ boundary, reduces to $\csc(2\theta)\epsilon$; the same happens for $j_A^\perp = \csc(2\theta')\epsilon$ on the $N_{\theta'}$ boundary. Several properties of these conformal defects [10,37] are also summarized in Table I. Notice that while their g function is constant along the conformal manifold—equaling the quantum dimension of the underlying topological line—the reflection coefficient changes. This is in contrast to the fact [10] that $\mathcal{R}$ is invariant under fusion with topological defects. This is intimately related to the breaking of a phantom symmetry.

Phantom symmetry and defect conformal manifolds—We now present the general construction of phantom symmetries. We will focus on symmetries stemming from nonlocal currents j_A in the twisted Hilbert spaces $\mathcal{H}_{\eta_A}$, where η_A are invertible, abelian topological lines in the folded theory. Our construction relies on three ingredients: (1) a set of anomaly-free symmetry generators η_A in $\text{CFT}_L \times \text{CFT}_R$. Each η_A factorizes into topological lines of the single CFTs as $\eta_A = \eta_{A,L} \eta_{A,R}$. (2) Current operators $j_A = \psi_{L,A} \psi_{R,A} \in \mathcal{H}_{\eta_A}$, neutral under all η_A . The corresponding symmetries are absent in the individual CFTs. (3) A topological line $\mathcal{A}$, satisfying $\mathcal{A} \times \eta_A = \mathcal{A}$ for all η_A . We will take $\mathcal{A} = \bigoplus_{\eta \in \Gamma} \eta$, where Γ is the group generated by

the η_A .

The spin selection rule [39,54] for $\mathcal{H}_{\eta_A}$: $s_A \in (\alpha_A/N_A^2) + (1/N_A)\mathbb{Z}$ —here $\alpha_A \in H^3[\mathbb{Z}_{N_A}, U(1)]$ is the anomaly—implies that the lines η_A must be anomaly-free. Furthermore bosons are the only neutral operators in $\mathcal{H}_{\eta_A}$. The allowed form of Γ is strongly constrained by the OPE:

$$j_A(z)j_B(w) \sim \frac{k_{AB}}{(z-w)^2} \mathbf{1} + i f_{AB}^C \frac{j_C(w)}{z-w}. \quad (3)$$

This OPE determines the fusion of the η_A , and hence Γ . Let us give two illustrative examples: (a) The OPE of a single $\mathfrak{u}(1)$ current $j(z)$ requires the latter to either be genuine or to live in the twisted sector of a $\mathbb{Z}_2$ line $\eta^2 = 1$. (b) Three currents $j_{1,2,3}$ can close into an $SU(2)_k$ current algebra. The form of Γ depends on the basis in which $\mathfrak{su}(2)$ algebra is presented. If j_i are associated with the standard Pauli basis of $\mathfrak{su}(2)$, the lines η_1, η_2, η_3 give rise to the three $\mathbb{Z}_2$ subgroups of $\mathbb{Z}_2 \times \mathbb{Z}_2$. If instead the basis is $(\sigma_3, \sigma^\pm)$, then j_3 is genuine, while $j_\pm$ are attached to lines $\eta_\pm$ fulfilling $\eta_+ \eta_- = 1$.

Notice that the Lie algebra basis in which the phantom currents appear cannot be changed: it is meaningful and determines the structure of Γ . The same Lie algebra can support different phantom symmetries, depending on the basis in which currents are attached to simple lines.

We define the phantom symmetry defect U_ε by

$$U_\varepsilon[\gamma] = \mathcal{A}[\gamma] \exp\left(i\varepsilon^A \int_\gamma \star j_A\right), \quad j \bullet \overset{\eta}{- -} | \underset{\mathcal{A}[\gamma]}{|}, \quad (4)$$

where ε^A are continuous parameters. This describes a continuous symmetry of the folded theory, which is non-invertible whenever $\mathcal{A}[\gamma]$ is nontrivial. Alternatively, consider gauging Γ . In the orbifolded theory, the currents j^A become genuine and form a group G_{orb} . The orbifolded theory has a dual $\hat{\Gamma}$ symmetry [55], under which the j^A are charged. Gauging back, the symmetry operator is

$$U_\varepsilon[\gamma] = \bigoplus_{r \in \hat{\Gamma}} \exp \left(i\varepsilon^A \int_\gamma r(j^A) \right), \quad (5)$$

making its noninvertible nature manifest [13,41,45,56]. The parameters ε^A belong to the orbifold $G_{\text{orb}}/\hat{\Gamma}$.

Boundary conditions transform under the action of U_ε , generating a conformal manifold. The exactly marginal deformation is the *tilt operator*, obtained by pulling back the phantom current onto the boundary. Due to the non-invertibility, a boundary condition B transforms in a NIM rep: $U_\varepsilon|B\rangle = \sum_{B'} n_B'{}^{B'}|B'\rangle$, typically yielding a nonsimple boundary. Its simple components can be extracted from the fusion algebra. This constrains the global topology of the manifold.

If $\text{CFT}_L = \text{CFT}_R$ we can consider reference boundary conditions $|\mathcal{L}\rangle$ arising from folding along a topological line $\mathcal{L}$. The boundary condition $|\mathcal{L}\rangle$ relates left and right movers as $\mathcal{O}_L = \mathcal{L}(\tilde{\mathcal{O}}_R)$. We find that the tilt operator on the deformed defect is mapped to the marginal operator $\mathcal{O}_{\Lambda=1} \sim \psi_L \times \mathcal{L}(\overline{\psi}_R)$, which is then exactly marginal.

g function and reflection coefficient—Our approach also allows to completely characterize the *g* function [9] and the

reflection coefficient $\mathcal{R}$ [10,37,57] of defects related by a broken phantom symmetry.

g function: For a conformal line defect $\mathcal{D}$, the g -function is defined as the (scheme independent) finite part of the defect free energy on the circle $F_{\mathcal{D}} = \log(g)$. This quantity is monotonic along defect RG [9, 58–60] and thus a key defect observable. We show that the g function is left invariant by the action of a phantom symmetry. $F_{\mathcal{D}}$ is computed by the disk partition function in the folded theory ($H = H_L + H_R$), leading to: $(d/d\varepsilon)g(\varepsilon) \propto \langle 0|q^H \int \star j|B_\varepsilon\rangle = 0$. If the defect conformal manifold contains a topological line $\mathcal{L}$, then $g = \langle \mathcal{L} \rangle$.

Reflection coefficient: Another interesting set of observables for a defect CFT are the reflection and transmission coefficients $\mathcal{R}, \mathcal{T} = 1 - \mathcal{R}$ [10]. They are defined as the fraction of reflected/transmitted energy in a scattering process involving the defect and are computed by the two-point function of the stress tensors on the two sides of the defect [57]. $\mathcal{R} = 0$ signals the presence of a topological defect, while $\mathcal{R} = 1$ describes factorized interfaces [49]. Fusion with topological defects leaves $\mathcal{R}$ constant [10]. This is not the case for defects related by a phantom symmetry, as there is no corresponding global symmetry in the unfolded CFT. In folded theory the topological lines U_ε only commute with the total stress tensor $T = T_L + T_R$.

Consider, for simplicity, a $U(1)$ phantom symmetry generated by a current $j = \psi_L \psi_R$ in the twisted sector of a $\mathbb{Z}_2$ line η . The folded theory has a genuine $(2, 0)$ primary field [10] $W^+ \propto c_R T_L - c_L T_R$. We can also construct another, “ η -twisted” $(2, 0)$ primary $W^- \propto h_R \partial \psi_L \psi_R - h_L \psi_L \partial \psi_R$.

These operators form a closed multiplet under the $U(1)$ phantom symmetry (see Supplemental Material [61]) [63]:

$$[Q, W^+] = -inW^-, \quad [Q, W^-] = inW^+, \quad (6)$$

$n \in \mathbb{Z}$ being the phantom symmetry charge. The reflection and transmission coefficients are determined by the two-point function of W^+ in the folded theory $\langle 0|W^+(z)\bar{W}^+(w)|B\rangle = [\omega_B^+/2/(z-w)^4]$, as $\mathcal{R} = (c_L^2 + c_R^2 + 2c_L c_R \omega_B^+)/ (c_L + c_R)^2$. We find it natural to also introduce the coefficient ω_B^- via $\langle 0|W^-(z)\bar{W}^-(w)|B\rangle = \omega_B^-/(2(z-w)^4)$. The multiplet structure determines $\omega_B^\pm$ along the conformal manifold in terms of the coefficients $\omega_B^\pm$ computed at a reference boundary condition $|B_\epsilon\rangle = U_\epsilon|B_0\rangle$ (see Supplemental Material [61] for details):

$$\omega_B^+(\epsilon) = \cos^2(n\epsilon)\omega_{B_0}^+ + a \sin^2(n\epsilon)\omega_{B_0}^-, \quad (7)$$

where $a = \pm$ refers to a deformation by the vector/axial phantom symmetries, respectively. With a wise choice of $|B_0\rangle$ we can compute the coefficients $\omega_{B_0}^\pm$: (a) If $\text{CFT}_L = \text{CFT}_R$ we can take $|B_0\rangle = |\mathcal{L}\rangle$ to be the transmissive boundary condition defined by folding the theory over a topological line $\mathcal{L}$. This has $\omega_{\mathcal{L}}^+ = -1$. Moreover $\omega_{\mathcal{L}}^-$ can be

computed by unfolding the theory around $\mathcal{L}$. Denoting by $q_{\psi_{L/R}}^{\mathcal{L}}$ the eigenvalues of $\mathcal{L}$ on $\psi_{L/R}$ we find $\omega_{\mathcal{L}}^- = q_{\psi_L}^{\mathcal{L}} q_{\psi_R}^{\mathcal{L}}$, and

$$\mathcal{R}(\epsilon) = \frac{1 \pm q_{\psi_L}^{\mathcal{L}} q_{\psi_R}^{\mathcal{L}}}{2} \sin^2(n\epsilon). \quad (8)$$

Notice that $|q_{\psi_L}^{\mathcal{L}}| \leq 1$, so the reflection coefficient is always positive and strictly smaller than 1 if $q_{\psi_L}^{\mathcal{L}} q_{\psi_R}^{\mathcal{L}} \neq 1$. (b) If the boundary condition is factorized, $|B_0\rangle = |B_L\rangle |B_R\rangle$, we instead have $\omega_{B_0}^+ = 1$. As ψ_L and ψ_R are chiral fields, their two-point functions in the presence of a boundary take the form $\langle 0 | \psi_{L/R}(z) \bar{\psi}_{L/R}(\bar{w}) | B_0 \rangle = \beta_{L/R} / (z - w)^{2h_{L/R}}$. Hence $\omega_{B_0}^- = \beta_L \beta_R / k$, where k is the level of the $U(1)$ algebra generated by the phantom current. The reflection coefficient reads

$$\mathcal{R}(\epsilon) = 1 - \frac{2c_L c_R}{(c_L + c_R)^2} \left(1 \mp \frac{\beta_L \beta_R}{k} \right) \sin^2(n\epsilon). \quad (9)$$

The minimum reflection coefficient along the conformal manifold is $\mathcal{R}^* = 1 - 2(1 + |\omega_{B_0}^-|) c_L c_R / (c_L + c_R)^2$. As $|\omega_{B_0}^-| \leq 1$ the reflection coefficient is positive definite. In Supplemental Material [61] we show that our formulas reproduce the known results for the Ising defects [37].

Examples—

$c = 1$ theories: We begin by analyzing defects in the 2d $c = 1$ CFT of a free compact boson ϕ at radius R . Vertex operators $V_{n,w} = e^{in\phi + iw\tilde{\phi}} : n, w \in \mathbb{Z}$ have conformal dimensions $h = \frac{1}{4}((n/R) + wR)^2$, $\bar{h} = \frac{1}{4}((n/R) - wR)^2$. The allowed marginal deformations of the identity defect are

$$V_{\pm 2n,0} \quad \text{at } R = \sqrt{2}n, \quad V_{0,\pm n} \quad \text{at } R = \frac{\sqrt{2}}{n}, \quad (10)$$

and their T duals. At $R = \sqrt{2}$ both sets are allowed. Are these deformations also exactly marginal?

At these radii, the theory has nongenuine fermionic $(1/2, 0)$, $(0, 1/2)$ operators, summarized in Table II. Marginal operators (10) arise in the OPE of the chiral-antichiral operators.

TABLE II. Phantom symmetry for the free boson. The chiral operators lie in twisted sectors $(2\pi\alpha, 2\pi\beta)$ of the $U(1)_m \times U(1)_w$ symmetry.

	$R = \sqrt{2}n$	$R = \sqrt{2}/n$
$(\frac{1}{2}, \frac{1}{2})$	$V_{\pm 2n,0}$	$V_{0,\pm n}$
$(\frac{1}{2}, 0)$	$V_{n,(1/2n)}, V_{-n,-(1/2n)}$	$V_{(1/n),(n/2)}, V_{-(1/n),(n/2)}$
$(0, \frac{1}{2})$	$V_{-n,(1/2n)}, V_{n,-(1/2n)}$	$V_{(1/n),-(n/2)}, V_{-(1/n),-(n/2)}$
η	$(0, (1/2n)), (0, -(1/2n))$	$((1/n), 0), (-(1/n), 0)$ if n even $((1/n), \frac{1}{2}), (-(1/n), \frac{1}{2})$ if n odd

Upon folding, the (anti)chiral fields generate four nongenuine (anti)holomorphic currents from their products. The folded theory has an enhanced phantom symmetry at these radii. Decomposing the scalar fields ϕ and $\tilde{\phi}$ into (anti)chiral components X and $\bar{X}$ as $\phi = (1/\sqrt{2}R)(X + \bar{X})$, $\tilde{\phi} = (R/\sqrt{2})(X - \bar{X})$, (anti)chiral operators are $e^{\pm iX_{L/R}}, e^{\pm i\bar{X}_{L/R}}$. Denoting $X = X_L + X_R$ we have the OPE

$$e^{+iX}(z) \times e^{-iX}(w) = \frac{1}{(z-w)^2} + \frac{i}{z-w} \partial X(w) + \dots, \quad (11)$$

with an analogous expression for the antidiagonal combination $X' = X_L - X_R$. We conclude that the folded theory has a phantom symmetry with algebra $\mathfrak{su}(2) \times \mathfrak{su}(2) \cong \mathfrak{so}(4)$. This enhancement is manifest in the fermionized description. At $R = \sqrt{2}$, the $c = 1$ compact boson is the bosonization of a free Dirac fermion. Folding yields two Dirac fermions, whose global symmetry is $O(4)_L \times O(4)_R \supset U(1)^4$. A generic boundary condition breaks it to an $O(4)$ subgroup, leaving a six-dimensional defect conformal manifold. The $U(1)$ Cartan symmetries remain exact upon unfolding, the remaining four correspond to the marginal operators discussed earlier. The values $R = \sqrt{2}n, \sqrt{2}/n$ are obtained by gauging $\mathbb{Z}_n$ subgroups of $U(1)^m$ and $U(1)^w$. This breaks the $O(4)$ symmetry down to its $SU(2) \times U(1)$ subgroup, thereby eliminating two of the six marginal deformations of the $R = \sqrt{2}$ theory. An analog phantom symmetry algebra exists at $R = \sqrt{2}p/q$, the chiral operators being $V_{\pm p/q, \pm q/2p}$. At these radii there are, however, no marginal deformations of the identity defect. The phantom conformal manifold will appear upon deformation of suitably symmetry-reflecting [30] conformal/topological lines. Further examples can be found by considering interfaces between $c = 1$ theories at different points of the conformal manifold. We leave their study for future work.

Interfaces between RCFTs: We now study interfaces between G_k and G'_k WZW models. Interfaces preserving some of the chiral algebra have been studied extensively [10,64–66]. Our results will describe symmetry breaking interfaces. We consider a single phantom current j , attached to a $\mathbb{Z}_2$ line. Thus either η_L, η_R are anomaly-free and $h_L = h_R = \frac{1}{2}$ or η_L, η_R are anomalous and $h_L = \frac{1}{4}, h_R = \frac{3}{4}$. We classify WZW models with such operators assuming ψ_L and ψ_R are chiral primaries. Embedding $\mathbb{Z}_2$ in the center of G , chiral operators in the $\mathbb{Z}_2$ twisted sector are order 2 simple currents $(J, 0)$ [67]. It is immediate to compute their conformal dimensions: $h = 1/2$ appears in $SU(2)_2, SU(4)_1, \text{Spin}(n)_1$; $h = 1/4$ in $SU(2)_1$ and $h = 3/4$ in $SU(2)_3, SU(6)_1, \text{USp}(6)_1, \text{Spin}(12)_1, (E_7)_1$.

We can also consider interfaces with an $\mathfrak{su}(2)$ phantom symmetry. The simplest group that admits the correct structure is $\mathbb{Z}_3$. Twisted chiral operators with $h = \frac{1}{3}$ are present in $SU(3)_1$ only, while $h = \frac{2}{3}$ appears in $SU(3)_1$ and $(E_6)_1$. Interfaces between these models have an $\mathfrak{su}(2)$ phantom symmetry.

Finally consider the GKO cosets $G_k \times G_l / G_{k+l}$ [68] as examples without continuous symmetry. These can be thought of the IR of the theory in the numerator after gauging the vector symmetry [69]. Chiral simple currents J, J' are protected, but must combine into singlets under the vector symmetry. We find that the three-state Potts CFT $3\text{Potts} = SU(3)_1 \times SU(3)_1 / SU(3)_2$ has a $\mathbb{Z}_3$ -twisted chiral operator of dimension $h = 2/3$. Interfaces between three-state Potts and $SU(3)_1$ thus have an $\mathfrak{su}(2)$ phantom symmetry.

Higher-dimensional examples: Phantom symmetries also feature in $d > 2$ free theories. Consider a free real scalar: the $O(1) = \mathbb{Z}_2$ symmetry is enhanced to $O(2)$ upon folding, the phantom symmetry being the $U(1)$ symmetry of the complex scalar, that contains $\mathbb{Z}_2$ as a subgroup.

The general case of interfaces between p and q scalars is analogous. The two theories have symmetries $O(p)$ and $O(q)$, respectively, while the phantom symmetry is $O(p+q)/O(p) \times O(q)$. Boundary conformal manifolds of N free scalars have been studied in [3], where the authors noticed that their geometry is that of a coset (see also Ref. [74] for codimension 1 defects). Choosing two integers p', q' with $p' + q' = N = p + q$ and splitting the scalar fields into $\{\phi_i, \phi_a\}$, $i = 1, \dots, p'$, $a = 1, \dots, q'$, they consider boundary conditions

$$\Lambda_{ai} \partial_\perp \phi_a - \partial_\perp \phi_i = 0, \quad \Lambda_{ai} \phi_i + \phi_a = 0. \quad (12)$$

Unfolding this describes interfaces between p and q scalars with Grassmannian geometry $O(N)/(O(p') \times O(q'))$.

Different choices of p', q' label different connected components of the interface conformal manifold. If $p = q$ (defect case), the component with $p' = q' = p$ has geometry $O(2p)/(O(p) \times O(p))$ and contains a submanifold $O(p)$ corresponding to topological defects. A similar result holds multiple free Majorana fermions. We conclude with a no-go theorem for interacting examples. Recall the unitarity bounds $\Delta_\phi \geq (d-2/2)$, $\Delta_\psi \geq (d-1/2)$, $\Delta_s \geq d+s-2$. These bounds are saturated by free fields and conserved (higher-spin) currents, respectively. A phantom current $J \sim \mathcal{O}_L \mathcal{O}_R$ only exists for free scalars $J_\mu \sim \phi_L \partial_\mu \phi_R$ or free fermions $J_\mu \sim \bar{\psi}_L \gamma_\mu \psi_R$. Other spin-one composite operators have $\Delta > d-1$ by the unitarity bounds. Thus phantom symmetries in interacting CFTs are intrinsic to $(1+1)d$ [75]. Ways around this no-go theorem might come from nongauge invariant currents in the folded theory, such as in $\mathbb{R}$ Maxwell theory in $4d$.

Acknowledgments—We thank Ho Tat Lam, Marco Meineri, Yifan Wang, and Yunquin Zheng for illuminating discussions. The work of A. A. is supported by the UKRI Frontier Research Grant, underwriting the ERC Advanced Grant “Generalized Symmetries in Quantum Field Theory and Quantum Gravity.” C. C. is supported by STFC Grant No. ST/X000761/1. The research of G. G. is funded through an ARC advanced project, and further supported by IISN- Belgium (convention 4.4503.15). The research of A. A. and C. C. was supported in part by grant NSF PHY-2309135 to the Kavli Institute for Theoretical Physics (KITP). G. R. is supported by the Huawei Young Talent program at IHES.

Data availability—No data were created or analyzed in this study.

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