

Hitting the Thermal Target for Leptophilic Dark Matter at Future Lepton Colliders

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(Received 12 November 2024; revised 14 July 2025; accepted 18 September 2025; published 22 October 2025)

We study future lepton collider prospects for testing predictive models of leptophilic dark matter (DM) candidates with a thermal origin. We calculate experimental milestones for testing the parameter space compatible with freeze-out and the associated collider signals at past, present, and future facilities. This analysis places new limits on such models by leveraging the utility of lepton colliders. At e^+e^- machines, we make projections using precision Z-pole observables from $e^+e^- \rightarrow \ell^+\ell^- + E$ signatures at large electron-positron collider and future projections for future circular collider (ee) in these channels. Additionally, a muon collider could also probe new thermal relic parameter space in this scenario via $\mu^+\mu^- \rightarrow X + E$, where X is any easily identifiable standard model object. Collectively, these processes can probe much of the parameter space for which DM direct annihilation to $\ell^+\ell^-$ yields the observed relic density in Higgs-like models with mass-proportional couplings to charged leptons.

DOI: [10.1103/w1bw-w7wt](https://doi.org/10.1103/w1bw-w7wt)

Introduction—Thermal freeze-out is the only dark matter (DM) production mechanism that is insensitive to unknown high-energy physics. Historically, this mechanism has been associated with the weakly interacting massive particle (WIMP) paradigm, which predicts the DM mass to be ~ 10 GeV–10 TeV [1]. While there is a rich experimental program to search for WIMPs, no verified signal has yet been identified, which motivates broadening the search effort to a wider range of scenarios.

It is well known that high energy colliders are powerful probes of thermal DM. There are currently many limits on various DM models based on existing large electron-positron collider (LEP) [2], Tevatron [3], and LHC [4] data. There are also multiple studies of DM sensitivity at proposed future facilities including the ILC [5], FCC-ee [6], FCC-hh [7], CLIC [8], CCC [9], CEPC [10], high-luminosity LHC (large hadron collider) [11], and various muon collider run options [12–16].

In this Letter we consider the future collider discovery potential for models of leptophilic DM with mass-proportional couplings [17] to standard model (SM) leptons. In

the predictive parameter space for this class of models, the scalar particle φ that mediates dark-visible interactions is heavier than the DM χ ($m_\varphi > m_\chi$) so that, during freeze-out, the process $\chi\chi \rightarrow \varphi^* \rightarrow \ell^+\ell^-$ directly annihilates DM into SM leptons to yield the observed relic density [18]. Since the cross section for this process depends on the coupling between φ and charged leptons, there is a minimum value required for viable freeze-out.

Thus, a thermal origin through the direct annihilation channel predicts a *minimum* signal yield for DM collider production via $e^+e^-, \mu^+\mu^- \rightarrow \ell^+\ell^-\varphi$, and there is a corresponding experimental milestone for decisively testing models with this annihilation topology. For $m_\varphi > 2m_\chi$, the mediator decays predominantly to DM, so φ production complements traditional mono- X searches $e^+e^-, \mu^+\mu^- \rightarrow X\varphi$, which are also present in this part of parameter space and included in our analysis of the muon collider reach. Our Letter finds that lepton colliders with e^+e^- or $\mu^+\mu^-$ beams are powerful probes in this class of models and can test nearly all the remaining parameter space that is compatible with a thermal relic origin.

Theory—

Model: Our representative scenario consists of a Majorana DM candidate χ with mass m_χ coupled to a CP -even scalar mediator φ with mass m_φ via

$$\mathcal{L}_{\text{int}} = -\frac{g_\chi}{2} \varphi \chi \chi - \varphi \sum_{\ell=e,\mu,\tau} g_\ell \bar{\ell} \ell, \quad g_\ell = g_e \left(\frac{m_\ell}{m_e} \right), \quad (1)$$

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where g_χ and g_ℓ are the Yukawa couplings to DM and charged leptons ℓ , respectively. Throughout, we refer to this benchmark as the low-energy limit of a more fundamental theory with an extended electroweak sector to ensure that the ϕ - ℓ interactions respect SM gauge invariance.

There are many possible SM extensions that either fully realize the interactions in Eq. (1) or approximate their limiting cases. For example, the mass proportionality in g_ℓ can naturally arise in a Type-III two-Higgs doublet model supplemented with an additional singlet scalar φ with the Higgs-portal couplings:

$$\mathcal{L} \supset \varphi \left(\mu_{11} H_1^\dagger H_1 + \mu_{12} H_1^\dagger H_2 + \mu_{22} H_2^\dagger H_2 \right) + \text{H.c.}, \quad (2)$$

where the doublet H_1 gives mass to quarks, H_2 gives mass to charged leptons, and μ_{ij} are dimensionful mixing parameters. As we show in Supplemental Material [19], after electroweak symmetry breaking, φ can predominantly mass-mix with the heavy CP -even scalar and, after diagonalizing the mass matrix, the portal couplings from Eq. (2) induce mass-proportional couplings to leptons, as shown in Eq. (1).

Alternatively, more general leptophilic interactions can arise if φ couples to a heavy fourth generation of vectorlike leptons that mass-mix with the known charged leptons. If the vectorlike leptons are heavy, they can be integrated out to yield $\varphi \bar{\ell} \ell$ couplings in Eq. (1). Complete models that realize these interactions can be found in Refs. [20,21]. Although these interactions offer much more general flavor structure than in Eq. (1), they can easily generate important limiting cases (e.g., muon- or tau-philic couplings for φ) which capture most of the phenomenology of interest here—namely, leptophilic heavy-flavor interactions with the SM. The mass-proportionality of the Yukawa couplings can be recovered if the vectorlike leptons in this scenario have minimal flavor violating interactions [17].

Returning to the phenomenological model of interest, in order for thermal freeze-out to be predictive and testable with laboratory measurements, the cross section for DM annihilation must depend on the φ coupling to the SM, i.e., annihilation cannot occur strictly in the dark sector. This requirement is readily achieved by demanding $m_\varphi > m_\chi$ so that the $\chi\chi \rightarrow \ell^+ \ell^-$ topology is kinematically allowed for all ℓ satisfying $m_\chi > m_\ell$. By contrast, if $m_\chi > m_\varphi$, the dominant channel for annihilation will be secluded annihilation via $\chi\chi \rightarrow \varphi\varphi$, which only depends on g_χ , and there is no minimum requirement on g_ℓ so long as χ thermalizes with the SM in the early universe. Note that this secluded channel yields an s -wave annihilation cross section, which is excluded for thermal relics below ~ 20 GeV. Thus, for the remainder of this Letter, we only consider the case where φ is heavier than χ .

Furthermore, since experimental limits on g_ℓ are generically stronger than those on g_χ , we work in the $m_\varphi > 2m_\chi$ regime, where the scalar will decay predominantly via $\varphi \rightarrow \chi\chi$. Thus, we are interested in signatures involving heavy lepton production in association with missing energy rather than φ decaying back to SM particles as multilepton final states are well constrained at colliders.

Cosmological production: In the early universe for $T \gg m_\chi$, the DM is in chemical equilibrium with the SM, but as the temperature cools below $T \sim m_\chi$, the χ population becomes Boltzmann suppressed. The observed density is achieved for $\langle \sigma v \rangle \sim 10^{-26} \text{ cm}^3 \text{ s}^{-1}$ [22], where for $m_\varphi > m_\chi$, the thermally averaged annihilation cross section is $\langle \sigma v \rangle = \sum_{\ell=e,\mu,\tau} \langle \sigma v \rangle_{\chi\chi \rightarrow \ell\ell}$. To obtain the thermal average, we calculate the annihilation cross section and follow the prescription from Ref. [23], where for each $\chi\chi \rightarrow \ell^+ \ell^-$ channel, the total annihilation cross section is

$$\sigma(s) = \frac{g_\ell^2 g_\chi^2 (s - 4m_\chi^2)^{3/2}}{16\pi[(s - m_\varphi^2)^2 + m_\varphi^2 \Gamma_\varphi^2]} \sqrt{1 - \frac{4m_\chi^2}{s}}, \quad (3)$$

where Γ_φ is the total φ decay width.

It is useful to consider the form of σv in the non-relativistic limit prior to performing the thermal average. In this regime, each channel contributes

$$\sigma v_{\chi\chi \rightarrow \ell\ell} \approx \frac{g_\ell^2 g_\chi^2 m_\chi^2 v^2}{8\pi(m_\varphi^2 - 4m_\chi^2)^2} \propto g_\ell^2 g_\chi^2 \left(\frac{m_\chi}{m_\varphi} \right)^4 \frac{1}{m_\chi^2}, \quad (4)$$

where v is the relative velocity between initial state χ particles. Matching the convention in Refs. [24,25], we define

$$y \equiv \epsilon^2 \alpha_\chi \left(\frac{m_\chi}{m_\varphi} \right)^4, \quad \epsilon \equiv g_\ell/e, \quad (5)$$

where $\alpha_\chi \equiv g_\chi^2/(4\pi)$. This quantity is useful because, away from the fine-tuned resonance region $m_\varphi \approx 2m_\chi$, there is a one-to-one relationship between σv and y independent of the m_φ/m_χ or g_χ/g_ℓ ratios; here the relic density for each value of m_χ is simply set by the product that defines y . In Fig. 1, the black curves in the y vs m_χ plane represent the parameter points for which $\chi\chi \rightarrow \ell\ell$ annihilation yields the observed DM abundance through thermal freeze-out.

Importantly, the nonrelativistic expansion in Eq. (4) also shows that this process is p -wave, so the annihilation rate falls precipitously as the universe expands and the temperature cools. As discussed below, this feature sharply suppresses the annihilation to SM particles during and after the epoch of recombination.

Collider searches—

Belle II: Since φ has sizable couplings to heavy flavor, it can be produced via $e^+ e^- \rightarrow \mu^+ \mu^- \varphi$, ($\varphi \rightarrow \chi\chi$), where

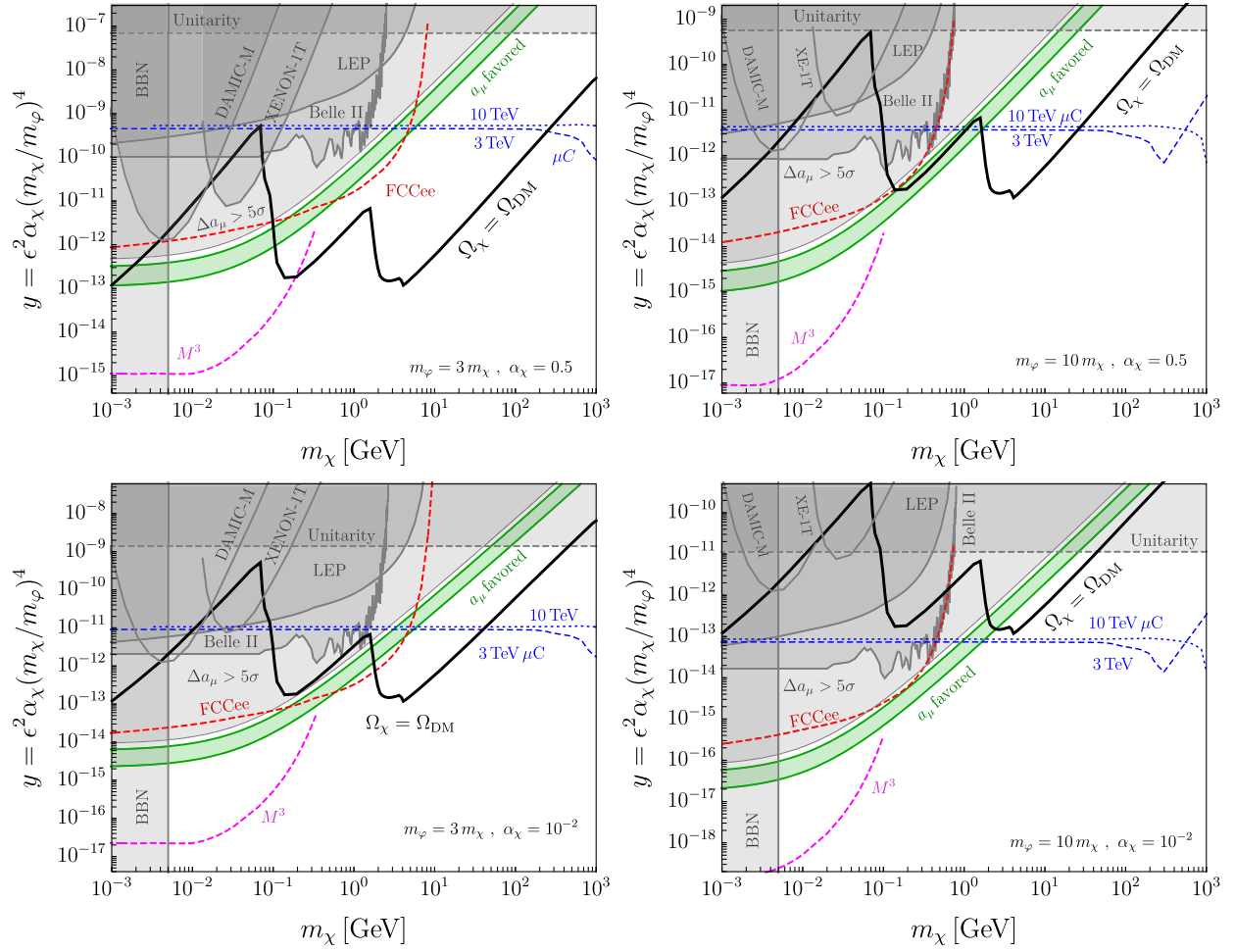


FIG. 1. Experimental milestone for $\chi\chi \rightarrow \ell^+\ell^-$ thermal freeze-out (black curve) plotted alongside gray-shaded constraints from the muon anomalous magnetic moment a_μ [26], electron recoil direct detection limits from DAMIC-M [27] and XENON-1T [28,29], and existing collider limits from LEP [30] and Belle II [31]. The green band indicates the favored parameter space for resolving the $(g-2)_\mu$ anomaly. The four plots show various parameter choices to illustrate a range of phenomenology: the ratio of $m_\phi/m_\chi = 3, 10$ and the coupling within the dark sector $\alpha_\chi = 0.5, 10^{-2}$. We choose these parameter values to illustrate how increasing the mass hierarchy (m_ϕ/m_χ) or decreasing the dark-sector coupling α_χ reduces the unconstrained model space. For resonant production of DM, e.g., $m_\phi \sim 2m_\chi$, the thermal target is well below couplings reachable by current collider projections (see Ref. [32] for details). The dashed curves are explained in the collider discussion in the main text.

the DM χ in the final state is observed as missing energy. There is an irreducible background from $e^+e^- \rightarrow \mu^+\mu^-\bar{\nu}\nu$ from the missing energy in the final state neutrinos, as well as reducible backgrounds from $e^+e^- \rightarrow \mu^+\mu^-\gamma\gamma$. Belle II has placed constraints on an $L_\mu - L_\tau$ gauge boson through this channel [31]. We recast these limits for our scalar scenario by using MadGraph5_aMC@NLO [33] to calculate the cross section for $e^+e^- \rightarrow \mu^+\mu^-\phi$ production and rescale the limits for the $L_\mu - L_\tau$ vector model with the corresponding cross section for vectors. We neglect any signal acceptance differences between scalars and vectors.

LEP: The LEP and LEP 2 e^+e^- colliders at CERN [30] have impressive sensitivity to DM production through mono- γ and missing energy $\cancel{E}$ signatures [2]. While these canonical signatures can also arise in our scenario from the

couplings in Eq. (1), they are not the dominant channel as the production cross section for $e^+e^- \rightarrow \gamma\phi \rightarrow \gamma E$ is suppressed by the small electron coupling g_e^2 . Since the ϕ coupling to the τ is enhanced by a factor of $(m_\tau/m_e)^2 \sim 10^6$, here we instead consider the signature $e^+e^- \rightarrow \tau^+\tau^-\phi$, ($\phi \rightarrow \chi\chi$), where the invisibly decaying mediator is produced as final state radiation and predicts deviations within well-measured SM uncertainties. Since SM τ decays always produce missing energy via ν , our signal process will be constrained by any previous measurement of Z decays that result in τ production [34]. Up to small differences in geometric acceptance and final state τ kinematic distributions, $Z \rightarrow \tau^+\tau^-\phi$ production is experimentally indistinguishable from $Z \rightarrow \tau^+\tau^-\phi$ for ϕ decaying invisibly.

One of the primary motivations for a high-energy electron-positron collider is a run at the Z pole where $\sqrt{s} \approx m_Z$, so the Z production cross section is resonantly enhanced, and with copious production, precision measurements can be made. Therefore, LEP is mainly sensitive to φ production through modifications to the well-measured properties of the Z .

Of particular interest is the radiative process $Z \rightarrow \tau^+ \tau^- \varphi$, ($\varphi \rightarrow \chi\chi$), where the φ decay to DM yields additional missing energy in the final state. We calculate the partial width for this three-body process in Supplemental Material [19]. From LEP, we obtain a limit on φ production from the measurement of the di-tau partial width [35] $\Gamma(Z \rightarrow \tau^+ \tau^-) = 84.08 \pm 0.22$ MeV. From the uncertainty we extract a 2σ limit on the $\text{BR}(Z \rightarrow \tau^+ \tau^- \varphi)$, we find the value of g_τ at each value of m_φ that satisfies $\Gamma(Z \rightarrow \tau^+ \tau^- \varphi) = 2(0.22 \text{ MeV})$, and we translate this into a constraint on y for a given choice of α_χ and m_χ/m_φ ratio. We also include a highly simplified estimate for the acceptance of $Z \rightarrow \tau\tau\varphi$ events due to analysis cuts on kinematics, which is discussed in Supplemental Material [19]. In Fig. 1 this bound is shaded in gray and labeled LEP.

Future Circular Collider (FCC): The FCC at CERN is a proposed high-energy collider to succeed the LHC at CERN. The first stage of the project is the FCC-ee, an e^+e^- collider to run at similar run configurations as LEP but with much greater luminosity, which aims to take data in the early 2040s [6,36–39]. As part of its overall physics program, FCC-ee will run at the Z pole and produce over one trillion Z bosons for precision electroweak measurements (called the *Tera-Z run*), thereby exceeding the previous LEP record of approximately 17 million [30]. This allows for a diverse physics program that involves precision measurements of Z decays (e.g., Refs. [40,41]).

To make sensitivity projects of the FCC-ee at the Z pole, we assume that statistical uncertainties dominate the FCC-ee error budget and rescale the uncertainty on the Z width measurement by $\sqrt{N_{\text{LEP}}/N_{\text{FCC-ee}}}$ for N_X the total number of Z bosons produced. With this estimation, we predict sensitivities to Z -boson branching ratios comparable to what is stated in Ref. [36]. In Fig. 1 we show projections of the improvement of FCC-ee over LEP with the red dashed curve.

Future muon collider: Another possible option for a future high-energy collider is a multi-TeV muon collider [42–47]. Because of their moderate mass and fundamental nature, muons can theoretically be accelerated to collide at $\sqrt{s} \sim 10$ TeV energies without insurmountable losses due to synchrotron radiation and without enormous hadronic backgrounds [48]. Although this technology needs a full demonstration program to prove its feasibility, considerable theoretical, experimental, and accelerator work has already gone into its design (see Refs. [49,50] for recent summaries), and its construction is already partially funded.

As a muon collider is still in its very early stages, the run configurations have not yet been fixed, and a multitude of possible center-of-mass energies and corresponding luminosities are considered.

In this Letter, we utilize the high energy of the muon collider to probe complementary parameter space to what can be done at the FCC-ee. We consider the process $\mu^+ \mu^- \rightarrow \gamma \varphi \rightarrow \gamma E$ with a mono- γ signature. Because of the kinematics of the process, the γ will have energy roughly $E_\gamma \sim \sqrt{s}/2$ depending on the width of the φ and energy resolution σ_E . The primary SM background for this process is $\mu^+ \mu^- \rightarrow \gamma \bar{\nu} \nu$. (We also considered the 3γ background, but as two photons, one with $E \sim \text{TeV}$, have to be missed, this background is comparatively negligible.) We simulate both the signal and background events in MadGraph5_aMC@NLO [33]. To mitigate this background, we utilize the fully constrained kinematics of the two-body final state of the signal and cut on the energy of the visible photon:

$$E_\gamma \in [E_0(1 - 2\sigma_E), E_0(1 + 2\sigma_E)], \quad (6)$$

where $E_0 \equiv [(E_{\text{CM}}^2 - m_\varphi^2)/2E_{\text{CM}}]$ and σ_E is the energy resolution that we fix nominally at $\sigma_E = 3\%$. We also take a conservative photon reconstruction efficiency of 98%, although we expect that this number for TeV photons might be higher. The search strategy in this channel is generic as long as the mediator decays invisibly via $\varphi \rightarrow \chi\chi(\cancel{E})$ and the other SM particles X recoil in a diametrically opposed direction to the missing energy and momentum in the final state. A similar strategy with partially visible decays was considered in Ref. [51].

The dashed blue curves in Fig. 1 are sensitivity projections for $\mu^+ \mu^- \rightarrow X \varphi \rightarrow X \cancel{E}$ at a 3 TeV muon collider concept with 1 ab^{-1} of integrated luminosity. (We do not consider a lower energy center of mass, like the Higgs pole, as higher-energy configurations have a broader physics program and can even produce upward of 10^6 to 10^8 Higgs.)

Other searches—

Muon g-2: Since the leptophilic mediator φ couples to the muon, at one loop it modifies its anomalous magnetic moment by [52,53]

$$\Delta a_\mu = \frac{g_\mu^2}{16\pi^2} \int_0^1 dz \frac{(1-z)(1-z^2)}{(1-z)^2 + r^2 z}, \quad (7)$$

where $r = m_\varphi/m_\mu$, which constrains the thermal relic parameter space shown in Fig. 1. The gray region labeled “ $\Delta a_\mu > 5\sigma$ ” is where model parameters make the tension with observation worse by five standard deviations. For parameters within the green band labeled “ a_μ favored,” the φ coupling to muons accounts for the discrepancy between the measured world average [26,54] and the R -ratio

prediction within the standard model [55], assuming the difference is due to new physics.

Fixed target experiments: For the sub-GeV DM mass range, proposed fixed target accelerators are powerful laboratory probes of light weakly coupled particles [56]. The mediator φ can be radiatively produced in muon-nucleus fixed target interactions via $\mu N \rightarrow \mu N \varphi$, ($\varphi \rightarrow \chi\chi$), where N is a target nucleus. For $m_\varphi > m_\mu$, the radiated φ typically acquires a large fraction of the incident beam energy, which serves as a trigger for a DM search if other visible particles produced in μ - N scattering can be vetoed. In Fig. 1 we show projections for the proposed M^3 experiment [51,53] based on this search strategy. Comparable projections could also be computed for the CERN NA64 μ experiment, which performs a similar search at higher energy [57], but translating the publicly available projections into the scalar model considered here is beyond the scope of this Letter.

Electron recoils: Since the mediator φ couples to electrons, this scenario is constrained by direct detection experiments sensitive to scattering off electrons. The non-relativistic cross section for $\chi e \rightarrow \chi e$ scattering is

$$\sigma_{\chi e} = \frac{g_\chi^2 g_e^2 \mu_{\chi e}^2}{\pi[(\alpha m_e)^2 + m_\varphi^2]^2} \rightarrow \frac{g_e^2 g_\chi^2 \mu_{\chi e}^2}{\pi m_\varphi^4} \approx \left(\frac{4e^2 m_e^2}{m_\chi^4} \right) y, \quad (8)$$

where $\mu_{\chi e}$ is the χ - e reduced mass and in the last step we took the $m_e \ll m_\varphi, m_\chi$ limit, where $\sigma_{\chi e} \propto y$ and the direct detection cross section has the same parametric dependence as the $\chi\chi \rightarrow e^+e^-$ annihilation cross section from Eq. (4). Although the electron scattering sensitivities of various experiments are comparable in this mass range, in Fig. 1 we show the current best limits from the DAMIC-M [27] and XENON-1T [28,29] experiments.

Nuclear recoils: Although φ has no tree-level coupling to quarks, at one-loop there is induced mass-mixing between φ and the SM Higgs, which yields the interactions

$$\mathcal{L}_{\text{int}} \supset \theta y_q \varphi \bar{q} q \equiv g_q \varphi \bar{q} q, \quad (9)$$

where q is any SM quark, $y_q = m_q/v_h$ is its SM Yukawa coupling to the Higgs field, and θ is the φ - h mixing angle. The leading contribution to this mixing is given by integrating out the τ loop $\theta \sim g_\tau y_\tau / (16\pi^2)$. Based on this mixing, the elastic spin-independent direct-detection cross section for χ - n scattering is $\sigma_{\chi n} = g_\chi^2 g_n^2 \mu_{\chi n}^2 / (\pi m_\varphi^4)$, where n is a target nucleon, $\mu_{\chi n}$ is the DM-nucleon reduced mass, and the induced φ nucleon coupling $g_n \propto \theta$ can be written in terms of the mediator couplings to quarks in Eq. (9) following the procedure in Ref. [58]—see also the Appendix of Ref. [59] for a modern treatment. Writing $\sigma_{\chi n}$ in terms of y from Eq. (5) is

$$y \approx 10^{-8} \left(\frac{\sigma_{\chi n}}{10^{-46} \text{ cm}^2} \right) \left(\frac{m_\chi}{20 \text{ GeV}} \right). \quad (10)$$

Thus, for cross sections of order $\sigma_{\chi n} \approx 10^{-46} \text{ cm}^2$ at the maximum sensitivity of XENONnT [60] near $m_\chi \approx 20 \text{ GeV}$, direct detection does not meaningfully constrain the thermal relic values of y as shown in the black curve of Fig. 1. A similar argument can be used to show that the two-loop QED scattering processes lead to similar results. Thus, we omit nuclear recoil limits through loop induced elastic scattering in Fig. 1. However, we note that, if the Higgs-mixing parameter θ is enhanced beyond the τ loop that we use as a reference value (e.g., from additional tree level contributions), the model dependent constraint from this process could be competitive.

Big bang nucleosynthesis (BBN): For $m_\chi \lesssim \text{MeV}$, the χ freeze-out occurs around the epoch of BBN and can modify light chemical yields relative to SM predictions. In this light mass range, the DM has a large thermal abundance at the onset of BBN, which modifies the Hubble expansion rate and affects the timing of BBN events [61,62]. Using the latest nuclear cross sections and data on the deuterium fraction, BBN observables require $m_\chi \gtrsim 5 \text{ MeV}$ for Majorana DM candidates that transfer entropy to photons upon freeze-out [63] and this defines the leftmost regions in the panels of Fig. 1.

Cosmic Microwave Background (CMB): Thermal DM below the GeV scale faces stringent bounds from CMB energy injection from out-of-equilibrium DM annihilation just after recombination [64]. For an s -wave annihilation cross section, thermal WIMPs are excluded below $\sim 20 \text{ GeV}$ if their annihilation yields visible energy into the CMB [18]. However, in our scenario the annihilation cross section in Eq. (4) is p -wave and therefore suppressed by v^2 in the nonrelativistic regime. Thus, the CMB limit from Planck does not constrain the thermal relic parameter space for this model.

Discussion—In this Letter we have studied a predictive leptophilic DM scenario in which the thermal relic abundance arises from direct annihilation to SM particles via $\chi\chi \rightarrow \ell^+\ell^-$ with mass-weighted couplings to charged leptons. Since this scenario has no tree-level couplings to quarks, LHC and nuclear-recoil direct detection signals are suppressed compared to traditional WIMP DM models. We have set new limits on this model based on various probes, including measurements of the muon anomalous magnetic moment, electron recoil direct detection, B -factory production, and Z -pole measurements at LEP. Collectively, these limits leave much of the thermal relic parameter space unexplored, particularly for DM masses above the GeV scale where the relic density arises predominantly through $\chi\chi \rightarrow \tau^+\tau^-$ annihilation in the early universe.

Future accelerators will play a key role in probing the remaining thermal relic parameter space in this scenario.

In the sub-GeV mass range, the proposed fixed-target missing-momentum experiment M^3 can comprehensively cover the full thermal target and complement ongoing efforts currently underway with the NA64 μ experiment. At higher mass, FCC-ee e^+e^- collider can probe large regions of viable parameter space with a Tera-Z run, which can improve sensitivity to $Z \rightarrow \tau^+\tau^-$ decays, currently limited by the luminosity of the LEP dataset. And at the largest masses compatible with a viable thermal history $m_\chi \sim \text{TeV}$ in this framework, a future TeV-scale muon collider can probe nearly all of the remaining freeze-out parameter space with mono- γ/Z searches.

Acknowledgments—We would like to thank André de Gouvêa, Rodolfo Capdevilla, Bertrand Echenard, Maxim Pospelov, Jesse Thaler, Simone Pagan Griso, Jessie Shelton, and Andrea Wulzer for helpful conversations. C.C. is supported by the U.S. Department of Energy (DOE) Office of High Energy Physics under Grant Contract No. DE-SC0012567. Fermilab is operated by the Fermi Research Alliance, LLC under Contract No. DE-AC02-07CH11359 with the U.S. Department of Energy. This material is based partly on support from the Kavli Institute for Cosmological Physics at the University of Chicago through an endowment from the Kavli Foundation and its founder Fred Kavli.

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