

Deterministic Double-Zero Media and Robust Wave Manipulation Using a Phononic Weyl Semimetal

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Double-zero media exhibit unparalleled impedance-independent wave manipulation properties. However, their reliance on accidental degeneracy makes them fragile to disturbances in structural geometry and material parameters. Here, we propose a topological approach to address this limitation. By constructing a Weyl semimetal with uniaxial screw symmetry, which exhibits a charge-2 Weyl point at the Brillouin zone center, we propose a deterministic directional double-zero medium characterized by linear dispersion along one direction and quadratic dispersion along the perpendicular directions, leading to impedance-independent total transmission and divergent impedance mismatch along different directions. By experimentally constructing this Weyl semimetal, we have observed the existence of a charge-2 Weyl point and its unique wave manipulation capabilities as directional double-zero media, including frequency-tunable wave collimation and a configurable transition between positive and negative refraction. These findings open new pathways for realizing topology-protected metamaterials with unprecedented capabilities for bulk-wave manipulation.

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The rapid development of metamaterials over the past decades have opened up unprecedented possibilities for flexibly manipulating electromagnetic and acoustic waves [1,2]. Among the various types of metamaterials, zero-index media—characterized by one or more vanishing constitutive parameters—have garnered significant interest due to their unique functionalities, including wavefront shaping [3–8], cloaking [6–12], wave tunneling [13–20], asymmetric transmission [21–23], photonic doping [24–32] and “antidoping” [10,33], and unusual emission properties [34,35]. Particularly, double-zero-index media (DZIM) [36–39] are very attractive due to their perfect

impedance-independent wave tunneling functionalities, which are absent in any other materials. However, realizing DZIM requires the two constitutive parameters to simultaneously approach zero at the same frequency, resulting in linear dispersions at the Brillouin zone center [40]. This dependence on accidental degeneracy makes DZIM highly sensitive to the disturbances in material parameters or structure geometry, significantly hindering their practical applications. To date, a robust DZIM against material or geometrical perturbations has yet to be demonstrated.

Recently, artificial topological materials [41–48], such as topological insulators and semimetals, have demonstrated robust topology-protected functionalities, including defect-immune and scattering-free transport. While most previous studies have focused on topological edge states, recent works are emerging to explore unconventional control of bulk wave propagation [49]. In the context of DZIM, a linear dispersion at the Brillouin zone center is desired. Charge-1 Weyl points [50–55] exhibit linear dispersions in all directions of momentum space. However, they do not exist at the Brillouin zone center. On the other hand, while charge-4 Weyl points can occur at the Brillouin zone center, they lack linear dispersion. Therefore, neither charge-1 nor

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charge-4 Weyl points can be associated with deterministic DZIM. To achieve a robust DZIM, both linear dispersion and deterministic degeneracy at the Brillouin-zone center are necessary.

In this Letter, we demonstrate the realization of a robust DZIM associated with a charge-2 Weyl point [56–59], which satisfies the aforementioned two conditions, in a tetragonal phononic lattice with uniaxial screw symmetry. Near the Weyl point, the dispersion is linear along one direction and quadratic along the perpendicular ones, perfectly matching the typical characteristics of a directional DZIM. Consequently, the phononic Weyl semimetal exhibits double-zero-index properties along the direction of linear dispersion and single-zero-index properties along the direction of quadratic dispersion. Notably, both the Weyl point and the associated deterministic double-zero characteristic are robust against variations in the material or structure parameters, ensuring stable and reliable performance. By fabricating the phononic Weyl semimetal using aluminum rods, we experimentally validate the existence of the charge-2 Weyl point and demonstrate some unique functionalities, including frequency-tunable robust wave collimation and configurable transitions between positive and negative refractions. Such phenomena were fragile against parameters or disturbances previously. Thus, this work paves the way for robust metamaterials with topology-protected properties and advanced functionalities.

Figure 1(a) shows the typical dispersion of the charge-2 Weyl point located at the Brillouin-zone center. To confirm

its topological nature, we plot the spin structure of the Weyl point in Fig. 1(b), and the Berry phase θ_B around it as a function of the polar angle θ in Fig. 1(c) [40]. Both of them indicate that the Weyl point has a topological charge of +2, which equals the winding number at the equator of spin structure or the number of times θ_B winds around $[-\pi, \pi]$ [57]. If artificial material with such a charge-2 Weyl point can be characterized as an acoustic effective medium (we will demonstrate that it indeed can be), the effective mass density $\rho_{\text{eff}z}$ along the direction of linear dispersion and compressibility β_{eff} should cross zero simultaneously at the frequency of the Weyl point, as plotted in Fig. 1(d). Under variations in geometrical parameters, the charge-2 Weyl point maintains as well as the degeneracy of zero $\rho_{\text{eff}z}$ and β_{eff} , with only a shift in their frequency, e.g., from ω_1 to ω_2 in Fig. 1(d). In other words, the double-zero property is deterministic, i.e., robust against any material or structural disturbances, if the lattice symmetry is unchanged. This is in sharp contrast to previous realizations of DZIM based on accidental degeneracy [5–8]. On the other hand, the effective parameters $\rho_{\text{eff}x}$ and $\rho_{\text{eff}y}$ along the direction of quadratic dispersion are usually nonzero, thus this DZIM is extremely anisotropic, leading to unusual phenomena absent in previous isotropic DZIMs, which shall be demonstrated later.

To realize the Weyl point shown in Fig. 1, we have designed and fabricated a phononic crystal (PC) composed of aluminum rods with rectangular cross sections 8 mm $\times$ 6 mm arranged in air. A photo of the fabricated sample is shown in Fig. 2(a). To enable geometric tunability, the aluminum rods are connected at their ends with bolts, nuts, and washers. By changing the washers with different thickness, the lattice constant a_z in the z direction is adjustable. For airborne acoustics, aluminum can be simply treated as sound-hard material. With a uniaxial screw symmetry $S_{4z} = (x, y, z) \rightarrow [y, -x, z + (a_z/4)]$, a layered stack of rods forms a tetragonal arrangement of unit cells, as shown in Fig. 2(b). The lattice constants in the x, y directions are fixed at 20 mm. With $a_z = 30$ mm, we calculate the band structure of this PC along high symmetry lines of the Brillouin zone [shown in Fig. 2(c)] using COMSOL Multiphysics and plot it in Fig. 2(d). At the Γ point, a charge-2 Weyl point with linear dispersion along the k_z direction and quadratic dispersion along the k_x (and k_y) direction is induced by the lattice symmetry (space group No. 76: $P4_1$), and can be described by the following Hamiltonian:

$$H = (c_1 + c_2 \mathbf{k}^2 + c_3 k_z^2) \sigma_0 + \sum_{i=1}^2 \left[c_{i,1} k_x k_y + c_{i,2} (k_x^2 - k_y^2) \right] \sigma_i + c_4 \sigma_3 k_z, \quad (1)$$

where c_i ($i = 1, 2, 3, 4$) and $c_{i,j}$ ($i, j = 1, 2$) are coefficients, k_x, k_y , and k_z are components of wave vector $\mathbf{k}$, σ_0 and σ_i ($i = 1, 2, 3$) are the identity matrix and Pauli matrix,

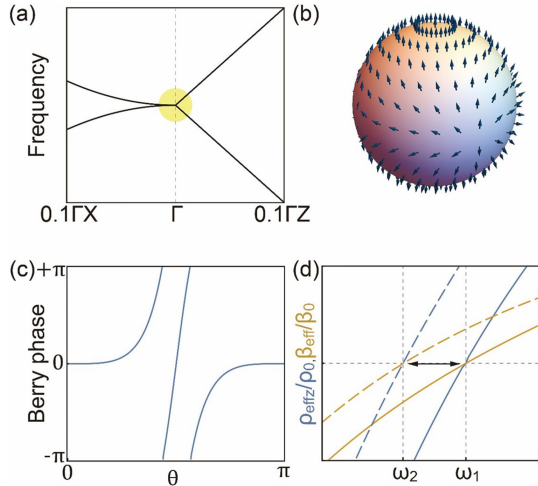


FIG. 1. Charge-2 Weyl point at the center of Brillouin zone. (a) Band dispersions of a Weyl point (in yellow circle), with linear dispersions along ΓZ and quadratic dispersions along ΓX . (b) Spin structure of the charge-2 Weyl Point. (c) Berry phase independent of polar angle in the yellow circle in (a) for upper band around the Γ point. (d) Schematic diagrams of the effective mass density and compressibility along the direction of linear dispersion for a PC with a charge-2 Weyl point. Solid and dashed lines represent the results under changes in geometric parameters. The horizontal dashed line indicates zero.

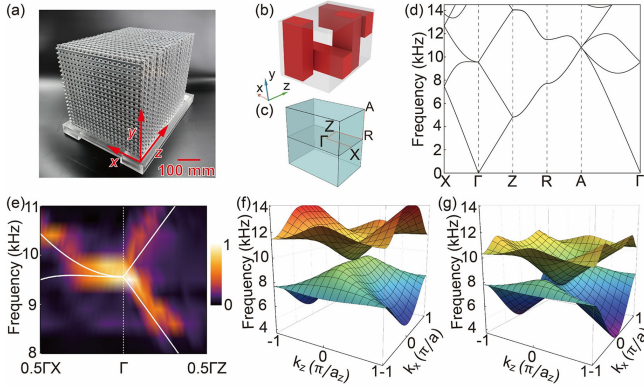


FIG. 2. A phononic Weyl semimetal with a charge-2 Weyl point at the center of Brillouin zone. (a) The structure, (b) the unit cell with side length $20 \text{ mm} \times 20 \text{ mm} \times a_z$, and (c) the Brillouin zone of the PC. The host medium (gray) in (b) is air. Red rectangular blocks in (b) are aluminum blocks with cross section $8 \text{ mm} \times 6 \text{ mm}$. (d) Simulated band structure of the PC with $a_z = 30 \text{ mm}$. (e) Simulated (line) and experimental results (color) of band structure near the Weyl point. (f), (g) Dispersion surface in $k_x - k_z$ plane with (f) $a_z = 30 \text{ mm}$ and (g) $a_z = 40 \text{ mm}$.

respectively [58]. At the center of Brillouin zone, the dispersions along k_x , k_y , and k_z are derived as

$$\begin{cases} c_1 + (c_2 \pm \sqrt{c_{12}^2 + c_{22}^2})k_x^2 \\ c_1 + (c_2 \pm \sqrt{c_{12}^2 + c_{22}^2})k_y^2 \\ c_1 \pm c_4 k_z + (c_2 + c_3)k_z^2 \end{cases} \quad (2)$$

indicating linear dispersion along the k_z direction and quadratic dispersion along the k_x and k_y directions.

To verify the existence of the charge-2 Weyl point, an acoustic speaker is applied to generate an acoustic beam propagating toward a PC sample with 20 periods in the x , y , and z directions, from 8 to 11 kHz. A 1/4-inch microphone is inserted into the PC to scan the pressure field distribution inside the sample. Fourier transform is applied to the measured data to obtain the band structure of this PC [40]. The measured band structure near the charge-2 Weyl point along the $X - \Gamma - Z$ high symmetry lines is presented in Fig. 2(e), which agrees well with the theoretical results (white line). To illustrate the robustness of the Weyl point, dispersion surfaces in $k_x - k_z$ plane with $a_z = 30 \text{ mm}$ and $a_z = 40 \text{ mm}$ are plotted in Figs. 2(f) and 2(g), respectively. With different a_z , the existence of Weyl points is preserved as well as the linear dispersion along the k_z direction and quadratic dispersion along the k_x direction. This is because charge-2 Weyl points are dictated by the underlying lattice symmetries of space group No. 76 rather than specific geometric parameters. In our system, breaking the uniaxial screw symmetry alters the lattice symmetry of the PC and

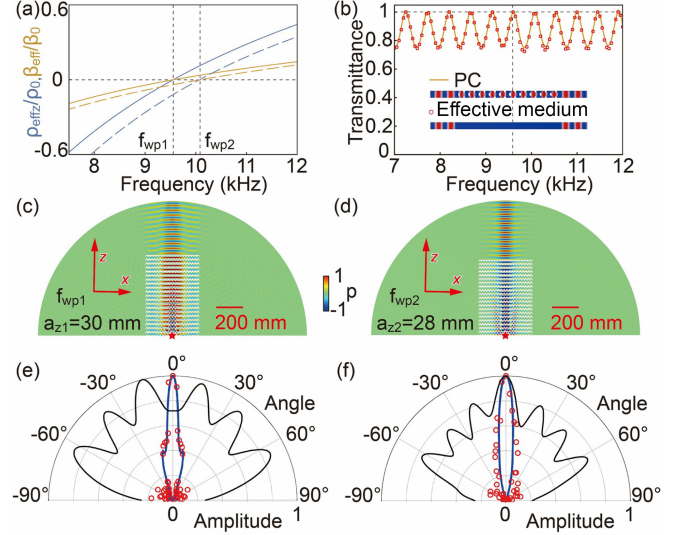


FIG. 3. Frequency-tunable robust wave collimation. (a) Effective mass density $\rho_{\text{eff}z}$ (blue lines) and compressibility β_{eff} (yellow lines) along the direction of linear dispersions for the PC with $a_{z1} = 30 \text{ mm}$ (solid lines) and $a_{z2} = 28 \text{ mm}$ (dashed lines) near the frequencies of Weyl points $f_{\text{WP1}} = 9.57 \text{ kHz}$ and $f_{\text{WP2}} = 10.08 \text{ kHz}$. (b) The transmittance spectrum when a planar wave normally incident to the PC (yellow solid line) with a_{z1} along the direction of linear dispersion and its effective medium (red circle). The vertical dashed line represents the frequency f_{WP1} . (c), (d) Simulated acoustic pressure fields radiated by a line source (red star) behind the PC at the frequency of Weyl point with (c) a_{z1} and (d) a_{z2} . The amplitude of line sources is 1 Pa. (e), (f) Simulated (blue lines) and measured (red circles) far-field amplitude distributions of the outgoing wave to transmission angles with (e) a_{z1} and (f) a_{z2} . Black lines are the far-field amplitudes when the PC is removed.

eliminates the charge-2 Weyl point at the Brillouin-zone center [40,58]. Nevertheless, charge-2 Weyl points can also emerge at the Brillouin-zone center in other space groups that do not possess screw symmetry [58].

In Fig. 3(a), we show the z -directional effective mass density $\rho_{\text{eff}z}$ and compressibility β_{eff} of the PC with different lattice constants $a_{z1} = 30 \text{ mm}$ (solid lines) and $a_{z2} = 28 \text{ mm}$ (dashed lines), by using the effective parameter method based on eigenfield averaging [38,40]. For both a_{z1} and a_{z2} , the $\rho_{\text{eff}z}/\rho_0$ and $\beta_{\text{eff}}/\beta_0$ approach zero simultaneously at the frequencies of Weyl point $f_{\text{WP1}} = 9.57 \text{ kHz}$ and $f_{\text{WP2}} = 10.08 \text{ kHz}$, respectively. This indicates that the directional double-zero property is indeed robust against the variation of geometrical parameters. Figure 3(b) shows the transmission spectrum of the PC near f_{WP1} under z -directional normal incidence, with the lattice constant a_{z1} . The transmittance of the PC (yellow solid line) and the effective medium (red circle) are highly consistent with each other, demonstrating the reliability of the effective parameters. On the other hand, along the $x(y)$ direction, we obtain $\rho_{\text{eff}x(y)}/\rho_0 = 0.81$ slightly above f_{WP1} and $\rho_{\text{eff}x(y)}/\rho_0 = -0.81$ slightly below f_{WP1} while

$\beta_{\text{eff}}/\beta_0 = 0$. This behavior, along with the z -directional effective parameters, is consistent with the dispersion of an extremely anisotropic zero-index material [40]. Along the z direction, this DZIM exhibits impedance-independent total transmission due to double-zero property [40]. While along the $x(y)$ direction, this DZIM exhibits divergent impedance mismatch, thus effectively preventing wave coupling through the corresponding boundaries.

In the following, we demonstrate two unique applications of this extremely anisotropic DZIM. The first one is frequency-tunable wave collimation. The isotropic double-zero property ensures impedance-independent total transmission and the wavefront of radiation can be manipulated by shaping the boundary of DZIM [4–8,38]. But in order to limit wave radiation to one direction, wave-blocking mechanisms such as perfect electric or magnetic conductors [8], sound-hard walls [38], or bound state in the continuum [25] are also required. Here, the DZIM with extreme anisotropy exhibit impedance-independent total transmission in one direction, and divergent impedance mismatch in other directions, thereby enabling perfect wave collimation. To demonstrate this phenomenon, an out-of-plane line source is placed at the boundary of the PC. The details of experimental setup are shown in Ref. [40]. The simulated acoustic pressure field distributions at the frequencies of two Weyl points, i.e., f_{WP1} and f_{WP2} in Fig. 3(a), are shown in Figs. 3(c) and 3(d), which correspond to different lattice constants a_{z1} and a_{z2} , respectively. In both cases, the cylindrical radiation emitted by the line source is collimated into the z direction, the outgoing wave has a planar wavefront perpendicular to the exit surface of the PC. We note that there is no propagation along the x and y directions, which is contrary to the case of isotropic DZIM. Because of the robustness of the Weyl point, the functionality of wave collimation is maintained when the geometrical or material parameters of the PC are changed, as long as the lattice symmetry is preserved. This fundamental characteristic of the directional DZIM differs from previous DZIM based on accidental degeneracies that are highly dependent on material and structural parameters [4–8,38]. We further plot the far-field radiation of the outgoing waves in Figs. 3(e) and 3(f) for the cases of a_{z1} and a_{z2} , respectively. In both cases, the simulated and the measured results agree excellently with each other. The half-power beam widths in Fig. 3(e) are 10.6° for the collimated wave beam and 160° when the PC is absent. In Fig. 3(f), the half-power beam widths are 12.8° in the collimated wave beam and 145° without the PC. Using a half-power beam width of $\leq 15^\circ$ as the criterion for collimation, the bandwidth of wave collimation in our deterministic DZIM is approximately 0.21 kHz. On the contrary, when we replace the deterministic directional DZIM with a nontopological DZIM, the wave-collimation effect disappears [40]. This validates the wave collimation functionality enabled by deterministic directional DZIM. The topologically

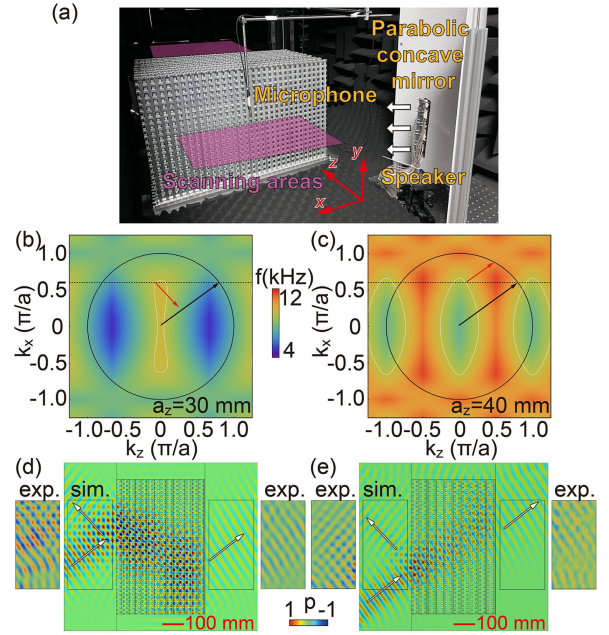


FIG. 4. Configurable transition between positive and negative refractions. (a) Scheme for the experimental setup. (b),(c) Iso-frequency contours of the PC with (b) $a_z = 30$ mm and (c) $a_z = 40$ mm. Black circles represent the isofrequency contour in air at 9.25 kHz. Black arrows represent the wave vector at the incident angle of 34° . Red arrows are the direction of the group velocity of the refracted wave. (d),(e) Simulated and experimental measured (inset graphs) acoustic pressure fields when a Gaussian beam incident to the PC at frequency 9.25 kHz and incident angle 34° with (d) $a_z = 30$ mm and (e) $a_z = 40$ mm.

protected collimation is robust against local perturbations such as structural defects [40].

Another interesting application is a configurable transition between positive and negative refractions. Figure 4(a) shows the experimental setup. A PC sample is constrained to a grooved base. A Gaussian beam with frequency 9.25 kHz is generated by a waveform generator (Tektronix AFG3022C), a speaker array (Five HiVi B1S), and a parabolic concave mirror. The beam is incident upon the PC at an angle of 34° . A microphone mounted on a motorized translation stage scans the acoustic pressure field on both the incident and transmitted regions with an 8 mm step size. The scanning areas are positioned 3 cm away from the sample, as marked by the purple rectangular boxes (40 cm $\times$ 20 cm).

By tuning a_z , the Weyl point can be redshifted or blueshifted along with the nearby dispersions. The isofrequency contours (IFC) of the PC with $a_z = 30$ mm and $a_z = 40$ mm are plotted in Figs. 4(b) and 4(c), respectively. The black circle represents the IFC of air at the frequency 9.25 kHz. The black arrow shows the wave vector of the incident wave at the incident angle of 34° , and the red arrow shows the direction of the group velocity of the refracted wave in the PC. For $a_z = 30$ mm, the IFC implies negative

refraction, where the incident and refracted beams are on the same side of the surface normal. For $a_z = 40$ mm, the IFC implies positive refraction, where the incident beam and refracted beam are on different sides of the surface normal. Therefore, there is a transition from negative to positive refraction when the lattice constant a_z is tuned from 30 to 40 mm. Figures 4(d) and 4(e) show the simulated acoustic pressure field distributions of the cases of negative refraction and positive refraction, respectively. The measured results are plotted in the corresponding inset graphs after normalization. Both the simulated and measured results show that the output beam is indeed down-shifted for $a_z = 30$ mm and up-shifted for $a_z = 40$ mm, confirming the transition from negative to positive refraction.

In summary, we have presented an approach to combine the robustness of topological materials and the wave manipulation capabilities of zero-index metamaterials. We prove that a charge-2 Weyl point located at the Brillouin-zone center of a tetragonal PC with uniaxial screw symmetry can function as a directional DZIM exhibiting impedance-independent total transmission along the k_z direction and divergent impedance mismatch along the k_x and k_y directions. The linear dispersion along the k_z direction and quadratic dispersions along the k_x and k_y directions perfectly match the characteristics of directional DZIM. Since the lattice symmetry protects the Weyl point at the Brillouin-zone center, the double-zero property is very robust against variations in geometric and material parameters. The robustness of deterministic DZIM enables unique phenomena, such as frequency-tunable wave collimation and configurable transition between positive and negative refractions. The deterministic DZIM is fundamentally distinct from nontopological DZIM, which are highly sensitive to geometric parameters and the shape of acoustic sources even when the lattice symmetry is maintained [8,38,40]. Thus, in nontopological DZIM, the double-zero property is typically confined to a single normalized frequency and is susceptible to disruption by coexisting flat bands and variations in geometric parameters, limiting functionalities such as frequency-tunable wave collimation and refraction switching [40]. We should also emphasize that our strategy for achieving the deterministic DZIM is also applicable for photonic crystals that follow electromagnetic (vector) wave equations [40]. Our findings pave the way for the development of robust metamaterials with topological singularities for advanced wave manipulation. For example, this topological approach offers the potential to realize functionalities such as frequency-tunable planar lens [40].

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Data availability—The data that support the findings of this Letter are not publicly available. The data are available from the authors upon reasonable request.

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