Erratum: All-Electrical Determination of Crystal Orientation in Anisotropic Two-Dimensional Materials [Phys. Rev. Lett. 120, 086801 (2018)]

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(Received 3 May 2026; published 5 November 2026)

DOI: 10.1103/vnm2-dh2r

A method for measuring anisotropic resistivity tensors in sheet conductors was introduced in this Letter. Given knowledge of the sample geometry including peripheral contact positions and at least three independent four-point resistance measurements, this all-electrical technique can determine the crystal orientation and electrical anisotropy ratio of two-dimensional materials, quantum wells, or sheet conductors of arbitrary shape. Using the standard Van der Pauw method along with scaling and rotation transformations, this method extracts the anisotropy ratio $\rho_{aa}/\rho_{cc} > 1$, angle θ of the low resistance ρ_{cc} direction relative to the sample frame x axis, and the geometric average of the sheet resistance $\rho_s = \sqrt{\rho_{aa}\rho_{cc}}$.

We wish to correct two errors in this Letter that affect the implementation of this anisotropic Van der Pauw method. These corrections do not affect the figures, experimental results, or conclusions of the Letter, as the correct equations were used in the actual data analysis.

First, although the diagram in Fig. 1(a) is correct as originally presented, the transformation in Eq. (2) should be corrected as follows:

$$\begin{pmatrix} c' \\ a' \end{pmatrix} = \begin{pmatrix} \alpha & 0 \\ 0 & 1/\alpha \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}. \tag{2}$$

The rightmost matrix represents a clockwise rotation of the x-y sample frame by the angle θ to the c-a crystal frame (Fig. 1b). The leftmost matrix is corrected, and represents an anisotropic scaling to the c'-a' frame that compresses the sample along the c axis by a factor α and stretches the sample along the a axis by a factor $1/\alpha$, where $\alpha = \sqrt[4]{\rho_{cc}/\rho_{aa}} < 1$ (Fig. 1c).

An additional example of the transformations is provided here in Fig. 5 to validate the revised equation. Assuming a square sample with $\alpha = 0.8$ and $\theta = 30°$, after rotation and scaling transformations the square sample in the x-y frame with anisotropic resistivity is mapped to an equivalent parallelogram with isotropic resistivity in the c'-a' frame. The geometric illustration in the example is drawn to scale and quantitatively matches the specified anisotropy parameters.

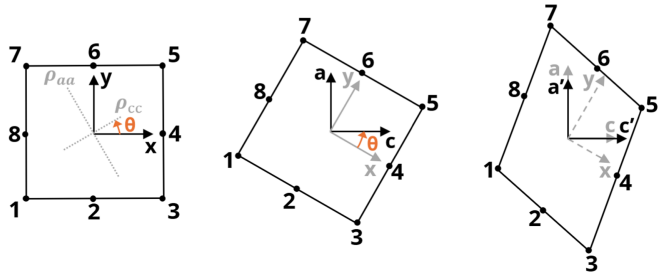


FIG. 5. Example of the transformations applied to a square sample with low-resistance direction ρ_{cc} at a counterclockwise angle $\theta = 30°$ relative to the sample x axis. The square with anisotropic resistivity in the x-y frame (left) is rotated to the c-a crystal axes (middle) and scaled into an equivalent parallelogram with isotropic resistivity in the c'-a' basis (right).

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The second correction concerns Eq. (4) of the Letter. The numerator and denominator in the argument of the natural logarithm should be swapped, as follows [1]:

$$f_{lm,np} = \frac{1}{\pi} \ln \left[\frac{(u_p - u_l)(u_n - u_m)}{(u_n - u_l)(u_p - u_m)} \right] \equiv f_k. \tag{4}$$

Acknowledgments—This work was funded in part by the QSI RENU Catalyst Research Award and the Trienens Institute for Sustainability and Energy at Northwestern University.

[1] L. J. Van der Pauw, A method of measuring specific resistivity and Hall effect of discs of arbitrary shape, Philips Res. Rep. **13**, 1 (1958).