

Erratum: Critical Probability Distributions of the Order Parameter from the Functional Renormalization Group [Phys. Rev. Lett. **129**, 210602 (2022)]

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In the Letter, we incorrectly treated the overall normalization of the rate function as an additional nonuniversal amplitude when comparing the functional renormalization group results with Monte Carlo simulations. More precisely, besides the nonuniversal amplitudes associated to the correlation length and the normalization of the order parameter, we allowed for an independent multiplicative normalization of the rate function. This is not consistent with two-scale-factor universality.

Indeed, according to the finite-size scaling form of Privman and Fisher [1], the singular part of the free-energy density can be written in terms of a universal function $\tilde{f}(x, y)$,

$$f_{\text{sing}}(t, h, L) = L^{-d} \tilde{f}(L/\xi_{\infty}, L^{(d+2-\eta)/2} h/h_0),$$

up to the two usual nonuniversal amplitudes, one associated with a length, or equivalently with the amplitude of ξ_{∞} , and one associated with the magnetic scaling field h . Taking the Legendre transform with respect to h , or equivalently considering the constrained free energy defining the rate function, gives

$$I(s, \xi_{\infty}, L) = L^{-d} I_{\zeta}(L^{(d-2+\eta)/2} h_0 s), \quad \zeta = L/\xi_{\infty}.$$

Thus, once the two nonuniversal amplitudes have been fixed, the scaling function I_{ζ} has a universal normalization. There is no third independent nonuniversal amplitude multiplying the rate function.

This correction does not affect the formal functional renormalization group construction of the rate function, its scaling form, or the qualitative conclusions of the Letter. It does, however, modify the quantitative interpretation of the comparison between the local potential approximation (LPA) result and Monte Carlo simulations. In the Letter, the rate function was normalized so as to match the Monte Carlo data at $\zeta = 0$. The good agreement shown there therefore demonstrates that the LPA accurately captures the shape of the scaling function as a function of ζ and of the scaled order parameter, but it does not test the universal overall amplitude.

A convenient universal measure of this amplitude is

$$\Delta I = L^d [I_{\zeta=0}(\tilde{s}_0) - I_{\zeta=0}(0)],$$

where $\tilde{s}_0$ denotes the position of the minimum of the scaling rate function at $\zeta = 0$. The LPA gives

$$\Delta I_{\text{LPA}} = -0.983,$$

whereas Monte Carlo simulations give

$$\Delta I_{\text{MC}} = -0.78(1).$$

Thus, at the LPA level, the universal amplitude is not reproduced quantitatively, although the functional shape is. A recent calculation at the next order of the derivative expansion gives

$$\Delta I = -0.73(6),$$

showing a substantial improvement over the LPA result [2].

[1] V. Privman and M. E. Fisher, *Phys. Rev. B* **30**, 322 (1984).

[2] F. Rose, A. Rançon, and I. Balog, *Phys. Rev. E* **113**, 054106 (2026).