

# Unitarity Bounds and Sum Rules in the Standard Model Effective Field Theory

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We present a comprehensive reassessment of perturbative unitarity bounds in the dimension-6 standard model effective field theory, exploiting a new formalism based on spinor-helicity techniques to derive partial-wave unitarity bounds for generic  $N \rightarrow M$  scattering amplitudes. We find that, in several cases, these theoretical constraints are already competitive with, or even stronger than, the corresponding experimental bounds for energy scales above a few TeV. This is especially the case for four-fermion operators under realistic flavor assumptions, where unitarity bounds can be further strengthened by exploiting sum rules. These results provide a valuable and timely contribution toward the ambitious program of new-physics searches with minimal theoretical priors supplemented by restrictive, yet general, first-principles criteria.

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**Introduction**—The discovery of a scalar particle consistent with the standard model (SM) Higgs boson at the Large Hadron Collider (LHC) has firmly established the validity of the SM. Since no experimental data at present significantly contradict SM predictions, it is customary to assume the emergence of new physics (NP) at energies much above the electroweak scale. In this context, effective field theories (EFTs) provide a systematic framework to parametrize possible deviations from the SM by introducing higher-dimensional operators constructed from SM fields and respecting its symmetries (SMEFT) [1–5].

A key theoretical consistency requirement for any EFT is perturbative unitarity, derived from the fundamental properties of the scattering  $S$  matrix [6]. The resulting bounds are inherently perturbative, being derived at tree level, and therefore do not represent sharp exclusions: their saturation signals the breakdown of the low-energy description, indicating the energy scale at which new degrees of freedom or a strongly coupled regime must appear. Historically, unitarity considerations in  $WW$  scattering led to an upper bound on the Higgs boson mass (the so-called no-lose Higgs theorem), which played a crucial role in motivating the construction of the LHC [7,8]. More recently, unitarity bounds have been extensively investigated in a wide variety of frameworks beyond the SM,

including SMEFT, Higgs EFT, axionlike particle EFTs, and gravitational theories [9–27]. These studies provide constraints that are complementary to those obtained from direct and indirect experimental searches for NP particles.

Traditionally, unitarity bounds are derived from  $2 \rightarrow 2$  scattering processes using partial-wave decomposition and diagonalization of the scattering matrix [6]. While this approach has proven effective, it suffers from significant limitations: it cannot account for  $2 \rightarrow N$  (with  $N > 2$ ) processes, which are particularly relevant at the high energies accessible at the LHC and future colliders, nor is it easily applicable to spin-2 or higher-spin theories. By contrast, as recently demonstrated in [28], on-shell methods—which have proved to be highly efficient in capturing the ultraviolet (UV) behavior of EFTs through renormalization group equations [29–43]—also emerge as an ideal tool for deriving unitarity bounds for EFTs at high energies.

Partial-wave unitarity bounds within the SMEFT have already been partially explored in the literature. In particular, Refs. [10–12] considered dimension-6 bosonic operators and two-fermion-two-boson operators, but restricted the analysis to operators without gluon fields. In these works, four-fermion operators were not considered. Earlier studies addressed unitarity in specific four-fermion interactions [44], but a systematic SMEFT treatment was still lacking. Moreover, dimension-6 operators  $\psi^2\phi^3$  modifying Yukawa interactions were neglected, and no unitarity bounds were derived for the Higgs operator  $(\phi^\dagger\phi)^3$ . In this Letter, we fill these gaps by providing the first complete coupled-channel implementation of perturbative unitarity for the baryon-number-conserving dimension-6 SMEFT. To this end, we compute all relevant scattering amplitudes including every allowed helicity, gauge, and flavor

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configuration, consistently retaining the correlations among Wilson coefficients entering the same partial-wave amplitudes. We then marginalize over each Wilson coefficient. This is especially relevant in flavor scenarios with dominant couplings to third-generation fermions, as naturally realized in models addressing the flavor puzzle at the TeV scale, which have attracted considerable interest in the recent literature [45–61].

The analyticity of the S matrix allows one to derive spin-dependent sum rules [62–68], providing additional theoretical constraints on effective four-fermion interactions beyond partial-wave unitarity bounds. Remarkably, the resulting pattern of SMEFT inequalities can be used to infer characteristic features of the underlying NP dynamics. Therefore, a further goal of this Letter is to explore the complementarity of unitarity and sum-rule constraints, as well as their competitiveness with both current and projected experimental limits.

In the high-energy particle physics community, there is a growing consensus on performing NP searches with minimal theoretical priors. The huge and otherwise unmanageable parameter space must be supplemented by restrictive, yet general, first-principles criteria to make such searches tractable. In this context, our Letter provides a systematic consistency framework that can be directly incorporated into present and future global SMEFT analyses.

#### Theoretical bounds—

**Partial-wave unitarity bounds:** To compute the unitarity bounds, we adopt the method developed in [28], which

allows one to project a generic scattering amplitude  $|\mathcal{A}_{i \rightarrow f}\rangle$  onto a kinematic basis  $|\mathcal{B}_{i \rightarrow f}^J\rangle$  with definite angular momentum  $J$ , as follows:

$$|\mathcal{A}_{i \rightarrow f}\rangle = \sum_J a_{i \rightarrow f}^J |\mathcal{B}_{i \rightarrow f}^J\rangle, \quad (1)$$

where  $a_{i \rightarrow f}^J$  denote the partial-wave coefficients and the elements of the kinematic basis are normalized as

$$\langle \mathcal{B}_{i \rightarrow f}^J | \mathcal{B}_{i \rightarrow f}^{J'} \rangle = \int d\Phi_i d\Phi_f \mathcal{B}_{i \rightarrow f}^J (\mathcal{B}_{i \rightarrow f}^{J'})^* = (2J+1) \delta^{JJ'}, \quad (2)$$

where  $d\Phi_{i,f}$  refers to the phase-space measure associated with the initial and final states. In terms of  $a_{i \rightarrow f}^J$ , the partial-wave unitarity bounds take the form

$$|\text{Re} a_{i \rightarrow i}^J| \leq 1, \quad 0 \leq \text{Im} a_{i \rightarrow i}^J \leq 2, \quad |a_{i \rightarrow f}^J| \leq 1, \quad (3)$$

with  $i \neq f$ . Further details on the construction of the basis and on the derivation of these bounds can be found in [27,28,69,70].

Using this framework, we systematically computed the unitarity bounds for all dimension-6 SMEFT Wilson coefficients in the Warsaw basis [2] reported in Appendix A. The resulting bounds are summarized in Table I.

TABLE I. Unitarity bounds on the modulus of the baryon-number-conserving dimension-6 SMEFT Wilson coefficients in the Warsaw basis [2], obtained by marginalizing over all other SMEFT coefficients (see Table II). The factors  $s/\Lambda^2$  are left implicit, i.e., the reported values constrain the combination  $s|C|/\Lambda^2$ . Fermionic operators correspond to the single-flavor limit.

| $X^3$                   | $\varphi^6$ and $\varphi^4 D^2$ |                   |                  | $\psi^2 \varphi^3$      |                    | $(\bar{L}L)(\bar{L}L)$ |                | $(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$ |                       |
|-------------------------|---------------------------------|-------------------|------------------|-------------------------|--------------------|------------------------|----------------|---------------------------------------------------|-----------------------|
| $C_G$                   | $4\pi/(9g_s)$                   | $C_\varphi$       | $32\pi^3/3$      | $C_{e\varphi}$          | $32\pi^2/\sqrt{3}$ | $C_{\ell\ell}$         | $2\pi$         | $C_{\ell edq}$                                    | $8\pi/\sqrt{3}$       |
| $C_{\tilde{G}}$         | $4\pi/(9g_s)$                   | $C_{\varphi\Box}$ | $8\pi/3$         | $C_{u\varphi}$          | $32\pi^2/3$        | $C_{qq}^{(1)}$         | $6\pi/5$       | $C_{quqd}^{(1)}$                                  | $16\pi(1+\sqrt{2})/9$ |
| $C_W$                   | $2\pi/(3g)$                     | $C_{\varphi D}$   | $32\pi/3$        | $C_{d\varphi}$          | $32\pi^2/3$        | $C_{qq}^{(3)}$         | $4\pi/3$       | $C_{quqd}^{(8)}$                                  | $4\pi(2+7\sqrt{2})/3$ |
| $C_{\tilde{W}}$         | $2\pi/(3g)$                     |                   |                  |                         |                    | $C_{\ell q}^{(1)}$     | $2\pi\sqrt{3}$ | $C_{\ell equ}^{(1)}$                              | $8\pi/\sqrt{3}$       |
|                         |                                 |                   |                  |                         |                    | $C_{\ell q}^{(3)}$     | $2\pi$         | $C_{\ell equ}^{(3)}$                              | $(2+\sqrt{2})\pi/3$   |
|                         |                                 |                   |                  |                         |                    |                        |                |                                                   |                       |
| $X^2 \varphi^2$         | $\psi^2 X \varphi$              |                   |                  | $\psi^2 \varphi^2 D$    |                    | $(\bar{R}R)(\bar{R}R)$ |                | $(\bar{L}L)(\bar{R}R)$                            |                       |
| $C_{\varphi G}$         | $\pi$                           | $C_{eW}$          | $4\pi\sqrt{2/3}$ | $C_{\varphi\ell}^{(1)}$ | $8\pi$             | $C_{ee}$               | $2\pi$         | $C_{\ell e}$                                      | $4\pi$                |
| $C_{\varphi\tilde{G}}$  | $\pi$                           | $C_{eB}$          | $4\pi$           | $C_{\varphi\ell}^{(3)}$ | $8\pi/3$           | $C_{uu}$               | $3\pi/2$       | $C_{\ell u}$                                      | $4\pi$                |
| $C_{\varphi W}$         | $2\pi\sqrt{2/3}$                | $C_{uG}$          | $2\pi\sqrt{3}$   | $C_{\varphi e}$         | $8\pi$             | $C_{dd}$               | $3\pi/2$       | $C_{\ell d}$                                      | $4\pi$                |
| $C_{\varphi\tilde{W}}$  | $2\pi\sqrt{2/3}$                | $C_{uW}$          | $4\pi\sqrt{2/3}$ | $C_{\varphi q}^{(1)}$   | $2\pi\sqrt{6}$     | $C_{eu}$               | $4\pi$         | $C_{qe}$                                          | $4\pi$                |
| $C_{\varphi B}$         | $2\pi\sqrt{2}$                  | $C_{uB}$          | $4\pi$           | $C_{\varphi q}^{(3)}$   | $8\pi/3$           | $C_{ed}$               | $4\pi$         | $C_{qu}^{(1)}$                                    | $2\pi\sqrt{2}$        |
| $C_{\varphi\tilde{B}}$  | $2\pi\sqrt{2}$                  | $C_{dG}$          | $2\pi\sqrt{3}$   | $C_{\varphi u}$         | $4\pi\sqrt{3}$     | $C_{ud}^{(1)}$         | $4\pi$         | $C_{qu}^{(8)}$                                    | $3\pi(1+1/\sqrt{2})$  |
| $C_{\varphi WB}$        | $4\pi$                          | $C_{dW}$          | $4\pi\sqrt{2/3}$ | $C_{\varphi d}$         | $4\pi\sqrt{3}$     | $C_{ud}^{(8)}$         | $8\pi$         | $C_{qd}^{(1)}$                                    | $2\pi\sqrt{2}$        |
| $C_{\varphi\tilde{W}B}$ | $4\pi$                          | $C_{dB}$          | $4\pi$           | $C_{\varphi ud}$        | $8\pi$             |                        |                | $C_{qd}^{(8)}$                                    | $3\pi(1+1/\sqrt{2})$  |

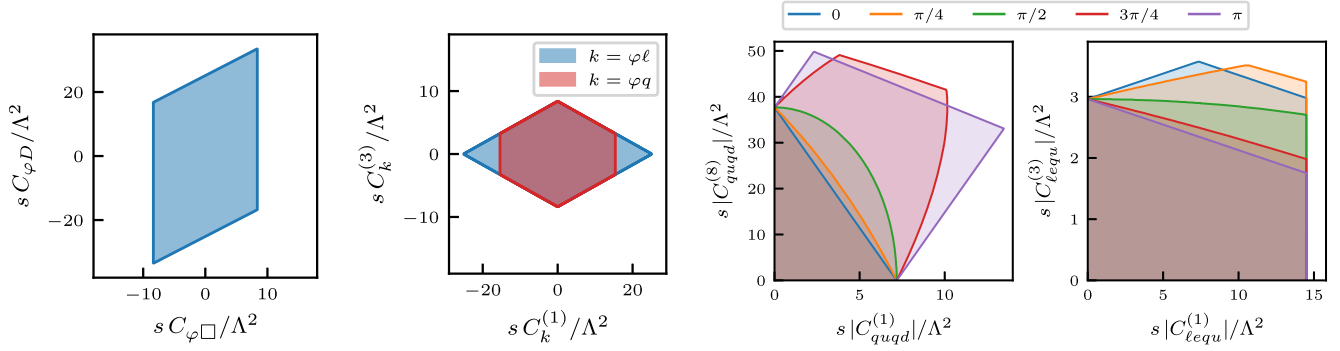


FIG. 1. Parameter space allowed by unitarity bounds in the plane of Wilson coefficients in the single-flavor limit. In the last two plots, we also show the dependence on the relative complex phases  $\arg(C_{quqd}^{(1)}/C_{quqd}^{(8)})$  and  $\arg(C_{\ell equ}^{(1)}/C_{\ell equ}^{(3)})$ .

Several comments are in order. In contrast to the common practice in SMEFT analyses, where only one or a few operators are considered at a time, the unitarity bound associated with each Wilson coefficient is obtained by marginalizing over all other SMEFT coefficients. In practice, the marginalized bound on each coefficient is its largest value allowed by unitarity with all remaining coefficients free to vary. Since the partial-wave amplitudes depend linearly on the Wilson coefficients, Eq. (3) carves out a convex region in coefficient space, and each marginalized bound is the projection of this region onto the corresponding axis. Moreover, since all scattering channels are considered, the region is bounded. Indeed, by the amplitude/operator correspondence, the number of independent amplitudes equals that of independent operators, so that the unitarity conditions constrain every possible direction in coefficient space. Thus, the numbers in Table I should be interpreted as conservative one-dimensional projections of the full coupled-channel unitarity region, rather than as single-operator bounds. This procedure is particularly relevant for coefficients that contribute to the same scattering amplitudes. This occurs for the following classes of operators: (i)  $(O_{\varphi\Box}, O_{\varphi D})$ ; (ii) all pairs involving either a field-strength tensor  $X$  or its dual  $\tilde{X}$ ; and (iii) all pairs containing fermionic bilinears in the

singlet or triplet (octet) representations of SU(2) (SU(3)). Representative examples are shown in Figs. 1 and 2.

To fully exploit the power of this method, we performed a fully coupled-channel analysis for each operator. In particular, to compute the amplitude of a process mediated by a given SMEFT operator, we considered all possible initial and final states in all allowed helicity, gauge, and flavor configurations. At dimension 6 the leading numerical constraints often arise from the lowest-multiplicity channels. Nevertheless, the general  $N \rightarrow M$  formalism [28] is essential for a uniform treatment of all operator classes, including  $\varphi^6$  and  $\psi^2\varphi^3$ , and makes the inclusion of higher-multiplicity channels automatic whenever they yield the leading energy growth. For practical purposes, we present in Table I the unitarity bounds assuming single-flavor fermions, due to the large number of independent components arising from Wilson coefficients with multiple flavor indices. In addition, Supplemental Material [71] includes an interactive Mathematica file containing both analytic and numerical unitarity bounds for four-fermion operators in the three-flavor scenario, under the minimal flavor violation and  $U(2)^5$  flavor assumptions [72,73].

Lastly, for operators of the type  $X^3$  involving three gauge field strengths, the unitarity bounds were computed using amplitudes that include both the SM contribution

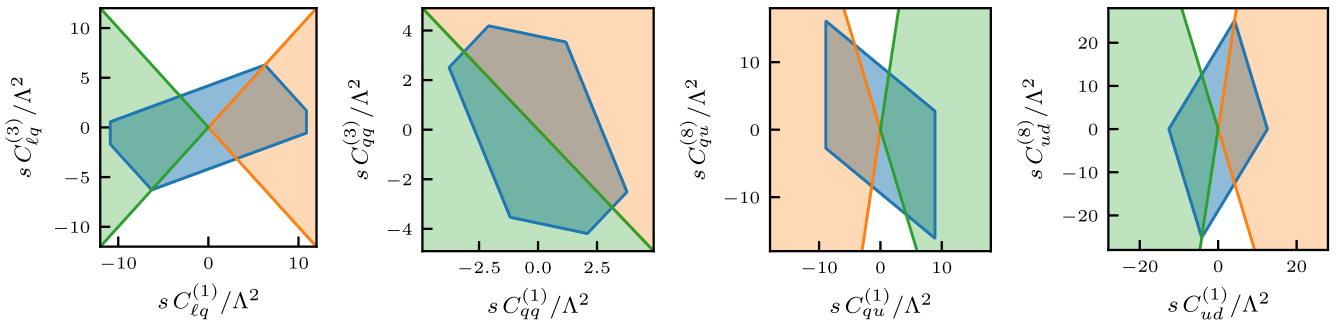


FIG. 2. Parameter space allowed by unitarity bounds (blue) and spinning sum rules in the cases of scalar (orange) and vector (green) dominance, in the single-flavor limit. The missing plot for the Wilson coefficients  $C_{qd}^{(1,8)}$  is identical to that of  $C_{qu}^{(1,8)}$ .

and at most one SMEFT insertion, effectively truncating the calculation at  $\mathcal{O}(1/\Lambda^2)$  at the amplitude level. Contributions of order  $\mathcal{O}(1/\Lambda^4)$  could, in principle, yield stronger bounds, especially when the gauge coupling is small; however, including them without simultaneously accounting for dimension-8 operators would be inconsistent. Indeed, from positivity constraints, the presence of a dimension-6 operator of type  $X^3$  necessarily implies the existence of a dimension-8 operator of type  $X^4$ , which would inevitably affect the resulting unitarity bounds; see Ref. [74].

Comparing the results in Table I with previous SMEFT analyses [10–12], we derive unitarity bounds for the complete set of four-fermion operators and for operators involving at least one gluon field, which were not covered in those works. With this method, such bounds are straightforward to obtain compared to alternative approaches. Moreover, the fully exploited coupled-channel analysis allows us to derive stronger bounds than those previously reported.

Overall, our bounds on purely bosonic operators agree with those in Ref. [12], except for a factor-of-2 difference arising from Eq. (3), as that work conventionally imposed  $|a'_{i \rightarrow f}| \leq 2$  for the partial-wave coefficients. In contrast, the bounds on operators involving fermions exhibit larger discrepancies, which we attribute to the combined effects of the coupled-channel analysis and the marginalization procedure. In particular, operators of the type  $\psi^2 X \varphi$  and  $\psi^2 \varphi^2 D$  show case-by-case differences of up to a factor of 2. Overall, our bounds are always more stringent, except for  $O_{\varphi ud}$ , for which our bound is weaker by a factor of  $\sqrt{2}$ . Further details will be presented elsewhere.

**Spinning sum rules:** The analyticity of the S matrix, together with partial-wave unitarity, allows one to derive additional theoretical constraints on effective four-fermion interactions beyond the unitarity bounds discussed above. In particular, Refs. [62–68] have introduced new dispersion relations that yield either spin-dependent sum rules or constraints on the high-energy behavior of scattering amplitudes. Remarkably, the resulting pattern of SMEFT inequalities can be used to infer characteristic features of the underlying UV dynamics, indicating whether it is dominated by scalar or vector particles. A detailed derivation of such constraints is provided in Ref. [67], where the authors used a basis that is different from the more conventional Warsaw basis. For this purpose, we report in Appendix B some technical details on sum rules, as well as the corresponding inequalities regarding  $\psi^4$  SMEFT Wilson coefficients in the Warsaw basis.

As an illustration, Fig. 2 shows the parameter space allowed by the unitarity bounds (blue) and the spinning sum rules under scalar (orange) and vector (green) dominance for four-fermion Wilson coefficients in the single-flavor limit. Stronger bounds generally arise in the

three-flavor scenario and can be explored using the accompanying interactive Mathematica file. We stress, however, that unlike the partial-wave unitarity bounds, which follow solely from the consistency of the EFT, the spinning sum rules are considerably more model-dependent and hold only under a specific set of assumptions whose violation invalidates the corresponding inequalities [65–68]. In particular: (i) they assume tree-level UV completions and can be violated at one loop [66]; (ii) they assume dominance by states of definite spin, so that completions involving a nontrivial interplay of spin-0 and spin-1 states can evade them, since the relevant dispersion relation in Eq. (B1) ceases to be sign definite; and (iii) they rely on the vanishing of boundary terms in the dispersion relation, which fails in particular for vector  $t$ -channel UV completions [66–68].

This residual model dependence is, however, physically informative: a coefficient lying outside the scalar- and vector-dominated regions does not exclude a UV completion but points to a more structured one, such as a vector  $t$ -channel mediator, a loop-level origin, or a mixed-spin spectrum. In Appendix C we have verified this picture explicitly by matching to the dictionary of tree-level UV completions of Ref. [75]: all scalar completions satisfy the sum-rule constraints, whereas vector  $t$ -channel completions can violate them, in agreement with limitation (iii).

**Phenomenology**—To assess the phenomenological impact of the theoretical bounds discussed above, it is useful to examine their complementarity with both current and anticipated future experimental constraints.

We first analyze the unitarity bounds for purely bosonic operators. Figure 3 shows the minimum energy scales  $E_*$  at which the bounds in Table I are violated, assuming Wilson coefficients are taken from the 95% CL marginalized intervals of Ref. [76], using linear (blue) and quadratic (orange) fits. Moreover, Fig. 4 (left) presents both the unitarity bounds and the experimental constraints from LEP and (HL-)LHC diboson data on the Wilson coefficients of the operators  $O_{\varphi q}^{(1)}$  and  $O_{\varphi q}^{(3)}$ . The experimental limits correspond to 95% CL marginalized intervals obtained from a linear fit [76]. As shown, the unitarity bounds—scaling as  $(\sqrt{s}/4 \text{ TeV})^2$ —are significantly stronger than those from LEP alone, but remain weaker than the constraints derived from the combined LEP and (HL-)LHC data.

As a second example, we consider the global analysis of LHC Run II data in the top sector induced by the four-quark operators  $O_{Qq}^{1,8}$  and  $O_{Qq}^{3,8}$ , where the experimental limits are obtained from a quadratic fit [77]. Most of the measurements involve final states with a top-quark pair, including kinematic distributions, the charge asymmetry, and associated top-pair production with a weak boson. As shown in Fig. 4 (middle), the unitarity bounds are quite competitive with the LHC experimental limits. Interestingly, the inclusion of sum rules—corresponding to an underlying scalar

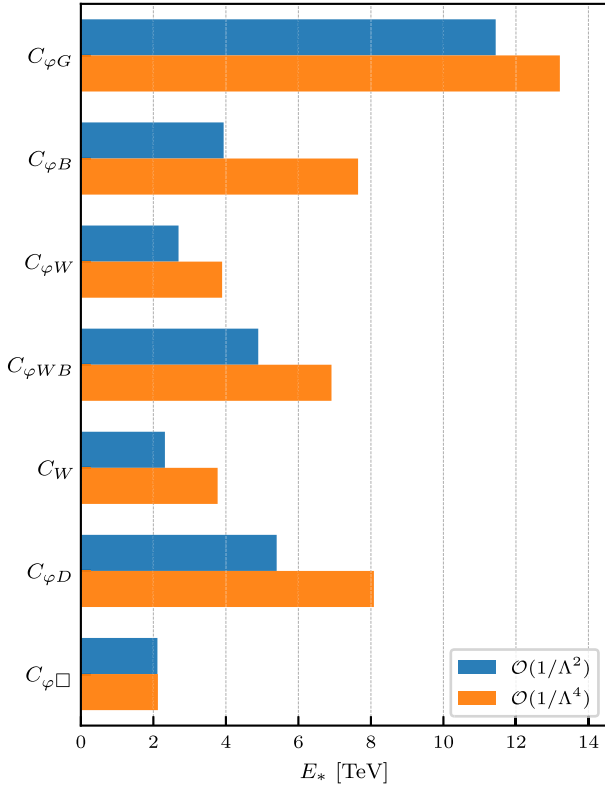


FIG. 3. Minimum energy scales  $E_*$  at which the unitarity bounds in Table I are violated for each purely bosonic Wilson coefficient in [76], using the 95% CL marginalized intervals from linear (blue) and quadratic (orange) fits.

or vector tree-level mediator of the four-fermion interactions—selects complementary regions of the parameter space, thereby providing insight into the nature of the underlying NP dynamics.

As a last example, we consider the comprehensive analysis of electroweak, flavor, and collider constraints on semileptonic four-fermion operators  $[O_{\ell q}^{(1)}]_{3333}$  and  $[O_{\ell q}^{(3)}]_{3333}$  in the  $U(2)^5$  scenario of Ref. [78], where NP predominantly couples to the third generation. Also in this case, as shown in Fig. 4 (right), the unitarity bounds and the sum-rule constraints are highly complementary and competitive with the experimental limits. In particular, it is noteworthy that the  $1\sigma$  region of the global fit lies outside the regions preferred by the sum rules. If the current experimental situation is confirmed and further strengthened, this would favor, for instance, an underlying  $t$ -channel UV completion.

As a final remark, the unitarity bounds constrain the combination  $\hat{s}|C|/\Lambda^2$ , so that the corresponding limits become increasingly stringent at larger values of the partonic center-of-mass energy  $\sqrt{\hat{s}}$ . Since perturbative unitarity determines the range of validity of the EFT, the relevant scale entering the bound is  $\sqrt{\hat{s}}$ , rather than the collider center-of-mass energy itself. This is particularly relevant for the HL-LHC: although it operates at the same collider energy as the LHC, its much larger integrated luminosity enables precision measurements deeper into the high-energy tails of differential distributions, thereby

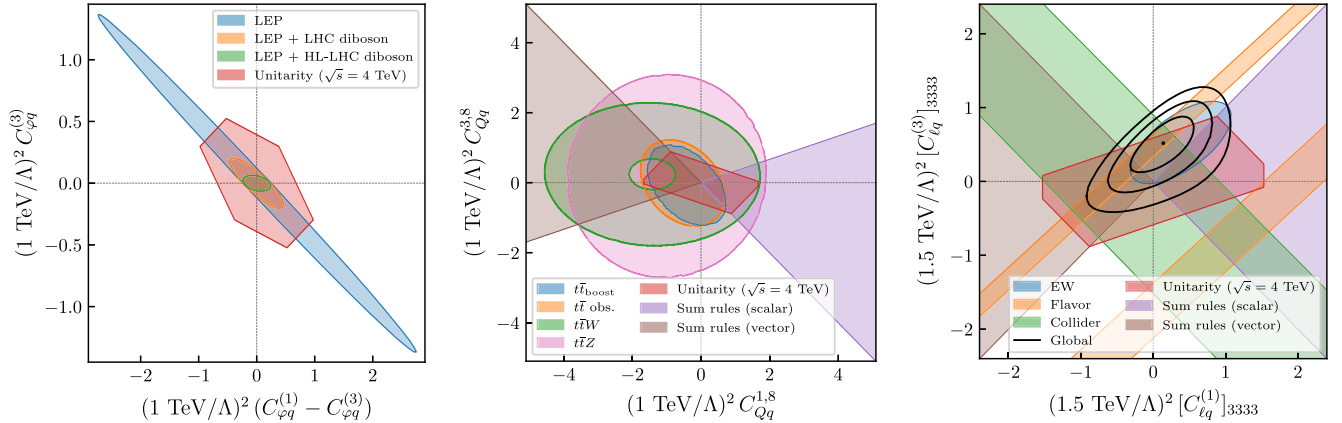


FIG. 4. Left: experimental and unitarity constraints on the Wilson coefficients of the operators  $O_{\varphi q}^{(1)} = \sum_{p=1,2} i(\varphi^\dagger \vec{D}_\mu \varphi)(\bar{q}_p \gamma^\mu q_p)$  and  $O_{\varphi q}^{(3)} = \sum_{p=1,2} i(\varphi^\dagger \vec{D}_\mu \varphi)(\bar{q}_p \sigma^I \gamma^\mu q_p)$ . Experimental limits correspond to 95% CL marginalized intervals [76]. Middle: experimental and unitarity constraints on the Wilson coefficients of the operators  $O_{Qq}^{1,8} = \sum_{p=1,2} (\bar{q}_3 \gamma_\mu T^A q_3)(\bar{q}_p \gamma^\mu T^A q_p)$  and  $O_{Qq}^{3,8} = \sum_{p=1,2} (\bar{q}_3 \gamma_\mu T^A \sigma^I q_3)(\bar{q}_p \gamma^\mu T^A \sigma^I q_p)$ . Experimental limits correspond to 68% CL and are derived from a quadratic fit [77]. Here, the sum rules select the region where  $C_{Qq}^{1,8} - (1 \pm 2)C_{Qq}^{3,8}$  are positive (negative) for scalar (vector) completions. Right: experimental and unitarity constraints on the Wilson coefficients of the operators  $[O_{\ell q}^{(1)}]_{3333} = (\bar{\ell}_3 \gamma^\mu \ell_3)(\bar{q}_3 \gamma_\mu q_3)$  and  $[O_{\ell q}^{(3)}]_{3333} = (\bar{\ell}_3 \gamma^\mu \sigma^I \ell_3)(\bar{q}_3 \gamma_\mu \sigma^I q_3)$ . Experimental limits correspond to 68% CL [78]. These experimental contours are not recomputed by imposing an additional unitarity cut; the unitarity bounds should instead be interpreted as a theoretical consistency overlay.

extending the range of partonic scales over which the SMEFT description can be meaningfully tested. Accordingly, while most collider constraints discussed here probe partonic center-of-mass energies of order  $\sqrt{\hat{s}} \sim 2$  TeV, we display the unitarity bounds at  $\sqrt{\hat{s}} = 4$  TeV as a representative benchmark for the multi-TeV regime accessible in HL-LHC measurements. In realistic analyses with a broad partonic energy spectrum, the unitarity condition should instead be evaluated at the value of  $\sqrt{\hat{s}}$  relevant for each event or kinematic bin, thereby defining the maximum hard scale up to which the SMEFT description remains perturbatively self-consistent.

**Conclusions**—Nonrenormalizable interactions induce scattering amplitudes that grow with energy, potentially leading to violations of perturbative unitarity at high scales. A consistent interpretation of experimental data, particularly at the (High-Luminosity) LHC and future high-energy colliders, therefore requires quantitative theoretical criteria to assess the range of validity of the SMEFT description.

In this Letter, we have performed a comprehensive reassessment of perturbative unitarity bounds on all dimension-6 SMEFT Wilson coefficients (see Table I), significantly extending and sharpening the partial results previously reported in the literature. In particular, the unitarity bound associated with each Wilson coefficient was obtained here by marginalizing over all other SMEFT coefficients (see Figs. 1 and 2). We performed a fully coupled-channel analysis, including all allowed initial and final states across helicity, gauge, and flavor configurations, thereby deriving stronger bounds than previously reported. We derived unitarity bounds for the complete set of four-fermion operators as well as for operators involving gluon fields, which were not covered in previous SMEFT studies [10–12].

As shown in Fig. 4, these theoretical constraints are already competitive with—and in some cases stronger than—the corresponding experimental bounds at energy scales above a few TeV. This is particularly relevant for four-fermion operators under realistic flavor assumptions motivated by the SM flavor puzzle, where the unitarity limits can be further strengthened by imposing sum rules.

More broadly, the framework developed here establishes the first systematic, perturbativity-based criterion that naturally incorporates operator correlations and can be directly implemented in present and future global SMEFT analyses, collider phenomenology, and Monte Carlo event generation. The results derived in this Letter strengthen the connection between theoretical and experimental constraints in future searches for physics beyond the standard model.

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**Data availability**—There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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## End Matter

*Appendix A: SMEFT dimension-6 operators*—In Table II we present the baryon-number-conserving dimension-6 operators of the SMEFT in the Warsaw basis [2]. The relevant conventions are as follows:  $\tilde{\varphi}^j = \epsilon_{jk}(\varphi^k)^*$  (with  $\epsilon_{12} = 1$ ),  $\varphi^\dagger \overleftrightarrow{D}_\mu \varphi = \varphi^\dagger D_\mu \varphi - (D_\mu \varphi)^\dagger \varphi$ , and  $\varphi^\dagger \overleftrightarrow{D}_\mu^I \varphi = \varphi^\dagger \sigma^I D_\mu \varphi - (D_\mu \varphi)^\dagger \sigma^I \varphi$ , dual field-strength tensors are defined as  $\tilde{X}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} X^{\rho\sigma}$  (with  $X = B, W^I, G^A$  and  $\epsilon_{0123} = 1$ ),  $\lambda_{\alpha\beta}^A$  are the Gell-Mann matrices and  $\sigma_{jk}^I$  are the Pauli matrices,  $T_{\alpha\beta}^A = \frac{1}{2} \lambda_{\alpha\beta}^A$  are the SU(3) generators, and  $\sigma_{\mu\nu} = (i/2)[\gamma_\mu, \gamma_\nu]$ . The metric, whose signature is relevant for the spinning sum rules discussed in Appendix B, is  $g_{\mu\nu} = \text{diag}(+1, -1, -1, -1)$ .

*Appendix B: Further details on spinning sum rules*—In this appendix, we provide further technical details regarding the spinning sum rules discussed in the main

text. Following Ref. [67], we consider the amplitude  $\mathcal{A}_{\alpha\beta}(s, t)$ , where  $s$  and  $t$  are the usual Mandelstam variables associated with an elastic scattering process of two fermions whose quantum numbers (e.g., flavor and helicity) are collectively denoted by  $\alpha$  and  $\beta$ . Expanding the UV amplitude in partial waves of fixed total angular momentum,  $a_{\alpha\beta}^J(s)$ , and assuming vanishing boundary terms at infinity, one finds [67]

$$\lim_{s \rightarrow 0} \partial_s \mathcal{A}_{\alpha\beta}(s, 0) = -\lim_{s \rightarrow 0} \partial_s \mathcal{A}_{\alpha\beta}(s, 0) = 8 \int_{s_0}^{\infty} \frac{ds}{s^2} (\text{Im} a_{\alpha\beta}^0(s) - 3 \text{Im} a_{\alpha\beta}^1(s)), \quad (\text{B1})$$

where  $s_0$  is the characteristic mass-squared scale of the UV completion. (Note that our partial-wave coefficients are normalized differently from those of Ref. [67]). Dominance of spin-0 (spin-1) states in the UV theory

TABLE II. Dimension-6 operators of the SMEFT in the Warsaw basis [2] that conserve baryon number. Non-Hermitian operators are denoted with  $^\dagger O$  and the corresponding Hermitian conjugates must be included in the Lagrangian.

| $X^3$                | $\phi^6$ and $\phi^4 D^2$                                         |                   |                                                                      | $\psi^2 \phi^3$      | $(\bar{L}L)(\bar{L}L)$                                                                      |                        | $(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$                              |                               |                                                                                       |
|----------------------|-------------------------------------------------------------------|-------------------|----------------------------------------------------------------------|----------------------|---------------------------------------------------------------------------------------------|------------------------|--------------------------------------------------------------------------------|-------------------------------|---------------------------------------------------------------------------------------|
| $O_G$                | $f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$                | $O_\phi$          | $(\phi^\dagger \phi)^3$                                              | $^\dagger O_{e\phi}$ | $(\phi^\dagger \phi)(\bar{\ell}_p e_r \phi)$                                                | $O_{\ell\ell}$         | $(\bar{\ell}_p \gamma_\mu \ell_r)(\bar{\ell}_s \gamma^\mu \ell_t)$             | $^\dagger O_{\ell edq}$       | $(\bar{\ell}_p^j e_r)(\bar{d}_s q_{tj})$                                              |
| $O_{\tilde{G}}$      | $f^{ABC} \tilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$        | $O_{\phi\Box}$    | $(\phi^\dagger \phi)\Box(\phi^\dagger \phi)$                         | $^\dagger O_{u\phi}$ | $(\phi^\dagger \phi)(\bar{q}_p u_r \tilde{\phi})$                                           | $O_{qq}^{(1)}$         | $(\bar{q}_p \gamma_\mu q_r)(\bar{q}_s \gamma^\mu q_t)$                         | $^\dagger O_{quqd}^{(1)}$     | $(\bar{q}_p^j u_r) \epsilon_{jk}(\bar{q}_s^k d_t)$                                    |
| $O_W$                | $\epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$         | $O_{\phi D}$      | $(\phi^\dagger D^\mu \phi)^*(\phi^\dagger D_\mu \phi)$               | $^\dagger O_{d\phi}$ | $(\phi^\dagger \phi)(\bar{q}_p d_r \phi)$                                                   | $O_{qq}^{(3)}$         | $(\bar{q}_p \gamma_\mu \sigma^I q_r)(\bar{q}_s \gamma^\mu \sigma^I q_t)$       | $^\dagger O_{quqd}^{(8)}$     | $(\bar{q}_p^j T^A u_r) \epsilon_{jk}(\bar{q}_s^k T^A d_t)$                            |
| $O_{\tilde{W}}$      | $\epsilon^{IJK} \tilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$ |                   |                                                                      |                      |                                                                                             | $O_{\ell q}^{(1)}$     | $(\bar{\ell}_p \gamma_\mu \ell_r)(\bar{q}_s \gamma^\mu q_t)$                   | $^\dagger O_{\ell equ}^{(1)}$ | $(\bar{\ell}_p^j e_r) \epsilon_{jk}(\bar{q}_s^k u_t)$                                 |
|                      |                                                                   |                   |                                                                      |                      |                                                                                             | $O_{\ell q}^{(3)}$     | $(\bar{\ell}_p \gamma_\mu \sigma^I \ell_r)(\bar{q}_s \gamma^\mu \sigma^I q_t)$ | $^\dagger O_{\ell equ}^{(3)}$ | $(\bar{\ell}_p^j \sigma_{\mu\nu} e_r) \epsilon_{jk}(\bar{q}_s^k \sigma^{\mu\nu} u_t)$ |
|                      |                                                                   |                   |                                                                      |                      |                                                                                             |                        |                                                                                |                               |                                                                                       |
| $X^2 \phi^2$         | $\psi^2 X \phi$                                                   |                   |                                                                      | $\psi^2 \phi^2 D$    |                                                                                             | $(\bar{R}R)(\bar{R}R)$ |                                                                                | $(\bar{L}L)(\bar{R}R)$        |                                                                                       |
| $O_{\phi G}$         | $(\phi^\dagger \phi) G_\mu^A G^{A\mu}$                            | $^\dagger O_{eW}$ | $(\bar{\ell}_p \sigma^{\mu\nu} e_r) \sigma^I \phi W_{\mu\nu}^I$      | $O_{\phi\ell}^{(1)}$ | $(\phi^\dagger i \overleftrightarrow{D}_\mu \phi)(\bar{\ell}_p \gamma^\mu \ell_r)$          | $O_{ee}$               | $(\bar{e}_p \gamma_\mu e_r)(\bar{e}_s \gamma^\mu e_t)$                         | $O_{\ell e}$                  | $(\bar{\ell}_p \gamma_\mu \ell_r)(\bar{e}_s \gamma^\mu e_t)$                          |
| $O_{\phi\tilde{G}}$  | $(\phi^\dagger \phi) \tilde{G}_\mu^A G^{A\mu}$                    | $^\dagger O_{eB}$ | $(\bar{\ell}_p \sigma^{\mu\nu} e_r) \phi B_{\mu\nu}$                 | $O_{\phi\ell}^{(3)}$ | $(\phi^\dagger i \overleftrightarrow{D}_\mu \phi)(\bar{\ell}_p \sigma^I \gamma^\mu \ell_r)$ | $O_{uu}$               | $(\bar{u}_p \gamma_\mu u_r)(\bar{u}_s \gamma^\mu u_t)$                         | $O_{\ell u}$                  | $(\bar{\ell}_p \gamma_\mu \ell_r)(\bar{u}_s \gamma^\mu u_t)$                          |
| $O_{\phi W}$         | $(\phi^\dagger \phi) W_\mu^I W^{I\mu}$                            | $^\dagger O_{uG}$ | $(\bar{q}_p \sigma^{\mu\nu} T^A u_r) \tilde{\phi} G_{\mu\nu}^A$      | $O_{\phi e}$         | $(\phi^\dagger i \overleftrightarrow{D}_\mu \phi)(\bar{e}_p \gamma^\mu e_r)$                | $O_{dd}$               | $(\bar{d}_p \gamma_\mu d_r)(\bar{d}_s \gamma^\mu d_t)$                         | $O_{\ell d}$                  | $(\bar{\ell}_p \gamma_\mu \ell_r)(\bar{d}_s \gamma^\mu d_t)$                          |
| $O_{\phi\tilde{W}}$  | $(\phi^\dagger \phi) \tilde{W}_\mu^I W^{I\mu}$                    | $^\dagger O_{uW}$ | $(\bar{q}_p \sigma^{\mu\nu} u_r) \sigma^I \tilde{\phi} W_{\mu\nu}^I$ | $O_{\phi q}^{(1)}$   | $(\phi^\dagger i \overleftrightarrow{D}_\mu \phi)(\bar{q}_p \gamma^\mu q_r)$                | $O_{eu}$               | $(\bar{e}_p \gamma_\mu e_r)(\bar{u}_s \gamma^\mu u_t)$                         | $O_{qe}$                      | $(\bar{q}_p \gamma_\mu q_r)(\bar{e}_s \gamma^\mu e_t)$                                |
| $O_{\phi B}$         | $(\phi^\dagger \phi) B_{\mu\nu} B^{\mu\nu}$                       | $^\dagger O_{uB}$ | $(\bar{q}_p \sigma^{\mu\nu} u_r) \tilde{\phi} B_{\mu\nu}$            | $O_{\phi q}^{(3)}$   | $(\phi^\dagger i \overleftrightarrow{D}_\mu \phi)(\bar{q}_p \sigma^I \gamma^\mu q_r)$       | $O_{ed}$               | $(\bar{e}_p \gamma_\mu e_r)(\bar{d}_s \gamma^\mu d_t)$                         | $O_{qu}^{(1)}$                | $(\bar{q}_p \gamma_\mu q_r)(\bar{u}_s \gamma^\mu u_t)$                                |
| $O_{\phi\tilde{B}}$  | $(\phi^\dagger \phi) \tilde{B}_{\mu\nu} B^{\mu\nu}$               | $^\dagger O_{dG}$ | $(\bar{q}_p \sigma^{\mu\nu} T^A d_r) \phi G_{\mu\nu}^A$              | $O_{\phi u}$         | $(\phi^\dagger i \overleftrightarrow{D}_\mu \phi)(\bar{u}_p \gamma^\mu u_r)$                | $O_{ud}^{(1)}$         | $(\bar{u}_p \gamma_\mu u_r)(\bar{d}_s \gamma^\mu d_t)$                         | $O_{qu}^{(8)}$                | $(\bar{q}_p \gamma_\mu T^A q_r)(\bar{u}_s \gamma^\mu T^A u_t)$                        |
| $O_{\phi WB}$        | $(\phi^\dagger \sigma^I \phi) W_\mu^I B^{\mu\nu}$                 | $^\dagger O_{dW}$ | $(\bar{q}_p \sigma^{\mu\nu} d_r) \sigma^I \phi W_{\mu\nu}^I$         | $O_{\phi d}$         | $(\phi^\dagger i \overleftrightarrow{D}_\mu \phi)(\bar{d}_p \gamma^\mu d_r)$                | $O_{ud}^{(8)}$         | $(\bar{u}_p \gamma_\mu T^A u_r)(\bar{d}_s \gamma^\mu T^A d_t)$                 | $O_{qd}^{(1)}$                | $(\bar{q}_p \gamma_\mu q_r)(\bar{d}_s \gamma^\mu d_t)$                                |
| $O_{\phi\tilde{W}B}$ | $(\phi^\dagger \sigma^I \phi) \tilde{W}_\mu^I B^{\mu\nu}$         | $^\dagger O_{dB}$ | $(\bar{q}_p \sigma^{\mu\nu} d_r) \phi B_{\mu\nu}$                    | $^\dagger O_{qud}$   | $(\tilde{\phi}^\dagger i D_\mu \phi)(\bar{u}_p \gamma^\mu d_r)$                             |                        |                                                                                | $O_{qd}^{(8)}$                | $(\bar{q}_p \gamma_\mu T^A q_r)(\bar{d}_s \gamma^\mu T^A d_t)$                        |

implies that the left-hand side of Eq. (B1) is positive (negative) definite for any choice of  $\alpha$  and  $\beta$ . This property can be exploited to derive sum rules among the flavor-dependent entries of the Wilson coefficients of  $\psi^4$  SMEFT operators, or between different  $\psi^4$  operators with the same fermionic content. For completeness, we report in the following the sum-rule bounds associated with SMEFT operators in the Warsaw basis, since in Ref. [67] different conventions are used. In particular, the combinations of Wilson coefficients reported in Table III, when contracted with  $\alpha_p \alpha_r^* \beta_s^* \beta_t$ , are positive (negative) if the completion is dominated by scalars (vectors). As a final remark, we stress that these bounds are subject to some limitations that may lead to their violation, as already discussed in the main text.

*Appendix C: Matching to UV completions*—To illustrate both the diagnostic power and the limitations of the spinning sum rules discussed in the main text and Appendix B, we match a representative set of single-field, tree-level UV completions [75] onto the semileptonic operators  $[O_{\ell q}^{(1)}]_{3333}$  and  $[O_{\ell q}^{(3)}]_{3333}$ , whose unitarity and sum-rule constraints are shown in Fig. 4 (right). The results are collected in Table IV.

The two scalar leptoquarks ( $\omega_1$ ,  $\zeta$ ) and the two vector leptoquarks ( $\mathcal{U}_2$ ,  $\mathcal{X}$ ) generate these operators through  $s$ -channel exchange in the elastic  $\ell q \rightarrow \ell q$  amplitude, and therefore enter the dispersion relation of Eq. (B1) with their physical spin. Explicitly, the relevant combinations of

Table III,  $[C_{\ell q}^{(1)}]_{3333} \pm [C_{\ell q}^{(3)}]_{3333}$ , are both nonnegative for  $\omega_1$  and  $\zeta$ , and both nonpositive for  $\mathcal{U}_2$  and  $\mathcal{X}$ , so that the

 TABLE III. Combinations of four-fermion Wilson coefficients that, when contracted with  $\alpha_p \alpha_r^* \beta_s^* \beta_t$ , are positive (negative) if the UV completion is dominated by scalars (vectors).

| Operator class         | Spinning sum rules                                                                                                                                                                                                                                                                                                                                                          |
|------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $(\bar{R}R)(\bar{R}R)$ | $[C_{ee}]_{prst}$<br>$[C_{uu}]_{prst}, [C_{uu}]_{prst} + [C_{uu}]_{ptsr}$<br>$[C_{dd}]_{prst}, [C_{dd}]_{prst} + [C_{dd}]_{ptsr}$<br>$[C_{eu}]_{prst}, [C_{ed}]_{prst}$<br>$[C_{ud}^{(1)}]_{prst} + \frac{1}{3}[C_{ud}^{(8)}]_{prst}$<br>$[C_{ud}^{(1)}]_{prst} - \frac{1}{6}[C_{ud}^{(8)}]_{prst}$                                                                         |
| $(\bar{L}L)(\bar{L}L)$ | $[C_{\ell\ell}]_{prst}, [C_{\ell\ell}]_{prst} + [C_{\ell\ell}]_{ptsr}$<br>$[C_{qq}^{(1)}]_{prst} + [C_{qq}^{(1)}]_{ptsr} + [C_{qq}^{(3)}]_{prst} + [C_{qq}^{(3)}]_{ptsr}$<br>$[C_{qq}^{(1)}]_{prst} - [C_{qq}^{(3)}]_{prst} + 2[C_{qq}^{(3)}]_{ptsr}$<br>$[C_{\ell q}^{(1)}]_{prst} + [C_{\ell q}^{(3)}]_{prst}$<br>$[C_{\ell q}^{(1)}]_{prst} - [C_{\ell q}^{(3)}]_{prst}$ |
| $(\bar{L}L)(\bar{R}R)$ | $-[C_{\ell e}]_{prst}, -[C_{\ell u}]_{prst}$<br>$-[C_{\ell d}]_{prst}, -[C_{qe}]_{prst}$<br>$-[C_{qu}^{(1)}]_{prst} - \frac{1}{3}[C_{qu}^{(8)}]_{prst}$<br>$-[C_{qu}^{(1)}]_{prst} + \frac{1}{6}[C_{qu}^{(8)}]_{prst}$<br>$-[C_{qd}^{(1)}]_{prst} - \frac{1}{3}[C_{qd}^{(8)}]_{prst}$<br>$-[C_{qd}^{(1)}]_{prst} + \frac{1}{6}[C_{qd}^{(8)}]_{prst}$                        |

TABLE IV. Tree-level matching of single-field UV completions [75] onto  $[O_{\ell q}^{(1)}]_{3333}$  and  $[O_{\ell q}^{(3)}]_{3333}$ . The penultimate column reports the associated perturbative unitarity bound, obtained as the most stringent of the constraints for  $[C_{\ell q}^{(1)}]_{3333}$  and  $[C_{\ell q}^{(3)}]_{3333}$  evaluated on the matched Wilson coefficients. The last column shows the plane  $(s[C_{\ell q}^{(1)}]_{3333}/\Lambda^2, s[C_{\ell q}^{(3)}]_{3333}/\Lambda^2)$ : the thick dark line denotes where each model lies, the medium-gray polygon is the region allowed by unitarity, and the light-gray wedges on the right and on the left are the regions allowed by the spinning sum rules for scalar and vector dominance, respectively. The scalar and vector leptoquarks are correctly classified by their spin, whereas the  $t$ -channel vectors evade the classification.

| Field           | Representation          | Spin | Channel | $[C_{\ell q}^{(1)}]_{3333}/\Lambda^2$                                      | $[C_{\ell q}^{(3)}]_{3333}/\Lambda^2$                                         | Unitarity bound                                                                                 | Plot |
|-----------------|-------------------------|------|---------|----------------------------------------------------------------------------|-------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------|------|
| $\omega_1$      | $(3, 1)_{-\frac{1}{3}}$ | 0    | $s$     | $ [y_{\omega_1}^{q\ell}]_{33} ^2/(4M_{\omega_1}^2)$                        | $- [y_{\omega_1}^{q\ell}]_{33} ^2/(4M_{\omega_1}^2)$                          | $ [y_{\omega_1}^{q\ell}]_{33} ^2 s \leq 4\pi M_{\omega_1}^2$                                    |      |
| $\zeta$         | $(3, 3)_{-\frac{1}{3}}$ | 0    | $s$     | $3 [y_{\zeta}^{q\ell}]_{33} ^2/(4M_{\zeta}^2)$                             | $ [y_{\zeta}^{q\ell}]_{33} ^2/(4M_{\zeta}^2)$                                 | $ [y_{\zeta}^{q\ell}]_{33} ^2 s \leq 4\pi M_{\zeta}^2$                                          |      |
| $\mathcal{U}_2$ | $(3, 1)_{\frac{2}{3}}$  | 1    | $s$     | $- [g_{\mathcal{U}_2}^{\ell q}]_{33} ^2/(2M_{\mathcal{U}_2}^2)$            | $- [g_{\mathcal{U}_2}^{\ell q}]_{33} ^2/(2M_{\mathcal{U}_2}^2)$               | $ [g_{\mathcal{U}_2}^{\ell q}]_{33} ^2 s \leq 4\pi M_{\mathcal{U}_2}^2$                         |      |
| $\mathcal{X}$   | $(3, 3)_{\frac{2}{3}}$  | 1    | $s$     | $-3 [g_{\mathcal{X}}]_{33} ^2/(8M_{\mathcal{X}}^2)$                        | $ [g_{\mathcal{X}}]_{33} ^2/(8M_{\mathcal{X}}^2)$                             | $ [g_{\mathcal{X}}]_{33} ^2 s \leq (16/3)\pi M_{\mathcal{X}}^2$                                 |      |
| $\mathcal{B}$   | $(1, 1)_0$              | 1    | $t$     | $-[g_{\mathcal{B}}^{\ell}]_{33}[g_{\mathcal{B}}^q]_{33}/M_{\mathcal{B}}^2$ | 0                                                                             | $ [g_{\mathcal{B}}^{\ell}]_{33}[g_{\mathcal{B}}^q]_{33}  s \leq 2\sqrt{3}\pi M_{\mathcal{B}}^2$ |      |
| $\mathcal{W}$   | $(1, 3)_0$              | 1    | $t$     | 0                                                                          | $-[g_{\mathcal{W}}^{\ell}]_{33}[g_{\mathcal{W}}^q]_{33}/(4M_{\mathcal{W}}^2)$ | $ [g_{\mathcal{W}}^{\ell}]_{33}[g_{\mathcal{W}}^q]_{33}  s \leq (16/3)\pi M_{\mathcal{W}}^2$    |      |

scalar (vector) leptoquarks are correctly placed in the scalar- (vector-) dominated region of Fig. 4 (right).

By contrast, the color-singlet vectors  $\mathcal{B} \sim (1, 1)_0$  and  $\mathcal{W} \sim (1, 3)_0$  couple separately to the lepton and quark currents and are exchanged in the  $t$  channel. The boundary terms then no longer vanish, modifying Eq. (B1), and the sum-rule classification fails:  $\mathcal{W}$  yields combinations of opposite sign and thus lies outside both the scalar- and

vector-dominated regions, while  $\mathcal{B}$  can fall in the region associated with the opposite (scalar) spin, depending on the sign of  $[g_{\mathcal{B}}^{\ell}]_{33}[g_{\mathcal{B}}^q]_{33}$ . These cases provide explicit examples of the statement made in the main text that vector  $t$ -channel completions can violate the spinning sum-rule constraints, whereas all tree-level scalar completions satisfy them.