

Nonreciprocity as a Generic Mechanism for Demixing in Flocking Mixtures

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(Received 16 February 2026; accepted 2 June 2026; published 29 June 2026)

We show that even weak nonreciprocal alignment leads to large-scale structure formation in flocking mixtures. By combining numerical simulations of a binary Vicsek model and the analysis of coarse-grained continuum equations, we demonstrate that nonreciprocity destabilizes the ordered phase formed by mutually aligning or antialigning species in a large part of the phase diagram. For aligning populations, this instability results in one species condensing in a single band that travels within a homogeneous liquid of the other species. When interactions are antialigning, both species self-assemble into polar clusters with large-scale chaotic dynamics. In both cases, the emergence of structures is accompanied by the demixing of the two species, despite the absence of repulsive interactions. Our theoretical analysis allows us to elucidate the origin of the instability, and show that it is generic to nonreciprocal flocks.

DOI: [10.1103/v435-qnhl](https://doi.org/10.1103/v435-qnhl)

In recent years, nonreciprocal interactions have been recognized as an essential feature of systems that evolve far from equilibrium, such as active matter [1–3]. Nonreciprocity is particularly relevant to describe biological populations competing for resources through complex social interactions [2,4,5], while it arises naturally in simpler contexts, including chemotactic mixtures [6–12] and heterogeneous populations of robots [13] or colloidal rollers [14]. One of the most spectacular consequences of nonreciprocity is the emergence of collectively induced dynamical states [15–17].

Since the seminal works of Vicsek *et al.* [18] and Toner and Tu [19,20], the study of flocking agents has become a paradigmatic example of active matter [21,22]. Nonreciprocity can be introduced into flocking models by limiting the particles' range of perception, so that they interact, e.g., mainly with neighbors in front of them. This geometric nonreciprocity significantly affects the collective behavior of flocks, leading to the formation of structures such as condensates [23], lanes, or vortex states [24–26]. Another natural way in which nonreciprocity arises is when heterogeneous populations interact asymmetrically [13,17,27–37]. In this case, nonreciprocity can

induce dynamical chiral states through the crossing of exceptional points [17,31,37,38]. Importantly, such collective behaviors require *strongly* nonreciprocal interactions, resulting in antisymmetric couplings between the two populations.

In this Letter, we focus on a comparatively less explored regime, and investigate the fate of flocks subjected to weak population nonreciprocity. Considering a Vicsek model (VM) of two populations that mutually align or antialign their velocities, we demonstrate that nonreciprocal interactions generally induce large-scale structure formation associated with the spatial segregation of the two species. Unlike in previous studies, the mechanism underlying such demixing behavior solely relies on nonreciprocal alignment and self-propulsion, and does not require additional features such as population heterogeneity [14] or pairwise repulsion [29,39]. We further show, using a coarse-grained continuous model, that this phenomenology originates from a generic long-wave instability, which we trace back to the coupling between density and orientational order inherent to the physics of flocks [21].

Microscopic model—The binary VM consists of pointlike self-propelled particles in two dimensions, aligning their direction of motion with neighbors within unit distance. The position $\mathbf{r}_k^S$ and orientation θ_k^S of particle k from species $S = A, B$ evolve in discrete time according to

$$\mathbf{r}_k^S(t+1) = \mathbf{r}_k^S(t) + v_0 \hat{\mathbf{u}}[\theta_k^S(t+1)], \quad (1a)$$

$$\theta_k^S(t+1) = \arg \left[\sum_{u,j} \chi^{su} e^{i\theta_j^u(t)} + \eta^S n_k(t) e^{i\zeta_k(t)} \right]. \quad (1b)$$

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Here, v_0^S and η^S denote the self-propulsion speed and noise strength associated with species S , respectively. $\hat{\mathbf{u}}(\theta) = (\cos \theta \sin \theta)^T$ and ξ_k is a delta-correlated white noise uniformly drawn from $[-\pi, \pi]$, while $n_k(t)$ is the total number of interacting neighbors of particle k (including itself) at time t .

The sum in Eq. (1b) runs over $U = A, B$ and particles j satisfying $|\mathbf{r}_k^S - \mathbf{r}_j^U| \leq 1$, while the coupling $-1 \leq \chi^{SU} \leq 1$ defines how particles from species S align ($\chi^{SU} > 0$) or antialign ($\chi^{SU} < 0$) their velocities with particles from species U . The average coupling $\bar{\chi} \equiv \frac{1}{2}(\chi^{AB} + \chi^{BA})$ then describes the tendency of the two species to mutually align ($\bar{\chi} > 0$) or anti-align ($\bar{\chi} < 0$) their velocities. In turn, enforcing reciprocal interspecies alignment imposes that $\Delta\chi \equiv \frac{1}{2}(\chi^{AB} - \chi^{BA}) = 0$, while interactions are nonreciprocal otherwise.

When $\chi^{SU} = 1$ for all $S, U \in \{A, B\}$, the model reduces to the classical VM, while other limiting cases such as fully antialigning mixtures [40], reciprocally anti-aligning populations ($\chi^{AA, BB} > 0$, $\bar{\chi} < 0$, $\Delta\chi = 0$) [41–43] and self-antialigning species ($\chi^{AA, BB} < 0$) with aligning interspecies interactions ($\bar{\chi} > 0$) [44,45] have been addressed in the literature. Here, we let particles of the same species align ($\chi^{AA, BB} = 1$) while varying the interspecies couplings χ^{AB} and χ^{BA} . In the regime of strong nonreciprocity, where $|\Delta\chi| > |\bar{\chi}|$, we observe oscillatory behavior consistently with results reported in the literature [17,31,35]. Hereafter, however, we focus on the weakly nonreciprocal regime, where $|\Delta\chi| \leq |\bar{\chi}|$.

As we focus on the effects induced by nonreciprocity alone, we discard the possible influence of population heterogeneity [46] by setting $\eta^{A,B} = \eta$ and $v_0^{A,B} = v_0$ [47]. We consider equal densities $\rho^{A,B} = \rho_0 = N/L^2$, where N denotes the size of each population and L is the linear size of the square domain. Unless specified otherwise, we use $\rho_0 = \frac{1}{2}$, $v_0 = 1$, $\bar{\chi} = \pm \frac{1}{2}$, and vary the angular noise η and the nonreciprocity parameter $\Delta\chi$. Since the problem is symmetric under the exchange $A \leftrightarrow B$, we further restrict the analysis to $\Delta\chi \geq 0$. To construct the phase diagrams of Figs. 1(a) and 1(b), we numerically integrated Eq. (1) in periodic square domains of size $L = 512$. Regardless of the sign of $\bar{\chi}$, the system is disordered at high noise and transitions to a symmetry broken phase with nonzero global polar orders $\mathbf{\Pi}^S = \langle \hat{\mathbf{u}}(\theta_k^S) \rangle_k$ when η decreases. For all values of $\Delta\chi$, this transition is mediated by a phase-coexistence region, reminiscent of the microphase separation scenario at play in the single-species VM [49,50].

Mutually aligning populations—For $\bar{\chi} > 0$ at the onset of order, both species self-organize into dense bands propagating through a dilute, gaseous background. As shown in Fig. 1(c) and in Supplemental Material [48], bands are mostly composed of a single species and are systematically arranged in pairs, with particles having the lower interspecies coupling (here B, in blue) at the rear.

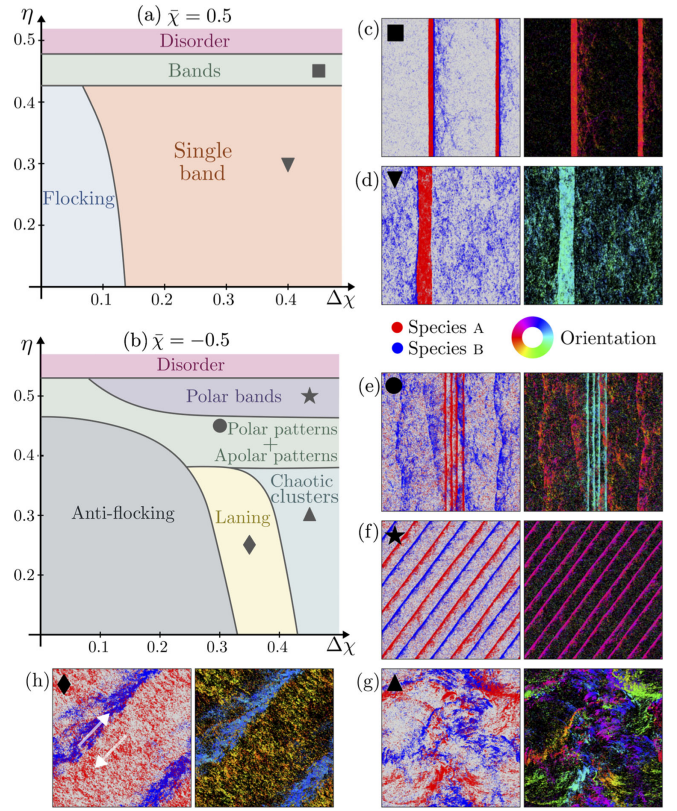


FIG. 1. Phase behavior of weakly nonreciprocal flocks. (a), (b) Stylized finite-size phase diagrams in the $(\Delta\chi, \eta)$ plane for aligning (a) and antialigning (b) intraspecies interactions. (c)–(h) Representative snapshots of the observed phases. Particles are color-coded according to their species (orientation) in the left (right) panel [caption below (d)], while the corresponding points in (a), (b) are indicated with solid symbols. Parameters: $(\eta, \Delta\chi) = (0.45, 0.45)$ (c), $(0.30, 0.40)$ (d), $(0.45, 0.30)$ (e), $(0.50, 0.45)$ (f), $(0.30, 0.45)$ (g), and $(0.25, 0.35)$ (h). Additional details on the construction of (a), (b) are provided in [48].

Such spatial synchronization by nonreciprocal alignment—also reported for discrete symmetry flocks [34,35]—can be qualitatively understood from a simple argument detailed in Appendix A.

Deeper in the ordered phase and for sufficiently low $\Delta\chi$, the species form a homogeneous flock, characterized by aligned mean polarities $\langle \mathbf{\Pi}^{A,B} \rangle_t$. However, this phase becomes unstable for sufficiently high $\Delta\chi$, where nonreciprocal alignment gives rise to traveling bands primarily made of A particles, which here align more strongly. After a coarsening process, the steady state eventually consists of a single band [Fig. 1(d)]. To characterize this structure, we use density profiles averaged along the band and over time, as depicted in Figs. 2(a) and 2(b). Contrary to the microphase coexistence region at higher noise, the density profiles reveal that the A-species band coincides with a local depletion of B particles, and moves within a homogeneous polar liquid of the B species.

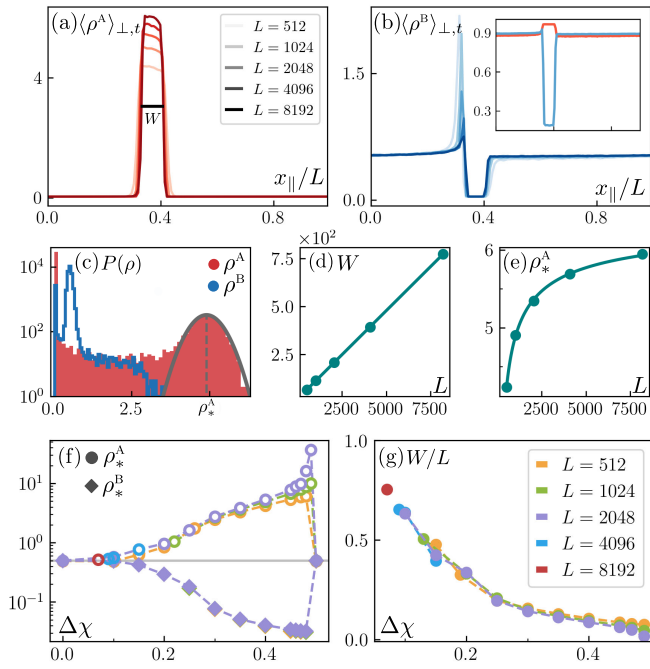


FIG. 2. Characterization of the single band phase. (a), (b) ρ^A (a) and ρ^B (b) profiles averaged along the band and over time for different system sizes. Inset of (b): the component of the local polarity profiles along the global order. (c) Density histograms at $L = 1024$. ρ_*^A is shown with a vertical dashed line, while ρ_*^B is defined as the position of left peak of the blue histogram. (d), (e) Band width (d) and density (e) as functions of system size. Symbols show the numerical data and solid lines indicate fits (see text). (f), (g) The densities in the band (f) and its normalized width (g) as functions of $\Delta\chi$ for various system sizes. The horizontal gray line in (f) indicates ρ_0 . In all panels, $\eta = 0.3$ and for (a)–(e) $\Delta\chi = 0.4$.

To determine the characteristic densities ρ_*^A and ρ_*^B within the band, we use the position of the right and left peak of the A and B density histograms, respectively, as illustrated in Fig. 2(c). The band width W is then defined from the half maximum of time-averaged A density profiles. Performing simulations in systems of increasing size, we show in Fig. 2(d) that W grows linearly with L . Moreover, ρ_*^A exhibits an unusually slow convergence to its asymptotic value, which we best fit as a power law: $\rho_*^A \simeq \rho_\infty^A - AL^{-\nu}$ with $\nu \simeq \frac{1}{2}$ [Fig. 2(e)]. These measures indicate that, up to strong finite size effects, the single-band phase displays characteristics of a phase-separated state in which both species spontaneously demix, despite the absence of repulsive interactions. Figures 2(f) and 2(g) illustrate how the properties of this phase vary with the amount of nonreciprocity. For small $\Delta\chi$, the width of the band W is comparable to the system size L , while it induces weak density modulations. As L grows, the homogeneous flocking phase becomes more susceptible to the presence of nonreciprocity, such that the demixed phase is found for increasingly smaller values of $\Delta\chi$, down to $\Delta\chi = 0.07$ for

$L = 8192$ ($N \approx 67 \times 10^6$ particles). Despite our computational efforts, we were unable to determine whether the transition from a homogeneous flock to the demixed phase arises at a finite value of $\Delta\chi$, or if the latter can survive an infinitesimal amount of nonreciprocity. Increasing $\Delta\chi$ rapidly enhances demixing [note the vertical logarithmic scale in Fig. 2(f)], such that for $\Delta\chi \lesssim \bar{\chi}$, the densities $\rho_*^{A,B}$ are orders of magnitude apart and W takes values close to zero. Remarkably, the resulting thin and dense band exists even for values of $\chi^{BA} = \bar{\chi} - \Delta\chi$ as low as 10^{-2} , while it naturally disappears when $\Delta\chi = \bar{\chi}$.

Mutually antialigning populations—For $\bar{\chi} < 0$ and low nonreciprocity, the coexistence phase at the onset of order may consist of trains of bands moving in opposite directions [Fig. 1(e)]. In these apolar arrangements, bands appear in equal number and are regularly spaced for $\Delta\chi = 0$ [48], while their structure becomes irregular at higher nonreciprocity. Consistently with results reported for $\Delta\chi = 0$ [43], these configurations coexist in a large part of the coexistence phase with polar arrangements of bands of alternating species that travel in the same direction [Fig. 1(f)]. We do not observe dynamical transitions between polar and apolar patterns, which are selected by the initial condition. Apolar configurations occur more frequently at low nonreciprocity and noise, while increasing $\Delta\chi$, polar trains arise more often [48]. In fact, the latter become the only observed configuration for sufficiently large η and $\Delta\chi$ [purple region in Fig. 1(b)].

Similarly to aligning populations, the homogeneous liquid with antialigned mean polar orders $\langle \Pi^{A,B} \rangle_t$ at $\Delta\chi = 0$ is only stable up to a finite amount of nonreciprocity. Beyond a $\Delta\chi$ threshold, the two species self-organize in a laning pattern consisting of alternating domains with opposite polarities [Fig. 1(h) and Supplemental movie SMov 1 [48]]. This pattern is highly dynamic, as the lanes persistently rotate and drift over long timescales (SMov 2 [48]), while they also intermittently break and reform. Increasing $\Delta\chi$, breaking events become more frequent, eventually preventing the pattern from repairing itself. At sufficiently high $\Delta\chi$, the populations demix by arranging into polar clusters mainly made of a single species, which move chaotically and lack macroscopic spatial order [Fig. 1(g) and SMov 3 [48]]. The lane breaking events, which are associated with a sharp decay of polar order for both species [see Fig. 4(a) in Appendix B], happen more often in larger systems (SMov 4 [48]). As a result, the system becomes progressively disordered as revealed by the distributions of $|\Pi^S|$ shown in Fig. 4(b). We therefore expect the laning pattern to eventually disappear in the thermodynamic limit, resulting in a disordered polar cluster phase. In contrast to mutually aligning flocks, chaotic clusters are not characterized by the bimodal density histograms typically expected in a phase-separated regime. Nevertheless, they correspond to a demixed state, as evidenced by

negatively correlated density fluctuations of the two species [Figs. 4(c) and 4(d)].

Hydrodynamic theory—To gain a theoretical understanding of the above findings, we coarse grained the microscopic model (1) into a continuous theory. Because of its success at qualitatively capturing the physics of the single-species VM, we followed the Boltzmann-Ginzburg-Landau approach [51–53]. As detailed in Appendix C, we first write the Boltzmann equation governing the evolution of the single-body distribution function $f^s(\mathbf{r}, \theta, t)$ associated with species s . By expanding f^s in angular Fourier modes, we then obtain infinite coupled hierarchies of nonlinear equations. The zeroth and first modes correspond to the local particle densities ($\rho^{A,B}$) and polarities ($\mathbf{p}^{A,B}$), respectively, which are the physical fields of interest. Using a standard scaling ansatz describing the relative amplitude of the modes and their spatio-temporal variations at the onset of order, we truncate and close the hierarchies at the first nontrivial order. The resulting hydrodynamic equations for ρ^s and $\mathbf{p}^s$ read,

$$\partial_t \rho^s + v_0 \nabla \cdot \mathbf{p}^s = 0, \quad (2a)$$

$$\begin{aligned} \partial_t p_i^s + \sum_{U,V} [\psi_{UV}^s (\mathbf{p}^V \cdot \nabla) p_i^U + \lambda_{UV}^s (\nabla \cdot \mathbf{p}^V) p_i^U + \nu_{UV}^s \mathbf{p}^U \cdot \partial_i \mathbf{p}^V] \\ = \sum_U \left[\alpha_U^s (\rho^A, \rho^B) - \sum_{V,W} \xi_{UVW}^s (\mathbf{p}^V \cdot \mathbf{p}^W) \right] p_i^U - \frac{v_0}{2} \partial_i \rho^s \\ + \sum_U D_U^s \nabla^2 p_i^U. \end{aligned} \quad (2b)$$

Except for α_U^s , which varies linearly with the densities, all coefficients in Eq. (2) are constant. In [48], we provide their formal expressions in terms of integrals involving the microscopic model parameters, as well as a *Mathematica* notebook for computing them numerically.

Expressing the polarities as $\mathbf{p}^s = s^s \hat{\mathbf{u}}(\theta^s)$, we rewrite Eq. (2b) in terms of their magnitudes s^s and orientations θ^s . The homogeneous, stationary solutions of Eq. (2) correspond to uniform $\rho^s = \rho_0^s$ and $s^s = s_0^s$, which solve for $U \neq S$

$$F_S^s s_0^s + F_U^s s_0^U \cos(\Delta\theta_{SU}) = 0, \quad F_U^s s_0^U \sin(\Delta\theta_{SU}) = 0, \quad (3)$$

where $F_U^s \equiv \alpha_U^s (\rho_0^A, \rho_0^B) - \sum_{V,W} \xi_{UVW}^s s_0^V s_0^W \cos(\Delta\theta_{VW})$ and $\Delta\theta_{SU} \equiv \theta^S - \theta^U$. When $|\Delta\chi| < |\bar{\chi}|$, Eq. (3) admit three types of solutions. For sufficiently high noise, $s_0^{A,B} = 0$ indicating a disordered state. Decreasing η below a threshold value marked by the cyan lines in Fig. 3, the solutions bifurcate to an ordered state with $s_0^{A,B} > 0$, corresponding to a flocking ($\Delta\theta_{AB} = 0$, $\bar{\chi} > 0$) or anti-flocking ($\Delta\theta_{AB} = \pi$, $\bar{\chi} < 0$) configuration.

To analyze the stability of these solutions, we linearize Eq. (2) for small perturbations, which we express in Fourier space: $(\delta\rho^s, \delta\theta^s, \delta s^s)(\mathbf{r}, t) \rightarrow (\delta\hat{\rho}^s, \delta\hat{\theta}^s, \delta\hat{s}^s)(\mathbf{q}, t)$.

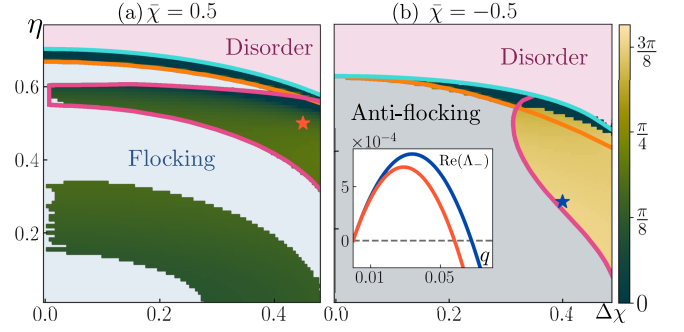


FIG. 3. Phase diagrams in the $(\Delta\chi, \eta)$ plane obtained from the linear stability of the stationary homogeneous solutions of Eq. (2) in the flocking (a) and antiflocking (b) regimes. Stable disordered, flocking, and antiflocking regions appear in pink, blue, and gray, respectively. The other regions indicate an instability, and are colored according to the orientation of the most unstable wave vector with respect to the direction of order. The solid cyan lines mark the onset of order, while the pink and orange lines enclose regions where the matrices $\mathbb{V}$ and $\mathbb{D}$, respectively, predict an instability. Inset of (b): growth rate of the flocking (blue) and antiflocking (red) instabilities as functions of the wave number $q = |\mathbf{q}|$ along the most unstable direction.

We obtain a six-dimensional linear system $\partial_t \hat{\mathbf{X}} = \mathbb{M}(\mathbf{q}) \hat{\mathbf{X}}$, where $\hat{\mathbf{X}} \equiv (\delta\hat{\rho}^A \delta\hat{\rho}^B \delta\hat{\theta}^A \delta\hat{\theta}^B \delta\hat{s}^A \delta\hat{s}^B)^T$ and the matrix $\mathbb{M}(\mathbf{q})$ is given in [48] for completeness. We construct the linear stability diagrams for both $\bar{\chi} > 0$ [Fig. 3(a)] and $\bar{\chi} < 0$ [Fig. 3(b)] by numerically evaluating the eigenvalues of $\mathbb{M}(\mathbf{q})$ for various values of η and $\Delta\chi$, and wave vectors $\mathbf{q}$. In qualitative agreement with the VM phenomenology [21] and our microscopic model simulations, both the flocking and antiflocking solutions are unstable at the onset of order. Similarly to single species flocks, the resulting long-wavelength instability is characterized by a growth rate $\text{Re}(\Lambda) \simeq |\mathbf{q}|^2$ for $|\mathbf{q}| \rightarrow 0$ [51,54]. We uncover an additional long-wavelength instability, located deeper in the ordered phase, and which typically requires sufficiently high nonreciprocity. For both the flocking and antiflocking cases this instability exhibits a distinct scaling: $\text{Re}(\Lambda) \simeq |\mathbf{q}|$ as $|\mathbf{q}| \rightarrow 0$ [inset of Fig. 3(b)], indicating that it originates from a new mechanism [55].

To elucidate its origin, we consider the limit $|\mathbf{q}| \rightarrow 0$ where nonhydrodynamic modes can be enslaved. The hydrodynamic modes—for which $\partial_t(\cdot) = \mathcal{O}(|\mathbf{q}|)$ —correspond to the left eigenvectors of $\mathbb{M}(\mathbf{0})$ with zero eigenvalue. In the (anti)flocking region, we find three of them: the two density fields $\delta\hat{\rho}^{A,B}$, and $\bar{\theta} = s_0^A F_A^B \delta\hat{\theta}^A + s_0^B F_B^A \delta\hat{\theta}^B$. Enslaving the fast modes and keeping only leading order terms in $|\mathbf{q}|$ (details in Appendix D), the dynamics reduces to a three-dimensional linear system $\partial_t \hat{\mathbf{X}}_h = \mathbb{M}_h(\mathbf{q}) \hat{\mathbf{X}}_h$, where $\hat{\mathbf{X}}_h = (\delta\hat{\rho}^A \delta\hat{\rho}^B \bar{\theta})^T$ and $\mathbb{M}_h(\mathbf{q}) = i\mathbb{V}|\mathbf{q}| - \mathbb{D}|\mathbf{q}|^2 + \mathcal{O}(|\mathbf{q}|^3)$. In particular,

$$\mathbb{V}|\mathbf{q}| = \begin{pmatrix} V_{AA}q_{\parallel} & V_{AB}q_{\parallel} & V_{A\theta}q_{\perp} \\ V_{BA}q_{\parallel} & V_{BB}q_{\parallel} & V_{B\theta}q_{\perp} \\ V_{\theta A}q_{\perp} & V_{\theta B}q_{\perp} & V_{\theta\theta}q_{\parallel} \end{pmatrix}, \quad (4)$$

where $q_{\parallel}$ and $q_{\perp}$ denote the components of $\mathbf{q}$ along and transverse to order, respectively, while the V terms depend on the coefficients of Eq. (2) in a complex way [48]. The matrix $\mathbb{D}$ also depends on the orientation of $\mathbf{q}$ and its formal expression is provided in [48]. As expected, $\mathbb{D}$ captures the generic instability at the onset of order, as evidenced by the orange lines in Fig. 3, above which it exhibits an eigenvalue with negative real part.

To simplify the analysis of $\mathbb{V}$'s eigenvalues, we note from Fig. 3(a) that upon decreasing η the homogeneous flocking solution is firstly unstable to longitudinal perturbations, such that we set $q_{\perp} = 0$. The matrix $i\mathbb{V}$ then has a purely imaginary eigenvalue $\Lambda_{\theta} = iV_{\theta\theta}$, while the remaining two are given by

$$\Lambda_{\pm} = \frac{i}{2}(V_{AA} + V_{BB} \pm \sqrt{\mathcal{R}}), \quad (5)$$

where $\mathcal{R} \equiv 4V_{AB}V_{BA} + (V_{AA} - V_{BB})^2$. When $\mathcal{R} > 0$, the eigenvalues in Eq. (5) are purely imaginary, and the stability of the solution to long-wavelength perturbations is determined by $\mathbb{D}$. On the other hand, for $\mathcal{R} < 0$ —i.e., when $V_{AB}V_{BA} < -\frac{1}{4}(V_{AA} - V_{BB})^2$ —the eigenvalue Λ_{-} acquires a positive real part, indicating the emergence of an instability. For reciprocally aligning populations V_{AB} and V_{BA} share the same sign and $\mathcal{R} > 0$, while the condition $V_{AB}V_{BA} < 0$ requires a sufficient amount of nonreciprocity. Antiflocking mixtures are more difficult to analyze, as in this case the instability is not purely longitudinal or transverse [Fig. 3(b)]. Nevertheless, the solid pink lines in Fig. 3 enclosing regions for which $i\mathbb{V}$ has an eigenvalue with positive real part show excellent agreement with the direct numerical estimate based on the six-dimensional system. When the instability is longitudinal, these lines correspond to $\mathcal{R} = 0$.

It is clear from Eqs. (4) and (5) that the new instability relies on the coupling between the two density fields. Therefore, it cannot arise in the single-species Vicsek model, or when nonreciprocally aligning spins are fixed on a lattice [56–58]. This also explains why it was missed in Ref. [17], where fluctuations of the densities are discarded in the analysis of the continuum model. Although $\mathcal{R}$ cannot be compactly expressed in terms of the microscopic model parameters, the density couplings in Eq. (4) can be written as $V_{SU} = v_0^S \partial_{\rho_U} \mathcal{L}^S$, where $\mathcal{L}^S$ is a linear combination of the $\alpha_V^U(\rho^A, \rho^B)$ coefficients in Eq. (2b) (details in Appendix D). This formulation notably highlights that the instability is driven by a combination of two ingredients: the particles' self-propulsion encoded by v_0^S ,

and the local (anti)alignment of velocities, which is reflected in the density dependencies of the α_V^U coefficients.

We have shown that even a weak amount of nonreciprocity significantly impacts the collective dynamics of flocking active matter. Combining agent-based simulations and the analysis of a coarse-grained theory, our results demonstrate that nonreciprocal alignment alone constitutes a generic route to demixing, without resorting to other mechanisms such as population heterogeneity [14] or pairwise repulsion [29,39]. Our theoretical analysis further emphasizes that this phenomenology is induced by the coupling between density and orientational order, which shapes most of the characteristics of flocks. Our findings echo recent works highlighting the metastability of flocks [59,60] and their fragility to the presence of spatial anisotropy [61], inclusions and boundaries [59,62], quench [63–66], and chirality [67] disorder. Since they rely solely on weakly nonreciprocal alignment, we believe that our results will have implications for the modeling of a variety of systems, including mixtures of Quincke rollers [14] or Janus colloids [30], and migrating heterogeneous cellular tissues [36].

Acknowledgments—We thank Hugues Chaté and Yu Duan for their critical reading of our manuscript, as well as Suropriya Saha for insightful discussions. C.M. acknowledges funding from the International Max Planck Research School (IMPRS) for the Physics of Biological and Complex Systems.

Data availability—The data that support the findings of this article are publicly available [68].

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End Matter

Appendix A: Synchronization of Vicsek bands via nonreciprocal alignment—Here, we provide a qualitative argument explaining the synchronization of polar bands such as those shown in Fig. 1(c). We consider a pair of A and B bands traveling with the same velocity $v_b^A = v_b^B$. As in Fig. 1(c), the band mostly composed of the species with the largest interspecies coupling (here A, in red) is at the front. Increasing the separation between the two bands, particles interact with fewer neighbors of the other species. As a result, the local polarization in the A band decays faster than that of the B band (since $\chi^{AB} > \chi^{BA}$). Since the velocity of each band is proportional to the local polar order inside [from Eq. (1a)], this leads to $v_b^A < v_b^B$, causing the B band to catch up. Similarly, reducing the interband separation yields $v_b^A > v_b^B$, such that the initial configuration is stabilized by nonreciprocity. The reversed configuration with the B band in front is, in turn, always unstable.

Appendix B: Laning pattern and chaotic clusters—As discussed in the main text, when increasing the system size the laning pattern reported in Figs. 1(b) and 1(h) for mutually antialigning species becomes more vulnerable to breaking events (see also SMov 4 [48]). Each of these events appears as a sharp decay of the global polar order $|\Pi^S|$ for both species, as shown in Fig. 4(a) for $S = A$. Hence, the corresponding polarity distributions develop a fat tail at low order [Fig. 4(b)] which is more

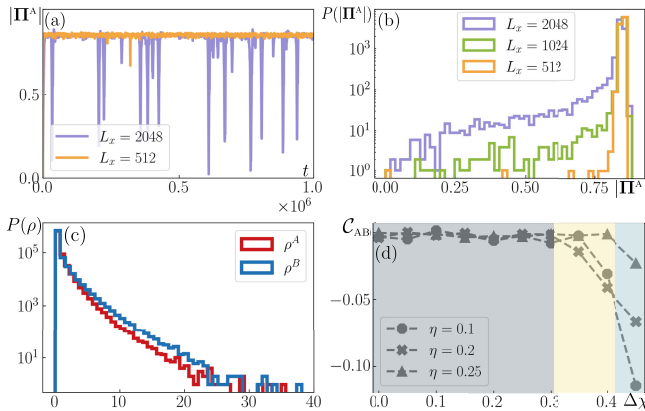


FIG. 4. (a),(b) Instability of the laning pattern for mutually antialigning species. (a) Exemplary time series of the global polar order $|\Pi^A|$ in the laning pattern for two system sizes. (b) The corresponding distributions. Parameters: $\eta = 0.30$ and $\Delta\chi = 0.35$. (c) Density histograms in the chaotic cluster phase for $\Delta\chi = 0.45$ and $\eta = 0.3$. (d) Normalized covariance C_{AB} of the densities ρ^A and ρ^B for different values of $\Delta\chi$ and η . The background color reflects the labelling of Fig. 1(b) for $\eta = 0.2$. In (c),(d) $L = 512$.

prominent in larger systems, indicating that the latter become progressively disordered. The resulting chaotic cluster phase is characterized by fat-tailed density histograms [Fig. 4(c)], as well as negatively correlated density fluctuations between the two species [Fig. 4(d)].

Appendix C: Derivation of the continuum model—We coarse-grained a multipopulation VM analogous to that defined in Eq. (1) via the Boltzmann-Ginzburg-Landau (BGL) approach [51–53]. To keep the derivation general, we consider n_s populations. Particles of species S self-propel at constant speed v_0^S while we model the influence of angular noise via stochastic tumbling events happening at a frequency λ^S and with a distribution $P_{\eta^S}(\theta)$ with zero mean and standard deviation η^S .

The BGL method consists in first writing a Boltzmann equation governing the evolution of the single-body distribution $f^S(\mathbf{r}, \theta, t)$ associated with species S ,

$$\partial_t f^S + v_0^S \hat{\mathbf{u}}(\theta) \cdot \nabla f^S = I_{sd}^S[f^S] + I_{al}^S[\{f^U\}_{U=1\dots n_s}], \quad (C1)$$

where the I_{sd}^S and I_{coll}^S account for rotational self-diffusion and (anti)alignment, respectively. These integrals read,

$$\begin{aligned} I_{sd}^S[f^S] &= \lambda^S \int_0^{2\pi} d\phi f^S(\mathbf{r}, \phi, t) P_{\eta^S}(\theta - \phi) - \lambda^S f^S(\mathbf{r}, \theta, t), \\ I_{al}^S[\{f^U\}] &= - \int_0^{2\pi} d\phi \sum_U f^S(\mathbf{r}, \theta, t) f^U(\mathbf{r}, \phi, t) K^{SU}(\theta - \phi) \\ &\quad + \int_0^{2\pi} d\phi_1 \int_0^{2\pi} d\phi_2 \sum_U f^S(\mathbf{r}, \phi_1, t) f^U(\mathbf{r}, \phi_2, t) \\ &\quad \times K^{SU}(\phi_2 - \phi_1) P_{\eta^S}[\theta - \phi_1 - H^{SU}(\phi_2 - \phi_1)]. \end{aligned} \quad (C2)$$

Equation (C2) is written under the standard assumptions behind the formulation of the Boltzmann equation. Namely, we neglect collisions involving more than two particles, while locality imposes that all distributions in (C2) depend on $\mathbf{r}$ only. In addition, we used the molecular chaos hypothesis to factorize the two-body particle distribution.

The kernel $K^{SU}(\phi_2 - \phi_1)$ is proportional to the rate of collisions between particles from species S and U with precollisional orientations ϕ_1 and ϕ_2 , respectively. A standard argument [54] yields $K^{SU}(\phi_2 - \phi_1) = 2r_0^S |v_0^U \hat{\mathbf{u}}(\phi_2) - v_0^S \hat{\mathbf{u}}(\phi_1)|$, where r_0^S is the radius of interaction associated with species S . Furthermore, $\phi_1 + H^{SU}(\phi_2 - \phi_1)$ is the postcollisional angle taken by both particles, such that $H^{SU}(\phi_2 - \phi_1) = \arg[1 + \chi^{SU} e^{i(\phi_2 - \phi_1)}]$.

Expanding the distribution f^S into angular Fourier modes as $f^S(\mathbf{r}, \theta, t) = (1/2\pi) \sum_k f_k^S(\mathbf{r}, t) e^{-ik\theta}$, the latter

obey [from Eq. (C1)]

$$\partial_t f_k^s + \frac{v_0^s}{2} [\nabla^* f_{k+1}^s + \nabla f_{k-1}^s] = \tilde{\lambda}_k^s f_k^s + \sum_{u,q} J_{kq}^{\text{SU}} f_q^u f_{k-q}^s, \quad (\text{C3})$$

where the complex gradient is defined as $\nabla \equiv \partial_x + i\partial_y$, while $\tilde{\lambda}_k^s \equiv \lambda^s(P_k^s - 1)$ with P_k^s the k^{th} Fourier mode of the distribution P_{η^s} . The mode coupling coefficients are given by $J_{kq}^{\text{SU}} = P_k^s I_{kq}^{\text{SU}} - I_{0q}^{\text{SU}}$ with

$$I_{kq}^{\text{SU}} \equiv \frac{1}{2\pi} \int_0^{2\pi} d\phi K^{\text{SU}}(\phi) e^{-iq\phi + ikH^{\text{SU}}(\phi)}. \quad (\text{C4})$$

The zeroth and first modes f_0^s and f_1^s are of particular interest, as they correspond to the complex representations of the local particle densities $\rho^s = f_0^s$ and polarities $\mathbf{p}^s = (\text{Re} f_1^s, \text{Im} f_1^s)^T$, respectively. Equation (C3) defines an infinite hierarchy, which we close by assuming a standard scaling ansatz [51,53] describing the relative amplitudes of the Fourier modes and their space and time derivatives at the onset of global order. Namely,

$$|f_k^s| \simeq \varepsilon^k (k > 0), \quad |f_0^s - \rho_0^s| \simeq \varepsilon, \quad \partial_t \simeq \nabla \simeq \varepsilon,$$

where ε is a small parameter. Formally, this scaling ansatz supposes that all species exhibit comparable global polarities. This should remain true in the presence of weak population heterogeneity.

We truncate the hierarchy at the first nontrivial order ε^3 . The equations for the particle densities $f_0^s = \rho^s$ are then exact, while f_1^s obeys

$$\begin{aligned} \partial_t f_1^s = & -\frac{v_0^s}{2} \nabla \rho^s - \frac{v_0^s}{2} \nabla^* f_2^s + \sum_u M_1^{\text{SU}}(\{\rho^u\}) f_1^u \\ & + \sum_u [J_{12}^{\text{SU}} f_2^u f_1^s + J_{1-1}^{\text{SU}} f_2^s f_1^u] + O(\varepsilon^5), \end{aligned} \quad (\text{C5})$$

where $M_k^{\text{SU}}(\{\rho^u\}) \equiv [\tilde{\lambda}_k^s + (\sum_v J_{k0}^{\text{SV}} \rho^v)] \delta^{\text{SU}} + J_{kk}^{\text{SU}} \rho^s$. The modes f_2^s appear in Eq. (C5) only through $O(\varepsilon^3)$ terms. Hence, we write the equation for f_2^s at $O(\varepsilon^2)$:

$$\sum_u M_2^{\text{SU}}(\{\rho^u\}) f_2^u - \frac{v_0^s}{2} \nabla f_1^s + \sum_u J_{21}^{\text{SU}} f_1^s f_1^u + O(\varepsilon^3) = 0.$$

We close the hierarchy by solving this linear system for f_2^s and inserting the solution into Eq. (C5). Considering two species with identical speed v_0 , interaction radius r_0 , and noise strength η , we then recover Eq. (2) of the main text after adopting the vector representation $f_1^s \rightarrow \mathbf{p}^s$. In all numerical calculations, we used a Gaussian noise distribution such that $P_k^s = \exp[-k^2 \eta^2 / 2]$.

For arbitrary alignment couplings χ , the integrals (C4) do not admit simple analytical expressions, such that the

coefficients in Eq. (2) cannot be simply expressed in terms of the parameters of the microscopic model. We thus provide in [48] a *Mathematica* notebook that can be used to numerically evaluate the coefficients of the continuum model.

Appendix D: Enslaving of the nonhydrodynamic modes—Here, we detail the enslaving procedure leading to Eq. (4). We consider an m -dimensional linear system of the form $\partial_t \mathbf{X} = \mathbb{M}(\mathbf{q}) \cdot \mathbf{X}$, where $\mathbb{M}(\mathbf{q})$ is a wave-vector dependent $m \times m$ matrix. As we are interested in the $q = |\mathbf{q}| \rightarrow 0$ limit, we expand $\mathbb{M}$ as

$$\mathbb{M}(\mathbf{q}) = \mathbb{M}_0 + iq\mathbb{M}_1 - q^2\mathbb{M}_2 + O(q^3), \quad (\text{D1})$$

where the $\mathbb{M}_i$ matrices are independent of q but may depend on the direction of $\mathbf{q}$.

We denote $\{\mathbf{Y}_j\}$ as the left eigenbasis of $\mathbb{M}_0$, such that for all $j = 1, \dots, m$, $\mathbf{Y}_j^T \cdot \mathbb{M}_0 = \Lambda_j \mathbf{Y}_j^T$, where $\{\Lambda_j\}$ are the corresponding eigenvalues. Assuming that the initial linear system describes fluctuations around a solution which is stable under $q = 0$ perturbations, all $\Lambda_j \leq 0$. We now suppose that the problem has n_h hydrodynamic modes, and we reorder the eigenvalues such that $\Lambda_1, \dots, \Lambda_{n_h} = 0$ and $\Lambda_{n_h+1}, \dots, \Lambda_m < 0$. Defining

$$\tilde{\mathbf{X}} = (\mathbf{Y}_1 \cdots \mathbf{Y}_m)^T \cdot \mathbf{X} \equiv \mathbb{P} \cdot \mathbf{X}, \quad (\text{D2})$$

the first n_h elements of $\tilde{\mathbf{X}}$ are the hydrodynamic modes, while the other $m - n_h$ are fast. This is straightforwardly shown from

$$\partial_t \tilde{\mathbf{X}} = \mathbb{P} \cdot \partial_t \mathbf{X} = \mathbb{P} \cdot \mathbb{M}_0 \cdot \mathbf{X} = \text{diag}(\lambda_1 \dots \lambda_m) \cdot \tilde{\mathbf{X}},$$

where we have used that $\mathbb{P} \cdot \mathbb{M}_0 \cdot \mathbb{P}^{-1} = \text{diag}(\lambda_1 \dots \lambda_m)$. While the slow modes are uniquely defined, the fast modes may be expressed differently so long as they are associated with a nonzero damping rate. The formulation (D2) nevertheless provides a compact change of basis formula between the slow and fast modes and the original variables. Going back to $q > 0$, the dynamics of $\tilde{\mathbf{X}}$ then follows $\partial_t \tilde{\mathbf{X}} = \tilde{\mathbb{M}}(\mathbf{q}) \cdot \tilde{\mathbf{X}}$, where $\tilde{\mathbb{M}} \equiv \mathbb{P} \cdot \mathbb{M} \cdot \mathbb{P}^{-1}$. In the regime of interest, $\mathbb{P}$ is always invertible. For convenience, we write separately the dynamics of the hydrodynamic ($\mathbf{X}_h$) and fast ($\mathbf{X}_f$) modes (dropping the tildes to lighten notations):

$$\partial_t \mathbf{X}_h - \mathbb{M}_h^h \cdot \mathbf{X}_h = \mathbb{M}_f^h \cdot \mathbf{X}_f, \quad (\text{D3})$$

$$\partial_t \mathbf{X}_f - \mathbb{M}_f^f \cdot \mathbf{X}_f = \mathbb{M}_h^f \cdot \mathbf{X}_h, \quad (\text{D4})$$

in terms of the block matrices $\mathbb{M}_y^x$. By construction, all elements of $\mathbb{M}_h^h$ and $\mathbb{M}_f^f$ are zero for $q = 0$, while $\mathbb{M}_f^f(q = 0)$ is diagonal with negative entries representing the finite damping rates of the fast modes. Expanding all block

matrices similarly to (D1), we then enslave the fast modes by solving Eq. (D4) in the limit $q \rightarrow 0$. Here, we focus on the leading order solution, while a derivation including $\mathcal{O}(q^2)$ corrections is presented in [48]. Therefore,

$$\mathbf{X}_f = -(\mathbb{M}_{f,0}^f)^{-1} \cdot \mathbb{M}_{h,0}^f \cdot \mathbf{X}_h + \mathcal{O}(q).$$

Replacing this solution in Eq. (D3), we then find that the hydrodynamic modes obey $\partial_t \mathbf{X}_h = \mathbb{M}_h(\mathbf{q}) \cdot \mathbf{X}_h$ with

$$\mathbb{M}_h(\mathbf{q}) = iq(\mathbb{M}_{h,1}^h - \mathbb{M}_{f,1}^h \cdot (\mathbb{M}_{f,0}^f)^{-1} \cdot \mathbb{M}_{h,0}^f) + \mathcal{O}(q^2). \quad (\text{D5})$$

Performing this analysis for the linear stability described in the main text, we find three slow modes: $\delta\hat{\rho}^{A,B}$ and $\bar{\theta} = s_0^A F_A^B \delta\hat{\theta}^A + s_0^B F_B^A \delta\hat{\theta}^B$. The expressions of the various matrices involved in the derivation are quite lengthy and not very informative. We can nevertheless express the coefficients that couple the density fluctuations in Eq. (4) as

$$\begin{aligned} V_{AA} &= v_0^A [C_{AA} E_A^A + C_{AB} E_A^B], & V_{AB} &= v_0^A [C_{AA} E_B^A + C_{AB} E_B^B], \\ V_{BA} &= v_0^B [C_{BA} E_A^A + C_{BB} E_A^B], & V_{BB} &= v_0^B [C_{BA} E_B^A + C_{BB} E_B^B], \end{aligned}$$

where the C_{SU} coefficients are constant and depend on the model parameters, while $E_U^S = (s_0^A \partial_{\rho^U} \alpha_A^S + \epsilon s_0^B \partial_{\rho^U} \alpha_B^S)_{\rho^S = \rho_0}$ with $\epsilon = \text{sign}(\bar{\chi})$ [48].