

Quasinormal Modes of Nonthermal Fixed Points

Matisse De Lescluze^{*} and Michal P. Heller[†]

Department of Physics and Astronomy, Ghent University, 9000 Ghent, Belgium



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Quasinormal modes play a prominent role in the relaxation of diverse physical systems to equilibria, ranging from astrophysical black holes to tiny droplets of quark-gluon plasma at the RHIC and LHC accelerators. We propose that a novel kind of quasinormal mode governs the direct approach to self-similar time evolution of nonthermal fixed points, whose relevance ranges from high-energy physics to cold atom gases. We utilize black hole perturbation theory techniques to compute the spectrum of these far-from-equilibrium quasinormal modes for a kinetic theory with a Fokker-Planck collision kernel in isotropic and homogeneous states. Our conclusion is that quasinormal modes of nonthermal fixed points give rise to a tower of progressively more decaying power-law contributions. A by-product of our analysis is a precise determination and improved understanding of the distribution function characterizing nonthermal fixed points.

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Introduction—Understanding thermalization in quantum many-body systems and quantum field theories (QFTs) is an important topic of contemporary research on both the theoretical and experimental frontiers [1,2]. When a QFT has many microscopic constituents that are strongly interacting, then thermalization is described in a dual gravitational description [3–5] by black hole formation and its subsequent relaxation with time [6]. As with astrophysical black holes, this relaxation is captured by an infinite tower of complex eigenfrequencies ω : quasinormal modes (QNMs) [7–9]. Their imaginary part governs the decay of observables such as the expectation value of the energy-momentum tensor to their thermal values, and their real part is responsible for generically present associated oscillations: $e^{-i\omega t}$. As time t progresses, the least damped QNM starts dominating physical observables.

In this Letter we recognize that QNMs should play another, as yet unexplored role in thermalization dynamics. In the course of the past 20 years, it has been predicted theoretically [10–13] and then verified experimentally [14–25] that overoccupied states are attracted to a transient, albeit long-lived, nonthermal fixed point (NTFP) regime characterized by a temporal self-similarity; see also [26,27] for reviews. In the simplest isotropic case, the weakly coupled dynamics is characterized by a distribution

function of particles $f(t, p = |\vec{p}|)$ that depends on the time t and the magnitude of spatial momentum p . The NTFP regime occurs in this setup when the temporal and momentum dependence effectively factorize

$$f(t, p) = A(t)f_s(B(t)p). \quad (1)$$

As a result, in the NTFP regime the form of a distribution function at a given time allows one to predict its form to the future by a mere rescaling of the distribution function and momentum. The Boltzmann equation

$$\partial_t f(t, p) = C[f](t, p), \quad (2)$$

then dictates that

$$A(t) = B(t)^\sigma, \quad B(t) = \left(\frac{t - t_*}{t_{\text{ref}}} \right)^\beta. \quad (3)$$

The number σ and the exponent β characterize the underlying dynamics, t_{ref} is a constant choosing normalization of f_s , and t_* is an initial condition-dependent offset picking the origin of time as 0 plays a special role for self-similarities such as (3) [24,28].

Our key insight aside from the aforementioned observed attractive nature of NTFPs is that $A(t)^{-1}f(t, p)$ becomes effectively time independent when viewed as a function of $\bar{p} \equiv B(t)p$ and renders this configuration effectively static in a close similarity to the static nature of the thermal state or a final black hole in a merger or collapse. This allows an eigenvalue problem to be set up for perturbations

$$A(t)^{-1}f(t, \bar{p}/B(t)) \approx f_s(\bar{p}) + \delta f(t, \bar{p}) \quad (4)$$

^{*}Contact author: matisse.delescluze@ugent.be

[†]Contact author: michal.p.heller@ugent.be

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around a NTFP regime. We show that the QNM ansatz in this case is given by

$$\delta f(t, \bar{p}) = B(t)^{i\Omega} \delta f_\Omega(\bar{p}). \quad (5)$$

The spectrum of eigenfrequencies Ω and eigenfunctions δf_Ω , the QNMs of a given NTFP, is determined by linearization of the Boltzmann equation (2) around the scaling solution (1).

Our general expectation is therefore that a NTFP is going to be approached at sufficiently late times in a power-law manner with a possible real part of Ω triggering slow oscillations that are logarithmic in time. At intermediate times, when deviations from the scaling form (1) are small, a sum of QNMs describes the signal. We expect that the lessons drawn in the context of black hole QNMs, including various recent developments [29–33], will generally be transferable to the novel setting of NTFPs.

To corroborate these general expectations, we explicitly compute the spectrum of QNMs in NTFPs arising for the collision kernel in the Boltzmann equation of the Fokker-Planck form [see Eq. (15)] relevant for, e.g., small angle elastic scattering of gluons at very weak coupling.

QNM eigenproblem around NTFPs—The emergence of exact NTFPs (1) is a consequence of the collision kernel C in Eq. (2) being a homogeneous functional of particle momenta [28,34]. This means that for a distribution function of the form in Eq. (4), the collision kernel behaves as

$$C[f](t, p) = B(t)^{\sigma\mu_\alpha + \mu_\beta} \{ \tilde{C}[f_s](\bar{p}) + \delta\tilde{C}[f_s, \delta f](t, \bar{p}) \}, \quad (6)$$

where we consider a small perturbation $\delta f \ll f_s$ to the scaling function f_s . The parametrization of δf was chosen to make it easy to capture time dependence in the static frame of f_s . μ_α is determined by the powers of f appearing in C , while μ_β is related to powers of the momentum p . In many cases this property of the collision kernel emerges in the case of overoccupation.

The other assumption necessary for the existence of NTFPs is conservation of energy or the particle number. This is ensured by relating $A(t)$ and $B(t)$ by a positive constant σ [28]; see Eq. (3). Conservation laws also constrain the behavior of perturbations around a NTFP, as will be explained later. We apply linear response theory by plugging the form of $f(t, p)$ in Eq. (4) into the Boltzmann equation (2) and considering only terms up to first order in δf . By making use of Eqs. (6) and (3), we obtain

$$\frac{B(t)^{\sigma\mu_\alpha + \mu_\beta}}{B(t)^{\sigma-1} \partial_t B(t)} = \frac{1}{D_1} = \frac{[\sigma + \bar{p} \partial_{\bar{p}}] f_s(\bar{p})}{\tilde{C}[f_s](\bar{p})}, \quad (7)$$

$$B(t) \partial_{B(t)} \delta f(t, \bar{p})|_{\bar{p}=\text{const}} = \frac{1}{D_1} \delta\tilde{C}[f_s, \delta f](t, \bar{p}) - \sigma \delta f(t, \bar{p}) - \bar{p} \partial_{\bar{p}} \delta f(t, \bar{p}), \quad (8)$$

with D_1 a separation of variables constant. Equation (7) was studied in [28], with the left equality determining the form of $B(t)$. The solution is given by Eq. (3) with $D_1 = \beta/t_{\text{ref}}$ and $1/\beta = (1 - \mu_\alpha)\sigma - \mu_\beta$. We used Eq. (7) to write the equation for δf in the form of (8). The latter is solved by ansatz (5). The sign convention for the imaginary unit was chosen so that later in the Letter the decaying modes will be characterized by $\text{Im}(\Omega) < 0$, as is the usual convention. Because Ω is constant, we can pull the $B(t)^{i\Omega}$ factor through the operators on the right-hand side and eliminate it, leading to

$$i\Omega \delta f_\Omega(\bar{p}) = \frac{1}{D_1} \delta\tilde{C}[f_s, \delta f_\Omega](\bar{p}) - \sigma \delta f_\Omega(\bar{p}) - \bar{p} \partial_{\bar{p}} \delta f_\Omega(\bar{p}). \quad (9)$$

Equations (5) and (9) are the main results of this Letter. They show that perturbations on top of a NTFP evolve in time in the same way as the self-similar distribution function f_s , but with different exponents $(\sigma + i\Omega)\beta$ of $t - t_*$ in the time-dependent amplitude; compare Eqs. (1) and (3) to Eq. (5). The perturbations through their imaginary part induce a power law in time approach (1) when $\text{Re}(i\Omega)\beta < 0$, which is a self-consistency condition for the attractive nature of NTFPs observed in *ab initio* simulations and experiments. Furthermore, real parts of Ω 's, if nonzero, would induce oscillatory behavior in the logarithm of B , i.e., in $\log(t - t_*)$. This is reminiscent of the near-equilibrium kinetic theory setup of [35].

The last key aspect that we want to highlight is that D_1 and the collision kernels $\tilde{C}[f_s]$ and $\delta\tilde{C}[f_s, \delta f]$ appear in the same combination in Eqs. (7) and (8). This means that all factors appearing as an overall amplitude in the collision kernel can simply be absorbed into the dummy variable t_{ref} , rendering the equations agnostic to them. In practical terms this implies that the QNM frequencies naturally turn out to be independent of the coupling since at leading order the coupling appears to be an overall factor in the collision kernel [36]; see also Eq. (15).

Two universally present QNMs—A corroboration of the discussion above and our later numerical studies is the existence of two special QNMs (5) around any NTFP. Looking back at Eqs. (1) and (3), there are two parameters with which we can perturb and investigate the response of the system, namely, t_* and t_{ref} .

Let us first consider $t_* \rightarrow t_* + \delta t_*$ in Eqs. (1) and (3) and expand to first order in δt_*

$$f(t, p) = A(t) f_s(\bar{p}) - \frac{\beta}{t_{\text{ref}}} A(t) B(t)^{-1/\beta} [\sigma f_s(\bar{p}) + \bar{p} \partial_{\bar{p}} f_s(\bar{p})]. \quad (10)$$

This has the same structure as predicted in (4) and (5), with $\Omega = i/\beta$ and $\delta f_\Omega(\bar{p}) \propto \sigma f_s(\bar{p}) + \bar{p} \partial_{\bar{p}} f_s(\bar{p})$. One can think

of this perturbation as arising at very late times when t_* is not included in A and B , as was the case before [24,28].

To understand the perturbation of t_{ref} one has to realize that f_s itself also contains a nontrivial dependence on t_{ref} ; see Eq. (7) with $D_1 = \beta/t_{\text{ref}}$. The correct form of $f(t, p)$ for $t_{\text{ref}} \rightarrow t_{\text{ref}} + \delta t_{\text{ref}}$ is

$$f(t, p) = A(t)f_s(\bar{p}) - \frac{\beta}{t_{\text{ref}}}A(t)(\sigma f_s(\bar{p}) + \bar{p}\partial_{\bar{p}}f_s(\bar{p})) + A(t)\partial_{t_{\text{ref}}}f_s(\bar{p})|_{\bar{p}=\text{const}}. \quad (11)$$

We found a zero mode $\Omega = 0$ with momentum dependence $\delta f_{\Omega}(\bar{p}) \propto \sigma f_s(\bar{p}) + \bar{p}\partial_{\bar{p}}f_s(\bar{p}) - t_{\text{ref}}/\beta \partial_{t_{\text{ref}}}f_s(\bar{p})|_{\bar{p}=\text{const}}$. This perturbation is the analog of a variation with respect to thermodynamic variables like temperature or chemical potential in equilibrium [37]. The fact that shifting t_{ref} leads to a zero mode can be easily understood. The reference time is a dummy constant and fixes the normalization of f_s . Using a different t_{ref} leads to a different scaling function in Eq. (1), while the physical distribution function $f(t, p)$ remains unchanged. The NTFP form is retained, and in the static frame of the scaling function nothing decays.

QNMs and conservation laws—We will now derive further important statements about NTFP QNMs in full generality. First, we discuss the importance of conservation laws. The dynamics of self-similar cascades for systems reaching a NTFP is dictated by conserved quantities, e.g., particle number density $n = \int d^d p f / (2\pi)^d$ and energy density $\epsilon = \int d^d p \omega_{\bar{p}} f / (2\pi)^d$. Here d the number of spatial dimensions and $\omega_{\bar{p}}$ is the dispersion relation of particles. We focus on $\omega_{\bar{p}} \propto |\bar{p}|^z = p^z$ as in [35]. The relation between $A(t)$ and $B(t)$ in Eq. (3) is the consequence of having one conserved quantity. Conservation of n leads to $\sigma = d$, while conservation of ϵ corresponds to $\sigma = d + z$. This only imposes the relevant conserved quantity to be constant when calculated from f_s in the static frame $\bar{p}$. We define the conserved quantity to be completely contained by f_s . Considering the example of energy density, this implies that

$$\epsilon = \int d^d p p^z f(t, p) = \int d^d \bar{p} \bar{p}^z f_s(\bar{p}). \quad (12)$$

One can always ensure this by choosing t_{ref} in such a way that the above equation holds. It is now clear that ϵ can be a constant only when the energy density of the perturbations defined in Eq. (4) in the static frame of f_s is time independent

$$\begin{aligned} \delta \epsilon(t) &= \int d^d \bar{p} \bar{p}^z \delta f(t, \bar{p}) \\ &= B(t)^{i\Omega} \int d^d \bar{p} \bar{p}^z \delta f_{\Omega}(\bar{p}) = \text{const}. \end{aligned} \quad (13)$$

There are two ways for the equation above to be constant. Either $\Omega = 0$ or $\delta \epsilon_{\Omega} \equiv \int d^d \bar{p} \bar{p}^z \delta f_{\Omega}(\bar{p}) = 0$. Both of these conditions are already clear from our predicted modes. As explained, the zero mode corresponds to a shift in t_{ref} . But the choice of t_{ref} ensures that f_s contains the same energy density as f . Changing t_{ref} thus changes the overall energy density, which is a different but allowed solution of the equations of motion near a NTFP. Since in a homogeneous and isotropic system there is only one conserved quantity dictating the dynamics for self-similar evolution, we expect there to be only one zero mode. The t_* mode for $\sigma = d + z$ makes the other condition $\delta \epsilon_{\Omega} = 0$ apparent,

$$\begin{aligned} \delta \epsilon_{\Omega} &\propto \int d\bar{p} \bar{p}^{d-1+z} \{ (d+z)f_s(\bar{p}) + \bar{p}\partial_{\bar{p}}f_s(\bar{p}) \} \\ &= \int d\bar{p} \bar{p} \left[\bar{p}^{d+z} f_s(\bar{p}) \right], \end{aligned} \quad (14)$$

which is zero if $\bar{p}^{d+z} f_s(\bar{p})$ vanishes at the boundaries of the integral. The effects of boundaries are discussed in more depth in the section on the scaling function. The discussion is completely analogous when considering conservation of particle number density. We will show that if conservation of the relevant quantity is violated, unstable modes appear.

We want to highlight that these ideas are completely general and do not assume specific collision kernels. Importantly, we did not make any additional assumptions to the ones that are needed to realize self-similar evolution near a NTFP as in Eq. (1).

Setup—We test the general lessons from the previous section in the explicit example of Fokker-Planck kinetic theory adopted from QCD applications given by

$$C_{\text{FP}}[f](t, p) = K \left\{ \frac{I_a}{p^2} \partial_p (p^2 \partial_p f) + \frac{I_b}{p^2} \partial_p [p^2 f(f+1)] \right\}, \quad (15)$$

where $K = \lambda^2 / 4\pi \mathcal{L}$, $I_a = \int [d^3 p / (2\pi)^3] f(f+1)$, and $I_b = 2 \int [d^3 p / (2\pi)^3] f/p$; see, e.g., [1]. λ is the ('t Hooft) coupling constant and the Coulomb logarithm $\mathcal{L}$ is given by $\log(\langle p \rangle / m_D)$, but we chose to set it equal to 1. In other words, we assume the argument of the logarithm to be independent of time. Doing this makes the FP kernel scale as the first term in Eq. (6). If the time dependence in K would be taken into account, it would induce small, slowly decaying logarithmic corrections to (3) [28,38]. This effect can be included in perturbation theory around the solutions we provide, which we leave for future investigations.

To allow a self-similar solution, we assume overoccupation and consider the Fokker-Planck (FP) kernel in the static frame of f_s . This boils down to replacing $f+1$ with f in the definitions of C_{FP} , I_a , and I_b above. Conservation of energy density ϵ leads to $\sigma = 4$ and the direct energy cascade NTFP $(\alpha, \beta) = (-4/7, -1/7)$ [39,40].

Accurate determination of the scaling function—To solve Eq. (9), we first need an accurate determination of the scaling function f_s . We leave the technical discussion to Sec. I of the Supplemental Material [41] and report on the key insights. To probe arbitrarily large momenta, we introduce a compactified variable $u = \bar{p}/(1 + \bar{p})$. This way $\bar{p} = 0$ coincides with $u = 0$ and $\bar{p} = \infty$ coincides with $u = 1$. For a system without cutoffs, u is numerically represented by a compact grid $[0, 1]$. The scaling solution for f_s goes $\propto 1/\bar{p}$ for $\bar{p} \ll 1$ and $\propto \bar{p}e^{-\bar{p}^2/2}$ for $\bar{p} \gg 1$. The numerical result is shown in Fig. 1 as the blue dots.

Let us now discuss the effect of cutoffs. The IR behavior of f_s remains unchanged, i.e., $\propto 1/\bar{p}$, under cutoffs at either boundary of the momentum grid; see Fig. 1. Introducing a UV cutoff does not significantly alter the limiting behavior of f_s . However, an IR cutoff changes the UV behavior from Gaussian, $\propto \bar{p}e^{-\bar{p}^2/2}$, to power law, $\propto 1/\bar{p}^4$. The fact that the UV behavior depends on the IR cutoff is not strange, considering that all momenta are coupled through the integrals I_a and I_b appearing in Eq. (7). The coefficient of the UV power law is proportional to the IR cutoff. This means that the Gaussian tail is recovered when the IR cutoff is taken to 0. A UV tail $\propto 1/\bar{p}^4$ leads to divergent energy densities. To mitigate this unphysicality, we impose a UV cutoff, a natural step in describing cascades near a NTFP. Since overoccupation cannot persist across an infinite momentum range in a finite system, we assume that it holds within two cutoffs, where an approximately conserved quantity exists. This enables self-similar evolution within that region, while dynamics outside this range are beyond our current scope. A more refined approach would incorporate dynamics beyond the grid boundaries, yielding a scaling solution consistent with the full momentum range. We leave this for future work and proceed by imposing boundary conditions where $f_s = 0$ beyond the cutoffs.

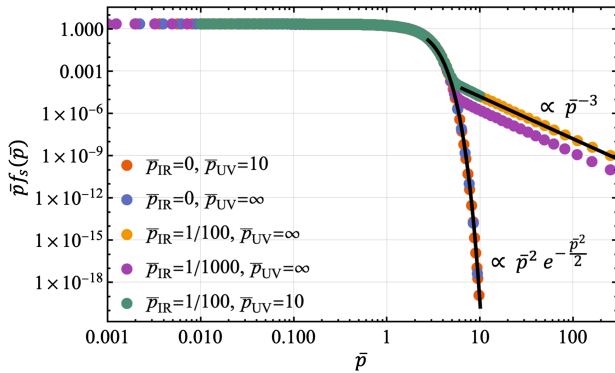


FIG. 1. Dependence of the scaling function $\propto f_s$ on UV and IR cutoffs. One sees that a UV cutoff does not alter the behavior significantly. However, an IR cutoff gets rid of the Gaussian tail and replaces it with a power law $\propto 1/\bar{p}^4$ visible as data points aligning along straight lines. The coefficient of the power law is proportional to the IR cutoff.

As a final remark, we want to highlight that our result for the scaling function remains the same for the Fokker-Planck collision kernel in Eq. (15) with $\mathcal{L}$ dependent on time, as would be dictated by the microscopics of QCD in the small angle scattering approximation. This can be understood by realizing that the right-hand side of Eq. (7) remains unchanged and $\mathcal{L}$ enters only on the time-dependent left-hand side [28].

Far-from-equilibrium QNMs spectrum—To find the spectrum of QNMs, we discretize Eq. (9) and solve it as an eigenvalue problem. We benchmarked this method against the analytical results of [37], where the spectrum of a linearized collision operator around equilibrium for a scalar field theory was considered; see Sec. VIII of the Supplemental Material [41] for more details. As input, we need the scaling function determined in the previous section. Solving the generalized eigenvalue equation on the full compact grid $u \in [0, 1]$ leads to perturbations that diverge for $u \rightarrow 1$ compared to f_s . The assumption of linearity $\delta f_\Omega \ll f_s$ thus does not hold over an infinite momentum range, and one should not consider the mode calculation for $\bar{p} \rightarrow \infty$. Complementary to our discussion of boundaries, with respect to the assumption of overoccupation in the previous section, it thus seems natural to include a UV cutoff in these mode calculations.

Figure 2 shows the resulting NTFP spectrum. There are seemingly unstable modes where $\text{Im} \Omega > 0$ when no boundary conditions are imposed (the blue squares). By imposing that the modes carry no energy density, $\delta\epsilon = 0$, we eliminate the growing perturbations and the spectrum becomes that of an attractor. This means that, on the linear level, the NTFP is completely stable when the assumptions that allow for the direct energy cascade remain valid, i.e., overoccupation and conservation of energy density. The remaining modes are purely imaginary—note the predicted one at $\Omega = -7i$; see also Fig. 3 (top panel). The nontrivial

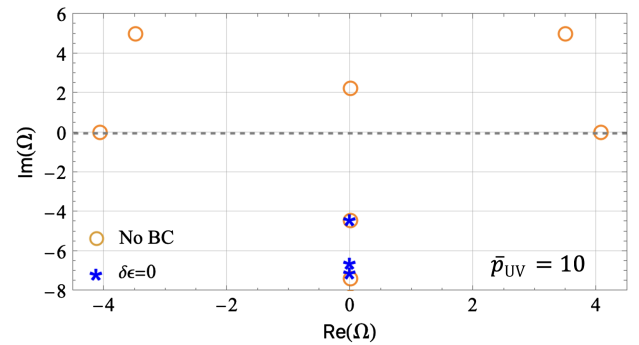


FIG. 2. Results for the QNM frequencies Ω calculated on a grid with cutoffs $\bar{p}_{\text{IR}} = 0$ and $\bar{p}_{\text{UV}} = 10$. When no boundary conditions are imposed, there appear to be unstable modes where $\text{Im}(\Omega) > 0$. By imposing that the perturbation carry no energy density $\delta\epsilon = 0$, all the modes are stable, indicating the stability of the NTFP when assuming overoccupation and conservation of energy density.

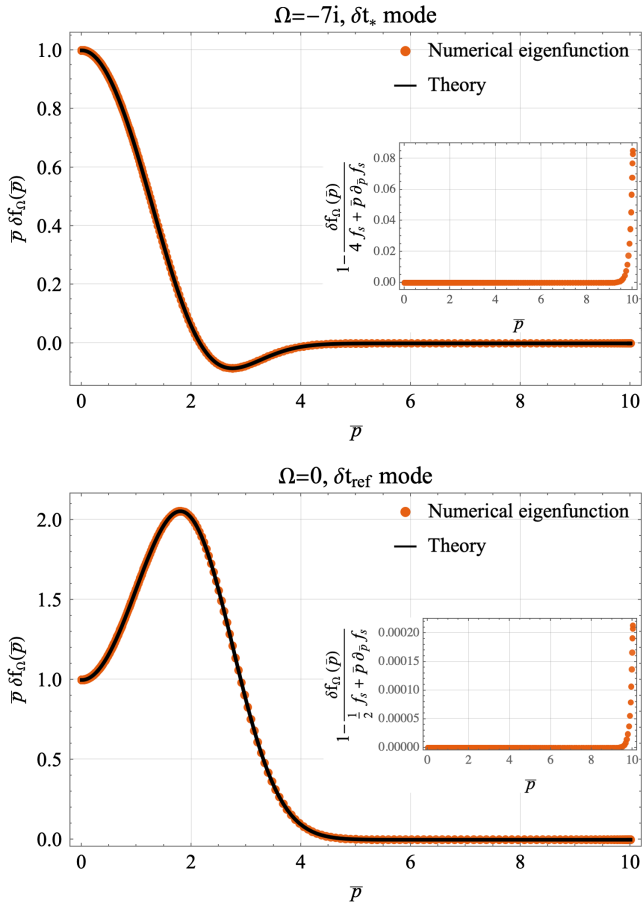


FIG. 3. Comparisons of the momentum dependences of the modes from the eigenvalue calculations to their predictions in Eqs. (10) and (11), respectively. Inset: their relative difference. There is excellent agreement, with the largest deviation being at the UV boundary due to nonvanishing boundary effects. This comparison to the theoretically predicted modes serves as a cross-check for our general claims.

modes specific to this theory are at likely irrational imaginary numbers, $\Omega \approx -4.29i$ and $\Omega \approx -6.49i$. They are very robust to changes in the UV cutoff and remain the same for the largest cutoff that we consider; see Sec. VI of the Supplemental Material [41] for their momentum dependence and stability with respect to $\bar{p}_{UV}$. We expect infinitely many QNM frequencies farther down in the lower half complex plane.

Introducing a nonzero IR cutoff $\bar{p}_{IR}$ has a more significant effect on the spectrum compared to the UV cutoff. Under the assumptions of overoccupation and energy conservation, the spectrum remains stable for small enough $\bar{p}_{IR}$, but the modes become complex instead of imaginary. In Sec. VII of the Supplemental Material [41], we show that for nonzero $\bar{p}_{IR}$ they exhibit the known Christmas tree structure of holographic QNMs [8]. When the cutoff is lowered, pairs of modes collide and the spectrum reduces to the $\bar{p}_{IR} = 0$ result. In holographic studies mode collisions have been observed at complex values of a continuously

varied parameter [29,30]. Interestingly, here we find mode collisions at real values of the continuous parameter $\bar{p}_{IR}$. The modes that lie deeper in the complex plane are more sensitive to changes in the IR cutoff (see Sec. VII of the Supplemental Material [41]), which is in agreement with the spectral stability analysis in a holographic setting in [31,32].

The boundary condition always disposes of the predicted zero mode $\Omega = 0$ since this mode carries energy density as explained in the third section. As a cross-check, we calculate and compare the corresponding eigenfunction by setting $\Omega = 0$ in the eigenvalue equation (9) and solving it as a linear equation for δf_{Ω} without imposing the boundary condition. The result is shown in Fig. 3 (bottom panel). The zero mode can be found in this way for all choices of IR and UV cutoffs that we considered. The theoretical form of the zero mode $1/2 f_S + \bar{p} \partial_{\bar{p}} f_S$ results from Eq. (11) when taking into account that t_{ref} appears to be $1/\sqrt{t_{\text{ref}}}$ in the amplitude of the scaling function; see Sec. III of the Supplemental Material [41].

Outlook—Our Letter aims to provide a bridge between far-from-equilibrium NTFPs and QNMs describing the transient approach to thermality. It is natural to envision a significant advancement of our understanding about the attractive nature of NTFP and phenomena in their vicinity coming from adopting the relevant knowledge and techniques from the fields of black hole perturbation theory and holography.

For the Fokker-Planck kinetic theory, we showed that the self-similar evolution is completely stable when the assumptions leading to a NTFP hold, i.e., overoccupation and conservation of energy density. This indicates that there are no unstable directions at the linear level via which the system can leave the NTFP.

There are many future directions that our work opens up. On the theoretical front, it is certainly gaining an understanding of the QNM spectrum for other collision kernels, particularly for the large- N kinetic theory that also encapsulates the infrared cascade [42]. As a preliminary step in this direction, in Supplementary Material I, III, and IV [41], we consider a toy model within the FP kinetic theory of the infrared particle number conserving cascade and the associated QNM spectrum, which now also contains modes with nontrivial $\text{Re}(\Omega)$. It would also be very interesting to connect our findings with earlier studies of stability of nonthermal fixed points using two-particle irreducible QFT techniques [43]. Finally, on the experimental front, it would be fascinating to directly observe QNMs of NTFPs using their experimental realisations in cold atomic gases. A natural way to proceed would be to study deviations of the distribution function from the scaling form in the static frame, as encapsulated by Eq. (4).

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Data availability—The data that support the findings of this article are openly available [44].

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