

Topological Chiral Superconductivity in the Triangular-Lattice Hofstadter-Hubbard Model

Feng Chen^{1,2,3,*} Wen O. Wang^{4,*} Jia-Xin Zhang^{5,4,†} Leon Balents^{4,5,6,‡} and D. N. Sheng^{3,§}

¹*School of Science and Engineering, The Chinese University of Hong Kong, Shenzhen, Guangdong, 518172, People's Republic of China*

²*The Hamburg Centre for Ultrafast Imaging and Institute for Quantum Physics, University of Hamburg, Luruper Chaussee 149, Hamburg 22761, Germany*

³*Department of Physics and Astronomy, California State University Northridge, Northridge, California 91330, USA*

⁴*Kavli Institute for Theoretical Physics, University of California, Santa Barbara, California 93106-4030, USA*

⁵*French American Center for Theoretical Science, CNRS, KITP, Santa Barbara, California 93106-4030, USA*

⁶*Canadian Institute for Advanced Research, Toronto, Ontario, M5G 1M1 Canada*



(Received 10 September 2025; revised 12 January 2026; accepted 2 February 2026; published 26 February 2026; corrected 28 August 2026)

Moiré materials provide exciting platforms for studying the interplay of strong electronic correlation and large magnetic flux effects. We study the lightly doped Hofstadter-Hubbard model on a triangular lattice through the large-scale density matrix renormalization group and determinantal quantum Monte Carlo simulations. We find strong evidence for a robust chiral superconducting (SC) phase with dominant power-law pairing correlations and a quantized spin Chern number. The SC phase emerges at very weak interaction and grows stronger at intermediate interaction strengths (U) for a wide range of hole doping. We also discuss the possible distinct nature of the normal state in different U regimes. Our Letter provides theoretical insights into the emergence of topological superconductivity from doping topological Chern bands or magnetic-flux-induced chiral spin liquid states of Moiré materials.

DOI: [10.1103/tm9q-w5y7](https://doi.org/10.1103/tm9q-w5y7)

Introduction—Understanding the mechanism of unconventional superconductivity (SC) in doped Mott insulators is a central topic in condensed matter physics [1,2]. Ever since Anderson's proposal of the resonating valence bond state as the undoped parent state of high-temperature SC in cuprates [3], quantum spin liquid and high- T_c SC have been naturally connected [4–9]. A particular class of quantum spin liquid that breaks time-reversal symmetry (TRS) and exhibits chiral edge modes is the chiral spin liquid (CSL) [10], which was predicted to give rise to $d + id$ -wave chiral SC through the condensation of fractional quasiparticles, known as anyon superconductivity [11–13].

Two-dimensional spin systems on geometric frustrated lattices are promising hosts for the CSL [14]. Numerical studies have established its existence in both the spin-1/2 kagome antiferromagnet [15–18] and the half filled triangular-lattice Hubbard model near the metal-insulator

transition [19–22]. However, non-SC charge density wave (CDW) [23] or chiral metal phases [24] have been reported by hole doping of the CSLs based on density matrix renormalization group (DMRG) studies of ladder systems. Meanwhile, numerical simulations have identified $d + id$ -wave SC besides topologically trivial nematic d -wave SC after doping the CSL or nearby magnetically ordered phases on a triangular-lattice system with TRS-breaking three-spin chiral interactions [25,26]. Interestingly, $d + id$ -wave SC also appears when doping the antiferromagnetic phase of the triangular-lattice $J_1 - J_2$ model through spontaneous TRS breaking [27] in a narrow parameter window. These observations demonstrate the competing orders in doped CSL and the rich interplay between geometrical frustration, hole dynamics, and spin fluctuations [28,29].

Experimentally, candidate materials for spin liquid are rare [30,31], and the route from spin liquid to unconventional SC remains illusive. Recently, the twisted transition metal dichalcogenide moiré system has emerged as a highly tunable platform for exploring strong correlation effects on a triangular lattice [32,33]. Particularly, the large moiré lattice constants make a large magnetic flux per unit cell accessible with currently available laboratory fields. Under a magnetic field, this system can be described by the Hubbard-Hofstadter model with spin or pseudospin SU(2) symmetry [34], which features strong correlation, non-trivial band topology, TRS breaking, and frustration. At

*These authors contributed equally to this work.

†Contact author: zhangjx.phy@gmail.com

‡Contact author: balents@spinsandelectrons.com

§Contact author: donna.sheng1@csun.edu

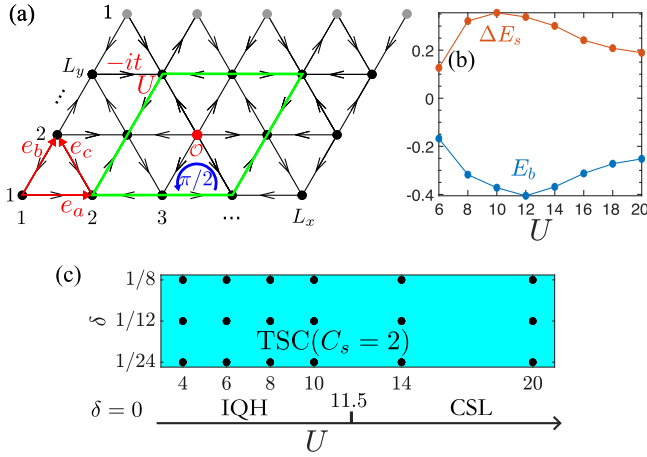


FIG. 1. Model and phase diagram. (a) The Hofstadter-Hubbard model on a triangular lattice. Each triangle is inserted $\pi/2$ flux for both spins. The arrow denotes the positive direction of τ_{ij} in the Hamiltonian. The green rectangular denotes the unit cell. (b) Cooper pair binding energy $E_b = E(N_e + 2, 0) + E(N_e, 0) - 2E(N_e + 1, 1/2)$ and spin excitation gap $\Delta E_s = E(N_e, 1) - E(N_e, 0)$ at different U s, obtained at $N = 36 \times 6$ and $\delta = 1/12$ through DMRG. Here $E(n, S_z^{\text{tot}})$ is the ground-state energy for the electron number n and total spin S_z^{tot} . (c) Phase diagram of the doped 6-leg Hubbard-Hofstadter model. Black dots represent the parameter points where DMRG calculations are performed. A topological SC (TSC) phase with spin Chern number $C_s = 2$ emerges after doping both IQH and CSL phases. A half filling phase diagram is also shown for comparison at the bottom.

half filling, a CSL phase has been found between the integer quantum Hall (IQH) phase at weak coupling and the 120° Néel ordered phase at strong coupling limit [34,35]. The binding between a pair of electrons has been numerically confirmed at half filling over a wide range of coupling strength on both sides of the IQH-CSL critical point [36]. It is argued through parton mean-field theory that topological SC emerges after doping near the topological quantum criticality associated with the closing of the charge gap while the spin gap remains open [36] (this argument may be generalized to fractional fillings [37]). However, direct numerical evidence for SC at finite doping remains lacking. This naturally leads to open questions such as whether the chiral SC survives competing orders at finite doping and what its topological nature is from doping either the IQH or CSL phases. Answering these questions will constitute a significant step toward establishing the mechanism of anyon superconductivity near topological criticality in realistic strongly correlated systems.

In this Letter, we address these questions through two unbiased and controlled numerical simulation methods. Through large-scale DMRG simulations [38] of the lightly doped triangular-lattice Hubbard-Hofstadter model with one-quarter flux quantum per triangle plaquette, we observe chiral SC over a broad range of the coupling strength U [Fig. 1(c)], corresponding to parent states ranging from IQH to CSL phases. The chiral SC is

characterized by quasi-long-range pairing correlations and a quantized spin Chern number $C_s = 2$ with possibly coexisting subdominant CDW. From a finite-temperature perspective, we apply a determinantal quantum Monte Carlo algorithm (DQMC) [39,40] to this model and find that SC pairing fluctuations become increasingly dominant as the temperature is lowered.

The Hamiltonian of the triangular-lattice Hubbard-Hofstadter model has the form

$$H = -it \sum_{\sigma} \sum_{\langle xy \rangle} \tau_{ij} (c_{i\sigma}^\dagger c_{j\sigma} - c_{j\sigma}^\dagger c_{i\sigma}) + U \sum_i n_{i\uparrow} n_{i\downarrow},$$

where $\sigma = \pm$ labels spins and $\tau_{ij} = +1$ or -1 if j to i follows or is opposite to the arrows in Fig. 1. This chosen gauge is called the imaginary C_{6z} gauge, and the Hamiltonian is symmetric under $\pi/3$ bare rotation around $\mathcal{O}$ in Fig. 1. We choose $t = 1$ as the energy unit. The hole doping level is denoted by δ , rendering the total electron number $N_e = N(1 - \delta)$. For DMRG, we consider a triangular lattice on a cylinder with $N = L_x \times L_y$ sites and impose the periodic (open) boundary condition in the y (x) direction [see Fig. 1(a)]. In the main text, we focus on $L_y = 6$ and present additional results for $L_y = 3, 4, 8$ in Supplemental Material [41]. The simulations enforce both charge-conservation $U(1)$ and spin- $SU(2)$ symmetries [42], and retain up to $M = 16000$ $SU(2)$ Schmidt states (bond dimension) for large $L_y = 6$ systems, resulting in a truncation error $\epsilon \approx 5.0 \times 10^{-6}$. For DQMC, we simulate the triangular lattice with periodic boundary conditions along both directions (torus geometry), using a 6×6 system size in the main text, with results for the smaller 4×4 and larger 8×8 systems in Supplemental Material [41], showing qualitatively consistent conclusions.

Quantum phase diagram—To systematically identify the phase diagram across a range of doping and interaction strengths, we first perform a comprehensive set of DMRG simulations. In Fig. 2(a), we show the electron density profiles corresponding to doping the IQH phase at $U = 8$ and the CSL phase near ($U = 14$) and far from ($U = 20$) criticality, respectively. In all three cases, there are charge stripes with a period of 4 sites along the x direction, corresponding to two holes per stripe, but the CDW amplitude is much weaker in the doped IQH phase. The phase structure of the nearest-neighbor pairing in Fig. 2(b) demonstrates that SC order is antisymmetric under magnetic translations along both e_a and e_b [41], corresponding to one of the four one-dimensional irreducible representations of the magnetic translational group when the flux per unit cell is π [43,44]. The pairing order gains $-\pi/3$ phase under the C_6 magnetic rotation, which takes its bare form when the center of rotation is $\mathcal{O}$. In Figs. 2(c) and 2(d), we show the dominant spin singlet pair-pair correlations $P_{\alpha\beta}(r) = \langle \hat{\Delta}_\alpha^\dagger(x_0, y_0) \hat{\Delta}_\beta(x_0 + r, y_0) \rangle$ with the pairing order $\hat{\Delta}_\alpha(x, y) = (\hat{c}_{(x,y),\uparrow} \hat{c}_{(x,y)+\mathbf{e}_a,\downarrow} - \hat{c}_{(x,y),\downarrow} \hat{c}_{(x,y)+\mathbf{e}_a,\uparrow}) / \sqrt{2}$,

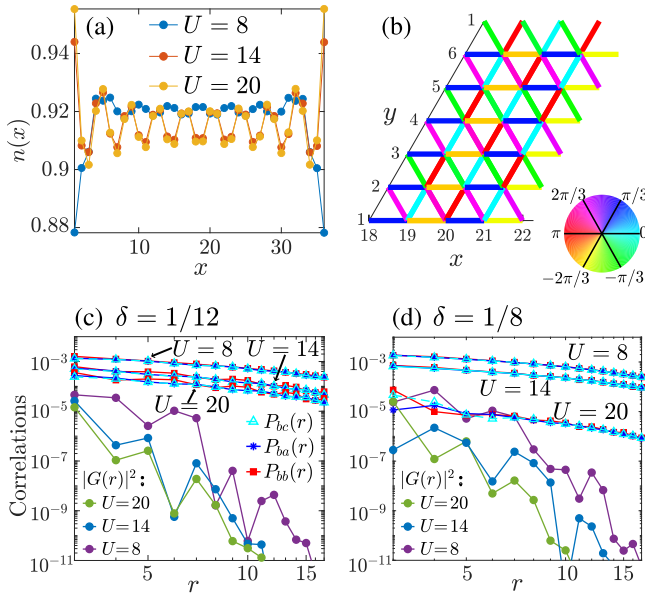


FIG. 2. Properties of the TSC phase based on DMRG simulations at $M = 14000$. (a) Rung-averaged electron density profiles $n(x) = \sum_{y=1}^{L_y} (n_{(x,y),\uparrow} + n_{(x,y),\downarrow}) / L_y$ for $N = 36 \times 6$ at various U s and $\delta = 1/12$. (b) The phase pattern of pairing on different bonds in the chiral SC phase represented by $U = 14$ and $\delta = 1/12$. The phase of the SC order on a bond α at $\mathbf{r}$ with respect to that of the reference bond along $\mathbf{e}_b$ at $\mathbf{r}_0 = (13, 1)$: $\arg[\langle \hat{\Delta}_b^\dagger(\mathbf{r}_0) \hat{\Delta}_\alpha(\mathbf{r}) \rangle] = \arg[\langle \hat{\Delta}_\alpha(\mathbf{r}) \rangle] - \arg[\langle \hat{\Delta}_b(\mathbf{r}_0) \rangle]$ are shown. (c) Pair-pair correlations $P_{\alpha\beta}(r)$ and single-particle Green's function $G(r) = \sum_\sigma \langle \hat{c}_\sigma^\dagger(x_0, y_0) \hat{c}_\sigma(x_0 + r, y_0) \rangle$ for various U s and $\delta = 1/12$ and $N = 36 \times 6$. (d) A similar plot for $\delta = 1/8$ and $N = 32 \times 6$.

where $\alpha = a, b, c$ and (x, y) denotes the location $x\mathbf{e}_a + y\mathbf{e}_b$. For both doping levels $\delta = 1/12$ and $1/8$, the pairing correlations are strong and decay slowly with distances for U from 8 to 14, but they are suppressed by one order of magnitude for $U = 20$ at $\delta = 1/8$, indicating weakened SC for very strong U . The isotropy of the chiral SC order is demonstrated by $P_{ba} \approx P_{bb} \approx P_{bc}$. The pair-pair correlations are much stronger than the amplitude squared of the single-particle Green's function $|G(r)|^2$ at longer distances, demonstrating the charge- $2e$ mode as the origin of SC. Consistently, the electrons show strong binding ($E_b < 0$) while the spin gap is finite ($\Delta E_s > 0$) [Fig. 1(b)]. The corresponding phase diagram is presented in Fig. 1(c) with the chiral SC phase present for a wide range of U and doping levels.

On the other hand, for smaller U (i.e., $U \lesssim 3$), SC becomes too weak to be distinguished from a possible metallic phase, as indicated by the absence of clearly dominant correlations (see Fig. S11 in Supplemental Material [41]), though a perturbative analysis suggests that an instability toward superconductivity may still emerge in the weak-coupling limit [41]. Taken together, the suppression of SC at both small and large U indicates that SC is

more robust at intermediate interaction strength, approximately near the topological criticality of the undoped parent system [36].

Dominant pairing correlations—Two representative points corresponding to doping the IQH ($U = 8$) and CSL ($U = 14$) states are characterized in Fig. 3 for systems size $N = 48 \times 6$ at doping level $\delta = 1/12$. The magnitude of pairing correlations at longer distances $|P_{bb}(r)|$ increases gradually as the DMRG bond dimension increases from $M = 8000$ to 16000 . Because the DMRG represents the ground state in the matrix product form [45] with finite bond dimensions, the scaling to $M \rightarrow \infty$ is needed to identify the true nature of long-distance correlations for wide cylinders. Using a second-order polynomial fitting of $1/M$, we find that the extrapolated $|P_{bb}(r)|$ shows a power-law decay with distance $|P_{bb}(r)| \sim r^{-K_{SC}}$, with the Luttinger exponent $K_{SC} \approx 0.70$, and 0.95 for $U = 8$ and 14 , respectively. Similar results are obtained for correlations with other bonds and for a smaller size $N = 36 \times 6$. $K_{SC} \lesssim 1$ indicates strong divergent SC susceptibility in the zero-temperature limit when $L_x \rightarrow \infty$ [46]. Similarly, quasi-long-range CDW orders are observed through the density-density correlations $D(r) = \langle \hat{n}_{x_0, y_0} \hat{n}_{x_0+r, y_0} \rangle - \langle \hat{n}_{x_0, y_0} \rangle \langle \hat{n}_{x_0+r, y_0} \rangle \sim r^{-K_{CDW}}$ with a larger Luttinger exponent ($K_{CDW} \approx 1.58$ and 1.74 for $U = 8$ and 14 , respectively), indicating the dominance of SC order. As shown in the inset of Fig. 3(d), the Luttinger exponent for SC decreases with the cylinder width according to $K_{SC} \propto 1/L_y$ for $L_y = 3-6$, suggesting long-range SC in the two-dimensional (2D) limit [47]. In contrast, K_{CDW} decreases much more slowly—or even nonmonotonically—with increasing L_y , which prevents a reliable extrapolation based on our results for $L_y \leq 6$ and leaves open the question of whether CDW coexists with SC in the 2D limit [48].

As a complement to the DMRG simulations, which detect the orders of ground state through equal-time correlation functions, the unequal-time correlation function $\chi(\beta/2) \equiv \beta \langle \mathcal{T}_\tau \hat{\mathcal{O}}^\dagger(\tau = \beta/2) \hat{\mathcal{O}}(\tau = 0) \rangle / N$ measured in DQMC provides a dynamic perspective on low-temperature order by probing the corresponding low-energy bosonic fluctuations within an energy window of order $k_B T$ [41]. Specifically, as shown in Fig. S13 in Supplemental Material [41], the pairing phase pattern at temperature $T = t/4$ for intermediate interaction strength $U = 8$ can be extracted by $\langle \mathcal{T}_\tau \hat{\Delta}_\alpha^\dagger(x_0, y_0, \tau = \beta/2) \times \hat{\Delta}_\beta(x_0 + r_x, y_0 + r_y, \tau = 0) \rangle$, and is in excellent agreement with that identified by DMRG. At the same temperature, the pattern becomes disrupted at significantly smaller U or at half filling, as shown in Fig. S14 in Supplemental Material [41]. Once the pairing phase pattern is determined, the unequal-time pairing susceptibility $\chi(\beta/2)$ can be defined by taking $\hat{\mathcal{O}} = \sum_{\alpha, x, y} \hat{\Delta}_\alpha(x, y) e^{-i\theta_\alpha(x, y)}$, where the sum runs over all inequivalent bonds (x, y, α) with their corresponding phase factors $\theta_\alpha(x, y)$. As illustrated in

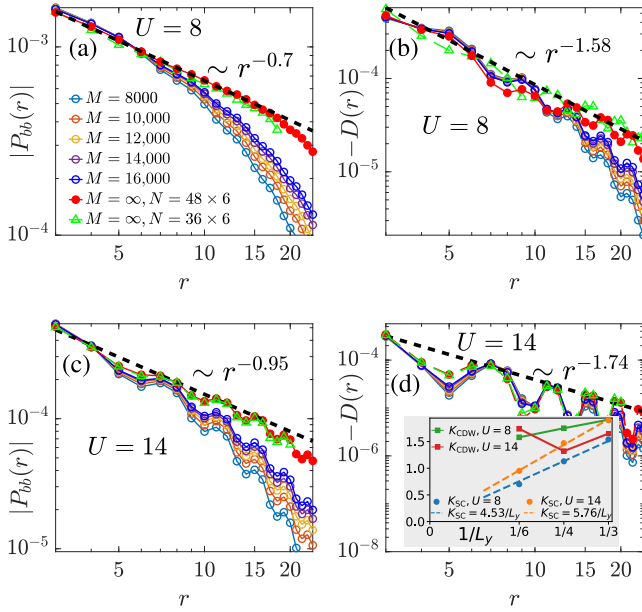


FIG. 3. CDW and SC correlation functions for doping the IQH ($U = 8$) and CSL ($U = 14$) states from DMRG simulations. (a), (b) The scalings and power-law fittings of pair-pair correlation $P_{bb}(r)$ and density-density correlation $D(r)$ for $U = 8$ and $\delta = 1/12$. (c), (d) Similar plots for $U = 14$. The second-order extrapolation to infinite M is applied. The results for $N = 48 \times 6$ and $N = 36 \times 6$ are consistent. The inset of (d) shows the system width dependence of the Luttinger exponents of CDW and SC using data given in Table I in Supplemental Material [41].

Fig. 4(a), $\chi(\beta/2)$ increases rapidly with decreasing temperature at finite doping $\delta \neq 0$, indicating that pairing fluctuations are increasingly transferred into the low-energy window bounded by the thermal scale $k_B T$. In contrast, for $\delta = 0$, this enhancement is absent, suggesting that pairing fluctuations remain irrelevant in the undoped case, as theoretically expected. Upon extrapolating the data to lower temperatures, we define a characteristic temperature T_c as the point where $\chi(\beta/2)$ diverges, signaling the proliferation of low-energy pairing modes (cf. Supplemental Material Fig. S15 [41]). This T_c serves as an alternative temperature scale for SC, physically analogous to the standard critical temperature defined by the divergence of the static (zero-frequency) susceptibility. The dependence of the estimated T_c on doping and interaction strength is summarized in Fig. 4(b). Notably, the dome-like doping dependence of T_c , which vanishes at zero doping, suggests that SC fluctuations are intimately connected to the nature of the undoped state. Moreover, in the DQMC-accessible range $U \leq 8$, which already approaches the intermediate-coupling regime near the DMRG-identified critical point at half filling [35], T_c increases with U , indicating enhanced pairing correlations.

Quantized spin Chern number—To characterize the topological nature of the TSC state, we perform the flux insertion through the cylinder in the $U(1) \times U(1)$ DMRG simulations [Fig. 5(a)] [16,49]. Specifically, we impose the

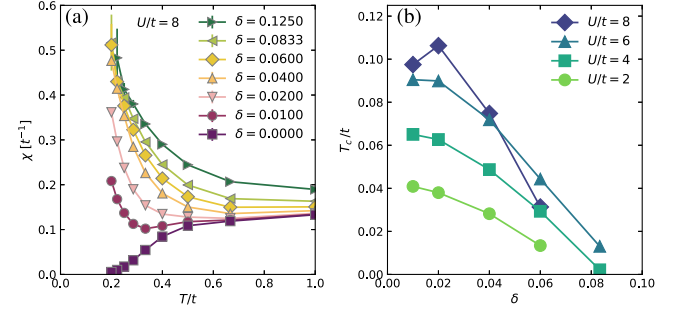


FIG. 4. Pairing susceptibility enhancement and estimation of T_c from DQMC simulations. (a) Temperature dependence of pairing susceptibility $\chi(\tau = \beta/2)$ for $U/t = 8$. (b) Superconducting characteristic temperature T_c estimated by linearly extrapolating $\chi^{-1}(\tau = \beta/2)$ to zero. Simulation lattice size is 6×6 .

spin-dependent boundary condition $\hat{c}_{(x,L_y+1),\sigma} = \hat{c}_{(x,1),\sigma} e^{i\sigma\theta_F}$ and measure the accumulated spins $\Delta Q_s = (n_\uparrow - n_\downarrow)|_0^{\theta_F}$ near the right edge while adiabatically increasing θ_F from 0 to 2π [Fig. 5(b)]. Similarly, a spin-independent boundary condition $\hat{c}_{(x,L_y+1),\sigma} = \hat{c}_{(x,1),\sigma} e^{i\theta_F}$ is used to calculate the accumulated charges $\Delta Q_c = (n_\uparrow + n_\downarrow)|_0^{\theta_F}$ near the right edge. The spin Chern number is given by $C_s = \Delta Q_s(\theta_F = 0 \rightarrow 2\pi)$ and has a well-quantized value of 2 in the TSC phase [Fig. 5(c)], similar to other TSC states in doped triangular Mott insulators without considering the orbital effect of complex hopping [26,27]. In contrast, the charge Chern number $C_c = \Delta Q_c(\theta_F = 0 \rightarrow 2\pi)$ vanishes (or is nearly zero) in the TSC phase but remains finite in the smaller U regime ($U \lesssim 2$), possibly consistent with chiral metal behavior in our finite size systems. The TSC state is further characterized by the nonzero spin chiral order $\langle \chi \rangle = \langle \hat{\mathbf{S}}_i \cdot (\hat{\mathbf{S}}_j \times \hat{\mathbf{S}}_k) \rangle$ (i, j, k are the sites of the elementary triangle plaquettes) (see Supplemental Material for details [41]).

Summary and discussion—To summarize, through ground-state DMRG and finite-temperature DQMC simulations of the triangular Hofstadter-Hubbard model, we identify a single chiral TSC phase with spin Chern number 2 and long-range spin chiral order across a wide range of interaction strength corresponding to doping both the IQH and CSL phases. Our results thus support the possibility of experimentally realizing TSC in the twisted transition metal dichalcogenide Moiré systems. The strongest electron binding is observed near the topological critical point between IQH and CSL. The quasi-long-range TSC is the dominant order and is significantly strengthened as the cylinder width increases. The SC order gains $-\pi/3$ phase under the magnetic C_6 rotation and is antisymmetric under unit magnetic translations.

In the future, it would be particularly interesting to study the normal state properties in more detail in order to distinguish between different pairing mechanisms. Our Letter demonstrates a single zero-temperature TSC phase over a wide range of interaction strengths [Fig. 1(c)], which

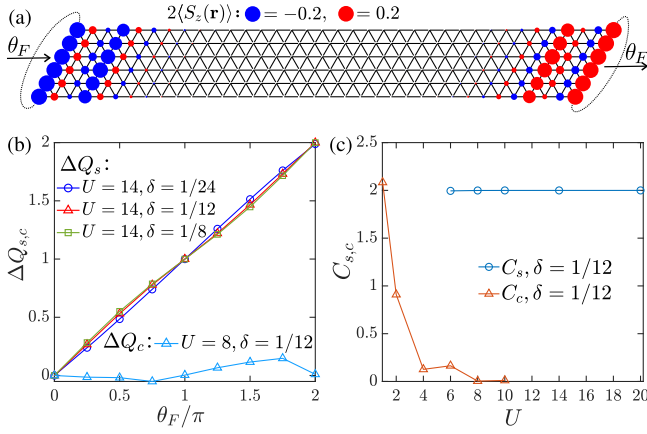


FIG. 5. Topological characterization of the TSC phase through flux insertion in DMRG simulations. (a) Real-space configuration of the spin magnetization after adiabatically inserting a quantized flux $\theta_F = 2\pi$. The area of the circle is proportional to the amplitude of $2\langle S_z(\mathbf{r}) \rangle$. The red (blue) color represents the positive (negative) value. (b) The net spin (charge) transferred $\Delta Q_{s,c}$ from the left edge to the right one after adiabatically threading spin-dependent (-independent) flux θ_F with steps of $\pi/4$. (c) Spin and charge Chern numbers $C_{s,c} = \Delta Q_{s,c}(\theta_F = 0 \rightarrow 2\pi)$ at different U and δ . The obtained results are well converged with a U(1) bond dimension $m = 10000$.

may be attributed to topological criticality, which coherently softens a charge- $2e$ bosonic mode and thereby energetically stitches the two sides together [36]. However, the normal states for $T > T_c$ may be quite distinct in different interaction ranges. In the small- U regime, the momentum-space electron distribution $n(\mathbf{k})$ of the doped system exhibits a sharp jump (Figs. 6(a), 6(c), and S12 in the Supplemental Material [41]), and second-order perturbative processes via interband transitions [41,50] provide a natural pairing route—basically similar to the Fermi surface instability induced by attractive channels in conventional metals. Therefore, an essentially Fermi-liquid metal with a nonquantized Hall response [51,52] is expected for $T > T_c$. On the other hand, as U increases, the sharp jump of $n(\mathbf{k})$ is significantly smeared out [Figs. 6(b) and 6(c)], electronic correlations become dominant, and the TSC appears to emerge directly from a doped CSL. This crossover is supported by the evolution of the ratio between the bare (bubble) susceptibility—constructed from single-particle Green’s functions in the DQMC simulations—and the fully renormalized susceptibility, as shown in Supplemental Material [41] (Fig. S18). While this ratio remains close to unity at weak coupling, it is strongly suppressed at large U , extrapolating to zero even at finite temperature, signaling that the pairing channel is driven by strong correlation effects. These qualitative differences in the nature of the normal state also imply clear and experimentally accessible diagnostics. For example, they can be distinguished by single-particle spectroscopy such as angle-resolved photoemission spectroscopy

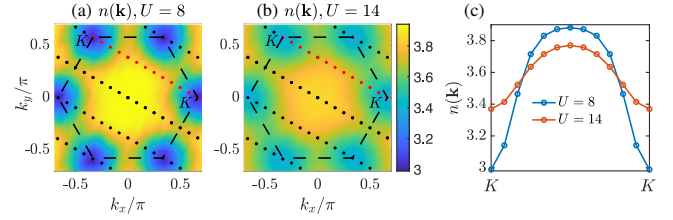


FIG. 6. Momentum-space electron density distribution $n(\mathbf{k}) = 1/N_{\text{cell}} \sum_{i,j=1}^{N_{\text{cell}}} \sum_{\alpha=1}^4 \sum_{\sigma} \langle c_{i\alpha\sigma}^\dagger c_{j\alpha\sigma} \rangle e^{i\mathbf{k} \cdot (\mathbf{R}_i - \mathbf{R}_j)}$ from DMRG simulations for (a) $U = 8$ and (b) $U = 14$, respectively, where N_{cell} is the number of 2×2 magnetic unit cells, α labels the sites within the cell, and $\mathbf{R}_{i,j}$ label the centers of the unit cells. The first magnetic Brillouin zone is shown by the dashed black line. The accessible wave vectors corresponding to the middle $N_{\text{cell}} = 12 \times 3$ cells (to exclude edge effects) of the cylinder are marked by black dots, and the pseudocolor plot is created using bilinear interpolations. (c) This shows explicitly $n(\mathbf{k})$ along the selected momentum path marked by the red dotted line in (a) and (b). Parameters: $N = 48 \times 6$ and $\delta = 1/12$.

above T_c , which directly probe whether coherent quasi-particles survive at low energies, which is expected in a weak coupling scenario but not in a doped strongly correlated parent state. Similarly, magnetic vortex cores, where the normal state physics is locally exposed, may host fractionalized excitations inherited from the parent chiral spin liquid. Future studies should probe the view of this normal state as a compressible anyon fluid [53–55], with an eye to experimentally measurable signatures. Our results may be a starting point for this endeavor.

Acknowledgments—We are grateful to the authors of Ref. [56] for sharing their manuscript with us and for coordinating submission; we reach similar conclusions. D. N. S. acknowledges helpful discussions with Clemens Kuhlenskamp and Ashvin Vishwanath. W. O. W. acknowledges support from the Gordon and Betty Moore Foundation through Grant No. GBMF8690 to the University of California, Santa Barbara, to the Kavli Institute for Theoretical Physics (KITP). This research was supported in part by NSF Grant No. PHY-2309135 to the KITP. DQMC simulations made use of computational facilities purchased with funds from the NSF (CNS-1725797) and administered by the Center for Scientific Computing (CSC). The CSC is supported by the California NanoSystems Institute and the Materials Research Science and Engineering Center (MRSEC; NSF DMR 2308708) at UC Santa Barbara. J. X. Z. was supported by the European Research Council (ERC) under the European Union’s Horizon 2020 research and innovation program (Grant Agreement No. 853116, acronym TRANSPORT). L. B. acknowledges support by the DOE through Grant No. DE-SC0020305 for theoretical analysis of superconductivity, and the Simons Collaboration on Ultra-Quantum Matter, which is a grant from the Simons Foundation (Grant No. 651440) for guidance of numerics. D. N. S. was

supported by the DOE, Office of BES under Grant No. DE-FG02-06ER46305 for large-scale simulations of interacting systems. We also acknowledge NSF Grant No. DMR-2406524 (D. N. S.) for instrument support.

F. C. performed the DMRG studies. W. O. W. conceptualized and performed the DQMC studies. J.-X. Z. carried out the analytical calculations and contributed to the interpretation of the DQMC results. All authors contributed to the development and completion of this work.

Data availability—The data that support the findings of this article are openly available [57], embargo periods may apply.

-
- [1] B. Keimer, S. A. Kivelson, M. R. Norman, S. Uchida, and J. Zaanen, From quantum matter to high-temperature superconductivity in copper oxides, *Nature (London)* **518**, 179 (2015).
 - [2] P. A. Lee, N. Nagaosa, and X.-G. Wen, Doping a Mott insulator: Physics of high-temperature superconductivity, *Rev. Mod. Phys.* **78**, 17 (2006).
 - [3] P. W. Anderson, The resonating valence bond state in La_2CuO_4 and superconductivity, *Science* **235**, 1196 (1987).
 - [4] M. Fabrizio, Superconductivity from doping a spin-liquid insulator: A simple one-dimensional example, *Phys. Rev. B* **54**, 10054 (1996).
 - [5] R. M. Konik, T. M. Rice, and A. M. Tsvelik, Doped spin liquid: Luttinger sum rule and low temperature order, *Phys. Rev. Lett.* **96**, 086407 (2006).
 - [6] D. S. Rokhsar, Pairing in doped spin liquids: Anyon versus d-wave superconductivity, *Phys. Rev. Lett.* **70**, 493 (1993).
 - [7] Z. A. Kelly, M. J. Gallagher, and T. M. McQueen, Electron doping a kagome spin liquid, *Phys. Rev. X* **6**, 041007 (2016).
 - [8] T. Senthil and P. A. Lee, Cuprates as doped $U(1)$ spin liquids, *Phys. Rev. B* **71**, 174515 (2005).
 - [9] M. Sigrist, T. M. Rice, and F. C. Zhang, Superconductivity in a quasi-one-dimensional spin liquid, *Phys. Rev. B* **49**, 12058 (1994).
 - [10] V. Kalmeyer and R. B. Laughlin, Equivalence of the resonating-valence-bond and fractional quantum Hall states, *Phys. Rev. Lett.* **59**, 2095 (1987).
 - [11] R. B. Laughlin, Superconducting ground state of noninteracting particles obeying fractional statistics, *Phys. Rev. Lett.* **60**, 2677 (1988).
 - [12] X. G. Wen, F. Wilczek, and A. Zee, Chiral spin states and superconductivity, *Phys. Rev. B* **39**, 11413 (1989).
 - [13] D.-H. Lee and M. P. A. Fisher, Anyon superconductivity and the fractional quantum Hall effect, *Phys. Rev. Lett.* **63**, 903 (1989).
 - [14] L. Balents, Spin liquids in frustrated magnets, *Nature (London)* **464**, 199 (2010).
 - [15] Y.-C. He, D. N. Sheng, and Y. Chen, Chiral spin liquid in a frustrated anisotropic kagome Heisenberg model, *Phys. Rev. Lett.* **112**, 137202 (2014).
 - [16] S.-S. Gong, W. Zhu, and D. N. Sheng, Emergent chiral spin liquid: Fractional quantum Hall effect in a kagome Heisenberg model, *Sci. Rep.* **4**, 6317 (2014).
 - [17] B. Bauer, L. Cincio, B. P. Keller, M. Dolfi, G. Vidal, S. Trebst, and A. W. W. Ludwig, Chiral spin liquid and emergent anyons in a kagome lattice Mott insulator, *Nat. Commun.* **5**, 5137 (2014).
 - [18] S.-S. Gong, W. Zhu, L. Balents, and D. N. Sheng, Global phase diagram of competing ordered and quantum spin-liquid phases on the kagome lattice, *Phys. Rev. B* **91**, 075112 (2015).
 - [19] A. Szasz, J. Motruk, M. P. Zaletel, and J. E. Moore, Chiral spin liquid phase of the triangular lattice Hubbard model: A density matrix renormalization group study, *Phys. Rev. X* **10**, 021042 (2020).
 - [20] B.-B. Chen, Z. Chen, S.-S. Gong, D. N. Sheng, W. Li, and A. Weichselbaum, Quantum spin liquid with emergent chiral order in the triangular-lattice Hubbard model, *Phys. Rev. B* **106**, 094420 (2022).
 - [21] A. Wietek, R. Rossi, F. Šimkovic, M. Klett, P. Hansmann, M. Ferrero, E. M. Stoudenmire, T. Schäfer, and A. Georges, Mott insulating states with competing orders in the triangular lattice Hubbard model, *Phys. Rev. X* **11**, 041013 (2021).
 - [22] Y. Zhou, D. N. Sheng, and E.-A. Kim, Quantum phases of transition metal dichalcogenide Moiré systems, *Phys. Rev. Lett.* **128**, 157602 (2022).
 - [23] C. Peng, Y.-F. Jiang, D.-N. Sheng, and H.-C. Jiang, Doping quantum spin liquids on the kagome lattice, *Adv. Quantum Technol.* **4**, 2000126 (2021).
 - [24] Z. Zhu, D. N. Sheng, and A. Vishwanath, Doped Mott insulators in the triangular-lattice Hubbard model, *Phys. Rev. B* **105**, 205110 (2022).
 - [25] Y.-F. Jiang and H.-C. Jiang, Topological superconductivity in the doped chiral spin liquid on the triangular lattice, *Phys. Rev. Lett.* **125**, 157002 (2020).
 - [26] Y. Huang and D. N. Sheng, Topological chiral and nematic superconductivity by doping Mott insulators on triangular lattice, *Phys. Rev. X* **12**, 031009 (2022).
 - [27] Y. Huang, S.-S. Gong, and D. N. Sheng, Quantum phase diagram and spontaneously emergent topological chiral superconductivity in doped triangular-Lattice Mott insulators, *Phys. Rev. Lett.* **130**, 136003 (2023).
 - [28] E. Fradkin, S. A. Kivelson, and J. M. Tranquada, Colloquium: Theory of intertwined orders in high temperature superconductors, *Rev. Mod. Phys.* **87**, 457 (2015).
 - [29] X.-Y. Song, A. Vishwanath, and Y.-H. Zhang, Doping the chiral spin liquid: Topological superconductor or chiral metal, *Phys. Rev. B* **103**, 165138 (2021).
 - [30] J. Khatua, B. Sana, A. Zorko, M. Gomilšek, K. Sethupathi, M. S. R. Rao, M. Baenitz, B. Schmidt, and P. Khuntia, Experimental signatures of quantum and topological states in frustrated magnetism, *Phys. Rep.* **1041**, 1 (2023).
 - [31] J. Knolle and R. Moessner, A field guide to spin liquids, *Annu. Rev. Condens. Matter Phys.* **10**, 451 (2019).
 - [32] F. Wu, T. Lovorn, E. Tutuc, and A. H. MacDonald, Hubbard model physics in transition metal dichalcogenide moiré bands, *Phys. Rev. Lett.* **121**, 026402 (2018).
 - [33] Y. Tang, L. Li, T. Li, Y. Xu, S. Liu, K. Barmak, K. Watanabe, T. Taniguchi, A. H. MacDonald, J. Shan, and

- K.F. Mak, Simulation of Hubbard model physics in WSe_2/WS_2 Moiré superlattices, *Nature (London)* **579**, 353 (2020).
- [34] C. Kuhlenskamp, W. Kadow, A. Imamoğlu, and M. Knap, Chiral pseudospin liquids in Moiré heterostructures, *Phys. Rev. X* **14**, 021013 (2024).
- [35] S. Divic, T. Soejima, V. Crépel, M. P. Zaletel, and A. Millis, Chiral spin liquid and quantum phase transition in the triangular lattice Hofstadter-Hubbard model, *arXiv:2406.15348*.
- [36] S. Divic, V. Crépel, T. Soejima, X.-Y. Song, A. J. Millis, M. P. Zaletel, and A. Vishwanath, Anyon superconductivity from topological criticality in a Hofstadter-Hubbard model, *Proc. Natl. Acad. Sci. U.S.A.* **122**, e2426680122 (2025).
- [37] F. Pichler, C. Kuhlenskamp, M. Knap, and A. Vishwanath, Microscopic mechanism of anyon superconductivity emerging from fractional Chern insulators, *Newton* **2**, 100340 (2025).
- [38] S. R. White, Density matrix formulation for quantum renormalization groups, *Phys. Rev. Lett.* **69**, 2863 (1992).
- [39] R. Blankenbecler, D. J. Scalapino, and R. L. Sugar, Monte Carlo calculations of coupled boson-fermion systems. I, *Phys. Rev. D* **24**, 2278 (1981).
- [40] S. R. White, D. J. Scalapino, R. L. Sugar, E. Y. Loh, J. E. Gubernatis, and R. T. Scalettar, Numerical study of the two-dimensional Hubbard model, *Phys. Rev. B* **40**, 506 (1989).
- [41] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/tm9q-w5y7> for detailed numerical results and discussions.
- [42] I. P. McCulloch, From density-matrix renormalization group to matrix product states, *J. Stat. Mech.* (2007) P10014.
- [43] D. Shaffer, J. Wang, and L. H. Santos, Unconventional self-similar Hofstadter superconductivity from repulsive interactions, *Nat. Commun.* **13**, 7785 (2022).
- [44] D. Shaffer, J. Wang, and L. H. Santos, Theory of Hofstadter superconductors, *Phys. Rev. B* **104**, 184501 (2021).
- [45] U. Schollwöck, The density-matrix renormalization group in the age of matrix product states, *Ann. Phys. (Amsterdam)* **326**, 96 (2011).
- [46] H.-C. Jiang, Superconductivity in the doped quantum spin liquid on the triangular lattice, *npj Quantum Mater.* **6**, 1 (2021).
- [47] Y. Gannot and S. A. Kivelson, How quantum phases on cylinders approach the two-dimensional limit, *Phys. Rev. B* **107**, 075127 (2023).
- [48] H. Xu, C.-M. Chung, M. Qin, U. Schollwöck, S. R. White, and S. Zhang, Coexistence of superconductivity with partially filled stripes in the Hubbard model, *Science* **384**, eadh7691 (2024).
- [49] M. P. Zaletel, R. S. Mong, and F. Pollmann, Flux insertion, entanglement, and quantized responses, *J. Stat. Mech.* (2014) P10007.
- [50] V. Crépel, T. Cea, L. Fu, and F. Guinea, Unconventional superconductivity due to interband polarization, *Phys. Rev. B* **105**, 094506 (2022).
- [51] A. A. Markov, G. Rohringer, and A. N. Rubtsov, Robustness of the topological quantization of the Hall conductivity for correlated lattice electrons at finite temperatures, *Phys. Rev. B* **100**, 115102 (2019).
- [52] X. Kou, L. Pan, J. Wang, Y. Fan, E. Choi, W.-L. Lee, T. Nie, K. Murata, Q. Shao, S.-C. Zhang, and K. Wang, Metal-to-insulator switching in quantum anomalous Hall states, *Nat. Commun.* **6**, 8474 (2015).
- [53] Z. D. Shi and T. Senthil, Doping a fractional quantum anomalous Hall insulator, *Phys. Rev. X* **15**, 031069 (2025).
- [54] M. Kim, A. Timmel, L. Ju, and X.-G. Wen, Topological chiral superconductivity beyond pairing in a Fermi liquid, *Phys. Rev. B* **111**, 014508 (2025).
- [55] Z. D. Shi and T. Senthil, Anyon delocalization transitions out of a disordered fractional quantum anomalous Hall insulator, *Proc. Natl. Acad. Sci. U.S.A.* **122**, e2520608122 (2025).
- [56] C. Kuhlenskamp, S. Divic, M. P. Zaletel, T. Soejima, and A. Vishwanath, Robust superconductivity upon doping chiral spin liquid and Chern insulators in a Hubbard-Hofstadter model, *arXiv:2509.02675*.
- [57] The DMRG data and code are available at https://github.com/cfengno1/Hofstadter_Hubbard_TSC and the DQMC data and code are available at [10.5281/zenodo.18476171](https://zenodo.org/record/18476171).

Correction: A support statement in the eighth sentence of the Acknowledgments has been corrected.