

Curvature Perturbations from First-Order Phase Transitions: Implications to Black Holes and Gravitational Waves

Gabriele Franciolini^{1,*}, Yann Gouttenoire^{2,3,†} and Ryusuke Jinno^{4,‡}

¹*CERN, Theoretical Physics Department, Esplanade des Particules 1, Geneva 1211, Switzerland*

²*School of Physics and Astronomy, Tel-Aviv University, Tel-Aviv 69978, Israel*

³*PRISMA+ Cluster of Excellence and MITP, Johannes Gutenberg University, 55099 Mainz, Germany*

⁴*Department of Physics, Graduate School of Science, Kobe University, 1-1 Rokkodai, Kobe, Hyogo 657-8501, Japan*



(Received 12 March 2025; revised 21 December 2025; accepted 11 February 2026; published 30 April 2026)

Understanding whether primordial black holes form during strong first-order phase transition (FOPT) is a crucial open question in cosmology. We address this using a fully covariant formalism to study cosmological perturbations, highlighting previously overlooked gauge dependencies. We show that non-covariant treatments can overestimate primordial black holes and scalar-induced gravitational waves. Once gauge dependencies are accounted for, both signals are strongly suppressed, with direct implications for the FOPT interpretation of the Pulsar Timing Array signal.

DOI: [10.1103/tfcx-kzqx](https://doi.org/10.1103/tfcx-kzqx)

Introduction—The Universe in the primordial epochs is expected to possess a higher degree of symmetry than today [1]. As the Universe cools, these symmetries may be broken through first-order phase transitions (FOPTs), a generic feature of many early-Universe scenarios, during which bubbles of the new vacuum nucleate and expand, converting the old vacuum energy into heat and bulk fluid motion (see Ref. [2] for a review). If the energy stored in the bubbles constitutes a large fraction of the total universe energy, FOPTs have been shown to leave sizable imprints on the spacetime metric, in terms of gravitational waves [3,4], primordial black holes (PBHs) [5–30], and curvature perturbations [20,31–37]. These curvature perturbations can source scalar-induced gravitational waves (SIGWs), studied in this context in [20] and more generally in [38–49].

Since cosmological perturbations generally exhibit gauge dependence [50], reliable predictions for these late-time observables require a covariant treatment of perturbations and their evolution. Such a treatment is required to assess whether FOPTs can produce observable PBHs and SIGWs, a question of central importance when evaluating the FOPT interpretation of the pulsar timing array signal [14,20,51].

In this Letter, we derive the statistical properties of cosmological perturbations produced during a strong

first-order phase transition, with a particular emphasis on the gauge choice and the calculation of gauge-invariant quantities. We evaluate the abundance of PBHs and the spectrum of SIGWs using the most appropriate quantities for each: the density contrast in the comoving gauge $\delta^{(C)}$ for the former, to match the convention used to report the threshold for collapse in numerical simulations, while the gauge-invariant curvature perturbation $\mathcal{R}$ for the latter.

Covariant linear perturbations from FOPTs—We model the nucleation rate by

$$\Gamma = H_n^4 e^{\beta(t-t_n)}, \quad (1)$$

where H_n is the Hubble factor ($H \equiv \dot{a}(t)/a$, with a being the scale factor) at $t = t_n$, defined as the nucleation time. Adopting a relativistic bag equation of state, the energy and pressure density decompose as

$$\rho = \rho_R + \rho_V, \quad p = \rho_R/3 - \rho_V, \quad (2)$$

where R and V stand for radiation and vacuum components, respectively. We consider the supercooled limit for which the latent heat ΔV is larger than the radiation energy density just before nucleation $\alpha \equiv \Delta V/\rho_R \gg 1$. At the background level, the radiation and vacuum energy density obeys the continuity equation

$$\overline{\rho}_R' + 4\mathcal{H}\overline{\rho}_R = -\overline{\rho}_V', \quad \text{with} \quad \overline{\rho}_V = \bar{F}\Delta V, \quad (3)$$

and the Friedmann equation $3M_{\text{pl}}^2 H^2 = \overline{\rho}_R + \overline{\rho}_V$. The prime denotes the derivative with respect to the conformal time η related to the cosmic time t by $\eta(t_2) = \eta(t_1) + \int_{t_1}^{t_2} d\tilde{t}/a(\tilde{t})$, and we defined $\mathcal{H} = aH$. The remaining fraction of false

*Contact author: gabriele.franciolini@cern.ch

†Contact author: yann.gouttenoire@gmail.com

‡Contact author: jinno@phys.sci.kobe-u.ac.jp

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

vacuum $\bar{F}(t)$ is given by

$$\bar{F}(t) = \exp \left[-\frac{4\pi}{3} \int_{-\infty}^t dt_n \Gamma(t_n) a(t_n)^3 \left(\int_{t_n}^t \frac{d\tilde{t}}{a(\tilde{t})} \right)^3 \right], \quad (4)$$

where we assumed bubble walls to expand at the speed of light $v_w \simeq 1$.

Due to the stochastic nature of bubble nucleation, the stress-energy tensor T_L^μ acquires fluctuations, assumed to be statistically isotropic, $T_0^0 = \bar{\rho} + \delta\rho$, $T_j^i = -(\bar{p} + \delta p)\delta_j^i + \Sigma_j^i$, and $T_0^i = (\bar{\rho} + \bar{p})v^i$, where $v^i = dx^i/dt$ is the coordinate velocity. In the following, we use $\delta \equiv \delta\rho/\bar{\rho}$. At the linear level, the 10 degrees of freedom of the symmetric 4×4 space-time metric $g_{\mu\nu}$ decomposes into four scalars, four vectors and two tensors. Keeping only the scalars (A , B , C , E), the spacetime metric reads

$$ds^2 = a^2 \left[(1 + 2A) d\eta^2 - 2\partial_i B dx^i d\eta - \left((1 - 2C)\delta_{ij} + 2\partial_{(i}\partial_{j)} E \right) dx^i dx^j \right], \quad (5)$$

where $\partial_{(i}\partial_{j)} E \equiv (\partial_i\partial_j - \frac{1}{3}\delta_{ij}\nabla^2)E$. In the absence of anisotropic stress, scalar metric perturbations contain only one physical degree of freedom. The apparent redundancy in their description is known as “gauge dependence” [50]. To extract physical predictions, one must fix the coordinate freedom by choosing a gauge. The assumption of isotropy, $\Sigma_j^i = 0$, imposes one constraint, leaving two additional scalar gauge degrees of freedom to fix. A common choice is the spatially flat gauge (F), defined by $C = E = 0$, in which we denote the density contrast by $\delta^{(F)}$. Linearized Einstein equations in Fourier space lead to [52]

$$\tilde{\delta}_{\mathbf{k}}^{(F)'} + 3\mathcal{H}(c_s^2 - \omega)\tilde{\delta}_{\mathbf{k}}^{(F)} = (1 + \omega)k^2\tilde{\mathcal{V}}_{\mathbf{k}} - 3\mathcal{H}\tilde{\delta}_{p,\text{nad},\mathbf{k}}, \quad (6)$$

where $\tilde{X}_{\mathbf{k}}(t)$ denotes the Fourier transform of $X(\mathbf{x}, t)$. The quantities $\omega \equiv \bar{p}/\bar{\rho}$ and $c_s^2 \equiv \bar{p}'/\bar{\rho}'$ are the equation of state and speed of sound, respectively. The quantity $\delta_{p,\text{nad}}$ is the nonadiabatic pressure perturbation

$$\delta_{p,\text{nad}} \equiv \frac{\delta p_{\text{nad}}}{\bar{p}}, \quad \text{with} \quad \delta p_{\text{nad}} \equiv \delta p^{(F)} - c_s^2 \delta\rho^{(F)}, \quad (7)$$

which is gauge-invariant in spite of the superscript (F) on the right hand-side. Using Eq. (2), Eq. (7) becomes

$$\delta p_{\text{nad}} = \frac{1 - 3c_s^2}{3} \bar{\rho} \delta^{(F)} - \frac{4}{3} \Delta V \delta F^{(F)}, \quad (8)$$

where $\delta F^{(F)} = F^{(F)} - \bar{F}$ with $F^{(F)} \equiv \rho_V^{(F)}/\Delta V$. The quantity $\mathcal{V}$ in Eq. (6) is the gauge-invariant scalar velocity $\mathcal{V} \equiv v + E'$ with $\partial_i(v) = \delta_{ij}v^j$. Einstein equation along $0i$ gives

$$\mathcal{V} = -\frac{2}{3(1 + \omega)} \frac{\mathcal{H}\Phi + \Phi'}{\mathcal{H}^2}, \quad (9)$$

where Φ is the gauge-invariant Newtonian potential [50]. A combination of Einstein equations along 00 and ij gives

$$\begin{aligned} \tilde{\Phi}_{\mathbf{k}}'' + 3(1 + c_s^2)\mathcal{H}\tilde{\Phi}_{\mathbf{k}}' + [3(c_s^2 - \omega)\mathcal{H}^2 + c_s^2 k^2]\tilde{\Phi}_{\mathbf{k}} \\ = \frac{3}{2}\mathcal{H}^2\tilde{\delta}_{p,\text{nad},\mathbf{k}}. \end{aligned} \quad (10)$$

The initial conditions for the system Eqs. (6) and (10) are

$$\tilde{\delta}_{\mathbf{k}}^{(F)}(0) = 0, \quad \tilde{\Phi}_{\mathbf{k}}(0) = 0, \quad \tilde{\Phi}_{\mathbf{k}}'(0) = 0, \quad (11)$$

where we have neglected the primordial curvature perturbations (i.e., generated during inflation), which we assume to be small at these scales. For more details on cosmological perturbation theory, we refer the reader to Supplemental Material (SM) [52] or Refs. [62–66].

In the spatially flat gauge, constant-time hypersurfaces are unperturbed $ds^2|_{\eta=\text{const}} = -a^2 dx^2$. This is particularly convenient since by plugging Eq. (8) into Eq. (6), we get a perturbation equation

$$\delta\tilde{\rho}_{R,\mathbf{k}}^{(F)'} + 4\mathcal{H}\delta\tilde{\rho}_{R,\mathbf{k}}^{(F)} = -\delta\tilde{\rho}_{V,\mathbf{k}}^{(F)'} + \frac{4}{3}\overline{\rho_R}k^2\tilde{\mathcal{V}}_{\mathbf{k}}, \quad (12)$$

which coincides with the background equation in Eq. (3) in the super-Hubble limit $k \ll \mathcal{H}$. Hence, in the spatially flat gauge and in the super-Hubble limit, different patches evolve as distinct FLRW universes [67], and we can identify

$$\tilde{F}_{\mathbf{k}}^{(F)} \underset{k \ll \mathcal{H}}{\simeq} \tilde{F}_{\mathbf{k}}^{(\text{FLRW})}, \quad (13)$$

where $F^{(\text{FLRW})}$ is the false vacuum fraction in a flat FLRW universe. Notice that such equality does not hold in other gauges. The code `deltaPT` developed in Ref. [20] calculates the false vacuum fraction $F_{\text{avg}}^{(\text{FLRW})}(R, t)$ averaged over a ball of radius R , in a flat FLRW universe. In this Letter, we rely on the approximation [68]

$$\tilde{F}_{\mathbf{k}}^{(\text{FLRW})}(t) \simeq V F_{\text{avg}}^{(\text{FLRW})}(R = k^{-1}, t), \quad (14)$$

with $V = 4\pi R^3/3$. The factor V ensures that Fourier transforms carry the usual volume dimension. Due to Eq. (14), it follows that $\tilde{X}_{\mathbf{k}}(t) \simeq V X_{\text{avg}}(R = k^{-1}, t)$ for any perturbations X derived in this Letter. This approximation should become exact in the super-Hubble limit $k \ll \mathcal{H}$ when gradient terms are negligible. We now denote perturbations by $X \equiv \tilde{X}_{\mathbf{k}}/V \simeq X_{\text{avg}}(R = k^{-1})$.

In practice, the evolution of perturbations can be derived as follows: from Eqs. (13) and (14), and `deltaPT`, we calculate δp_{nad} in Eq. (8), then we solve the closed system of

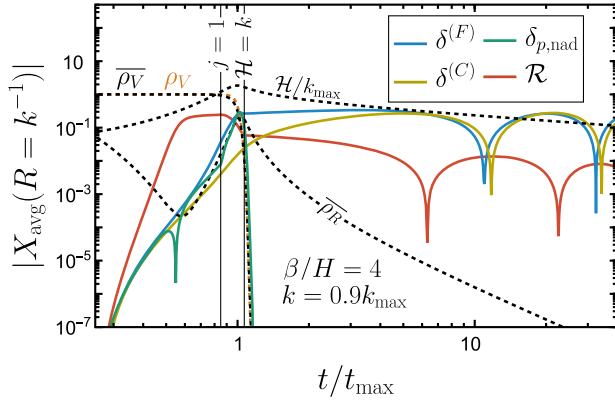


FIG. 1. Time evolution of the comoving curvature perturbation $\mathcal{R}$, and the density contrast in the spatially-flat $\delta^{(F)}$ and comoving $\delta^{(C)}$ gauge during a FOPT. We show perturbations X averaged over a comoving volume $4\pi k^{-3}/3$. We also report the non-adiabatic pressure δp_{nad} peaking close to Hubble crossing epoch $\mathcal{H} = k$. For presentation purposes we assume $\beta/H = 4$ and show the wave number $k = 0.9k_{\text{max}}$, where we identify k_{max} as the maximal $\mathcal{H}$. $j = 1$ indicates the time of first bubble nucleation, which is slightly delayed with respect to average (orange vs black dashed line).

equations in (6) and (10) to determine Φ and $\delta^{(F)}$; see SM [52] for more details. Another common choice of coordinate is the comoving gauge (C), in which the fluid velocity perturbation vanishes ($v + B = 0, E = 0$). The density contrast in the comoving gauge can be derived using the transformation rules

$$\delta^{(C)} = \delta^{(F)} + 3(1 + \omega)\mathcal{R}, \quad (15)$$

where $\mathcal{R}$ is the comoving curvature perturbation given by the gauge-invariant expression [50]

$$\mathcal{R} \equiv \Phi - \mathcal{H}\mathcal{V} = \frac{5 + 3\omega}{3 + 3\omega}\Phi + \frac{2\Phi'}{3(1 + \omega)\mathcal{H}}. \quad (16)$$

The transformation rule in Eq. (15) follows from the textbook relationship $\zeta = \mathcal{R} - \delta^{(C)}/3(1 + \omega)$ [65] where $\zeta \equiv -\delta^{(F)}/3(1 + \omega)$ is the curvature perturbation on uniform-density hypersurfaces; see SM for details [52]. Building on this framework, our public GitHub release deltaPT 2.0 [69] extends deltaPT [20] to compute the covariant cosmological perturbations analyzed in this Letter. We show the time evolution of $\delta^{(F)}$, $\delta^{(C)}$, $\mathcal{R}$, and δp_{nad} in Fig. 1, for a single realization of the bubble nucleation dynamics within a ball of radius $R = k^{-1}$ operated with deltaPT 2.0 [69]. We derive the perturbations statistics by making $N_{\text{sim}} = 10^6$ realizations of random bubble nucleation histories and storing the Hubble-crossing values of the perturbations in a histogram [70]. In Fig. 2, we show the resulting probability density function (PDF) of the density

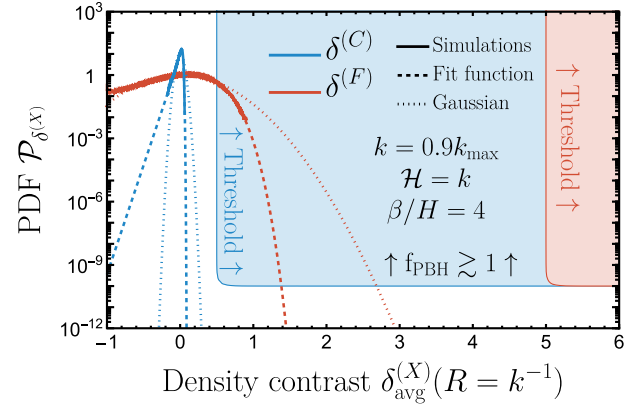


FIG. 2. Probability distribution of the density contrast in the comoving (blue) and spatially-flat (red) gauges, obtained from solving cosmological perturbations equations for $N_{\text{sim}} = 10^6$ distinct random nucleation histories inside a patch of comoving volume $4\pi k^{-3}/3$. The dashed lines show the best fits using the distribution in Eq. (24). The dotted lines shows the best fits using a Gaussian distribution. We choose $k = 0.9k_{\text{max}}$ where k_{max} is the scale entering at percolation. The colored boxes indicate the collapse condition in the comoving (blue) and spatially flat (red) gauges. We cut the box from below at the indicative amplitude for a stellar mass PBH.

contrast. The variance of the density contrast in the spatially-flat and comoving gauges, as well as the comoving curvature perturbation are shown in Fig. 3.

Primordial black hole formation—Once the density contrast in a Hubble patch exceeds a threshold, it collapses into a PBH [71–75]. Numerical relativity simulations [76] define the collapse threshold in terms of the compaction function evaluated at Hubble crossing t_H [77,78]. At this epoch, it reduces to the density contrast averaged over a

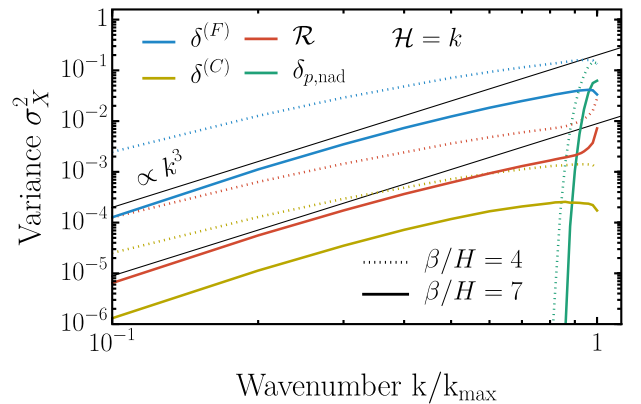


FIG. 3. Variance of the perturbations X averaged over a ball of comoving radius k^{-1} , $\sigma_X^2 \equiv \langle X_{\text{avg}}^2(R = k^{-1}, t_H) \rangle$, evaluated at Hubble crossing epoch t_H . We show the comoving curvature perturbation $\mathcal{R}$ (red), the density contrast in the spatially flat $\delta^{(F)}$ (blue) and comoving gauge $\delta^{(C)}$ (yellow), and the nonadiabatic pressure perturbation δp_{nad} (green).

Hubble-scale region, $\delta_{\text{avg}}(R = k^{-1}, t_H)$ [79,80], which in our notation corresponds to $\delta(k \simeq \mathcal{H})$. As we have seen, the latter is gauge dependent; see also Ref. [81]. Recent simulations have relied on the comoving gauge; see e.g., Ref. [79]. This gauge is advantageous since the 00 component of Einstein's equations reduces to Poisson's equation

$$\Delta\Phi = \frac{3}{2}\mathcal{H}^2\delta^{(C)}. \quad (17)$$

In this gauge, a patch collapses into a PBH if [76,79,80]

$$\delta^{(C)}(k \simeq \mathcal{H}) \gtrsim \delta_c^{(C)} \in [0.40, 0.67], \quad (18)$$

with the exact value depending on the curvature real space profile [82]. These simulations assume adiabatic evolution from super-Hubble scales [76,79,80]. In our scenario, nonadiabatic effects are present but decay rapidly after percolation (see Fig. 1), and we therefore adopt the standard collapse criterion evaluated at $k = \mathcal{H}$.

We proceed to estimate the abundance of primordial black holes. Denoting $P(\delta^{(C)})$ as the PDF for $\delta^{(C)}$ at $k = \mathcal{H}$, the PBH mass fraction at formation reads

$$\beta_k(M) = \int_{\delta_c^{(C)}}^{\infty} d\delta^{(C)} \left(\frac{M(\delta^{(C)})}{M_k} \right) P(\delta^{(C)}). \quad (19)$$

In the first line of Eq. (19), the PBH mass M is related to $\delta^{(C)}$ through the critical scaling law [83–86]

$$M(\delta^{(C)}) = \mathcal{K}M_k(\delta^{(C)} - \delta_c^{(C)})^\gamma, \quad (20)$$

with $\gamma = 0.36$, $\mathcal{K} \sim 3$, and $M_k = 4\pi M_{\text{pl}}^2/H_k$ representing the mass within the Hubble sphere when scale k reenters, $H_k = k/a$. Using that PBHs redshift like matter, we obtain the DM abundance composed of PBH f_{PBH}

$$\frac{df_{\text{PBH}}}{d\ln(M)} \simeq \int d\ln k \left(\frac{\beta_k(M)}{3 \times 10^{-10}} \right) \left(\frac{T_k}{\text{GeV}} \right), \quad (21)$$

where T_k is the temperature at Hubble crossing. Approximating the spectrum to a nearly monochromatic distribution close to $k \sim k_{\text{max}}$, one parametrically finds

$$f_{\text{PBH}} \sim \left(\frac{P(\delta_c^{(C)})}{10^{-10}} \right) \left(\frac{T_k}{\text{GeV}} \right), \quad (22)$$

where the Hubble crossing temperature is related to the Hubble mass as

$$M_k(T_k) = 4.8 \times 10^{-2} M_\odot \left(\frac{106.75}{g_*} \right)^{1/2} \left(\frac{\text{GeV}}{T_k} \right)^2, \quad (23)$$

and g_* is the number of relativistic degrees of freedom.

As shown in blue in Fig. 2, the distribution $P(\delta^{(C)})$, here plotted for $k = 0.9k_{\text{max}}$, is negatively skewed with an extended tail in the negative range and an exponential suppression of large overdensities, reflecting the exponentially low probability of forming late-blooming patches [12]. As in Refs. [20,87], the fit (with $\epsilon, \sigma > 0$)

$$P(\delta) \propto \exp \left[\frac{\epsilon}{2}(\delta - \mu) - \frac{2}{\epsilon^2 \sigma^2} (1 - e^{\frac{\epsilon}{2}(\delta - \mu)})^2 \right] \quad (24)$$

accurately describes the data (see SM [52]).

Due to the negative non-Gaussianity (NG), even for a completion rate as low as $\beta/H = 4$, $P(\delta^{(C)})$ drops sharply around $\delta^{(C)} \sim 0.1$ and is strongly suppressed above the optimistic PBH threshold $\delta_c^{(C)} \gtrsim 0.4$ in Eq. (18). We found similar results for $\beta/H = 2$, just above the no-completion boundary. We conclude that within the framework considered here, slow FOPTs do not appear to yield PBHs in observable amounts. The PDF for $\delta^{(F)}$ in the spatially flat gauge, shown in red in Fig. 2, is broader but still heavily skewed, and with the collapse threshold shifted to

$$\delta_c^{(F)} \simeq 10\delta_c^{(C)} \quad (25)$$

following from Eqs. (15) and (17) with $k = \mathcal{H}$, $\omega = 1/3$, and $\Phi' \simeq 0$ [88], the conclusion remains unchanged. Note that our analysis neglects nonlinear corrections to the density-curvature relation, which would slightly reduce the variance of $\delta^{(C)}$ further [89,90].

Induced gravitational waves—We now focus on the GW spectrum from FOPT, seizing the relevance of SIGWs from curvature perturbations. The GW abundance today can be written as

$$\Omega_{\text{GW}}(k) = \frac{1.7 \times 10^{-5}}{g_*^{1/3}(T_k)} \left[\Omega_{\text{PGW}}(k, T_k) + \Omega_{\text{SIGW}}(k, T_k) \right], \quad (26)$$

where Ω_{PGW} and Ω_{SIGW} are the spectrum of primary GWs (PGWs) and SIGWs. Primary GW are produced from bubble collision and relativistic shells. Assuming that the latter remain thin and conserve their energy after collision, their GW spectrum is approximated by the broken power-law bulk flow formula [91–93]

$$\Omega_{\text{PGW}} \simeq 0.06 \left(\frac{H}{\beta} \right)^2 \frac{(a+b)f_*^a f_*^b}{(af_*^{a+b} + bf_*^{a+b})} S_H(f), \quad (27)$$

with $f_* = 0.8(\beta/H)(\mathcal{H}/2\pi)$, $a = 0.9$, $b = 2.1$, and $S_H(f)$ imposing a f^3 behavior for $f < \mathcal{H}/2\pi$. SIGWs are given by [38–49]

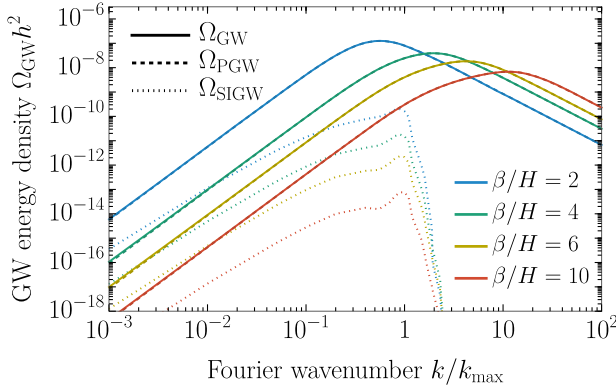


FIG. 4. SIGW contribution to the GWs from FOPT. The solid line reports the total Ω_{GW} , while the dotted line the subdominant SIGWs, which includes the (negligible) NG corrections.

$$\Omega_{\text{SIGW}}(k, \eta) = \frac{1}{24} \left(\frac{k}{\mathcal{H}(\eta)} \right)^2 \overline{\mathcal{P}_h(k, \eta)}, \quad (28)$$

where the power spectrum induced at second-order from curvature perturbations is

$$\begin{aligned} \overline{\mathcal{P}_h(k, \eta)} (2\pi)^3 \delta_D^{(3)}(\mathbf{k} + \mathbf{k}') &= \frac{16k^3}{2\pi^2} \sum_s \int \frac{d^3\mathbf{p}}{(2\pi)^3} \int \frac{d^3\mathbf{q}}{(2\pi)^3} \\ &\times \overline{J_s(\mathbf{k}, \mathbf{p}, \eta) J_s(\mathbf{k}', \mathbf{q}, \eta)} \\ &\times \langle \tilde{\mathcal{R}}_{\mathbf{p}} \tilde{\mathcal{R}}_{\mathbf{k}-\mathbf{p}} \tilde{\mathcal{R}}_{\mathbf{q}} \tilde{\mathcal{R}}_{\mathbf{k}'-\mathbf{q}} \rangle, \end{aligned} \quad (29)$$

having defined

$$J_s(\mathbf{k}, \mathbf{p}, \eta) \equiv \mathcal{Q}_s(\mathbf{k}, \mathbf{p}) I(\mathbf{k}, \mathbf{p}, \eta) \quad (30)$$

as the product of the polarization tensor contracted with the loop momenta $\mathcal{Q}_s(\mathbf{k}, \mathbf{p}) = e_s^{ij}(\hat{\mathbf{k}}) p_i p_j$, and the kernel function $I(\mathbf{k}, \mathbf{p}, \eta)$ being the convolution of source and Green function; see e.g., Ref. [94] and references therein for the explicit expressions. The overline denotes a time average over oscillations [95] and s labels the two GW polarizations. We take the Gaussian limit in which only disconnected contractions contribute to the four-point function in (29). We extract the curvature power spectrum, $\langle \tilde{\mathcal{R}}(\mathbf{k}) \tilde{\mathcal{R}}(\mathbf{k}') \rangle = (2\pi)^3 \delta_D^{(3)}(\mathbf{k} + \mathbf{k}') (2\pi^2/k^3) \mathcal{P}_{\mathcal{R}}(k)$, from the variance of the volume-averaged curvature perturbation $\sigma_{\mathcal{R}}^2$ shown in Fig. 3 by inverting the formula

$$\sigma_{\mathcal{R}}^2 \equiv \langle \mathcal{R}_{\text{avg}}^2 \rangle = \int \frac{dk}{k} |\tilde{W}(k, R)|^2 \mathcal{P}_{\mathcal{R}}(k), \quad (31)$$

where $\tilde{W}(k, R)$ is the Fourier-transform of the top-hat window function. Using $\mathcal{P}_{\mathcal{R}}(k) \propto k^3$ we obtain $\mathcal{P}_{\mathcal{R}}(k = R^{-1}) \simeq 2\sigma_{\mathcal{R}}^2/3\pi$. Figure 4 shows the resulting GW spectra for various β/H . We conclude that SIGWs are never strong enough to form a secondary peak in the total spectrum and at most affect the far-IR tail due to the logarithmic scaling $\Omega_{\text{SIGW}}(k \ll k_*) \propto k^3(1 + \tilde{A} \ln^2(k/\tilde{k}))$

[96]. In the SuM [52], we show that non-Gaussianities, quantified by the skewness $\langle \mathcal{R}^3 \rangle$ [97–114], have negligible contributions to the SIGW signal.

Discussion and implications of our results—In this Letter we have employed a covariant formalism to study cosmological perturbations originating from a FOPT, with implications for PBHs and SIGWs. Upon modeling bubble dynamics on a FLRW background, previous works have implicitly chosen the spatially flat gauge (F). However, the values of the density contrast at the PBH formation threshold $\delta_c^{(C)} \in [0.4, 0.67]$ and its relation to the comoving curvature perturbation at Hubble crossing $\mathcal{R} \simeq -9\delta^{(C)}/4$ [115]—widely quoted in the PBH literature—are valid in the comoving gauge (C). At Hubble crossing, the density contrast in the spatially flat and comoving gauges are related by $\delta^{(F)} \simeq 10\delta^{(C)}$; see Eq. (25). Misidentifying these two gauges leads to an underestimation of the PBH collapse threshold by a factor of 10, leading to a dramatic overestimation of the PBH abundance. The same gauge confusion overestimates the comoving curvature perturbation $\mathcal{R}$ by a factor 10 and therefore overestimates SIGWs by a factor of 10^4 . Additionally, we identified a factor of $2/3\pi$ between the variance of the spatially averaged $\mathcal{R}$ and its power spectrum; see below Eq. (31), further suppressing SIGWs by a factor of 20, leading to lower SIGWs amplitude with respect to Ref. [20] by a factor 2×10^5 .

Let us now discuss the implication of the fact that PBH and SIGW production from FOPTs appear to be far smaller than previously thought. First, this implies it would be harder to use PBHs to search for evidence of strong FOPTs having taken place in the primordial universe. Second, this result has important implications for the FOPT interpretation of the PTA signal: the loudest region of the posterior is not necessarily excluded by PBH overproduction constraints, particularly those from LIGO-Virgo-KAGRA [14,20,51], and FOPTs with a reheating temperature above a GeV might not explain the PTA signal [20].

Limitations and future directions—We have neglected gravitational effects on nucleation rates [116] and on the expansion of bubbles [33,37], as well as the gradient energy of bubble walls [11,19,26]. The actual distribution of radiation converted from the vacuum energy is also important in understanding perturbations on small scales $k \gtrsim \mathcal{H}$ [117,118]. While asphericities have a small impact on the threshold for PBH collapse in the standard scenario [119], it is not yet clear whether the same conclusion applies in the present case. Future work should incorporate these effects and perform full numerical simulations tracking non-linear curvature perturbations and non-adiabatic pressure effects to refine PBH formation criteria.

Acknowledgments—Y.G. thanks Caner Unal for his valuable contributions during early stages of this project, and Sokratis Trifinopoulos and Miguel Vanvlasselaer for interesting discussions. We thank Pedro Schwaller for

spotting a typo in Fig. 4 and Karsten Jedamzik for drawing our attention to typographical issues in Eqs. (6) and (8). Y. G. acknowledges support by the Cluster of Excellence “PRISMA+” funded by the German Research Foundation (DFG) within the German Excellence Strategy (Project No. 390831469), and by a fellowship awarded by the Azrieli Foundation. The work of R. J. is supported by JSPS KAKENHI Grants No. 23K17687, No. 23K19048, and No. 24K07013.

Data availability—The data are not publicly available. The data are available from the authors upon reasonable request.

-
- [1] D. A. Kirzhnits and A. D. Linde, *Phys. Lett.* **42B**, 471 (1972).
 - [2] Y. Gouttenoire, *Beyond the Standard Model Cocktail*, Springer Theses (Springer, Cham, 2022).
 - [3] C. Caprini *et al.*, *J. Cosmol. Astropart. Phys.* **04** (2016) 001.
 - [4] C. Caprini *et al.*, *J. Cosmol. Astropart. Phys.* **03** (2020) 024.
 - [5] H. Kodama, M. Sasaki, and K. Sato, *Prog. Theor. Phys.* **68**, 1799 (1982).
 - [6] S. D. H. Hsu, *Phys. Lett. B* **251**, 343 (1990).
 - [7] J. Liu, L. Bian, R.-G. Cai, Z.-K. Guo, and S.-J. Wang, *Phys. Rev. D* **105**, L021303 (2022).
 - [8] K. Hashino, S. Kanemura, and T. Takahashi, *Phys. Lett. B* **833**, 137261 (2022).
 - [9] T. H. Jung and T. Okui, *Phys. Rev. D* **110**, 115014 (2024).
 - [10] K. Kawana, T. Kim, and P. Lu, *Phys. Rev. D* **108**, 103531 (2023).
 - [11] M. Lewicki, P. Toczec, and V. Vaskonen, *J. High Energy Phys.* **09** (2023) 092.
 - [12] Y. Gouttenoire and T. Volansky, *Phys. Rev. D* **110**, 043514 (2024).
 - [13] I. Baldes and M. O. Olea-Romacho, *J. High Energy Phys.* **01** (2024) 133.
 - [14] Y. Gouttenoire, *Phys. Rev. Lett.* **131**, 171404 (2023).
 - [15] A. Salvio, *J. Cosmol. Astropart. Phys.* **12** (2023) 046.
 - [16] Y. Gouttenoire, *Phys. Lett. B* **855**, 138800 (2024).
 - [17] R. Jinno, J. Kume, and M. Yamada, *Phys. Lett. B* **849**, 138465 (2024).
 - [18] I. K. Banerjee and U. K. Dey, *J. High Energy Phys.* **07** (2024) 006; **08** (2024) 054(E).
 - [19] M. M. Flores, A. Kusenko, and M. Sasaki, *Phys. Rev. D* **110**, 015005 (2024).
 - [20] M. Lewicki, P. Toczec, and V. Vaskonen, *Phys. Rev. Lett.* **133**, 221003 (2024).
 - [21] M. Lewicki, P. Toczec, and V. Vaskonen, *Phys. Dark Universe* **50**, 102075 (2025).
 - [22] W.-Y. Ai, L. Heurtier, and T. H. Jung, *Phys. Rev. D* **113**, 023542 (2026).
 - [23] A. Conaci, L. Delle Rose, P. S. B. Dev, and A. Ghoshal, *J. High Energy Phys.* **12** (2024) 196.
 - [24] S. Kanemura, M. Tanaka, and K.-P. Xie, *J. High Energy Phys.* **06** (2024) 036.
 - [25] I. K. Banerjee, F. Rescigno, and A. Salvio, *J. Cosmol. Astropart. Phys.* **07** (2025) 007.
 - [26] K. Hashino, S. Kanemura, T. Takahashi, M. Tanaka, and C.-M. Yoo, *J. Cosmol. Astropart. Phys.* **09** (2025) 006.
 - [27] K. Murai, K. Sakurai, and F. Takahashi, *J. High Energy Phys.* **07** (2025) 065.
 - [28] J. Zou, Z. Zhu, Z. Zhao, and L. Bian, [arXiv:2502.20166](https://arxiv.org/abs/2502.20166).
 - [29] A. Ghoshal, E. Megias, G. Nardini, and M. Quiros, *J. High Energy Phys.* **07** (2025) 277.
 - [30] *Primordial Black Holes*, edited by C. Byrnes, G. Franciolini, T. Harada, P. Pani, and M. Sasaki, Springer Series in Astrophysics and Cosmology (Springer, New York, 2025).
 - [31] M. Sasaki, H. Kodama, and K. Sato, *Prog. Theor. Phys.* **68**, 1561 (1982).
 - [32] J. Liu, L. Bian, R.-G. Cai, Z.-K. Guo, and S.-J. Wang, *Phys. Rev. Lett.* **130**, 051001 (2023).
 - [33] L. Giombi and M. Hindmarsh, *J. Cosmol. Astropart. Phys.* **03** (2024) 059.
 - [34] G. Elor, R. Jinno, S. Kumar, R. McGehee, and Y. Tsai, *Phys. Rev. Lett.* **133**, 211003 (2024).
 - [35] M. R. Buckley, P. Du, N. Fernandez, and M. J. Weikert, *J. Cosmol. Astropart. Phys.* **07** (2024) 031.
 - [36] R.-G. Cai, Y.-S. Hao, and S.-J. Wang, *Sci. China Phys. Mech. Astron.* **67**, 290411 (2024).
 - [37] R. Jinno and J. Kume, *J. Cosmol. Astropart. Phys.* **02** (2025) 057.
 - [38] K. Tomita, *Prog. Theor. Phys.* **54**, 730 (1975).
 - [39] S. Matarrese, O. Pantano, and D. Saez, *Phys. Rev. D* **47**, 1311 (1993).
 - [40] S. Matarrese, O. Pantano, and D. Saez, *Phys. Rev. Lett.* **72**, 320 (1994).
 - [41] S. Matarrese, S. Mollerach, and M. Bruni, *Phys. Rev. D* **58**, 043504 (1998).
 - [42] V. Acquaviva, N. Bartolo, S. Matarrese, and A. Riotto, *Nucl. Phys. B* **667**, 119 (2003).
 - [43] S. Mollerach, D. Harari, and S. Matarrese, *Phys. Rev. D* **69**, 063002 (2004).
 - [44] C. Carbone and S. Matarrese, *Phys. Rev. D* **71**, 043508 (2005).
 - [45] K. N. Ananda, C. Clarkson, and D. Wands, *Phys. Rev. D* **75**, 123518 (2007).
 - [46] D. Baumann, P. J. Steinhardt, K. Takahashi, and K. Ichiki, *Phys. Rev. D* **76**, 084019 (2007).
 - [47] J. R. Espinosa, D. Racco, and A. Riotto, *J. Cosmol. Astropart. Phys.* **09** (2018) 012.
 - [48] K. Kohri and T. Terada, *Phys. Rev. D* **97**, 123532 (2018).
 - [49] G. Domènech, *Universe* **7**, 398 (2021).
 - [50] J. M. Bardeen, *Phys. Rev. D* **22**, 1882 (1980).
 - [51] J. Ellis, M. Fairbairn, G. Franciolini, G. Hütsi, A. Iovino, M. Lewicki, M. Raidal, J. Urrutia, V. Vaskonen, and H. Veermäe, *Phys. Rev. D* **109**, 023522 (2024).
 - [52] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/tfcx-kzqx> for further details and additional Refs. [53–61].
 - [53] J. M. Bardeen, P. J. Steinhardt, and M. S. Turner, *Phys. Rev. D* **28**, 679 (1983).
 - [54] V. De Luca, G. Franciolini, A. Kehagias, and A. Riotto, *J. Cosmol. Astropart. Phys.* **03** (2020) 014.
 - [55] G. Domènech and M. Sasaki, *Phys. Rev. D* **103**, 063531 (2021).

- [56] K. Inomata and T. Terada, *Phys. Rev. D* **101**, 023523 (2020).
- [57] J.-P. Li, S. Wang, Z.-C. Zhao, and K. Kohri, *J. Cosmol. Astropart. Phys.* **10** (2023) 056.
- [58] D. H. Lyth, K. A. Malik, and M. Sasaki, *J. Cosmol. Astropart. Phys.* **05** (2005) 004.
- [59] H. V. Ragavendra, *Phys. Rev. D* **105**, 063533 (2022).
- [60] C. Yuan, Z.-C. Chen, and Q.-G. Huang, *Phys. Rev. D* **101**, 063018 (2020).
- [61] S. Wang, Z.-C. Zhao, and Q.-H. Zhu, *Phys. Rev. Res.* **6**, 013207 (2024).
- [62] H. Kodama and M. Sasaki, *Prog. Theor. Phys. Suppl.* **78**, 1 (1984).
- [63] V. F. Mukhanov, H. A. Feldman, and R. H. Brandenberger, *Phys. Rep.* **215**, 203 (1992).
- [64] V. Mukhanov, *Physical Foundations of Cosmology* (Cambridge University Press, Oxford, 2005).
- [65] D. Baumann, *Cosmology* (Cambridge University Press, Cambridge, England, 2022).
- [66] P. Peter and J.-P. Uzan, *Primordial Cosmology*, Oxford Graduate Texts (Oxford University Press, New York, 2013).
- [67] D. Wands, K. A. Malik, D. H. Lyth, and A. R. Liddle, *Phys. Rev. D* **62**, 043527 (2000).
- [68] We leave a proper de-convolution from the top-hat window function $W(r, R) = \Theta(R - r)/V$ followed by inverse-Fourier transformation for future work.
- [69] G. Franciolini, Y. Gouttenoire, and R. Jinno, deltaPT 2.0: A C++ code to compute cosmological perturbations generated during strongly supercooled first-order phase transitions, GitHub, <https://github.com/YannGou/deltaPT2.0> (2025).
- [70] In each patch, we randomly sample the nucleation times t_j and positions d_j of the first $j_c = 50$ bubbles and assume a deterministic evolution for subsequent nucleation dynamics [20]. The probability distributions for t_j and d_j within a patch of radius R are derived analytically in Appendix A of Ref. [20] and are summarized in the SuM [52].
- [71] Y. B. Zel'dovich and I. D. Novikov, *Sov. Astron.* **10**, 602 (1967), <https://inspirehep.net/literature/1747265>.
- [72] S. Hawking, *Mon. Not. R. Astron. Soc.* **152**, 75 (1971).
- [73] B. J. Carr and S. W. Hawking, *Mon. Not. R. Astron. Soc.* **168**, 399 (1974).
- [74] B. J. Carr, *Astrophys. J.* **201**, 1 (1975).
- [75] G. F. Chapline, *Nature (London)* **253**, 251 (1975).
- [76] A. Escrivà, *Universe* **8**, 66 (2022).
- [77] M. Shibata and M. Sasaki, *Phys. Rev. D* **60**, 084002 (1999).
- [78] T. Harada, C.-M. Yoo, and Y. Koga, *Phys. Rev. D* **108**, 043515 (2023).
- [79] I. Musco, *Phys. Rev. D* **100**, 123524 (2019).
- [80] A. Escrivà, C. Germani, and R. K. Sheth, *Phys. Rev. D* **101**, 044022 (2020).
- [81] T. Harada, C.-M. Yoo, T. Nakama, and Y. Koga, *Phys. Rev. D* **91**, 084057 (2015).
- [82] I. Musco, V. De Luca, G. Franciolini, and A. Riotto, *Phys. Rev. D* **103**, 063538 (2021).
- [83] M. W. Choptuik, *Phys. Rev. Lett.* **70**, 9 (1993).
- [84] C. R. Evans and J. S. Coleman, *Phys. Rev. Lett.* **72**, 1782 (1994).
- [85] J. C. Niemeyer and K. Jedamzik, *Phys. Rev. Lett.* **80**, 5481 (1998).
- [86] I. Musco and J. C. Miller, *Classical Quantum Gravity* **30**, 145009 (2013).
- [87] E. Tomberg, *Phys. Rev. D* **108**, 043502 (2023).
- [88] As known from textbooks, e.g., Ref. [65], and rederived in SM [52], the approximation $\Phi' \simeq 0$ holds for superhorizon modes only in the absence of nonadiabatic pressure perturbations, $\delta_{p,\text{nad}} = 0$. In our scenario, bubble dynamics generate $\delta_{p,\text{nad}} \neq 0$, leading to departures from $\Phi' \simeq 0$ on superhorizon scales. Nevertheless, $\Phi' \simeq 0$ remains a good approximation near horizon crossing, $k \simeq \mathcal{H}$, justifying its use in evaluating the PBH collapse threshold. The time evolution of $\Phi(t)$ is shown in SM [52].
- [89] V. De Luca, G. Franciolini, A. Kehagias, M. Peloso, A. Riotto, and C. Ünal, *J. Cosmol. Astropart. Phys.* **07** (2019) 048.
- [90] S. Young, I. Musco, and C. T. Byrnes, *J. Cosmol. Astropart. Phys.* **11** (2019) 012.
- [91] R. Jinno and M. Takimoto, *J. Cosmol. Astropart. Phys.* **01** (2019) 060.
- [92] T. Konstandin, *J. Cosmol. Astropart. Phys.* **03** (2018) 047.
- [93] I. Baldes, M. Dichtl, Y. Gouttenoire, and F. Sala, *J. High Energy Phys.* **07** (2024) 231.
- [94] J. E. Gammal *et al.* (LISA Cosmology Working Group), *J. Cosmol. Astropart. Phys.* **05** (2025) 062.
- [95] M. Maggiore, *Phys. Rep.* **331**, 283 (2000).
- [96] V. Atal and G. Domènech, *J. Cosmol. Astropart. Phys.* **06** (2021) 001; **10** (2023) E01.
- [97] X.-c. Luo and D. N. Schramm, *Astrophys. J.* **408**, 33 (1993).
- [98] E. Komatsu and D. N. Spergel, *Phys. Rev. D* **63**, 063002 (2001).
- [99] T. Nakama, J. Silk, and M. Kamionkowski, *Phys. Rev. D* **95**, 043511 (2017).
- [100] J. Garcia-Bellido, M. Peloso, and C. Unal, *J. Cosmol. Astropart. Phys.* **09** (2017) 013.
- [101] C. Unal, *Phys. Rev. D* **99**, 041301 (2019).
- [102] R.-G. Cai, S. Pi, and M. Sasaki, *Phys. Rev. Lett.* **122**, 201101 (2019).
- [103] R.-G. Cai, S. Pi, S.-J. Wang, and X.-Y. Yang, *J. Cosmol. Astropart. Phys.* **05** (2019) 013.
- [104] H. V. Ragavendra, P. Saha, L. Sriramkumar, and J. Silk, *Phys. Rev. D* **103**, 083510 (2021).
- [105] C. Yuan and Q.-G. Huang, *Phys. Lett. B* **821**, 136606 (2021).
- [106] P. Adshead, K. D. Lozanov, and Z. J. Weiner, *J. Cosmol. Astropart. Phys.* **10** (2021) 080.
- [107] M. W. Davies, P. Carrilho, and D. J. Mulryne, *J. Cosmol. Astropart. Phys.* **06** (2022) 019.
- [108] K. T. Abe, R. Inui, Y. Tada, and S. Yokoyama, *J. Cosmol. Astropart. Phys.* **05** (2023) 044.
- [109] S. Garcia-Saenz, L. Pinol, S. Renaux-Petel, and D. Werth, *J. Cosmol. Astropart. Phys.* **03** (2023) 057.
- [110] S. Garcia-Saenz, Y. Lu, and Z. Shuai, *Phys. Rev. D* **108**, 123507 (2023).
- [111] J.-P. Li, S. Wang, Z.-C. Zhao, and K. Kohri, *J. Cosmol. Astropart. Phys.* **06** (2024) 039.
- [112] C. Yuan, D.-S. Meng, and Q.-G. Huang, *J. Cosmol. Astropart. Phys.* **12** (2023) 036.
- [113] G. Perna, C. Testini, A. Ricciardone, and S. Matarrese, *J. Cosmol. Astropart. Phys.* **05** (2024) 086.

- [114] Y. Gouttenoire, S. Trifinopoulos, and M. Vanvlasselaer, *J. Cosmol. Astropart. Phys.* **02** (2026) 072.
- [115] The relation $\mathcal{R} \simeq -9\delta^{(C)}/4$ follows from Eqs. (16) and (17) for $k = \mathcal{H}$, $\omega = 1/3$, and $\Phi' \simeq 0$.
- [116] S. R. Coleman and F. De Luccia, *Phys. Rev. D* **21**, 3305 (1980).
- [117] R. Jinno, H. Seong, M. Takimoto, and C. M. Um, *J. Cosmol. Astropart. Phys.* **10** (2019) 033.
- [118] M. Lewicki and V. Vaskonen, *Eur. Phys. J. C* **83**, 109 (2023).
- [119] C.-M. Yoo, T. Harada, and H. Okawa, *Phys. Rev. D* **102**, 043526 (2020); **107**, 049901(E) (2023).