

Quantum Tomography in Neutral Meson and Antimeson Systems

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The flavor space of particles produced in collider environments contains informative quantum correlations. We present a systematic approach for constructing the complete flavor density matrix for a meson and antimeson system ($M\bar{M}$) in the Bloch vector space at a given time t , which can be at or after production. We point out that the B_s^0 and K^0 systems are superior to the B_d^0 and D^0 systems for quantum tomography because of their flavor oscillation and decay properties. Performing quantum tomography for the $M\bar{M}$ system can facilitate the study of production mechanisms, decoherence phenomena, quantum information variables, and potential new sources of CP violation.

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Introduction—Ever since the original quantum mechanical formulation of $K^0\bar{K}^0$ mixing [1], systems of a neutral meson M and its antimeson $\bar{M}$ ($M\bar{M}$) have demonstrated many aspects of fundamental physics. Because of the mixing between a meson and an antimeson, a meson state may be intrinsically treated as a qubit in flavor space. The pairs of oscillating mesons correspond to two-qubit systems that have been measured for various mesons [2–6]. Attempts to observe entanglement and Bell nonlocality have been made, including in the $K^0\bar{K}^0$ system from resonant ϕ decay [7–29], in the $B_d^0\bar{B}_d^0$ system from the decay of the $\Upsilon(4S)$ particle [5,6,30–33], in $B_s^0\bar{B}_s^0$ from $\Upsilon(5S)$ [34], and quantum correlations in other heavy flavor oscillations [35–38], as well as in the two-pion and three-pion systems [39]. A temporal analogy of Bell inequality that tests hidden variable theory in a single particle system—the Leggett-Garg inequality [40]—has also been discussed in meson systems [41–43].

Going beyond the existing literature, in this Letter, we show how to reconstruct the most general two-particle flavor quantum state at an arbitrary time after production. Unlike Refs. [34–39], which assumes that the initial flavor state is known in a specific form and studies a few observables, we perform quantum tomography for a generally parametrized flavor state by identifying the full set of

observables. We focus on the semileptonic decay channels in both the $B_s^0\bar{B}_s^0$ and $K^0\bar{K}^0$ systems, using the total decay rate and the flavor asymmetry to form a complementary set of flavor state measurements. Based on the intrinsic properties of flavor oscillations and decays, we identify optimal systems such as B_s^0 and K^0 for quantum tomography. Using the observables we derive, we show that it is possible to fully measure the quantum properties of the flavor state of the meson pair, thus enabling, for the first time, systematic quantum state tomography in flavor space.

When a heavy-quark pair originates from open flavor production at colliders, the corresponding heavy hadrons are initially in flavor eigenstates. In general, however, the flavor correlation of the pair of hadronic states cannot be determined from first principles because of the nonperturbative hadronization process. Developing an approach to quantum tomography in this system may, therefore, provide an opportunity to study the underlying hadronization mechanisms [44].

The demonstration of flavor quantum tomography not only makes all quantum information measurements available in the flavor sector but also provides a crucial reference for theoretical predictions of the flavor state production from nonperturbative processes.

Flavor quantum states and time evolution—When a meson is neutral under gauge symmetries, the meson state $|M\rangle$ mixes with the antimeson state $|\bar{M}\rangle$. Examples of neutral mesons M include B_d^0 , B_s^0 , D^0 , and K^0 . In the flavor space spanned by $|M\rangle$ and $|\bar{M}\rangle$, a single meson can be analyzed as a two-level quantum system [1], known as a qubit, and described with a density matrix ρ_M . This matrix can be parameterized as

$$\rho_M = \frac{\mathbb{1}_2 + b_i \sigma_i}{2}, \quad (1)$$

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where $\mathbb{1}_2$ is the two-dimensional identity matrix and σ_i ($i = x, y, z$) are the Pauli matrices, with repeated indices summed. The real vector b_i , known as the Bloch vector, specifies a quantum state in the Bloch vector space [45,46].

With a common mass m and a common decay width Γ for the meson and the antimeson, the non-Hermitian effective Hamiltonian is

$$\mathcal{H} = \mathbf{M} - i\Gamma/2 = \begin{pmatrix} m - i\frac{\Gamma}{2} & H_{12} \\ H_{21} & m - i\frac{\Gamma}{2} \end{pmatrix}, \quad (2)$$

where H_{12} and H_{21} are complex parameters that encode the mixing of the meson-antimeson. Diagonalizing Eq. (2) yields the mass eigenstates $|M_1\rangle$ and $|M_2\rangle$, along with their masses and decay widths:

$$|M_1\rangle = p|M\rangle + q|\bar{M}\rangle, \quad \text{with } (m_1, \Gamma_1), \quad (3)$$

$$|M_2\rangle = p|M\rangle - q|\bar{M}\rangle, \quad \text{with } (m_2, \Gamma_2), \quad (4)$$

where $\Gamma_1 + \Gamma_2 = 2\Gamma$, $|q|^2 + |p|^2 = 1$ and $p/q = \sqrt{H_{12}/H_{21}}$. In the absence of CP violation, $H_{21} = H_{12} = (\Delta m/2) + i(\Delta\Gamma/4)$ and $p = q = 1/\sqrt{2}$, where $\Delta m \equiv m_2 - m_1$ and $\Delta\Gamma = \Gamma_1 - \Gamma_2$. We adopt the convention $CP|M\rangle = |\bar{M}\rangle$, implying that $|M_1\rangle$ is a CP -even eigenstate and $|M_2\rangle$ is CP odd. In the Bloch vector space, the flavor eigenstates $|M\rangle$ and $|\bar{M}\rangle$ are described by $\vec{b} = (0, 0, \pm 1)$, while the mass eigenstates $|M_1\rangle$ and $|M_2\rangle$ are described by $\vec{b} = (\pm 1, 0, 0)$. When CP is not conserved, the CP and mass eigenstates no longer coincide.

The oscillation and decay of the meson system can be described by a nonunitary time evolution $U(t)$ [47–50]. The fraction of remaining mesons at time t is

$$\frac{N(t)}{N_0} = \text{tr}(U(t)\rho_M U(t)^\dagger), \quad (5)$$

where N_0 is the number of mesons at $t = 0$ and $N(t)$ is the number of mesons left at time t . The explicit forms of $U(t)$ and $N(t)$ are given in Supplemental Material [51]. One can choose any starting point as $t = 0$ such as the time of production.

The time evolution of the quantum density matrix, in the Schrödinger picture, is

$$\rho_M(t) = \frac{U(t)\rho_M U(t)^\dagger}{\text{tr}(U(t)\rho_M U(t)^\dagger)}. \quad (6)$$

The normalization of $\rho_M(t)$ leads to a unitary evolution.

Oscillation and decay in the Bloch vector space—It is intuitive to analyze the state evolution in the Bloch vector space [45,46] because different components of the quantum density matrix ρ_M are probed by the various decays of the neutral meson, as we formulate explicitly below. The time

evolution of the Bloch vector is [49,52,53]

$$\frac{d}{dt}\vec{b}(t) = -2\vec{E} \times \vec{b}(t) + \vec{\Gamma} - [\vec{\Gamma} \cdot \vec{b}(t)]\vec{b}(t), \quad (7)$$

where the vectors $\vec{E}$ and $\vec{\Gamma}$ are defined from the decompositions of the Hermitian matrices $\mathbf{M}$ and $\mathbf{\Gamma}$ in Eq. (2) as

$$\mathbf{M} = E_0\mathbb{1}_2 + \vec{E} \cdot \vec{\sigma}, \quad \mathbf{\Gamma} = \Gamma_0\mathbb{1}_2 + \vec{\Gamma} \cdot \vec{\sigma}. \quad (8)$$

In this Letter, we work in the CP -conserving limit with both vectors $\vec{E} = (\Delta m/2, 0, 0)$ and $\vec{\Gamma} = (-\Delta\Gamma/2, 0, 0)$ along the $\vec{x}$ direction.

Project to flavor eigenstates—Consider a multiparticle final state $|f\rangle$ with $CP|f\rangle = |\bar{f}\rangle$ and $|f\rangle \neq |\bar{f}\rangle$. In the leading decay, the meson M only decays to f while the antimeson $\bar{M}$ only decays to $\bar{f}$. The semileptonic decays of neutral mesons are examples of such decays. The observation of such decays identifies the flavor of the neutral meson as $|M\rangle$ or $|\bar{M}\rangle$.

The probabilities of projecting the meson state to flavor eigenstates are

$$\langle M|\rho_M(t)|M\rangle = \frac{1 + b_z(t)}{2}, \quad \langle \bar{M}|\rho_M(t)|\bar{M}\rangle = \frac{1 - b_z(t)}{2}.$$

Let $N_{f/\bar{f}}(t)$ be the number of $f/\bar{f}$ final states produced in decays from $t = 0$ until time t . The rate of change is

$$\frac{d}{dt}N_{f/\bar{f}}(t) = N(t) \frac{1 \pm b_z(t)}{2} \Gamma_{M \rightarrow f}, \quad (9)$$

where $\Gamma_{M \rightarrow f}$ is the partial width of M decaying to f and $\Gamma_{M \rightarrow f} = \Gamma_{\bar{M} \rightarrow \bar{f}}$. Therefore, the decay asymmetry $N_f(t) - N_{\bar{f}}(t)$ directly corresponds to the expectation value of σ_z with $\text{Tr}(\sigma_z \cdot \rho_M(t)) = b_z(t)$,

$$\begin{aligned} \frac{d[N_f(t) - N_{\bar{f}}(t)]}{dt} &= N(t)b_z(t)\Gamma_{M \rightarrow f}, \\ &= N_0 e^{-\Gamma t} (b_z c_t - b_y s_t) \Gamma_{M \rightarrow f}, \end{aligned} \quad (10)$$

where $c_t = \cos(\Delta m t)$, $s_t = \sin(\Delta m t)$, and $b_i = b_i(0)$ are the components of the Bloch vector at $t = 0$. The decay asymmetry probes both b_y and b_z as they evolve into each other according to Eq. (7), which describes precession around the x direction in the limit of $\Delta m \gg \Gamma$. Therefore, for different events within the ensemble, decays occur at different times, allowing us to measure different directions in the y - z plane of the Bloch vector space.

Project to CP eigenstates: The neutral mesons can also decay to CP eigenstates with $|f_{\eta_{CP}}\rangle = CP|f_{\eta_{CP}}\rangle = \eta_{CP}|f_{\eta_{CP}}\rangle$, where $\eta_{CP} = \pm 1$ is the CP eigenvalue of the final state $|f_{\eta_{CP}}\rangle$. The decays of $B_s^0 \rightarrow J/\psi\eta$ and $K^0 \rightarrow \pi^+\pi^-$ are such examples. Instead of projecting the meson

state to $|M\rangle$ or $|\bar{M}\rangle$, the decay to $|f_{\eta_{CP}}\rangle$ projects the meson state to the CP eigenstates (or mass eigenstates) $|M_1\rangle$ or $|M_2\rangle$, whose Bloch vectors are along the x direction. Such decays allow the reconstruction of b_x .

Because of the uneven decay rates of the CP -odd and CP -even states, there is no simple asymmetric observable directly corresponding to $\langle\sigma_x\rangle$ as in Eq. (10). However, the decay can still be viewed as a projection to $|M_1\rangle$ and $|M_2\rangle$ with

$$\langle M_1|\rho_M(t)|M_1\rangle = \frac{1+b_x(t)}{2}, \quad \langle M_2|\rho_M(t)|M_2\rangle = \frac{1-b_x(t)}{2}.$$

[We assume CP -conserving $\langle M|f_+\rangle = \langle \bar{M}|f_+\rangle$ for illustration, while our result Eq. (11) does not rely on this.] Therefore, the x component of the Bloch vector can be reconstructed from decay to CP eigenstates. Furthermore, decay to CP eigenstates results in a decay width difference and a lifetime difference, leading to the dependence of b_x on the decay rate to total flavor eigenstates, both f and $\bar{f}$, as a function of time,

$$\begin{aligned} \frac{d[N_f(t) + N_{\bar{f}}(t)]}{dt} &= N(t)\Gamma_{M\rightarrow f} \\ &= N_0 e^{-\Gamma t} (\text{ch}_t - b_x \text{sh}_t) \Gamma_{M\rightarrow f}, \end{aligned} \quad (11)$$

where $\text{ch}_t = \cosh(\Delta\Gamma t/2)$ and $\text{sh}_t = \sinh(\Delta\Gamma t/2)$.

In summary, the components of the Bloch vector in the y - z plane are probed by the flavor asymmetry $N_f - N_{\bar{f}}$ through oscillations in Eq. (10), and the Bloch vector in the x direction is measured from $N_f + N_{\bar{f}}$ via total decay rates in Eq. (11). By fitting the distributions of $N_f \pm N_{\bar{f}}$ with respect to the decay time, the complete flavor density matrix is reconstructed for a single meson using the decay channels $M \rightarrow f/\bar{f}$.

Generalization to meson pairs—The quantum state of a two-meson system, with the two subsystems denoted as A and B , respectively, can be expressed by the density matrix,

$$\rho_{MM} = \frac{\mathbb{1}_4 + b_i^A \sigma_i \otimes \mathbb{1}_2 + b_i^B \mathbb{1}_2 \otimes \sigma_i + C_{ij} \sigma_i \otimes \sigma_j}{4}, \quad (12)$$

where b_i^A and b_i^B describe the flavor state of mesons A and B , respectively, and C_{ij} is the flavor correlation matrix between the two mesons. Repeated indices are summed over. Collectively, these are also called the Fano coefficients [54].

Like in the case of a single meson, the Fano coefficients of the two-meson system b_i^A , b_i^B , and C_{ij} are obtained by measuring the flavor asymmetry and the total decay rate to a flavor eigenstate for each meson. This leads us to define the following four observables:

$$N_{\text{tot}} = N_{ff} + N_{\bar{f}f} + N_{f\bar{f}} + N_{\bar{f}\bar{f}}, \quad (13)$$

$$A_{ff} = N_{ff} - N_{\bar{f}f} - N_{f\bar{f}} + N_{\bar{f}\bar{f}}, \quad (14)$$

$$A_f^A = N_{ff} - N_{\bar{f}f} + N_{f\bar{f}} - N_{\bar{f}\bar{f}}, \quad (15)$$

$$A_f^B = N_{ff} + N_{\bar{f}f} - N_{f\bar{f}} - N_{\bar{f}\bar{f}}, \quad (16)$$

where $N_{f_1 f_2}$ is the number of events in which the meson A decays to f_1 and the meson B decays to f_2 . N_{tot} is the total number of events from semileptonic decays, A_{ff} is the asymmetry between same-flavor and opposite-flavor decays, and $A_f^{A/B}$ is the decay flavor asymmetry of one meson on the condition that the other meson also decays to a flavor eigenstate. All the above quantities depend on the times t_1 and t_2 , the decay times of the first and second mesons.

The distributions of the four observables have the following dependence on the flavor density matrix of the meson pair [51],

$$\begin{aligned} \frac{d^2 N_{\text{tot}}}{dt_1 dt_2} &= N_0 \Gamma_{B_0 \rightarrow f}^2 e^{-\Gamma(t_1+t_2)} (\text{ch}_{t_1} \text{ch}_{t_2} - \text{ch}_{t_1} \text{sh}_{t_2} b_x^A \\ &\quad - \text{sh}_{t_1} \text{ch}_{t_2} b_x^B + \text{sh}_{t_1} \text{sh}_{t_2} C_{xx}) + \mathcal{O}(\epsilon), \end{aligned} \quad (17)$$

$$\begin{aligned} \frac{d^2 A_{ff}}{dt_1 dt_2} &= N_0 \Gamma_{B_0 \rightarrow f}^2 e^{-\Gamma(t_1+t_2)} (s_{t_1} s_{t_2} C_{yy} - c_{t_1} s_{t_2} C_{zy} \\ &\quad - s_{t_1} c_{t_2} C_{yz} + c_{t_1} c_{t_2} C_{zz}) + \mathcal{O}(\epsilon), \end{aligned} \quad (18)$$

$$\begin{aligned} \frac{d^2 A_f^A}{dt_1 dt_2} &= N_0 \Gamma_{B_0 \rightarrow f}^2 e^{-\Gamma(t_1+t_2)} (\text{ch}_{t_2} (c_{t_1} b_z^A - s_{t_1} b_y^A) \\ &\quad - \text{sh}_{t_2} (c_{t_1} C_{zx} - s_{t_1} C_{yx})) + \mathcal{O}(\epsilon), \end{aligned} \quad (19)$$

$$\begin{aligned} \frac{d^2 A_f^B}{dt_1 dt_2} &= N_0 \Gamma_{B_0 \rightarrow f}^2 e^{-\Gamma(t_1+t_2)} (\text{ch}_{t_1} (c_{t_2} b_z^B - s_{t_2} b_y^B) \\ &\quad - \text{sh}_{t_1} (c_{t_2} C_{xz} - s_{t_2} C_{xy})) + \mathcal{O}(\epsilon), \end{aligned} \quad (20)$$

where N_0 is the total number of $M\bar{M}$ events, and ϵ is the CP violation parameter in the $M\bar{M}$ mixing defined by $p/q = (1+\epsilon)/(1-\epsilon)$. For B_s^0 , $\epsilon \sim \mathcal{O}(10^{-5})$, and for K^0 , $\epsilon \sim \mathcal{O}(10^{-3})$. This validates our analysis as a good approximation in the CP -conserving limit. Other CP violation sources are irrelevant to the four observables in the semileptonic decay channel above.

By measuring the four distributions, we reconstruct all the Fano coefficients $\vec{b}^A$, $\vec{b}^B$, and C_{ij} . Deviations from the predictions made by the parameterization of the density matrix may indicate the presence of decoherence phenomena [3,4,50].

Simulations and experimental feasibility—From an observational perspective, an $M\bar{M}$ system needs to have a sizable value of $\Delta m/\Gamma$ and $\Delta\Gamma/\Gamma$, ideally of the order of unity or larger. This corresponds to rapid oscillations and an

appropriate decay lifetime difference. Both the B_s^0 and K_0 systems are such favorable systems with $(\Delta m/\Gamma, \Delta\Gamma/\Gamma)_{B_s^0} = (27, 0.14)$ and $(\Delta m/\Gamma, \Delta\Gamma/\Gamma)_{K^0} = (0.95, 1.99)$ [51]. We, therefore, adopt the B_s^0 and K^0 systems to quantitatively demonstrate the procedure of quantum tomography.

For both B_s^0 and K^0 systems, we choose two examples of density matrices for concreteness. The first is ρ_{Bell} , parameterized as

$$b_i^A = b_i^B = 0 \quad \text{and} \quad C_{ij} = -\delta_{ij}, \quad (21)$$

which corresponds to the CP -odd flavor state $(|M\bar{M}\rangle - |\bar{M}M\rangle)/\sqrt{2}$. The second is the Werner state ρ_κ [55] given by

$$\rho_\kappa = (1 - \kappa)\rho_{\text{Bell}} + \kappa \frac{\mathbb{1}_4}{4}, \quad (22)$$

where $\mathbb{1}_4/4$ is a maximally mixed state and we use $\kappa = 0.2$.

For the semileptonic events populated over (t_1, t_2) , we first extract N_0 , b_x^A , b_x^B , and C_{xx} from the distribution of N_{tot} with Eq. (17). The extracted value of N_0 is then used when fitting to A_{ff} , A_f^A , and A_f^B , in Eqs. (18)–(20) to measure the coefficients b_i^A , b_i^B , and C_{ij} .

The events of the B_s^0 meson pair are generated with $0 < t_1, t_2 < 3.5$ ps. The oscillatory time evolution versus t_1 is shown in Fig. 1(a), integrated over a range of t_2 for the ρ_{Bell} initial state. When fitting the Fano coefficients, we adopt a similar bin width as Ref. [56] with each period divided into 6 bins, leading to 60×60 bins in the $t_1 - t_2$ plane.

The event generation and analysis for K^0 meson pairs are similar. We generate events with $0 < t_1, t_2 < 0.59$ ns, within the first half-period of the damped oscillation as shown in Fig. 1(b). We divide the half-period into 20 bins to perform the fit, similar to [57], which yields a total of 20×20 bins. Although K^0 and $\bar{K}^0$ cannot completely evolve into each other before the quick decay of K_S , the oscillation of a quarter-period is still sufficient to perform quantum tomography.

Our results are shown in Table I from 1000 pseudoexperiments, with statistical uncertainties scaled to 10^7 events, which is aligned with the maximal number of semileptonically decayed $B_s^0\bar{B}_s^0$ events at Belle II [58]. We find that the density matrix can be reconstructed to high precision in our formalism. Furthermore, if we include a systematic error of 1% in each bin of our pseudo experiments, the systematic uncertainty remains smaller than the statistical errors in our fitting. For B_s^0 mesons, the measurement precision of the C_{xx} entry of the flavor correlation matrix is much lower than the other coefficients due to the relatively small $\Delta\Gamma/\Gamma$. The sensitivity of C_{xx} is suppressed by two factors of $\sinh(\Delta\Gamma t/2)$. For K_0 with nearly maximal $\Delta\Gamma/\Gamma$, the x

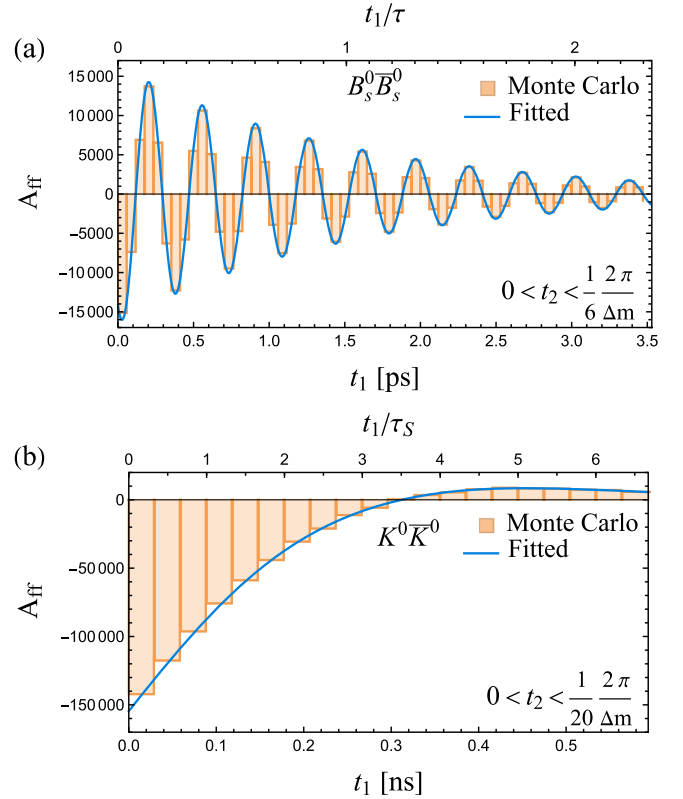


FIG. 1. Monte Carlo results and fitted distribution of $A_{ff}(t_1, t_2)$ for $B_s^0\bar{B}_s^0$ and $K^0\bar{K}^0$, in the direction of t_1 with t_2 interval specified in the figure. The labels of the upper axes are the time in units of the decay lifetime of B_s^0 and K_S^0 .

components of the Fano coefficients such as C_{xx} can be reconstructed equally well as the other coefficients. Collecting events with decays within several times the K_S lifetime is enough to perform quantum tomography, although studying the K_L events at later times is also fruitful in exploring quantum phenomena [3,4,59,60].

In practice, approximately 7×10^6 $B_s^0\bar{B}_s^0$ pairs have been produced in Belle [61,62], and around 3×10^8 are expected to be produced in Belle II [58], mostly in pure Bell states from the decay of $\Upsilon(5S)$.

In LHCb, a pair of B_s^0 mesons can be produced from the hadronization of $b\bar{b}$, which has a cross section of $\sigma(b\bar{b}X) = 0.56$ mb [63], and a fraction of around 1.4% [64] goes to a pair of B_s^0 mesons. With the Run 2 luminosity of 5.7 fb^{-1} , there are 8×10^{10} $B_s^0\bar{B}_s^0$'s produced. The fraction of both B_s^0 's decaying semileptonically is $\text{Br}(B_s^0 \rightarrow \ell^+ \nu_\ell X^-)^2 = 3.7\%$, which potentially yields as many events as our sample simulation. The reconstruction of a pair of b hadrons has been discussed in Ref. [65], and we expect a similar efficiency. Because of the complicated nature of production and hadronization in hadronic collisions, it is unclear to what extent a B^0 meson pair would be flavor entangled, which is very interesting for future work, both in theory and in experiment.

TABLE I. The central values and statistical uncertainties of the Fano coefficients in the $B_s^0 \bar{B}_s^0$ system and in the $K^0 \bar{K}^0$ system with 10^7 events, using the initial states ρ_{Bell} and ρ_κ with $\kappa = 0.2$. Last column: observables used to fit the corresponding Fano coefficients.

	$B_s^0 \bar{B}_s^0$ fitted $\rho_{\text{Bell}}(\rho_\kappa)$	$K^0 \bar{K}^0$ fitted $\rho_{\text{Bell}}(\rho_\kappa)$	Observable
b_x^A	$0 \pm 0.017(0 \pm 0.017)$	$0 \pm 0.0019(0 \pm 0.0020)$	N_{tot}
b_x^B	$0 \pm 0.017(0 \pm 0.017)$	$0 \pm 0.0019(0 \pm 0.0020)$	
b_y^A	$0 \pm 0.0009(0 \pm 0.0009)$	$0 \pm 0.0020(0 \pm 0.0023)$	A_f^A
b_z^A	$0 \pm 0.0009(0 \pm 0.0009)$	$0 \pm 0.0014(0 \pm 0.0015)$	
b_y^B	$0 \pm 0.0009(0 \pm 0.0009)$	$0 \pm 0.0020(0 \pm 0.0023)$	A_f^B
b_z^B	$0 \pm 0.0009(0 \pm 0.0009)$	$0 \pm 0.0014(0 \pm 0.0015)$	
C_{xx}	$-1 \pm 0.31(-0.8 \pm 0.31)$	$-1 \pm 0.0037(-0.8 \pm 0.0036)$	N_{tot}
C_{yx}	$0 \pm 0.016(0 \pm 0.016)$	$0 \pm 0.0024(0 \pm 0.0027)$	
C_{zx}	$0 \pm 0.016(0 \pm 0.016)$	$0 \pm 0.0017(0 \pm 0.0018)$	A_f^A
C_{xy}	$0 \pm 0.016(0 \pm 0.016)$	$0 \pm 0.0024(0 \pm 0.0027)$	
C_{xz}	$0 \pm 0.016(0 \pm 0.016)$	$0 \pm 0.0017(0 \pm 0.0018)$	A_f^B
C_{yy}	$-1 \pm 0.0011(-0.8 \pm 0.0010)$	$-1 \pm 0.0026(-0.8 \pm 0.0030)$	
C_{yz}	$0 \pm 0.0008(0 \pm 0.0009)$	$0 \pm 0.0019(0 \pm 0.0021)$	A_{ff}
C_{zy}	$0 \pm 0.0008(0 \pm 0.0009)$	$0 \pm 0.0019(0 \pm 0.0021)$	
C_{zz}	$-1 \pm 0.0011(-0.8 \pm 0.0009)$	$-1 \pm 0.0009(-0.8 \pm 0.0012)$	

K^0 is typically a pair produced from a ϕ meson decay. Around 8×10^9 $K^0 \bar{K}^0$ pairs have been produced at KLOE and KLOE-2 [66], and a fraction of 1.1% decays in $0 < t < \pi/\Delta m$, among which a fraction of $\text{BR}(K_S^0 \rightarrow \pi^\pm \ell^\mp \nu_\ell) \text{BR}(K_L^0 \rightarrow \pi^\pm \ell^\mp \nu_\ell) = 8 \times 10^{-4}$ decays to leptons on both sides.

The B_d^0 system, with $(\Delta m/\Gamma, \Delta\Gamma/\Gamma)_{B_d^0} = (0.77, 0.004)$, does have a suitable oscillation time, but the decay lifetime difference is too small to yield a usable measurement of b_x . However, in this system, the $(2 \times 2)_{yz}$ subset of the correlation matrix can be measured, which is sufficient to measure Bell nonlocality [5,6]. The sensitivity to the other Fano coefficients could be increased by a factor of a few by combining the decay to CP eigenstates such as $B_d^0 \rightarrow J/\psi K_S$. On the other hand, the D^0 system has small values of both $\Delta m/\Gamma$ and $\Delta\Gamma/\Gamma$, making quantum tomography much more challenging.

Applications to quantum information—Having performed quantum tomography for the $M\bar{M}$ system, we are in a position to study any quantum information observable of the system. We list some example quantities below and provide their definitions in the End Matter.

Concurrence is a measure of how much two subsystems in a system are entangled (not separable) [67]. The Bell variable is a normalized measurement of Bell’s inequality where a positive value signifies that a local hidden variable model cannot imitate this result [68]. Quantum discord captures a broader class of quantum correlations than entanglement; it can be nonzero even for separable states [69,70]. The steerability variable is a normalized measure of steerability where a positive value indicates that a state does not admit a local hidden-state model.

Conditional entropy quantifies the number of bits needed for the subsystem $\mathcal{A}$ to reconstruct the subsystem $\mathcal{B}$, which can have negative values for the quantum system [71]. The second stabilizer Rényi entropy (SSRE) is a measure of the “magic” of a quantum system that quantifies the computation advantage this system would have over a classical computer [72].

These quantities are listed in Table II for ρ_κ with $\kappa = 0.2$. Quantum correlations are indicated by a negative value of conditional entropy and a positive value of all other quantities. All quantities can be measured with high precision. For quantities that require the full density matrix for their calculation, their precision is limited by the largest uncertainty in Table I. Some quantities, however, only require a subset of the Fano coefficients, such as Bell nonlocality, in which case it can often be measured to higher precision.

TABLE II. Reconstructed quantum information quantities from Table I for ρ_κ . The statistical uncertainties correspond to 10^7 events. Positive numbers of discord, concurrence, steerability, and Bell variable indicate nonclassical correlation. Negative conditional entropy indicates a quantum system, and positive SSRE indicates nonzero magic.

	$B_s^0 \bar{B}_s^0$	$K^0 \bar{K}^0$
Quantum discord	0.58 ± 0.10	0.617 ± 0.003
Concurrence	0.69 ± 0.15	0.701 ± 0.003
Steerability variable	0.48 ± 0.07	0.4818 ± 0.0017
Bell variable	0.1856 ± 0.0015	0.187 ± 0.004
Conditional entropy	-0.16 ± 0.31	-0.154 ± 0.007
SSRE (magic)	0.38 ± 0.15	0.389 ± 0.003

Conclusion and outlook—The flavor space of particles produced in collider environments encodes quantum information as rich as that of the spin space [73–76]. Expressing the flavor density matrix for a neutral meson M in terms of the Pauli matrices in the Bloch vector space, we constructed the flavor correlation matrices for the $M\bar{M}$ system at a given time t . The crucial observations are that different components in flavor space are related to different physical observables. Defining the flavor eigenstates along the z direction, the y and z components are measured from the oscillation of the flavor asymmetry, with a sensitivity related to the mass difference Δm . The x components are measured from the time dependence of the total semileptonic decay rate, with a sensitivity related to the decay lifetime difference $\Delta\Gamma$.

Our analysis provides a systematic way to construct the complete flavor density matrix for an $M\bar{M}$ system and study its production mechanisms, decoherence phenomena, and quantum properties. From an observational point of view, meson systems with suitable $\Delta m/\Gamma$ and $\Delta\Gamma/\Gamma$ are required. We observed that the B_s^0 and K_S^0 systems are superior to the B_d^0 and D^0 systems due to their favorable oscillation and decay properties. Through numerical simulations, we demonstrated that the entire density matrix can be precisely reconstructed, as shown in Fig. 1 and Table I. Through the quantum tomography procedure, any quantum information variable can be constructed, as shown in Table II.

Some final remarks are in order. First, instead of aiming to exclude local hidden variable models [10,77–84], the objective of this Letter is to reconstruct the complete flavor density matrix of the meson pairs and study their quantum information properties. Second, although the overall effects are rather small, the CP -violating interactions in the mixing would change the precession properties in the state evolution and, therefore, introduce additional modulation in the observables. The flavor asymmetry observables would involve all of the 3×3 elements of the flavor correlation matrix instead of only the y and z components. Precision tomography could, thus, be sensitive to new CP -violating sources. Third, for meson pairs from quark hadronization, there is no first-principles calculation of the flavor state. Our formalism assumes the most general $M\bar{M}$ quantum state to begin with as in Eq. (12). This provides a new avenue for exploring hadronization mechanisms in terms of quantum correlations, both theoretically and experimentally.

Overall, we demonstrated that performing quantum tomography for the $M\bar{M}$ system can facilitate the study of production mechanisms, decoherence phenomena, quantum information variables, and potential new sources of CP violation.

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Data availability—The data are not publicly available. The data are available from the authors upon reasonable request.

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End Matter

In the following, we list the definition of quantum information quantities, together with the value of these quantities for Werner states in Fig. 2.

For a two-qubit system, the concurrence $\mathcal{C}$ [85] is given by

$$\mathcal{C} = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4), \quad (\text{A1})$$

where λ_i ($i = 1, 2, 3, 4$) are the eigenvalues, sorted by decreasing magnitude, of the matrix

$$R_\rho = \sqrt{\sqrt{\rho} \tilde{\rho} \sqrt{\rho}}, \quad \tilde{\rho} = (\sigma_2 \otimes \sigma_2) \rho^* (\sigma_2 \otimes \sigma_2). \quad (\text{A2})$$

$0 < \mathcal{C} \leq 1$ indicates an entangled state while $\mathcal{C} = 0$ indicates a separable state. For a separable state, the two

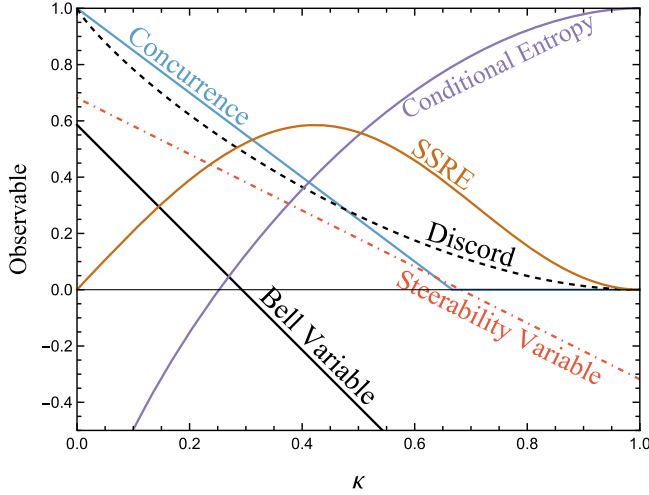


FIG. 2. Representative quantum information quantities versus the parameter κ for Werner states.

subsystems can be fully described independently, but this is not possible for an entangled state.

Bell's inequality is obeyed by all classical theories, including those with local hidden variables [68]. For two qubits, the inequality is given by the Clauser-Horne-Shimony-Holt (CHSH) inequality [86]. A state that obeys the CHSH inequality is called Bell local, while a state that does not is called Bell nonlocal. The linear approximation to the inequality is measured by the Bell variable $\mathcal{B}$ [76],

$$\mathcal{B} = |C_{yy} + C_{zz}| - \sqrt{2}. \quad (\text{A3})$$

With this normalization, $-\sqrt{2} < \mathcal{B} \leq 0$ indicates a Bell local state, and $0 < \mathcal{B} < 2 - \sqrt{2}$ indicates a Bell nonlocal state.

Quantum discord $\mathcal{D}$ is the difference between the total mutual information and the classical mutual information. The latter requires an extremization over all possible directions; however, for states where only the correlation matrix is nonzero and diagonal, there is a closed form [87,88],

$$\begin{aligned} \mathcal{D} = & 1 + \frac{1}{4}(1 - C_{zz} - C_{xx} - C_{yy})\log_2\left(\frac{1 - C_{zz} - C_{xx} - C_{yy}}{4}\right) \\ & + \frac{1}{4}(1 - C_{zz} + C_{xx} + C_{yy})\log_2\left(\frac{1 - C_{zz} + C_{xx} + C_{yy}}{4}\right) \\ & + \frac{1}{4}(1 + C_{zz} - C_{xx} + C_{yy})\log_2\left(\frac{1 + C_{zz} - C_{xx} + C_{yy}}{4}\right) \\ & + \frac{1}{4}(1 + C_{zz} + C_{xx} - C_{yy})\log_2\left(\frac{1 + C_{zz} + C_{xx} - C_{yy}}{4}\right) \\ & - \frac{1}{2}(1 + \lambda)\log_2\left(\frac{1 + \lambda}{2}\right) - \frac{1}{2}(1 - \lambda)\log_2\left(\frac{1 - \lambda}{2}\right), \quad (\text{A4}) \end{aligned}$$

where $\lambda = \max\{|C_{xx}|, |C_{yy}|, |C_{zz}|\}$. A discord of $\mathcal{D} = 0$ indicates a classical state, while a discord of $0 < \mathcal{D} \leq 1$ indicates the presence of quantum correlations.

Bell nonlocality is a relatively strict condition that forbids states with local hidden variable descriptions. A weaker condition is called steerability, which forbids states with a description where one subsystem can be described by a local hidden variable theory and the other subsystem has a quantum mechanical description.

The steerability variable $\mathcal{S}$ is derived from the steerability condition [89] and is given by

$$\mathcal{S} = \frac{1}{2\pi^2} \int d\hat{\mathbf{n}} \sqrt{\hat{\mathbf{n}}^T \mathbf{C}^T \mathbf{C} \hat{\mathbf{n}}} - \frac{1}{\pi}, \quad (\text{A5})$$

where $\mathbf{C}$ is the correlation matrix. This formulation applies when only the correlation matrix is nonzero. When $\mathcal{S} \geq 0$, the state is steerable.

The conditional entropy of a state ρ_{AB} is [88,90]

$$S(\rho_A|\rho_B) = S(\rho_{AB}) - S(\rho_B), \quad (\text{A6})$$

where $S(\rho_{AB})$ is the von Neumann entropy of the total quantum state ρ_{AB} , and ρ_B is the reduced density matrix obtained by performing the partial trace over the subsystem $\mathcal{A}$. The von Neumann entropy is

$$S(\rho) = -\text{tr}(\rho \log_2 \rho). \quad (\text{A7})$$

The conditional entropy is the number of bits that would need to be given to one subsystem to reconstruct the other subsystem. Classically, this ranges from 0, meaning one subsystem does not need any information to reconstruct the total system, to 1, meaning that the full information of the other subsystem is required. Quantum mechanically, the conditional entropy can be as small as -1 with the interpretation that a negative value indicates that a bit is available for future quantum communication between the subsystems [71].

In quantum computation, a state that has stronger quantum correlations, like a Bell state, does not necessarily lead to a genuine computational advantage over a classical computer. States that offer a true advantages are said to possess magic. A measure of the magic of a quantum state is the second stabilizer Rényi entropy (SSRE), given by [72]

$$\mathcal{M}_2 = -\log_2\left(\frac{1 + \sum_i (b_i^A)^4 + \sum_j (b_j^B)^4 + \sum_{i,j} C_{ij}^4}{1 + \sum_i (b_i^A)^2 + \sum_j (b_j^B)^2 + \sum_{i,j} C_{ij}^2}\right). \quad (\text{A8})$$

When $\mathcal{M}_2 = 0$, this indicates a state does not possess magic and does not have a computational advantage over a classical system. The larger the value of $\mathcal{M}_2$ of a state, the larger the computational advantage of the state has.