Krylov Spread Complexity as Holographic Complexity beyond Jackiw-Teitelboim Gravity

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One of the important open problems in quantum black hole physics is a dual interpretation of holographic complexity proposals. To date the only quantitative match is the equality between the Krylov spread complexity in the triple-scaled Sachdev-Ye-Kitaev (SYK) model and the complexity = volume proposal in classical Jackiw-Teitelboim gravity. Our Letter utilizes the recent connection between the double-scaled SYK model and sine dilaton gravity to show that the quantitative relation between Krylov spread complexity and complexity = volume extends to finite temperatures and to full quantum regime on the gravity side at disk level. Furthermore, we isolate the first quantum correction to the complexity = volume proposal and propose to view it as a complexity of quantum fields in the bulk. The key lesson from our Letter is that gravity demands to assign complexity also to Euclidean state preparation.

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Introduction—Understanding black hole interiors has been an important theme of research in quantum gravity ever since the foundational papers on black holes appeared. A major modern incarnation of this research front has been studies of holographic complexity over the course of the past decade [1,2]. This research direction stems from a heuristic yet convincing observation in [3] that connects perpetual growth of black hole interiors in spatial volume with the expected behavior (proven in random quantum circuits [4]) of quantum circuit complexity in chaotic systems. In the realm of holography, this property of black hole interiors is now formalized within the complexity = anything paradigm [5,6], which provides an infinite set of boundary-anchored bulk geometric objects that all grow linearly with the boundary time at late enough times. Arguably the simplest complexity = anything proposal is the one that was considered first: the complexity = volume proposal dealing with the spatial volume of extremal time slices of the bulk [7].

The key open question in the studies of holographic complexity is that of its interpretation in the microscopic description on the boundary. To date, despite a decade of research on the topic, there has been only one fully quantitative match discovered [8]. In 2023, the authors of [14] (see also Ref. [15]) showed that complexity = volume in the simplest holographic gravity theory [classical Jackiw-Teitelboim (JT) gravity] is given by the Krylov spread complexity [16] in the dual description in terms of a particular (triple-scaling) limit of the SYK model at infinite temperature. While this may come as a surprise, since Krylov complexity is *a priori* a distinct quantity from quantum circuit complexity originally proposed in this context, both quantities realize Susskind's expectation about linear growth behavior in chaotic quantum systems [4,16,17].

The so far isolated nature of [14] raises the crucial question: does the relation between holographic complexity and microscopic Krylov spread complexity extend to other gravity setups? The aim of our Letter is to present that the answer is affirmative within a freshly discovered duality between two-dimensional sine dilaton gravity and a double-scaled SYK model [18–21]. From the point of view of the parameter space of the double-scaled SYK model, the result of [14] lies at a single point and our Letter extends it to a two-dimensional plane spanned by the interaction strength (nonlocality of the model), in which case we make use of the quantum generalization of the complexity = volume proposal put forward in [22], and inverse temperature β. In the Outlook, we speculate on the implications of our Letter for the broader field of holographic complexity.

Setup—The SYK model [23,24] describes a system of N Majorana fermions, with dynamics governed by a p-body interaction of the form

$$H_{\mathrm{SYK}} = i^{p/2} \sum_{1\le i_1<\cdots<i_p\le N} J_{i_1\cdots i_p}\psi_{i_1}\cdots\psi_{i_p}, \tag{1}$$

where the couplings $J_{i_1\cdots i_p}$ are typically assumed to be Gaussian random variables. We will be focusing on the

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double-scaled SYK model (DSSYK), where both N and p are sent to infinity, while keeping the ratio $|\log q| = (p^2/N)$ finite. A striking feature of this regime is that all amplitudes in the DSSYK model are computable in an exact way, by evaluating the so-called chord diagrams, an intermediate combinatorial tool to solve the system [25,26]. Specifically, this is done by introducing an auxiliary quantum mechanical system governed by an effective Hamiltonian, the transfer matrix $\hat{T}$, which acts on a Hilbert space spanned by chord states $|n\rangle$. See, for instance, Ref. [27] for a recent comprehensive review.

Our gravitational setup is sine dilaton gravity, a recently proposed gravity dual to the double-scaled SYK model [18,19]. The gravitational path integral of sine dilaton gravity takes the form

$$\int \mathcal{D}g\mathcal{D}\Phi \exp\left[\frac{1}{2}\int \mathrm{d}^2x\sqrt{g}\left(\Phi R + \frac{\sin(2|\log q|\Phi)}{|\log q|}\right)\right], \quad (2)$$

where we suppressed the boundary terms. In (2) the potential for dilaton Φ exhibits a sine profile and thus corresponds to a deformation of the linear dilaton potential of JT gravity; see, e.g., Ref. [28] for a review of the latter. The main object of interest for us is the extremal volume of the complexity = volume proposal in this gravity theory, which in two dimensions is just the geodesic length. Specifically, to prove that DSSYK Krylov spread complexity is dual to length in this gravity theory, we will be interested in the gravitational two-point function in sine dilaton gravity, corresponding to the boundary to boundary propagator of a nonminimally coupled scalar field in the bulk [18]. The effective geometry this matter probe will experience is given by

$$\begin{aligned}\mathrm{d}s^2_{\mathrm{eff}} &= \mathrm{d}s^2_{\mathrm{sine}} e^{-2i|\log q|\Phi} \\ &= -(\rho^2 - \sin(\theta)^2)\mathrm{d}t_S^2 + \frac{\mathrm{d}\rho^2}{\rho^2 - \sin(\theta)^2}, \end{aligned} \quad (3)$$

which represents the Schwarzschild patch of a two-dimensional anti–de Sitter black hole with Hawking temperature $\beta_{\mathrm{BH}} = 2\pi/\sin(\theta)$, which reduces to the JT black hole when $\theta \ll 1$. For the remainder of this Letter, we will use the parameter θ to conveniently parametrize finite temperatures. The length in the Kruskal extension of the geometry (3), after holographic renormalization, is given by

$$L = 2\log[\cosh(t\sin(\theta)/2)] - 2\log[\sin(\theta)] \quad (4)$$

in terms of the two-sided boundary Lorentzian time t. The expression (4) represents the semiclassical limit of the full quantum expectation value of the length of the thermofield double state [29] in sine dilaton gravity as a function of t, which we will compute in (11) and match with DSSYK Krylov spread complexity. By introducing the canonical conjugate P of the length L and performing canonical quantization, the gravitational Hamiltonian $\hat{H}_{\mathrm{grav}}$ of sine dilaton gravity reads [18,20]

$$\hat{H}_{\mathrm{grav}} = \frac{1}{\sqrt{2|\log q|}\sqrt{1-q^2}}\left[-\cos(\hat{P}) + \frac{1}{2}e^{i\hat{P}}e^{-\hat{L}}\right], \quad (5)$$

which precisely matches the DSSYK transfer matrix $\hat{T}$ [15,25,26], making the duality manifest. Moreover, as discussed in detail in [20], the invariance of the Hamiltonian (5) under periodic shifts $P \to P + 2\pi$ of the momentum instructs us to consider this as a redundancy and to treat P as a compact variable. Upon doing so, the theory is projected onto discretized lengths and the Euclidean time periodicity β_{BH} of the black hole background (3) reduces to the true microscopic temperature of the DSSYK model $\beta = (2\pi - 4\theta)/\sin(\theta)$ [30]. We point out how the discretization of chord number is obvious from the point of view of the DSSYK model [15,25,26], but the quantization of the length is far from obvious from the gravity side. Upon projection, the physical Hilbert space $|L\rangle$ can then be identified with the chord Hilbert space of the DSSYK model, spanned by the chord number states $|n\rangle$ [15,25,26], via the holographic dictionary

$$L = 2|\log q|n. \quad (6)$$

More details on the presented duality can be found in Supplemental Material [32] in Sec. SM1.

In sine dilaton gravity, the two-point function of a massive probe is captured by the insertion of the bilocal operator $e^{-\Delta\hat{L}}$ of conformal weight Δ [18]. The structure of this expectation value takes a similar form to the JT gravity one [33,34] and reads

$$\langle e^{-\Delta\hat{L}}\rangle = Z_\beta^{-1}\langle L=0|e^{-\tau\hat{H}_{\mathrm{grav}}}e^{-\Delta\hat{L}}e^{-(\beta-\tau)\hat{H}_{\mathrm{grav}}}|L=0\rangle, \quad (7)$$

where the disk has been divided in two parts of length τ and $\beta - \tau$ corresponding to the Euclidean boundary times to which the semiclassical geodesic is anchored. An explicit evaluation of (7) is provided in Supplemental Material [32] in Sec. SM2, including definitions of the appearing special functions. Analytically continuing $\tau = \beta/2 + it$ in (7), we finally obtain the Lorentzian two-point function in sine dilaton gravity. Consistently, the latter is given by the expectation value of the bilocal operator in the state

$$|\psi(t)\rangle_\beta = e^{-it\hat{H}_{\mathrm{grav}}}|\psi_\beta\rangle, \quad (8)$$

which corresponds to the Lorentzian time evolution of the Hartle-Hawking state $|\psi_\beta\rangle = Z_\beta^{-1/2}e^{-\beta\hat{H}_{\mathrm{grav}}/2}|L=0\rangle$, obtained by the Euclidean half-disk preparation amplitude.

Deriving Krylov complexity from length—Krylov spread complexity [16] is defined as the expectation value of the position of a state $|\psi(t)\rangle$ spreading over the Krylov basis $|K_n\rangle$,

$$C_K(t) = \sum_{n=0}^{\infty} n|\langle K_n|\psi(t)\rangle|^2, \qquad |\psi(t)\rangle = e^{-iHt}|R\rangle. \quad (9)$$

The state $|\psi(t)\rangle$ follows Hamiltonian time evolution from the reference state $|R\rangle$ and the Krylov basis $|K_n\rangle$ is the unique basis that arises from Gram-Schmidt orthonormalization of the set of states constructed via repeated

application of the Hamiltonian to the reference state $\{H^n|R\rangle\}$, commonly referred to as the Lanczos algorithm.

We begin the derivation of the core statement of this Letter on the gravity side and study the expectation value of the length in sine dilaton gravity. Specifically, we extract the length from the two point function (7), where Δ is the scaling dimension of the bilocal operator, via

$$\langle \hat{L} \rangle = \left[-\partial_\Delta \langle e^{-\Delta \hat{L}} \rangle \right]_{\Delta=0}. \tag{10}$$

Note that this starting point coincides with the way the authors of [22] lay out their study of the black hole interior at very late times in JT gravity. The key observation we make is that, by leveraging the relation (10) to derive $\langle \hat{L} \rangle$ from (7) supplied with the analytic continuation $\beta/2 + it$, one obtains

$$\langle \hat{L} \rangle = \frac{|\log q^2|}{Z_\beta} \int_0^\pi d\theta_1 \rho(\theta_1) \int_0^\pi d\theta_2 \rho(\theta_2) e^{-(\frac{\beta}{2}+it)E(\theta_1)} \times e^{-(\frac{\beta}{2}-it)E(\theta_2)} \sum_{n=0}^\infty \frac{n}{(q^2;q^2)_n} H_n(\cos\theta_1|q^2) H_n(\cos\theta_2|q^2) \tag{11}$$

for the expectation value of the length. Importantly, a sum over n appears. This sum originates from the matrix element $\langle \theta_1 | e^{-\Delta \hat{L}} | \theta_2 \rangle$, provided explicitly in Eq. (S12) in Supplemental Material [32], and allows us to recast this result into the form of a Krylov spread complexity (9) as we will show now. On the way, crucially, we identify the state $|\psi(t)\rangle$ to which this Krylov spread complexity is associated.

As discussed in the setup and further detailed in Supplemental Material [32] in Sec. SM1, on the basis of the previously established duality with its holographic dictionary (6), which relates length in sine-dilaton gravity and chord number in the DSSYK model, we can now identify the components appearing in (11) as eigenvalues $E(\theta)$ and chord basis eigenfunctions $\langle n|\theta\rangle$ [explicitly provided in Supplemental Material [32] in Eq. (S6) [35] and (S9), respectively] of the transfer matrix $\hat{T}$, the effective Hamiltonian of the DSSYK chord Hilbert space [15,25,26] which governs the transition from chord states $|n\rangle$ to $|n \pm 1\rangle$. The second ingredient that facilitates recasting the sine dilaton bulk length (11) into a Krylov spread complexity is the crucial insight in the works [14,15] that the chord basis $|n\rangle$ coincides with the Krylov basis $|K_n\rangle$ for transfer matrix evolution starting from the zero chord state as reference state. The conceptual reason for this is that, when constructing the Krylov basis, each application of $\hat{T}$ to $|0\rangle$ adds a chord such that $|K_n\rangle = |n\rangle$, where importantly the built-in orthonormalization in the Lanczos algorithm cancels out the chord annihilation contribution in $\hat{T}$ at each step [36]. Analytic knowledge of the Krylov basis in this case is essential because it allows us to precisely identify the length (11), evolving in Lorentzian time t at nonzero DSSYK inverse temperature β, with the Krylov spread complexity $C_K(t)_\beta$,

$$\langle \hat{L} \rangle = \frac{|\log q^2|}{Z_\beta} \sum_{n=0}^\infty n \left| \int_0^\pi d\theta \rho(\theta) e^{-(\frac{\beta}{2}+it)E(\theta)} \langle n|\theta\rangle \langle \theta|0\rangle \right|^2 = |\log q^2| \sum_{n=0}^\infty n \left| \langle n| \frac{e^{-iT(t-i\beta/2)}}{\sqrt{Z_\beta}} |0\rangle \right|^2 = |\log q^2| C_K(t)_\beta \tag{12}$$

of the (normalized) state $|\psi(t)\rangle_\beta = Z_\beta^{-1/2} e^{-iT(t-i\beta/2)}|0\rangle$ that arises by mixed Lorentzian plus Euclidean transfer matrix evolution from the zero chord state $|0\rangle$ as reference state. The matching

$$2|\log q| C_K(t)_\beta = \langle \hat{L} \rangle = \left[-\partial_\Delta \langle e^{-\Delta \hat{L}} \rangle \right]_{\Delta=0} \tag{13}$$

is one of the main results of this Letter. Note that this is a match on the quantum level that is valid for all allowed values of $q \in (0,1)$ in the DSSYK model, as well as for all values of β. We refer to Sec. SM3 in Supplemental Material [32] for an operator perspective on this result: For transfer matrix evolution starting from the zero chord state, we can identify the associated Krylov operator with the length operator of sine dilaton gravity, making complexity equals volume manifest as a statement about linear operators. In Sec. SM4, we provide an interpretation of the identified evolution in the chord Hilbert space on the level of the double-scaled SYK model based on [14].

Two limits—Although our identification (13) holds at the quantum level, it is important to check that the correct semiclassical limit, as $q \to 1$ limit in the setup of this Letter [38], is recovered. Indeed, as one can observe in

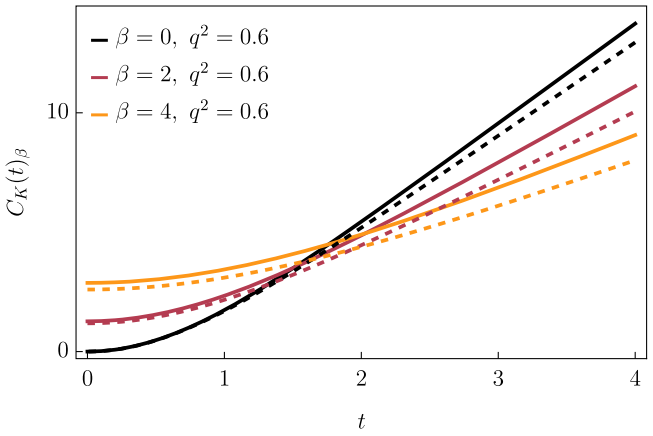


FIG. 1. Numerical values for the finite-temperature Krylov spread complexity $C_K(t)_\beta$ at various temperatures β and $q^2 = 0.6$ (solid). It is visible that, for larger β, the initial onset at $t = 0$ that accounts for higher complexity of the Euclidean state preparation is larger and the complexity growth slows down. The complexities are contrasted with the expectation value of the classical length in sine dilaton gravity at temperature β (dashed), normalized by $2|\log q|$.

Fig. 1, the numerical evaluation of (11) is well interpolated by the semiclassical effective length in sine dilaton gravity given in (4). Details on the numerical evaluation can be found in Supplemental Material [32] in Sec. SM5 and we defer the reader to the section about quantum corrections in the latter part of this Letter for a discussion of the deviations from (4) in the quantum regime.

The semiclassical limit was also checked for the exact two-point function in [39], leading, by extrapolation through (10), to the same result (4). We can verify that the correct JT limit is recovered by additionally considering small values of θ, where the spectral edge of the DSSYK spectrum is probed. In this regime, the semiclassical length simplifies to

$$L_{\mathrm{JT}} = 2\log(\cosh(t\theta/2)) - 2\log(\theta), \tag{14}$$

which corresponds to the length of the ER bridge in the JT black hole background [40], at temperature $\beta_{\mathrm{BH}} = 2\pi/\theta$ [41]. On the other hand, we can also investigate the regime where $\beta \to 0$ which, according to $\beta = \beta_{\mathrm{DSSYK}} = (2\pi - 4\theta)/\sin(\theta)$, corresponds to $\theta = \pi/2$, the midpoint of the DSSYK spectral support. In this regime, the length becomes

$$L_{\beta=0} = 2\log\left(\cosh(t/2)\right). \tag{15}$$

In [14], DSSYK Krylov spread complexity for $\beta = 0$ was connected to a JT classical length by performing a particular combination of limits. We clarify that taking purely the $\beta \to 0$ limit does not lead to JT gravity. At the level of sine dilaton gravity, the $\beta \to 0$ regime is different from the standard JT limit, as it corresponds to an expansion of the dilaton potential around $\Phi = \pi/2$. In this case, as shown in [20], the gravity theory reduces to a regularization of flat-space JT gravity, reproducing the results of [43] associated with the high-temperature limit of the DSSYK model. This clarification does not contradict the results of [14] because their combination of limits is different from the pure $\beta \to 0$ limit that we take.

Lessons for holographic complexity—There are two main lessons to be drawn from the result (13) with respect to its validity at finite temperatures, a regime of precise matching of bulk and boundary complexity that is established in this Letter for the first time.

The first lesson is that gravity demands to assign complexity to both Lorentzian evolution as well as Euclidean state preparation in order to match Krylov complexity to the Lorentzian evolution of the volume $\langle \hat{L} \rangle$ at nonzero inverse temperature β. Importantly, this notion of complexity is different from starting at a reference state proportional to $e^{-\beta T/2}|0\rangle$ and assigning complexity only to its Lorentzian evolution. This is because the different choice of reference state leads to a different Krylov basis and Krylov complexity,

$$C_K \text{ for } e^{-iT(t-i\beta/2)}\underbrace{|0\rangle}_{|R\rangle} \neq C_K \text{ for } e^{-iTt}\underbrace{e^{-\beta T/2}|0\rangle}_{|R\rangle}.$$

Our approach in this Letter is well motivated by the fact that it is known from the gravitational point of view that the length at finite temperature arises via the analytic continuation $\beta/2 + it$ in (11) and not by a structurally different calculation.

The second lesson concerns the dependence of the Krylov complexity $C_K(t)_\beta$ on the temperature. We observe two important features, both clearly visible in Fig. 1. The complexity (solid line) acquires a positive onset that increases with increasing β. This is the complexity associated with the Euclidean preparation of the Hartle-Hawking state $e^{-\beta T/2}|0\rangle$. The other distinct feature is that finite temperature slows down the complexity growth. This agrees with the observations in [44] and, as discussed there, is consistent with what has been confirmed for other instances of the SYK model. When contrasted with the analytic result for the classical length (dashed line) given in Eq. (4), we observe correlation but not a precise match for a generic choice of q and β. We will now show in the following section that this is expected and can be resolved by taking into account quantum corrections to the classical length.

The role of quantum corrections to the length—In [14], agreement between Krylov spread complexity in the triple-scaled SYK model and length in classical JT gravity was reported. This identification requires the limit $q \to 1$ as part of the triple-scaling procedure. Let us highlight that this is very different from what is report in this Letter. Here, we explicitly derive the dual Krylov spread complexity directly from the length in sine dilaton gravity, valid for all values of q. The strength of our result lies in the fact that we leverage a full gravitational dual to the DSSYK model, extending beyond the triple-scaling limit, and uncover a fully quantum bulk manifestation of complexity within the dual gravity theory. Let us focus on the region of q close to but not exactly 1 to exemplify this statement.

In this sector, the full quantum expression of the length in sine dilaton gravity is well approximated by the classical answer $\langle \hat{L} \rangle_{\mathrm{cl}}$, given in Eq. (4), modified by the first quantum correction in $|\log q|$. The analytical expression for this correction has been very recently derived by one of us and his collaborators in [45] and reads

$$\begin{aligned}\langle \hat{L} \rangle_{q\text{-corr}} = &-\frac{2|\log q|}{4[(\pi-2\theta)\cot(\theta)+2]} \\ &\times \Big\{ [-t^2\sin^2(\theta)][\cot^2(\theta)+\tanh^2(t\sin(\theta)/2)] \\ &+ \frac{4[t\sin(\theta)\tanh\left(\frac{1}{2}t\sin(\theta)\right)-2]}{(\pi-2\theta)\cot(\theta)+2} \\ &- [(\pi-2\theta)\cot(\theta)+2]^2\operatorname{sech}^2(t\sin(\theta)/2) \\ &+ [t\tanh(t\sin(\theta)/2)-2\csc(\theta)]^2+4 \Big\}. \end{aligned} \tag{16}$$

Numerically, we can now compare the Krylov spread complexity $C_K(t)_\beta$ we derived in (12) to check whether

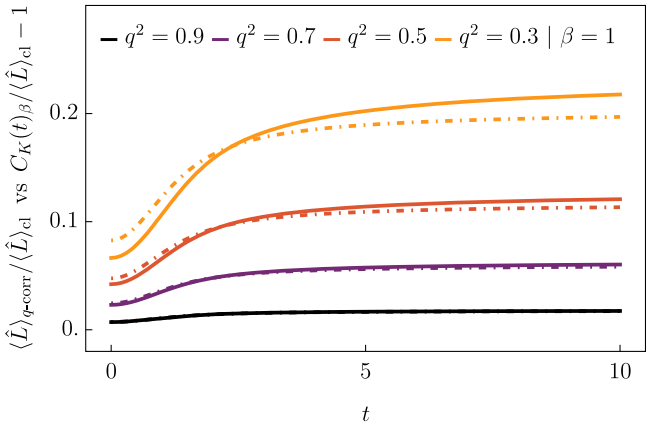


FIG. 2. Numerically evaluated Krylov complexity $C_K(t)_\beta$ (solid), normalized by the classical length $\langle\hat{L}\rangle_{\rm cl}$ (and a factor of $2|\log q|$ that is omitted in the label) for $\beta = 1$ and various q (color coding) vs the leading order quantum correction $\langle\hat{L}\rangle_{q\text{-corr}}$ (dot-dashed), also normalized by $\langle\hat{L}\rangle_{\rm cl}$, for the same values of q and β. For q close to 1, the curves fall on top of each other, signaling perfect agreement; For q further away from $q = 1$, the deviation systematically increases because of higher order corrections that were not taken into account.

it matches with $\langle\hat{L}\rangle_{\rm cl}$ in (4) plus $\langle\hat{L}\rangle_{q\text{-corr}}$ in (16) for values of q close to $q = 1$. This comparison is visualized in Fig. 2. We find excellent agreement around $q = 1$, highlighting that the derived complexity indeed matches the quantum length. As expected, the agreement gets worse for lower q; this is because, here, higher order corrections to the length that are not known analytically become relevant.

Furthermore, the functional form of the first quantum correction to the length (16) suggests a particular interpretation: In the early-time limit, the correction itself behaves quadratically in t, supplemented by an onset in the case of finite inverse temperature β, while in the late-time limit it exhibits linear growth. The functional behavior of this first quantum correction to the bulk length coincides with the characteristic behavior of Krylov spread complexities. On the basis of this observation, we raise the question whether the correction individually has a dual manifestation as a particular notion of Krylov spread complexity. Reminiscent of the case of holographic entanglement entropy where the first quantum correction is given by entanglement between bulk quantum fields [46], we speculate that the first length quantum correction has a dual interpretation as Krylov spread complexity of bulk quantum fields.

Summary—We presented a precise duality between DSSYK Krylov spread complexity and length in sine dilaton gravity. This is accomplished by leveraging the freshly discovered duality between the DSSYK model and sine dilaton gravity to recast the bulk length as a Krylov spread complexity. In particular, to establish such a duality at finite temperature for the first time, gravity demands to assign complexity to the Euclidean preparation of the Hartle-Hawking state. The derived duality holds on the quantum level, meaning for all DSSYK parameters $q \in (0, 1)$. This is exemplified by a numerical comparison of the derived Krylov spread complexity with the first quantum correction to the length.

Outlook—Krylov complexity is expected to saturate at times exponential in the entropy. *Ab initio*, this seems to conflict with the linear growth that is observed in [14] and this Letter as the asymptotic behavior of the studied Krylov spread complexities. The resolution of this puzzle lies in taking into account nonperturbative corrections. From the gravitational perspective, the two-point function is expected to receive contributions from higher topologies in the gravitational path integral of sine dilaton gravity. In particular, the saturation of the two-point function—manifesting as a late-time plateau that halts the linear growth—is likely driven by nonperturbative effects in the topological expansion. This mirrors the behavior observed in JT gravity, where the dual matrix model [47] provided a framework to explain such nonperturbative late-time phenomena [22]. Remarkable progress in this direction was very recently also reported in [37,48]. We conjecture that similar mechanisms should govern sine dilaton gravity, whose dual matrix model is expected to correspond to the eigenstate thermalization hypothesis matrix model for q-deformed JT gravity [49], as recently proposed in [21]. On the boundary side, any potential incarnation of Krylov complexity is thus expected to go beyond the double-scaled SYK model, which corresponds to the strict $N \to \infty$ limit.

Moreover, it is important to state that, for Krylov spread complexity, a proof of the presence of the switchback effect—i.e., scrambling time delay of the linear growth associated with the presence of shock waves in the bulk [7]—would complete its understanding as a genuine holographic complexity. There are strong reasons to believe that this characteristic can be made manifest in the context of the presented model. We expect that insights from JT gravity, such as the observation of Shapiro time delay in the two-point function of a heavy bilocal operator via a boundary clock analysis [28] and the connection of the crossed four-point function with shock wave scattering [50], translate to sine dilaton gravity. These characteristics are very closely tied to what would be interpreted as the switchback effect from the boundary complexity viewpoint, justifying the conjecture of existence of the switchback effect in DSSYK Krylov complexity. Recently, important boundary insights connecting to this line of argument have been reported for the triple-scaled DSSYK model in [44,51]. A finite β adaptation of these methods, together with relaxation of the triple-scaling limit, could facilitate the proposed study of the switchback effect in the general case that we elaborate on in this Letter.

Furthermore, the analysis of the quantum correction to the length leads us to two important questions to be investigated. First, it is important to gain a geometric understanding of the first quantum correction to the length

in sine dilaton gravity. This carries the potential to obtain a refined understanding of the complexity equals volume conjecture in the full quantum regime where quantum corrections to the volume have to be taken into account. Second, the previously stated speculation that the first quantum correction to the length has a dual interpretation as Krylov spread complexity of bulk quantum fields deserves further investigation.

Finally, the most important question is whether the paradigm presented in this Letter extends to higher dimensions. An interesting lesson seems to be that a crucial ingredient in the match is to account for the Euclidean segment of time evolution.

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Data availability—The data are not publicly available. The data are available from the authors upon reasonable request.

[1] L. Susskind, *Three Lectures on Complexity and Black Holes*, SpringerBriefs in Physics (Springer, Cham, 2018), p. 10.
[2] S. Chapman and G. Policastro, Quantum computational complexity from quantum information to black holes and back, Eur. Phys. J. C **82**, 128 (2022).
[3] L. Susskind, Computational complexity and black hole horizons, Fortschr. Phys. **64**, 24 (2016); **64**, 44(A) (2016).
[4] J. Haferkamp, P. Faist, N. B. T. Kothakonda, J. Eisert, and N. Y. Halpern, Linear growth of quantum circuit complexity, Nat. Phys. **18**, 528 (2022).
[5] A. Belin, R. C. Myers, S.-M. Ruan, G. Sárosi, and A. J. Speranza, Does complexity equal anything?, Phys. Rev. Lett. **128**, 081602 (2022).
[6] A. Belin, R. C. Myers, S.-M. Ruan, G. Sárosi, and A. J. Speranza, Complexity equals anything II, J. High Energy Phys. 01 (2023) 154.
[7] D. Stanford and L. Susskind, Complexity and shock wave geometries, Phys. Rev. D **90**, 126007 (2014).
[8] Note that other efforts, including [9–13], reported a quantitative match in the context of a broad complexity research, but never directly with any standard holographic complexity proposal.
[9] B. Czech, Einstein equations from varying complexity, Phys. Rev. Lett. **120**, 031601 (2018).
[10] B. Chen, B. Czech, and Z.-z. Wang, Query complexity and cutoff dependence of the CFT2 ground state, Phys. Rev. D **103**, 026015 (2021).
[11] N. Chagnet, S. Chapman, J. de Boer, and C. Zukowski, Complexity for conformal field theories in general dimensions, Phys. Rev. Lett. **128**, 051601 (2022).
[12] J. Erdmenger, A.-L. Weigel, M. Gerbershagen, and M. P. Heller, From complexity geometry to holographic spacetime, Phys. Rev. D **108**, 106020 (2023).
[13] P. Caputa, B. Chen, R. W. McDonald, J. Simón, and B. Strittmatter, Spread complexity rate as proper momentum, arXiv:2410.23334.
[14] E. Rabinovici, A. Sánchez-Garrido, R. Shir, and J. Sonner, A bulk manifestation of Krylov complexity, J. High Energy Phys. 08 (2023) 213.
[15] H. W. Lin, The bulk Hilbert space of double scaled SYK, J. High Energy Phys. 11 (2022) 060.
[16] V. Balasubramanian, P. Caputa, J. M. Magan, and Q. Wu, Quantum chaos and the complexity of spread of states, Phys. Rev. D **106**, 046007 (2022).
[17] V. Balasubramanian, M. Decross, A. Kar, and O. Parrikar, Quantum complexity of time evolution with chaotic Hamiltonians, J. High Energy Phys. 01 (2020) 134.
[18] A. Blommaert, T. G. Mertens, and J. Papalini, The dilaton gravity hologram of double-scaled SYK, J. High Energy Phys. 06 (2025) 050.
[19] A. Blommaert, T. G. Mertens, and S. Yao, Dynamical actions and q-representation theory for double-scaled SYK, J. High Energy Phys. 02 (2024) 067.
[20] A. Blommaert, A. Levine, T. G. Mertens, J. Papalini, and K. Parmentier, An entropic puzzle in periodic dilaton gravity and DSSYK, J. High Energy Phys. 07 (2025) 093.
[21] A. Blommaert, A. Levine, T. G. Mertens, J. Papalini, and K. Parmentier, Wormholes, branes and finite matrices in sine dilaton gravity, arXiv:2501.17091.
[22] L. V. Iliesiu, M. Mezei, and G. Sárosi, The volume of the black hole interior at late times, J. High Energy Phys. 07 (2022) 073.
[23] S. Sachdev and J. Ye, Gapless spin fluid ground state in a random, quantum Heisenberg magnet, Phys. Rev. Lett. **70**, 3339 (1993).
[24] A. Kitaev, A simple model of quantum holography, http://online.kitp.ucsb.edu/online/entangled15/kitaev/ and http://online.kitp.ucsb.edu/online/entangled15/kitaev2/ (2015).
[25] M. Berkooz, M. Isachenkov, V. Narovlansky, and G. Torrents, Towards a full solution of the large N double-scaled SYK model, J. High Energy Phys. 03 (2019) 079.
[26] M. Berkooz, P. Narayan, and J. Simon, Chord diagrams, exact correlators in spin glasses and black hole bulk reconstruction, J. High Energy Phys. 08 (2018) 192.
[27] M. Berkooz and O. Mamroud, A cordial introduction to double scaled SYK, Rep. Prog. Phys. **88**, 036001 (2025).
[28] T. G. Mertens and G. J. Turiaci, Solvable models of quantum black holes: A review on Jackiw–Teitelboim gravity, Living Rev. Relativity **26**, 4 (2023).

[29] J. M. Maldacena, Eternal black holes in anti-de Sitter, J. High Energy Phys. 04 (2003) 021.
[30] Indeed, gauging $P \to P + 2\pi$ is also responsible for selecting $|L = 0\rangle$ as the true gravitational vacuum state of the theory, since all negative lengths states are removed from the physical Hilbert space [20]. This reduces the Euclidean periodicity $\beta_{\rm BH} = 2\pi/\sin(\theta)$ of the black hole background (3) to $\beta = \beta_{\rm DSSYK} = (2\pi - 4\theta)/\sin(\theta)$, reproducing the true microscopic temperature of the DSSYK model. In contrast, the Euclidean time periodicity of the solution (3) corresponds to the so called "fake temperature" $\beta_{\rm BH} = \beta_{\rm fake} = [2\pi/\sin(\theta)]$ [31].
[31] H. W. Lin and D. Stanford, A symmetry algebra in double-scaled SYK, SciPost Phys. **15**, 234 (2023).
[32] See Supplemental Material at http://link.aps.org/supplemental/10.1103/spcr-jgm6 where, in Sec. SM1, more details on sine dilaton gravity and the duality with the DSSYK model are given. In Sec. SM2, the analytic expression for the two-point function of a massive probe in sine dilaton gravity is provided. In Sec. SM3, an operator perspective on complexity = volume in the setup of this Letter is laid out. In Sec. SM4, an SYK interpretation of the chord Krylov problem is discussed. In Sec. SM5, details of the numerical evaluation of DSSYK Krylov spread complexity are described.
[33] A. Blommaert, T. G. Mertens, and H. Verschelde, The Schwarzian theory—a Wilson line perspective, J. High Energy Phys. 12 (2018) 022.
[34] L. V. Iliesiu, S. S. Pufu, H. Verlinde, and Y. Wang, An exact quantization of Jackiw-Teitelboim gravity, J. High Energy Phys. 11 (2019) 091.
[35] Without loss of generality, we set the DSSYK coupling J that appears as a multiplicative factor in $E(\theta)$ to $J = 1$ for the rest of this Letter. Hence, all times are understood to be measured in units of J.
[36] While recent arguments suggest that the identification $|K_n\rangle = |n\rangle$ holds only for times shorter than those exponential in the entropy [37], we argue that any corrections to this correspondence lie beyond the strict $N \to \infty$ limit of the DSSYK model. This perspective aligns with the expectation that late-time deviations arise from higher-genus contributions to the gravitational path integral of sine dilaton gravity. Since such contributions are not considered in our analysis, the identification of the Krylov and chord bases remains robust within the scope of our Letter.
[37] V. Balasubramanian, J. M. Magan, P. Nandi, and Q. Wu, Spread complexity and the saturation of wormhole size, arXiv:2412.02038.
[38] We point out that the $q \to 1$ limit in our framework is different from the triple-scaling limit $(q \to 1, \theta \ll 1)$, since we are keeping θ fixed. We regard it as a semiclassical limit because, after rescaling the dilaton in the sine dilaton action (2) as $2|\log q|\Phi \to \Phi$ which is our convention in this Letter, $|\log q|$ stands on the same footing as $G_{\rm N}$.
[39] A. Goel, V. Narovlansky, and H. Verlinde, Semiclassical geometry in double-scaled SYK, J. High Energy Phys. 11 (2023) 093.
[40] D. Harlow and D. Jafferis, The factorization problem in Jackiw-Teitelboim gravity, J. High Energy Phys. 02 (2020) 177.
[41] The answer (14) is not the full quantum mechanical length in JT gravity, which can be obtained by acting as in (10) on the exact JT gravity two-point function [33,34] and would be given by the triple-scaling limit of (11). Equation (14) then only follows after the $G_{\rm N} \to 0$ limit of the exact JT answer [42].
[42] L. Griguolo, J. Papalini, and D. Seminara, On the perturbative expansion of exact bi-local correlators in JT gravity, J. High Energy Phys. 05 (2021) 140.
[43] A. Almheiri, A. Goel, and X.-Y. Hu, Quantum gravity of the Heisenberg algebra, J. High Energy Phys. 08 (2024) 098.
[44] J. Xu, On chord dynamics and complexity growth in double-scaled SYK, J. High Energy Phys. 06 (2025) 259.
[45] L. Bossi, L. Griguolo, J. Papalini, L. Russo, and D. Seminara, Sine-dilaton gravity vs double-scaled SYK: Exploring one-loop quantum corrections, J. High Energy Phys. 06 (2025) 152.
[46] T. Faulkner, A. Lewkowycz, and J. Maldacena, Quantum corrections to holographic entanglement entropy, J. High Energy Phys. 11 (2013) 074.
[47] P. Saad, S. H. Shenker, and D. Stanford, JT gravity as a matrix integral, arXiv:1903.11115.
[48] P. Nandy, Tridiagonal Hamiltonians modeling the density of states of the double-scaled SYK model, J. High Energy Phys. 01 (2025) 072.
[49] D. L. Jafferis, D. K. Kolchmeyer, B. Mukhametzhanov, and J. Sonner, Jackiw-Teitelboim gravity with matter, generalized eigenstate thermalization hypothesis, and random matrices, Phys. Rev. D **108**, 066015 (2023).
[50] H. T. Lam, T. G. Mertens, G. J. Turiaci, and H. Verlinde, Shockwave S-matrix from Schwarzian quantum mechanics, J. High Energy Phys. 11 (2018) 182.
[51] M. Ambrosini, E. Rabinovici, A. Sánchez-Garrido, R. Shir, and J. Sonner, Operator K-complexity in DSSYK: Krylov complexity equals bulk length, J. High Energy Phys. 08 (2025) 059.