

Dynamic Redundancy and Mortality in Stochastic Search

Samantha Linn^{1,*} and Aanjaneya Kumar^{2,3,†}

¹*Department of Mathematics, Imperial College London, London SW7 2AZ, United Kingdom*

²*Santa Fe Institute, 1399 Hyde Park Road, Santa Fe, New Mexico 87501, USA*

³*High Meadows Environmental Institute, Princeton University, Princeton, New Jersey 08544, USA*

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Stochastic search processes are a fundamental part of natural and artificial systems. In such settings, the number of searchers is rarely constant: new agents may be recruited while others can abandon the search. Despite the ubiquity of these dynamics, their combined influence on search efficiency remains unexplored. Here we present a general framework for stochastic search in which independent agents progressively join and leave the process, a mechanism we term “dynamic redundancy and mortality” (DRM). Under minimal assumptions on the underlying search dynamics, this framework yields exact first-passage time statistics. It further reveals surprising connections to stochastic resetting, including a regime in which the resetting mean first-passage time emerges as a universal lower bound for DRM, as well as regimes in which DRM search is faster. We illustrate our results through a detailed analysis of one-dimensional Brownian DRM search. Altogether, this Letter provides a rigorous foundation for studying first-passage processes with a fluctuating number of searchers, with direct relevance across physical, biological, and algorithmic systems.

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Introduction—Stochastic search processes pervade natural and artificial systems—from molecules locating binding sites and immune cells finding antigens to animals foraging and algorithms exploring vast landscapes. In all such settings, performance depends on how efficiently a system locates a target under uncertainty. This efficiency is often quantified by first-passage times (FPTs), defined as the time taken by a searcher to find a target for the first time [1], which has received attention in applications ranging from chemical reaction kinetics to stochastic optimization [2–11].

However, in all the above examples and many other real-world search processes, the population of searchers is not fixed—new agents can be recruited to join the search (“dynamic redundancy”) while existing ones drop out, die, or decay (“mortality”). The acceleration of search through redundancy, or the presence of multiple searchers, has received wide attention [12–18] and its dynamic counterpart, where searchers are stochastically injected into the system, has recently been introduced [19–23]. Moreover, search processes under mortality have a long history, where the effect of finite searcher lifetime has been shown to

substantially impact FPTs [24–31]. These two ubiquitous features—dynamic redundancy and mortality—fundamentally reshape search statistics and rarely act in isolation, but their combined influence on stochastic search remains unexplored. This key gap is the focus of this Letter.

In this Letter, we characterize the combined impact of dynamic redundancy and mortality (hereafter referred to as DRM) on FPT statistics. Under minimal assumptions on the underlying search process, we explicitly express the DRM survival probability in terms of the survival statistics of a single mortal searcher. In doing so, we reveal a subtle

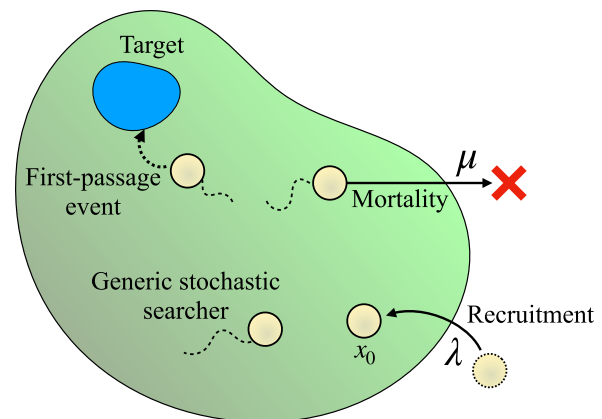


FIG. 1. A schematic for the stochastic search process where searchers are recruited to the search at rate λ (dynamic redundancy) and abandon the search process at rate μ (mortality). The central focus of this Letter is to characterize the time taken for the target to be found for the first time under DRM.

*Contact author: s.linn@imperial.ac.uk

†Contact author: aanjaneya@santafe.edu

relationship between DRM and stochastic resetting. We find, in particular, that these processes, which are macroscopically identical, have nuanced differences on the scale of individual trajectories that manifest in the FPT statistics. The stochastic resetting mean FPT (MFPT) moreover serves as a universal lower bound for the DRM process with equal recruitment and mortality rates. We also establish an optimal upper bound for general recruitment and mortality rates and use this bound to show when DRM is faster than resetting. We conclude with a detailed case study of a Brownian DRM search process in one dimension (1D).

The setting—Consider a stochastic search process in an arbitrary domain Ω that is either confined or unbounded. One searcher, initially positioned at $x_0 \in \Omega$, seeks a target fixed at $\Omega^* \subset \Omega \setminus x_0$. At rate λ , new searchers are recruited to the search process, initialized at x_0 , and independently search for the target. Each searcher independently drops out of the search at rate μ . Figure 1 illustrates a schematic of this DRM search process. We note that this model was first introduced in Ref. [19] by Campos and Méndez who analyzed the case $\Omega = \mathbb{R}$ with the target at the origin and $\mu = 0$.

The central quantity of interest in this Letter is the time taken for the target to be found for the first time by any of the searchers, which we denote by the random variable $T_{\lambda,\mu}$. Note that when $\lambda = \mu = 0$, then $T_{\lambda,\mu} \equiv T_{0,0} \equiv \tau$ is the classical FPT—the random time taken by a single searcher initially located at x_0 to find the target. Throughout this Letter, we assume τ is strictly positive and not always infinite. These constitute our only assumptions on the underlying search process, which we emphasize are extremely mild and broadly applicable.

Failure of conventional FPT approach—A natural starting point for many first-passage problems is to study the evolution of the searcher density. As an illustration, consider a Brownian DRM process in 1D. The mean number density of searchers $\rho(x, t)$ evolves according to the equation

$$\frac{\partial \rho(x, t)}{\partial t} = D \frac{\partial^2 \rho(x, t)}{\partial x^2} - \mu \rho(x, t) + \lambda \delta(x - x_0), \quad (1)$$

where the first term on the right-hand side corresponds to the diffusive motion of the particles, and the second and third terms correspond to the loss and gain of density through mortality at x and recruitment at x_0 , respectively. To obtain the first-passage statistics, one conventionally introduces an absorbing boundary at the target and computes the flux through the target. In single searcher problems, the flux is exactly the FPT density. This is true, for instance, when $\lambda = \mu = r$ in Eq. (1), which describes a single Brownian particle undergoing stochastic resetting to x_0 at rate r . In DRM, however, the number of searchers dynamically fluctuates and so the FPT density does not

equal, and cannot easily be obtained from, the flux through the target.

An alternative approach to FPTs—The inability to extract the FPT density via standard boundary value methods, exemplified through the aforementioned example of 1D Brownian motion, necessitates an alternative approach, which we develop here. We return to the more general DRM search setting. To obtain FPT statistics, consider the survival probability $S_{\lambda,\mu}(t) := \mathbb{P}(T_{\lambda,\mu} > t)$ defined as the probability that the target is not found by any searcher until time $t > 0$. In Supplemental Material [32], we elaborate on methods introduced in Refs. [19,21] to show that $S_{\lambda,\mu}(t)$ satisfies

$$S_{\lambda,\mu}(t) = S_{0,\mu}(t) \exp\left(-\lambda \int_0^t [1 - S_{0,\mu}(t')] dt'\right), \quad (2)$$

where $S_{0,\mu}(t)$ denotes the probability that a single mortal searcher that dies at rate μ has not found the target by time t . We can express $S_{0,\mu}(t)$ as

$$S_{0,\mu}(t) := \mathbb{P}(T_{0,\mu} > t) = 1 - \int_0^t e^{-\mu t'} \mathbb{P}(\tau = t') dt', \quad (3)$$

where $\mathbb{P}(\tau = t)$ denotes the FPT density of a single searcher without mortality. While we focus on DRM as described by Eqs. (2) and (3), one could easily generalize the mortality distribution via substitution of the exponential function or generalize the recruitment position by integrating over said position density within the integrand of Eq. (3).

Equation (2) provides an exact expression for the DRM survival probability of any search process whose underlying FPT satisfies the minimal assumptions on τ . While Eq. (2) determines the full FPT statistics, we focus our analysis on the MFPT $\mathbb{E}[T_{\lambda,\mu}]$ which can be obtained as

$$\mathbb{E}[T_{\lambda,\mu}] = \int_0^\infty S_{\lambda,\mu}(t) dt. \quad (4)$$

In several practically relevant scenarios, the MFPT alone may not fully characterize the search process since FPT distributions can be broad or strongly skewed [33–35]. However, it is shown in Supplemental Material [32] that for large values of λ and μ the coefficient of variation of the FPT is close to 1, and for moderate to large values of λ and μ the mean serves as a natural timescale of the DRM FPT distribution.

Universal bounds on the DRM MFPT—We prove in Ref. [32] that the MFPT satisfies the universal bounds

$$\frac{1 - p_\mu}{\lambda p_\mu} \leq \mathbb{E}[T_{\lambda,\mu}] \leq \frac{1 - p_\mu}{\lambda p_\mu} - \frac{\partial}{\partial \mu} \log p_\mu \quad (5)$$

for all $\lambda, \mu > 0$, where we define $p_\mu \in (0, 1)$ by

$$p_\mu := \int_0^\infty e^{-\mu t} \mathbb{P}(\tau = t) dt. \quad (6)$$

The quantity p_μ can be interpreted as the probability that an individual mortal searcher finds the target before dying. The bounds in Eq. (5) admit a simple pathwise interpretation. Suppose the search process is not terminated when the target is first found and index searchers in order of recruitment. The lower bound in Eq. (5) is the mean recruitment time of the first successful searcher by recruitment index, while the upper bound is obtained by additionally waiting for this searcher's hitting time, i.e., its MFPT conditioned on success [32]. In the true DRM process, since another searcher could find the target between this first-born successful searcher's recruitment and hitting times, the right-hand side is indeed an upper bound (and not an exact equality). In Supplemental Material [32], we further demonstrate how the same bounds can be obtained from Eq. (2).

An important consequence of the upper bound is that the MFPT is finite for all $\lambda, \mu > 0$. Even when $\lambda < \mu$, and hence there is less than one searcher on average at any given time, the MFPT remains finite due to the continuous recruitment of (new) searchers whose inter-recruitment times are almost surely finite. We further note that the upper bound expressed in Eq. (5) is optimal—it is attained for deterministic first-passage processes. If $\mathbb{P}(\tau = t) = \delta(t - t^*)$, then $\mathbb{E}[T_{\lambda,\mu}] = [(1 - p_\mu)/\lambda p_\mu] + t^*$ and thus the upper bound in Eq. (5) holds with equality.

Stochastic resetting as a universal DRM lower bound when $\lambda = \mu$ —Stochastic resetting is the process by which prescribed dynamics are randomly repositioned in the state space, often instantaneously and to the initial condition (hence “resetting”) [36–44]. Consider a single stochastic searcher that undergoes such resetting at rate $r > 0$. The corresponding FPT τ_r is known to satisfy [45]

$$\mathbb{E}[\tau_r] = \frac{1 - p_r}{r p_r}. \quad (7)$$

Now consider when $\lambda = \mu = r$, which we call a “balanced” DRM process. In this case, the lower bound to the DRM MFPT in Eq. (5) is exactly the resetting MFPT in Eq. (7). This bound is rather surprising; when $\lambda = \mu$ there is an average of one searcher at any given time, so one might expect the two processes to simply be equivalent. Moreover, we have already seen through Eq. (1) the macroscopic equivalence between the DRM and resetting processes. Despite these similarities, resetting always outperforms balanced DRM. The discrepancy between their performances can, however, become arbitrarily small in the large r limit. In particular, we show in Supplemental Material [32] that the second term in the upper bound of Eq. (5) vanishes as $\mu = r \rightarrow \infty$ so long as a single searcher without mortality can reach the target arbitrarily quickly

with positive probability (as is the case in Brownian motion). In this case, it follows that

$$\lim_{r \rightarrow \infty} r p_r \mathbb{E}[T_{r,r}] = \lim_{r \rightarrow \infty} r p_r \mathbb{E}[\tau_r] = 1. \quad (8)$$

This set of observations is one of the main results of this Letter.

Case study of Brownian motion in 1D—We now illustrate our results through the example of Brownian motion in 1D. Consider a Brownian DRM search on the real line ($\Omega = \mathbb{R}$) with a searcher initially located at $x_0 > 0$ and a target placed at the origin ($\Omega^* = 0$). Additional searchers are recruited to x_0 at rate λ and each independently dies at rate μ . We construct this DRM survival probability using Eq. (2) and the known density of τ [1],

$$\mathbb{P}(\tau = t) = \frac{x_0}{\sqrt{4D\pi t^3}} e^{-\frac{x_0^2}{4Dt}}, \quad (9)$$

where $D > 0$ is the diffusion coefficient. Substituting Eq. (9) into Eq. (6) yields $p_\mu = e^{-x_0 \sqrt{\mu/D}}$, and the DRM survival probability $S_{\lambda,\mu}(t)$ is determined via further substitution into Eq. (2). The full expression for $S_{\lambda,\mu}(t)$ is rather involved even for this simple case and is therefore contained in Supplemental Material [32]. However, when this DRM process is balanced, one can check that the large r behavior of the survival probability exhibits the same exponential decay as that of stochastic resetting [32,45],

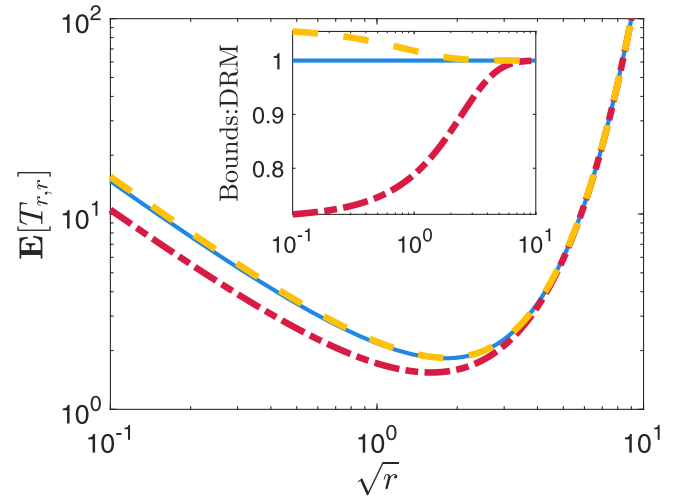


FIG. 2. Balanced ($\lambda = \mu = r$) DRM MFPT $\mathbb{E}[T_{r,r}]$ for Brownian motion on the half line with $x_0 = 1$ and $D = 1$. The solid blue curve indicates the solution computed from Eq. (4) via quadrature; the dash-dotted red curve indicates the exact solution in resetting theory and the dashed yellow curve indicates the algebraically computed upper bound as stated in Eq. (12). The inset shows ratios of each bound to the MFPT curve.

$$\lim_{r \rightarrow \infty} e^{r p_r t} S_{r,r}(t) = \lim_{r \rightarrow \infty} e^{r p_r t} \mathbb{P}(\tau_r > t) = 1. \quad (10)$$

One also finds through substitution the explicit bounds on the MFPT in this setting,

$$\frac{e^{x_0 \sqrt{\mu/D}} - 1}{\lambda} \leq \mathbb{E}[T_{\lambda,\mu}] \leq \frac{e^{x_0 \sqrt{\mu/D}} - 1}{\lambda} + \sqrt{\frac{x_0^2}{4D\mu}}. \quad (11)$$

Setting $\lambda = \mu = r$ yields

$$\frac{e^{x_0 \sqrt{r/D}} - 1}{r} \leq \mathbb{E}[T_{r,r}] \leq \frac{e^{x_0 \sqrt{r/D}} - 1}{r} + \sqrt{\frac{x_0^2}{4Dr}}. \quad (12)$$

We illustrate the results from Eqs. (12) and (8) in Fig. 2. Clearly, the lower bound diverges both when $r \rightarrow 0$ and $r \rightarrow \infty$. Moreover, since $\mathbb{E}[T_{r,r}]$ is finite for all finite $r > 0$, we infer the existence of an “optimal” rate that minimizes the balanced DRM MFPT.

For general, uncorrelated $\lambda, \mu > 0$, reducing the MFPT merely involves maximizing λ while minimizing μ . Setting $\lambda = \mu = r$ introduces a trade-off between increasing redundancy and decreasing mortality and we see in Eq. (12) that stochastic resetting always outperforms DRM. These observations raise a natural question: how much must redundancy dominate a DRM system such that it outperforms stochastic resetting?

When is DRM faster than resetting?—Suppose $\mu = r$ and $\lambda = \alpha r$, where $\alpha > 0$ and $r > 0$ is the “turnover rate,” which sets a timescale for the DRM process. If $\alpha < 1$, mortality dominates but the DRM MFPT remains finite. For $\alpha > 1$, redundancy dominates and we find that mean

DRM search times can be faster than those of stochastic resetting. In particular, we show in Supplemental Material [32] that for any $\alpha > 1$ there exists a $r^* > 0$ such that for all $r > r^*$ the DRM MFPT outpaces the corresponding resetting MFPT with resetting rate r . Furthermore, when the second term in the upper bound of Eq. (5) vanishes,

$$\lim_{r \rightarrow \infty} \frac{\mathbb{E}[T_{\alpha r, r}]}{\mathbb{E}[\tau_r]} = \lim_{r \rightarrow \infty} r p_r \mathbb{E}[T_{\alpha r, r}] = 1/\alpha. \quad (13)$$

Equation (13) asserts that for any $\alpha > 1$ the DRM MFPT outpaces stochastic resetting by a factor α in the high-turnover limit.

Additionally, for the specific case of 1D Brownian motion, we show in Supplemental Material [32] that in the low-turnover limit,

$$\lim_{r \rightarrow 0} \frac{\mathbb{E}[T_{\alpha r, r}]}{\mathbb{E}[\tau_r]} = \lim_{r \rightarrow 0} \sqrt{\frac{Dr}{x_0^2}} \mathbb{E}[T_{\alpha r, r}] = \frac{\sqrt{\alpha + 1}}{\alpha}. \quad (14)$$

Consequently, for $\alpha > \phi := (1 + \sqrt{5})/2 \approx 1.618$, 1D Brownian DRM search outperforms stochastic resetting. We note that this critical value ϕ is exactly the golden ratio. Figure 3(a) demonstrates this set of results summarized by Eqs. (13) and (14). Interestingly, we see that only moderately high turnover is required for minimally redundant-dominant (e.g., $\alpha \approx 1.2$) DRM systems to outperform stochastic resetting, while for $\alpha > \phi$, DRM outperforms resetting for all r . Figure 3(b) further illustrates the dependence of $\mathbb{E}[T_{\alpha r, r}]$ on α and r . Evident from the heat map is that for any α there exists an optimal rate r to minimize the

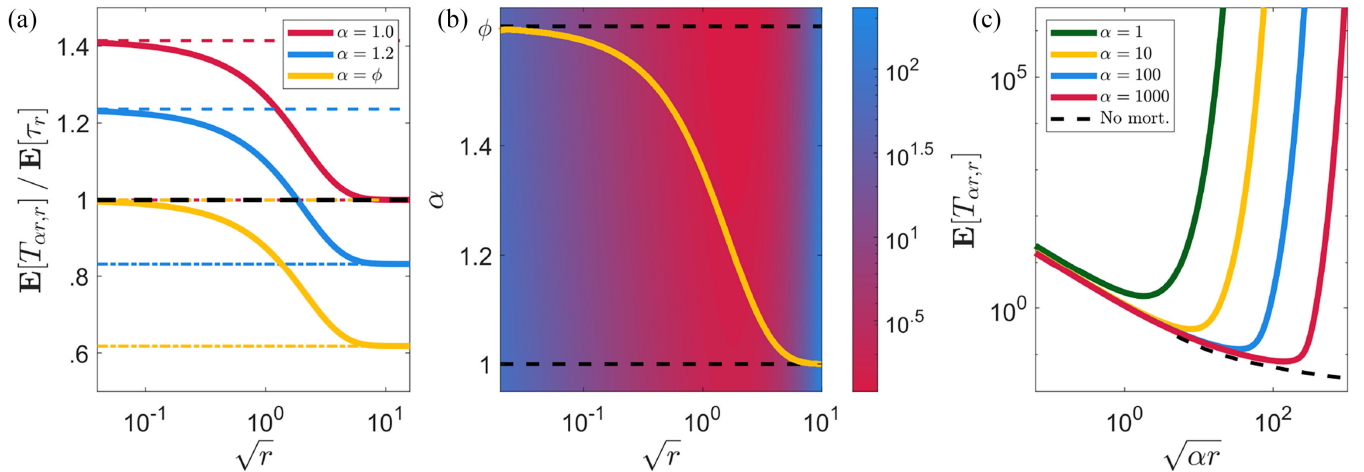


FIG. 3. DRM for 1D Brownian motion with $\alpha > 0$ where $\mu = r$ and $\lambda = \alpha r$. (a) Ratio of DRM to resetting MFPTs denoted by solid curves computed numerically. The dash-dotted lines are the high-turnover limits as in Eq. (13). The dotted lines are the low-turnover limits as in Eq. (14). (b) Phase diagram and heat map of the DRM MFPT. The solid yellow curve denotes the separatrix for the DRM and resetting MFPT; above (below) this curve the DRM MFPT is less (greater) than the stochastic resetting MFPT. The letter ϕ denotes the golden ratio. In Supplemental Material [32], we prove convergence of the small and large r limits. (c) The no-mortality result is an asymptotic result for the frequent recruitment limit recently derived in Ref. [21] and is extrapolated here for finite but large recruitment rates (dashed black line). Throughout, $x_0 = 1$ and $D = 1$.

MFPT. Moreover, the yellow curve acts as a separatrix in the phase space: for all pair values (r, α) that lie above it, DRM outpaces stochastic resetting, whereas, for all values below, the resetting MFPT is a lower bound to the DRM MFPT.

Redundant dominance differs from no mortality—Setting $\lambda/\mu = \alpha$ also allows us to establish the nontrivial role of mortality on the MFPT. Consider the large α limit (i.e., $\lambda \gg \mu$). One may initially expect the limiting results to resemble the case of frequent recruitment without mortality ($\mu = 0$), which was recently studied under the nomenclature of “fast immigration” [21]. In fact, this behavior is not realized. Rather, as illustrated in Fig. 3(c), the influence of mortality always eventually manifests, causing the MFPT to diverge. However, for modest r and large α , the MFPT trend mimics that of fast immigration.

Discussion—In this Letter, we developed a probabilistic framework to study FPTs of stochastic search with dynamic redundancy and mortality. By expressing the full FPT survival probability in closed form, we obtained optimal model-independent upper and lower bounds on the MFPT under mild assumptions on the underlying search dynamics. A central result is that when recruitment and mortality occur at equal rates, stochastic resetting is a global lower bound for DRM. However, DRM can always outpace stochastic resetting even when recruitment only slightly dominates mortality for sufficiently high searcher turnover. We moreover found the redundant-dominant limit to exhibit remarkably different behavior than the corresponding no-mortality system. Altogether, this Letter provides the first exact treatment of FPTs of stochastic search processes characterized by the continuous recruitment and mortality of searchers, with direct relevance across physical, biological, and algorithmic systems.

Our results open several directions for future work. An exciting next step would be to show that the FPT of *any* balanced DRM search process satisfying our minimal assumptions does not just have mean $1/(rp_r)$, but is, in fact, a rate- rp_r exponential random variable in the high-turnover limit. More broadly, the framework developed here is readily extensible to other practically relevant first-passage problems, including stochastic gating [46–50], spatially distributed recruitment [13,51], non-Markovian mortality processes [52], and to higher-order statistics such as the k th fastest FPT [22,53–57]. Finally, while we focused on independent recruitment and mortality, it would be of interest to model them as correlated, where perfect correlation corresponds to stochastic resetting and noncorrelation is akin to the work herein.

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Data availability—There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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