

Direct Measurement of Diffusion Coefficients: Evidence for Diffusive Stochastic Heating in Collisionless Plasmas

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Open questions in collisionless plasma dissipation can be addressed using space-based observations in different astrophysical environments, with implications for both astrophysical and laboratory plasma systems. We study a low- β , highly imbalanced, sub-Alfvénic stream observed by Parker Solar Probe (PSP) to identify and distinguish between signatures of stochastic heating (SH) and resonant heating (RH) by parallel ion cyclotron waves ($\parallel$ -ICWs). Prior work studying this stream [Trevor A. Bowen *et al.*, Stochastic heating in the sub-Alfvénic solar wind, *Phys. Rev. Lett.* **135**, 255201 (2025)] showed that the SH rate, accounting for intermittency, matched the amplitude of the local energy transfer (LET) rate, while the RH rate did not. This comparison relied on a number of assumptions regarding the nature of the diffusive process and the calculation of the LET rate. We introduce a novel technique of inverting the proton guiding center equation to empirically measure velocity-space diffusion coefficients using three-dimensional proton velocity distribution functions, from the ion electrostatic analyzer (the Solar Probe Analyzer for Ions) on PSP. Measured diffusion coefficients are used to determine phase-space heating rates, leading to a calculation of a fully kinetic heating rate independent of assumptions made in prior work. We show that scale-dependent analytic expressions for SH via noncoherent fluctuations match the empirical measurements from PSP data, provided that we account for intermittency in the heating calculation. In contrast, the derived heating rates for SH that accounts for the effects of the helicity barrier and heating rates for RH via $\parallel$ -ICWs do not peak in the same region of velocity space as the empirical measurements, nor do they reach the required magnitude. Our approach provides novel methodology to uniquely identify and constrain heating processes in collisionless plasmas and shows evidence of a Fokker-Planck-like diffusive process in the near-Sun solar wind.

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Introduction—As the solar wind expands outward from the Sun’s corona and through the inner heliosphere, it cools more slowly than would be expected via adiabatic expansion [1]. Double adiabatic theory, which describes the evolution of the distribution function in the presence of a background magnetic field, predicts strong parallel temperature anisotropies ($T_{\parallel}/T_{\perp} \gg 1$, where $T_{\perp}$ and $T_{\parallel}$ are defined relative to the background field) as the wind evolves. However, observations of proton velocity distribution functions (VDFs) show large perpendicular temperature anisotropies ($T_{\perp}/T_{\parallel} > 1$), especially near the Sun

(e.g., Refs. [2,3]). There must be additional heating as the wind evolves to explain the decades of observations of solar wind temperatures as a function of heliocentric distance (e.g., Ref. [4]), and the heating mechanisms at work must preferentially accelerate particles perpendicular to the background field to explain the observations of large perpendicular anisotropies [5].

Various heating mechanisms have been proposed to explain these observational signatures: nonresonant (stochastic) processes heating the distribution via uncorrelated kicks due to fluctuations in the fields [stochastic heating (SH)] [6–9], resonant heating (RH) [10–12], magnetic reconnection [13], and others [14]. Recent work has suggested that the heating mechanisms may be sensitive to the level of balance between counterpropagating Alfvénic fluctuations [15,16], leading to a helicity barrier (HB) in the wave number spectrum that prevents strongly imbalanced turbulence from cascading to small scales [17,18].

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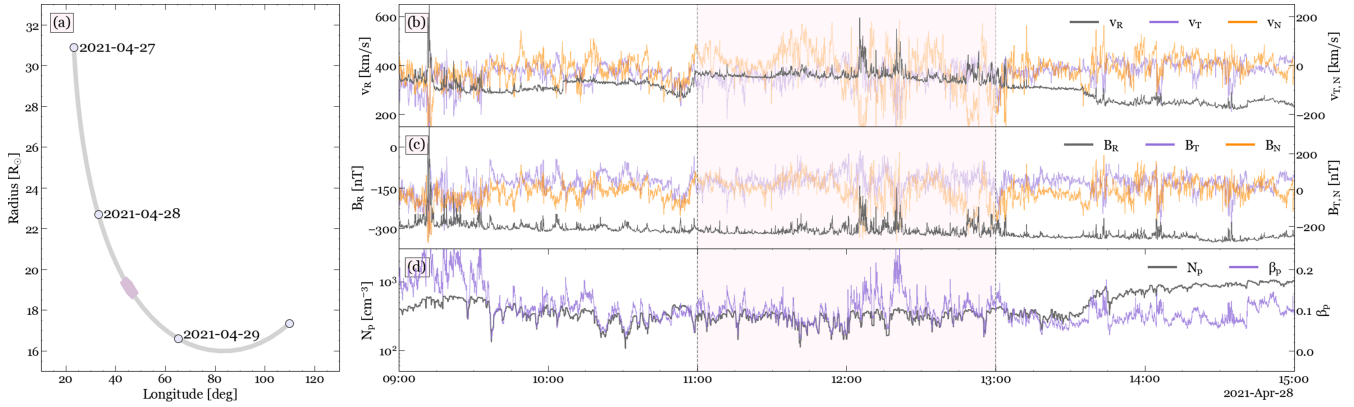


FIG. 1. An overview of the time period of interest in this Letter. (a) PSP trajectory (longitude vs radius). (b) Velocity measurements from PSP-SPANi. (c) Fluxgate magnetic field measurements from PSP-FIELDS. (d) Proton density measurements from PSP-SPANi (partial moment). We take the maximum value across a rolling window of ten observations (~ 35 s) to account for density dropouts due to FOV effects. Proton plasma β is shown in purple, $\beta_p = nk_B T / [B^2 / (2\mu_0)]$, calculated from the PSP-SPANi density shown in (d). The highlighted portion is our time period of interest.

Observational work has directly connected levels of parallel ($\parallel$) cyclotron resonant damping with turbulent heating in the super-Alfvénic solar wind [11,12]. Additionally, a numerical study of forced imbalanced turbulence found that RH dominates as the HB prevents the cascade of fluctuations to the smallest perpendicular scales resulting in oblique-cyclotron RH that drives $\parallel$ propagating waves via an Alfvén ion cyclotron wave instability [18]. In the solar wind, such $\parallel$ propagating waves can propagate outward from the Sun and may result in net heating via cyclotron damping, seen both observationally [19] and in simulations. However, in the sub-Alfvénic solar wind, Bowen *et al.* [6] (B25 moving forward) has shown that cyclotron resonant heating does not provide the required heating to match observed cascade rates, but rather suggest SH as a viable mechanism to provide the requisite energy to heat the sub-Alfvénic wind, provided intermittent fluctuations are accounted for. This previous work assumed perpendicular diffusion via intermittent structures to calculate the SH rate, without any evidence that this process was actually occurring. Simulations show evidence for SH that can perpendicularly heat the distribution as it evolves through the inner heliosphere [7,8,20,21]. Observational work has estimated coefficients associated with the SH rate at different heliocentric distances [22] and calculated dissipation rates throughout the heliosphere [23,24]. These works were limited in their heating rate calculations to computation from scalar moments and spectra rather than a calculation of the kinetic phase-space heating rate from the full distribution function.

Each of these heating mechanisms is associated with a diffusive process and a distinctive set of diffusion coefficients. Here, we use a fully kinetic framework to make an empirical measurement of diffusion coefficients and heating rates as a function of velocity space from measured VDFs in a sub-Alfvénic solar wind stream observed by Parker Solar Probe (PSP) [25]. We directly compare our

empirical heating rates with proposed heating rate expressions for SH [7–9,15,26] and RH via $\parallel$ ion cyclotron waves (ICWs) [27]. Our results show that scale-dependent analytic expressions for SH rates, including intermittent effects, match empirical measurements in the sub-Alfvénic solar wind. In contrast, a recent SH expression that assumes heating to be maximized at a particular scale to account for the effects of a HB [15] peaks in the incorrect region in velocity space to match our empirical measurements. Similarly, expressions for RH via $\parallel$ -ICWs do not match observations. These comparisons allow us to test the validity of assumptions made in prior works (e.g., B25), as our methodology enables direct computation of diffusion coefficients and heating rates from empirical observation of the VDF evolution, without assumption of a specific mechanism. This Letter provides a novel method to calculate phase-space diffusion coefficients and fully kinetic heating rates for identification of dominant heating mechanisms at work in collisionless plasmas and constraining coefficients associated with theoretical expressions with wide applicability in both observations and simulations of space and laboratory plasma environments.

Data—PSP provides measurements of the electromagnetic fields and charged particles in the inner heliosphere, allowing us to measure and constrain the heating and acceleration mechanisms powering the solar wind. We focus on a portion of the previously studied first sub-Alfvénic stream from PSP encounter 8 (e.g., Refs. [6,28–30]), where PSP enters the magnetically dominated corona and the spacecraft is flying radially toward the Sun with little change in Carrington longitude. We study the 2-h interval from 11:00 to 13:00 on April 28, 2021 (highlighted in Fig. 1) when we estimate the source connectivity (following [31]) to be stable, even with varied model input parameters, and connected to a single negative polarity coronal hole. This allows for study of the evolution sub-Alfvénic stream from a stable source. This is a low- β ($\beta \sim 0.1$), highly imbalanced

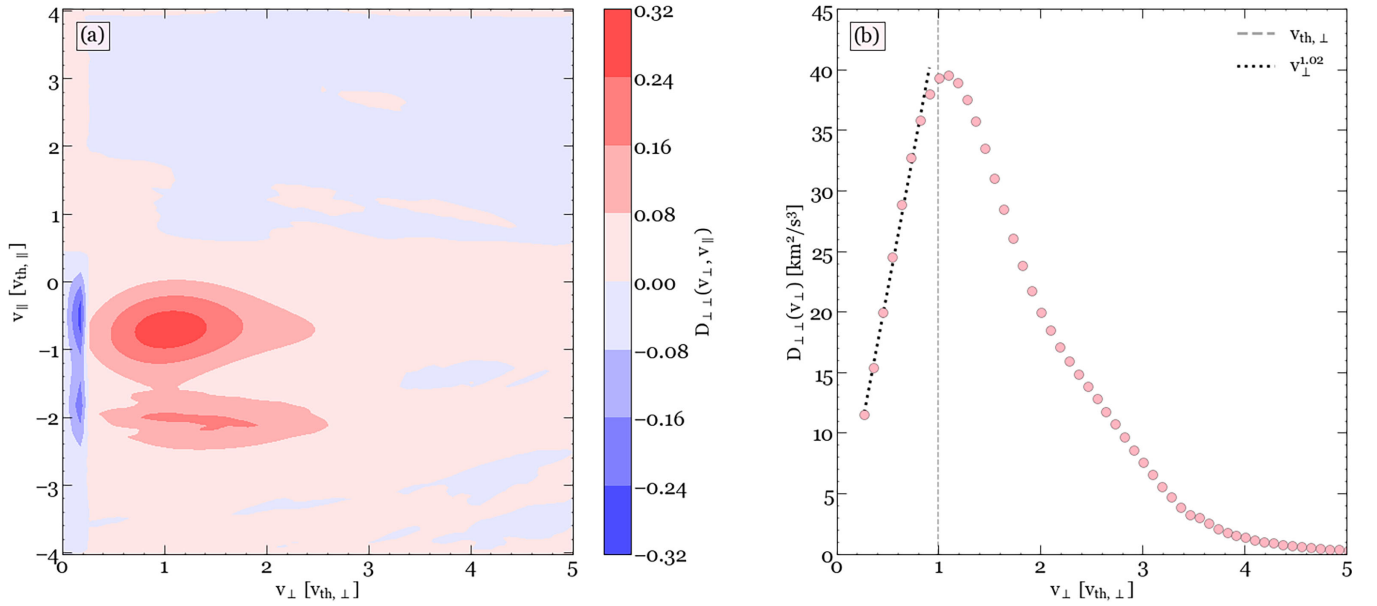


FIG. 2. Example of the mean perpendicular diffusion coefficients ($D_{\perp\perp}$) calculated over the time period of interest. (a) Mean 2D $D_{\perp\perp}$ over time period of interest. (b) Mean 1D $D_{\perp\perp}(v_{\perp}) = \int D_{\perp\perp}(v_{\perp}, v_{\parallel}) dv_{\parallel}$ over period. The integration of each calculated 2D coefficient is preformed prior to taking the mean. The dotted black line shows the fit to the portion below the average $v_{th,\perp}$ (vertical dashed gray line) for the interval.

($\sigma_C \sim 0.9$) stream, indicating applicability of SH and RH mechanisms.

We use observations of proton VDFs from the Ion Solar Probe Analyzers (SPANi) [32,33] on PSP which provides 3D measurements of the velocity distribution at a cadence of ~ 3.5 s. This period has well-resolved proton distribution functions where the bulk of the distribution falls within the instrument's field of view (FOV; see End Matter). Figure 1(b) shows velocity measurements (from SPANi partial moments) in the radial-tangential-normal (RTN) coordinate system and Fig. 1(d) shows the partial proton density moment from SPANi and proton plasma beta (β_p). High-resolution magnetic field measurements using the merged fluxgate and search coil data product (SCaM) [34] from the FIELDS suite [35] are used to make measurements of the *in situ* fluctuations at a high frequency. Figure 1(c) shows the fluxgate magnetic field measurements in RTN coordinates.

Empirical calculation of diffusion coefficients—We use the proton guiding center equation (1), which describes the steady-state evolution of a gyrotropic particle distribution [36,37], to calculate diffusion coefficients in the sub-Alfvénic wind stream,

$$\begin{aligned} (U + v_{\parallel}) \frac{\partial f}{\partial r} + \frac{v_{\perp}}{2} \frac{d \ln A}{dr} \left(v_{\perp} \frac{\partial f}{\partial v_{\parallel}} - (U + v_{\parallel}) \frac{\partial f}{\partial v_{\perp}} \right) \\ = \frac{\partial}{\partial v_{\parallel}} \left(D_{\parallel\parallel} \frac{\partial f}{\partial v_{\parallel}} + D_{\parallel\perp} \frac{\partial f}{\partial v_{\perp}} \right) \\ + \frac{1}{v_{\perp}} \frac{\partial}{\partial v_{\perp}} \left[v_{\perp} \left(D_{\parallel\perp} \frac{\partial f}{\partial v_{\parallel}} + D_{\perp\perp} \frac{\partial f}{\partial v_{\perp}} \right) \right]. \end{aligned} \quad (1)$$

The equation is written in the bulk plasma reference frame, moving outward at the radial flow speed $U(r)$, and phase-space variables ($v_{\perp}, v_{\parallel}$) are defined relative to this frame. The left-hand side contains the advection term and double adiabatic (Chew-Goldberger-Low) terms, respectively, while the right-hand side accounts for diffusion by small-scale fluctuations. D_{ii} are the diffusion coefficients, r is heliocentric radial position, and $A(r)$ is the flux-tube area as a function of radial distance. For this essentially local analysis, we ignore large-scale radial forces such as gravity and the ambipolar electric field. We define the flux-tube area as [26] $A(r) \sim [R^6 / (R^4 + 6)]$, where $R = r/R_{\odot}$.

All partial derivatives are taken using centered finite differences to conserve total particle density in the distribution functions [38–41]. To take these centered finite differences of the distribution functions (e.g., $\partial f / \partial v_i$), the VDFs must be fit and interpolated onto a uniformly spaced grid, in velocity space. We use radial basis functions (RBFs) to fit the measured SPANi VDFs [42,43] and interpolate them onto a uniformly spaced 5 km/s grid. The RBF method better captures the non-Maxwellian features of the distribution functions produced by collisionless processes, in comparison to typical bi-Maxwellian fitting methods [12], and has been used extensively in the solar wind to study heating (e.g., Ref. [42]).

With the calculation of gradients both in phase space ($\partial f / \partial v_i$) and between spacecraft observations ($\partial f / \partial r$) and the *in situ* measurements of the bulk flow U , the left-hand side of Eq. (1) and all terms on the rhs other than D_{ii} can be determined. We require a set of nine data points in ($r, v_{\perp}, v_{\parallel}$) space to create a fully constrained system of equations. We invert the system using singular value

decomposition [44] to solve for $D_{\perp\perp}$, $D_{\parallel\parallel}$, and $D_{\perp\parallel}$ as a function of $(v_{\perp}, v_{\parallel})$, assuming cylindrical symmetry ($D_{\perp\parallel} = D_{\parallel\perp}$). This method produces 2D measurements of $D_{ii}(v_{\perp}, v_{\parallel})$ over time that can be used to compute heating rates and compared to theoretical diffusion coefficients. As nine VDF data points are required to constrain this calculation, we use a rolling window across the interval to get 116 measurements of the diffusion coefficients over the 2-h interval. Supplemental Material [45] includes a full derivation of the inversion process, including the offset between the VDF grid and the D_{ii} grid, leading to the necessity for nine observation points to constrain the equation. Here, we focus on $D_{\perp\perp}(v_{\perp}, v_{\parallel})$ and derived quantities (e.g., $dQ_{\perp}$). Future work will examine $D_{\parallel\parallel}$ and $D_{\perp\parallel}$.

In Fig. 2(a), we show the mean (over the full interval) 2D perpendicular diffusion coefficient as a function of phase space. $D_{\perp\perp}(v_{\perp}, v_{\parallel})$ peaks near $v_{th,\perp}$, $v_{th,\parallel}$, and $2v_{th,\parallel}$. We numerically integrate $D_{\perp\perp}(v_{\perp}, v_{\parallel})$ to calculate a 1D velocity diffusion coefficient, $D_{\perp\perp}(v_{\perp}) = \int_{-\infty}^{\infty} D_{\perp\perp}(v_{\perp}, v_{\parallel}) dv_{\parallel}$. We calculate $D_{\perp\perp}(v_{\perp})$ associated with each $D_{\perp\perp}(v_{\perp}, v_{\parallel})$ measurement as shown in Fig. 2(b). A fit of $D_{\perp\perp}(v_{\perp})$ where $v_{\perp} < \langle v_{th,\perp} \rangle$ shows $D_{\perp\perp} \propto v_{\perp}^{1.02}$. The diffusion coefficient peaks near $v_{th,\perp}$ as to be expected for SH [9].

With these empirical measurements of $D_{\perp\perp}(v_{\perp})$ we directly compare to proposed analytic expressions for SH [7,8], RH via $\parallel$ -ICWs [42], and a formulation that accounts for the effects of the HB [15]. We calculate a perpendicular heating rate from $D_{\perp\perp}(v_{\perp})$ through integration by parts [8,21],

$$Q_{\perp} = \int dQ_{\perp} = -2\pi m_i \int_0^{\infty} v_{\perp}^2 D_{\perp\perp}(v_{\perp}) \frac{\partial f_{\perp}}{\partial v_{\perp}} dv_{\perp}, \quad (2)$$

where $f_{\perp} = \int f(v_{\perp}, v_{\parallel}) dv_{\parallel}$ is the reduced distribution function from the RBF fits to the observed VDFs.

Comparison with stochastic and cyclotron resonant heating—Analytic expressions for SH have been proposed by a variety of authors (e.g., Refs. [7–9,15]), referring to a process whereby large-amplitude fluctuations can disrupt a particle’s gyromotion, breaking the conservation of the magnetic moment, leading to perpendicular heating of the particle distribution. SH typically depends on a “stochasticity parameter” ($\varepsilon = \delta v/v$) that describes the efficiency of the process. There are various definitions of ε depending on where theory predicts heating to be most efficient, as well as methods that use scale-dependent versions of ε [6,8,21].

Regardless of the definition of δv and the corresponding stochasticity parameter ε , these SH formulations typically provide an analytic expression for $D_{\perp\perp}^{SH}$ in low- β Alfvénic turbulence [7],

$$D_{\perp\perp}^{SH} = c_1 \frac{\Omega_i}{v_i} \delta v^3 e^{-c_2/\varepsilon}, \quad (3)$$

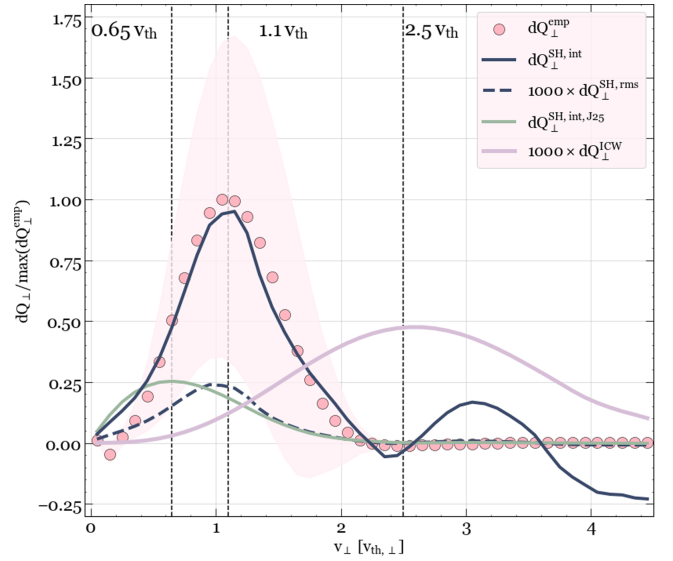


FIG. 3. A comparison of the heating rates calculated from the empirical measurement (pink), intermittent SH (blue), rms SH (dashed blue), J25 method with intermittency (green), and $1000 \times$ RH via ICWs (purple). The versions of the SH expression use canonical coefficients from prior work: $c_1 = 0.75$ and $c_2 = 0.34$ [7]. The dashed line at $0.65v_{th,\perp}$, $1.1v_{th,\perp}$, $2.5v_{th,\perp}$ shows the location of the peak in $dQ_{\perp}^{SH,int,J25}$, $dQ_{\perp}^{emp}$, and $dQ_{\perp}^{ICW}$, respectively. The pink shaded region shows the error from the empirical calculation. All measurements are normalized to the maximum value in the $dQ_{\perp}^{emp}$ calculation.

where differences between formulations lie in the definitions of δv and ε (see End Matter). c_1 and c_2 are (order unity) coefficients that account for assumptions in the physics of these models or can be fit parameters [23]. We use $c_1 = 0.75$ and $c_2 = 0.34$, determined via test-particle simulations [7]. $D_{\perp\perp}^{SH}$ is used in Eq. (2) to determine $Q_{\perp}^{SH}$. $D_{\perp\perp}^{SH}$ includes an exponential term ($e^{[-c_2]/\varepsilon}$) that accounts for the suppression of the heating where ε is small. This suppression factor was added semiempirically in many formulations and recent work has shown that this term holds through a derivation from first principles [9].

We compare different proposed expressions for δv in the SH diffusion coefficient: a scale-dependent formulation [8] and a formulation using fluctuations at a particular scale to account for HB effects, $\delta v(v_{\perp}^m)$ (J25 moving forward) [15]. The versions of δv are calculated from the PSP SCAm data (see End Matter).

As $D_{\perp\perp}^{SH} \propto \delta v^3$, large-amplitude, intermittent fluctuations can have a dramatic effect on the SH diffusion coefficient. We calculate $dQ_{\perp}^{SH}$ with “rms” ($D_{\perp\perp}^{SH,rms}$) and “intermittent” ($D_{\perp\perp}^{SH,int}$) velocity diffusion coefficients [6]. $D_{\perp\perp}^{SH,rms}$ is defined using the rms fluctuation amplitude $\delta v^{rms}(v_{\perp}) = \sqrt{\langle \delta v(v_{\perp}, t)^2 \rangle}$ over the time interval, thus averaging out the effects of large-amplitude intermittent fluctuations. $D_{\perp\perp}^{SH,int}(v_{\perp}) = \langle |D_{\perp\perp}(v_{\perp}, t)| \rangle$ to account for

intermittency. We produce a scale-dependent $D_{\perp\perp}^{\text{SH}}(v_{\perp})$ from which we can calculate $dQ_{\perp}^{\text{SH}}(v_{\perp})$ [Eq. (2)]. We note that the J25 formulation relies on the fluctuations at a particle scale $[\delta v_{\lambda}(v_{\perp}^m)]$ associated with the steepening of the turbulent power spectrum (see End Matter). Thus, $D_{\perp\perp}^{\text{SH,int,J25}}$ is not a function of scale, but rather a single value at all $v_{\perp}$. The velocity-space dependence of the intermittent J25 heating rate, $dQ_{\perp}^{\text{SH,int,J25}}$, comes from Eq. (2).

Figure 3 compares $dQ_{\perp}^{\text{emp}}$ with the three SH formulations and the perpendicular heating rate for RH via $\parallel$ -ICWs ($dQ_{\perp}^{\text{ICW}}$; see End Matter). We find that $dQ_{\perp}^{\text{emp}}$ peaks near $1.1v_{\text{th},\perp}$, consistent with results from simulations in low- β plasmas [20,21] and theoretical curves [9], then rapidly drops off after. While both the $dQ_{\perp}^{\text{SH,rms}}$ and $dQ_{\perp}^{\text{SH,int}}$ curves peak near $1.1v_{\text{th},\perp}$, only the case that includes intermittency reaches the required heating magnitude. We see a secondary peak in the intermittent SH rate at $\sim 3v_{\text{th},\perp}$ that we believe could be attributed to a breakdown in the assumptions of the SH model at these perpendicular speeds, where fluctuations cause a smooth rather than chaotic drift to the ion's guiding center motion. The J25 heating rate peaks at $\sim 0.65v_{\text{th},\perp}$ and is too small to reach the required magnitude. $dQ_{\perp}^{\text{ICW}}$ is significantly smaller and peaks at $2.5v_{\text{th},\perp}$.

Discussion—The interplay between turbulent dissipation and collisionless heating is important in a range of plasma systems (e.g., Refs. [46–50]). This Letter highlights a novel method to empirically measure diffusion coefficients that can serve as a fundamental constraint on collisionless heating mechanisms. We have applied this method in the sub-Alfvénic solar wind to constrain SH, which various theoretical [7–9], simulation [20,21], and observational [6,23,24] works have shown is viable in low- β plasmas. Prior work calculating SH rates assumed a diffusive process without any evidence for one existing (e.g., B25). Our empirical measurement, with minimal assumptions, of a physically meaningful diffusion coefficient suggests the presence of a Fokker-Planck-type process, underlying collisionless heating mechanisms that are responsible for plasma heating in the outer corona.

Our results show agreement of the scale-dependent analytic expression for the SH heating rate with empirical measurements, provided that intermittency, and thus the effect of infrequent, large-amplitude fluctuations, are accounted for, consistent with prior theoretical work [51]. Our direct estimate of diffusive heating is not subject to a number of critical assumptions made by B25 (see End Matter). The general agreement between these two studies largely validates many of these assumptions, particularly in constraints on c_2 , which is critical in understanding the efficiency and suppression of stochastic heating. We find our empirical measurement does not match with the recent formulation proposed by J25 to account for a variable level of balance in the turbulence, and thus effects of the helicity barrier [15], despite this stream being highly imbalanced.

While the c_1 parameter could be increased by a factor of ~ 4 in $dQ_{\perp}^{\text{SH,int,J25}}$ to reach the required heating magnitude, this heating rate peaks in the incorrect region in velocity space and thus is not a viable description of heating in this stream. Future work could include fitting the empirically measured coefficients and heating rates to determine values of c_1 and c_2 and comparing these measurements during periods of balanced and imbalanced turbulence to understand the impacts of steepening in the turbulent spectrum.

During the time period of interest, PSP was traversing the outer corona in the sub-Alfvénic wind. This time period is ideal to study as the VDFs are well resolved and we observe a radially evolving plasma parcel, both necessary for the inversion method. These results thus provide direct insight into coronal heating processes and showcase the importance of SH in this regime. Future statistical work comparing sub-Alfvénic intervals with their super-Alfvénic counterparts could help determine if and where the transition from SH to RH, which has been reported at larger distances [11], takes place. Martinović *et al.* [23,24] quantified SH rates in the solar wind, not including intermittent effects, and found that the SH rate increases closer to the Sun. Understanding the origin of this transition and how it is modulated also remains an open question that could be addressed via the novel method described here.

Future work using this methodology to calculate solar wind heating rates across discontinuities, in super-Alfvénic streams, and in reconnection exhausts would allow us to understand and constrain the phase-space signatures of heating mechanisms in various regions of the heliosphere, which can be used to drive global models [52]. Similar calculations in different environments (e.g., planetary shocks, the high- β plasma sheet) with varying dimensionless parameters (e.g., β , m_A , σ_C) have implications for our understanding of heating processes relevant to astrophysical plasmas in a range of environments that lack fully kinetic measurements or cannot be directly measured by spacecraft. These results could be used to test theoretical predictions for dominant heating mechanisms based on fundamental plasma parameters (e.g., Ref. [53]).

This methodology could be applied to multispacecraft constellations (e.g., the Magnetospheric Multiscale Mission) to calculate heating rates over spatial structures, such as Earth's bow shock, and compared with proposed heating mechanisms to understand the phase-space dependence of the observed heating. Future multispacecraft missions could utilize these methods and would require fewer time steps to calculate the spatial gradients necessary to constrain the calculation. Understanding the phase-space signatures of dissipation mechanisms and the evolution of the distribution function can inform theories for entropy growth in collisionless systems [54,55]. The field particle correlation (FPC) technique [56], uses single-point measurements of the electric field and particle distributions to directly measure local energy transfer rates between fields and particles and characterize the mechanisms enabling the

transfer. Comparisons between our methods and the FPC technique when high fidelity electric field measurements are available could provide further constraints on velocity-space signatures of collisionless damping mechanisms.

Through this novel methodology for calculating heating rates as a function of velocity space, we show the ability to observationally constrain which heating mechanisms operate in the solar wind and which analytical formulations properly describe the velocity-space signatures, independent of assumptions made in many prior works (e.g., Refs. [6,23]). We show evidence for a Fokker-Planck-type diffusive process in the sub-Alfvénic stream of study and that scale-dependent SH rates, which include intermittency, match our empirical measurements both in magnitude and in the region of phase space where these heating rates peak. This is a compelling constraint on turbulent heating and collisionless dissipation in the near-Sun environment. These methods provide a new pathway for identification of heating mechanisms in collisionless plasmas and constraining coefficients associated with analytic models of heating mechanisms.

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Data availability—The data that support the findings of this article are openly available [32–35].

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End Matter

SPANi field of view—In Fig. 4, we show a quantification of the SPANi FOV that combines both look directions, θ and ϕ [57]. The high metric throughout the interval indicates that the proton distribution function is well within the instrument's FOV and thus suitable for this analysis.

Calculation of the stochastic heating rate—There are various formulations for analytic expressions for stochastic heating, specifically different definitions of δv and the associated stochasticity parameter ($\varepsilon = \delta v/v$). We outline the methods we compare below: (1) In KC16 [8], $\varepsilon = \delta v_\rho/v_\perp$, where δv_ρ is the rms fluctuation amplitude at each scale $\rho = v_\perp/\Omega$, making ε scale dependent. (2) In J25 [15], define $\varepsilon = \hat{\xi}_i = \delta v_\lambda/v_{th,i}$, where δv_λ are the velocity fluctuations at the perpendicular scale corresponding to the smallest nonlinear timescale. This can account for the helicity barrier, which causes the

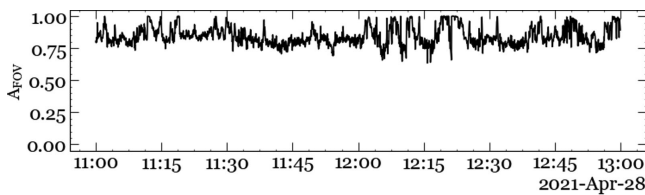


FIG. 4. PSP-SPANi FOV metric combining energy flux vs θ and ϕ over the period of interest as observed by PSP-SPANi [57].

power spectrum to steepen at a wave number $k_\perp^*$ that is significantly smaller than $\rho_{th,p}^{-1}$.

We use PSP SCAm data, which provides a two-axis measurement of the magnetic field at high fidelity to calculate δv over the same 116 time intervals as for the empirical calculation. We point the reader to B25 for additional discussion of the assumptions. We note that the inversion method described in the main text calculates the velocity-space heating rate independent of these assumptions. (1) $\delta \mathbf{B}(f, t)$: We calculate the scale-dependent $\delta \mathbf{B}$ using five-point increments with 88 time lags ranging from $0.85 \text{ ms} < \tau < 7.23 \text{ s}$ ($0.07 < f < 586 \text{ Hz}$) where $f = 1/2\tau$ [58]. (2) $\delta v(k_\perp, t)$: We assume Alfvénic turbulence, with the kinetic Alfvénic wave (KAW) dispersion relation, to convert $\delta B(f, t)$ to $\delta v(f, t)$ whereby $\delta v/v_{\text{phase}}^{\text{KAW}} = \delta B/B_0$ using $v_{\text{phase}}^{\text{KAW}} = v_A \sqrt{1 + k_\perp^2 \rho_{th}^2}$. This conversion ignores the $k_\parallel$ term (e.g., $k_\parallel = 0$) such that $k_\perp = 2\pi f/(v_{SW} \sin \theta_{VB})$, where v_{SW} is the solar wind speed, and θ_{VB} is the angle between the velocity vector ($\mathbf{v}$) and magnetic field ($\mathbf{B}$). We note this correction is valid in the frequency regime where $\omega < \Omega_p$ ($\Omega_p = qB/m_p$). (3) $\delta v(v_\perp, t)$: We use the Taylor hypothesis to determine corresponding physical scales. This allows us to relate each time lag (τ) to wave numbers $k_\perp$ and $k_\parallel$ assuming the wave vector is in the plane containing v_{SW} and $\mathbf{B}$,

$$2\pi f = \mathbf{k} \cdot \mathbf{V} = k_\parallel v_A + k_\parallel v_{SW} \cos \theta_{VB} + k_\perp v_{SW} \sin \theta_{VB}. \quad (\text{B1})$$

We use the condition that $k_{\perp}\rho = 1$ giving $k_{\perp} = \Omega_i/v_{\perp}$ to determine $\delta v(v_{\perp}, t)$ following the modified Taylor hypothesis (MTH) as relevant to flows where $v_{SW} < \sim v_A$ [59,60]. We assume critical balance where $k_{\parallel}v_A = \delta vk_{\perp} = \delta v\Omega_p/v_{\perp}$, giving

$$v_{\perp}^{\text{MTH}} = \frac{\Omega_p}{2\pi f} \left[\delta v \left(1 + \frac{v_{SW}}{v_A} \cos \theta_{VB} \right) + v_{SW} \sin \theta_{VB} \right]. \quad (\text{B2})$$

(4) $\delta v_{\lambda}(t)$: We identify the relevant perpendicular scale $k_{\perp}^m$, which is where the nonlinear growth rate $k_{\perp}\delta v(k_{\perp})$ is maximized. We determine $v_{\perp}^m$, assuming $k_{\perp}^m\rho = 1$ giving $v_{\perp}^m = \Omega_i/k_{\perp}^m$, leading to $\delta v_{\lambda}(t) = \delta v(v_{\perp}^m, t)$ for calculation of the J25 coefficient.

Upon calculation of δv , we calculate the rms, intermittent, and J25 diffusion coefficients [Eqs. (B3)–(B5)]. T is the length of the nonoverlapping window used to calculate δv from the measurements. The resulting heating rates calculated following Eq. (2) are shown in Fig. 3,

$$D_{\perp\perp}^{\text{SH,rms}}(v_{\perp}) = c_1 \frac{\Omega_i}{v_{\perp}} \langle \delta v(v_{\perp}, t) \rangle_T^3 e^{-c_2 v_{\perp} / \langle \delta v(v_{\perp}, t) \rangle_T}, \quad (\text{B3})$$

$$D_{\perp\perp}^{\text{SH,int}}(v_{\perp}) = \left\langle c_1 \frac{\Omega_i}{v_{\perp}} \delta v(v_{\perp}, t)^3 e^{-c_2 v_{\perp} / \delta v(v_{\perp}, t)} \right\rangle_T, \quad (\text{B4})$$

$$D_{\perp\perp}^{\text{SH,int,J25}} = \left\langle c_1 \frac{\Omega_i}{v_{\perp}} \delta v_{\lambda}(t)^3 e^{-c_2 v_{\perp} / \delta v_{\lambda}(t)} \right\rangle_T. \quad (\text{B5})$$

Calculation of the ion cyclotron heating rate—We calculate a perpendicular ion cyclotron heating rate for comparison with $dQ_{\perp}^{\text{emp}}$. We take the quasilinear expression for the resonant cyclotron effect on the proton distribution due to a spectrum of parallel propagating ICWs [27,42,61], form the product of this with $\frac{1}{2}mv^2$, and integrate over phase space to obtain a quasilinear heating rate,

$$Q^{\text{ICW}} = \int \frac{1}{2} m_p v^2 \frac{df}{dt} d^3v. \quad (\text{C1})$$

After integration by parts, we find the following:

$$Q^{\text{ICW}} = -\pi^2 \Omega_p^2 m_p \int dv_{\perp} \int dv_{\parallel} \int dk v_{\perp}^2 \left(\frac{\omega}{k} \right)^2 \times \delta(\omega - kv_{\parallel} - \Omega_p) I(k) \left[\left(1 - \frac{kv_{\parallel}}{\omega} \right) \frac{\partial f}{\partial v_{\perp}} + \frac{kv_{\perp}}{\omega} \frac{\partial f}{\partial v_{\parallel}} \right]. \quad (\text{C2})$$

Here the definition of the δ function imposes the resonance condition

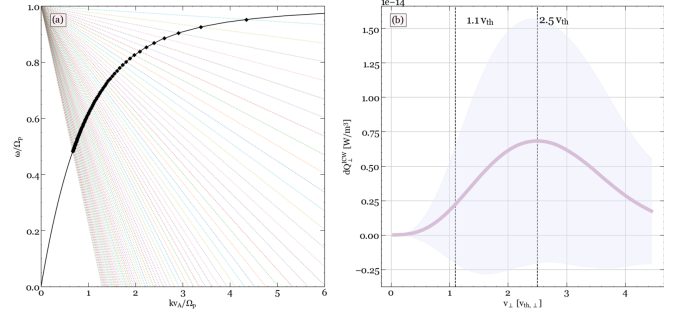


FIG. 5. (a) Representation of the cyclotron resonance condition used to determine k_{res} and ω_{res} for calculation of the phase and group speeds. The solid black line shows the dispersion relation for parallel propagating ICWs in a cold electron-proton plasma. The colored lines show the resonance condition $\omega(k) = kv_{\parallel} + \Omega_i$ for values of $v_{\parallel}$ in our grid (we note resonance only occurs for $v_{\parallel} < 0$, which is what is shown here). The diamonds represent the points corresponding to $(k_{\text{res}}, \omega_{\text{res}})$ for each $v_{\parallel}$. (b) Average differential heating rate ($dQ_{\perp}^{\text{ICW}}$) calculated from Eq. (C7) and $\parallel$ -ICW spectra [11,42]. The dashed line at $2.5v_{\text{th},\perp}$ ($1.1v_{\text{th},\perp}$) shows the location of the peak in $dQ_{\perp}^{\text{ICW}}$ ($dQ_{\perp}^{\text{emp}}$). The purple shaded region shows the error from the empirical calculation.

$$\omega(k) = kv_{\parallel} + \Omega_p \quad (\text{C3})$$

as a function of $v_{\parallel}$ of the proton and the dispersion relation for $\parallel$ -ICWs. The dispersion relation in a cold electron-proton plasma is

$$\frac{\omega}{k} = v_A \sqrt{1 - \frac{\omega}{\Omega_p}}, \quad (\text{C4})$$

where we assume only outward propagation. The proton resonance with these dispersive waves is illustrated in Fig. 5(a), depicting the one-to-one relationship between the proton $v_{\parallel}$ and the resonant wave. We define a $(v_{\perp}, v_{\parallel})$ grid that corresponds to the uniformly spaced VDF grid (see Supplemental Material [45]). For each $v_{\parallel}$, we define a resonant wave number (k_{res}) and frequency (ω_{res}) based on Eq. (C3). The diamonds in Fig. 5(a) indicate the values of ω_{res} and k_{res} where the resonance condition and the dispersion relation are both satisfied on this grid. k_{res} and ω_{res} are then used to determine the phase and group speeds of the resonant wave [27],

$$V_{\text{res}} = \frac{\omega_{\text{res}}}{k_{\text{res}}}, \quad (\text{C5})$$

$$W_{\text{res}} = \frac{2V_{\text{res}}^2}{V_{\text{res}}^2 + v_A^2}. \quad (\text{C6})$$

Using the δ function to substitute in k_{res} , we define a differential heating rate where $Q^{\text{ICW}} = \int dv_{\perp} \int dv_{\parallel} dQ^{\text{ICW}}$,

$$dQ^{\text{ICW}} = -\pi^2 \Omega_p^2 m_p \times \left\{ V_{\text{res}} \frac{v_{\perp}^2 I(k_{\text{res}})}{|W_{\text{res}} - v_{\parallel}|} \left[(V_{\text{res}} - v_{\parallel}) \frac{\partial f}{\partial v_{\perp}} + v_{\perp} \frac{\partial f}{\partial v_{\parallel}} \right] \right\}. \quad (\text{C7})$$

Here, $I(k_{\text{res}})$ is the ICW intensity associated with the resonance of proton at a specific $v_{\parallel}$. From PSP SCaM observations, we produce a spectrum $I(k)$ [11,42] at a 56-s cadence, producing 2064 spectra over the 2-h interval. Each $I(k)$ is interpolated to produce $I(k_{\text{res}})$, where k_{res} are the resonant wave numbers corresponding to each $v_{\parallel}$.

We formulate a differential perpendicular ICW heating rate in a manner similar to the empirical and SH rates shown in Fig. 3: $dQ_{\perp}^{\text{ICW}} = \int dv_{\parallel} dQ^{\text{ICW}} \Delta v_{\perp}$ shown in Fig. 5(b). The $v_{\perp}$ and $v_{\parallel}$ grids used here are equivalent to the grid used for the interpolation of the VDFs and calculation of $dQ_{\perp}^{\text{SH}}$. We find that the heating rate determined from the ICW calculation peaks at $\sim 2.5v_{\text{th}}$, in comparison to the peak near $1.1v_{\text{th}}$ in $dQ_{\perp}^{\text{emp}}$ and $dQ_{\perp}^{\text{SH,int}}$, and is far too small to account for our observed heating (Fig. 3).