

Cracking Down on Fracture to Functionalize Damage

Leo de Waal¹, Matthaios Chouzouris¹, and Marcelo A. Dias^{1*}

Institute for Infrastructure and Environment, School of Engineering, University of Edinburgh, Edinburgh, United Kingdom



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In this Letter we propose a novel relationship between topology and damage propagation in Maxwell lattices that redefines fracture as a functional design feature rather than mere degradation. We demonstrate that topologically protected modes, inherently robust against perturbations, localize along lattice discontinuities and govern the mechanical response. By precisely engineering the microstructure, we direct these modes to control stress distributions and trigger predictable, controlled damage. Our findings, validated through comprehensive numerical simulations and experiments, advance our understanding of nontrivial mechanical responses in Maxwell lattices and establish a clear framework for designing materials with improved fracture energy. This Letter paves the way for further exploration of topology-driven phenomena in mechanical systems and promises a new direction in the design of robust materials.

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Designing mechanism of damage in metamaterial provides a pathway towards tuneable energy dissipation. While the underlying microstructure of lattice-based metamaterials has been related to the initial fracture toughness [1–8], there is limited understanding of its effect on the subsequent crack growth, during which there is a great latent potential to tune the energy of fracture [9–13].

Efforts to design the damage path have generally focused on the inclusion of heterogeneities [14–18]. However, recently, Maxwell lattices have shown potential in providing an alternative avenue through their robust topological and geometric quantities [19–25]. Specifically, the underlying microstructure can be adjusted to manipulate the form of non-trivial zero-energy deformations *floppy modes* (FM) and *states of self-stress* [(SSS), internal stress distributions that balance without external loads] that localize along bulk discontinuities. This enables control over the deformation and stresses that develop within the lattice, particularly at larger *slenderness ratio's* [(SR), unit-cell size to strut thickness] where the energy associated with bending deformations is sufficiently low to prevent delocalization of the stress [24]. Furthermore, Maxwell lattices straddle the rigidity transition, providing a pathway to design toughening mechanisms through a change in the failure characteristic [26]. Such toughening mechanisms are observed in other lattices [13] where zones of bridging elements remain behind the damage path.

Building on earlier analyses of mechanisms of damage in topological lattices [24], this Letter shows how those robust, topology-protected features can be functionalized to program damage localization and propagation. We show that fracture energy, a central parameter in classical fracture mechanics, can be systematically tuned through the design of the lattice's microstructure. Critically, these predictions are validated experimentally, establishing a physics-informed design framework for robust control of material failure. In doing so, we bridge the gap between abstract theoretical principles and practical engineering, providing a concrete pathway toward the design of damage-resilient structures.

The core idea for this functionalization is presented in Fig. 1 for the mode I type tension loading setup. The form of the FM and SSS influences the stress that develops within the lattice. Under fixed boundary conditions, SSS tending towards the long-wavelength limit generally dominate the stresses developing within the lattice. Physically, this occurs as the activation of the mode at the boundary spreads over many unit cells, naturally favoring long-wavelength deformations. This results in a bulk response that is overall stretching dominated [24]. However, the bending stresses about the crack tip [Fig. 1(b)] tend to map onto the FM that localize on the precrack edge [Figs. 1(b) vs 1(c)]. Therefore, when the topology asymmetrically localizes modes (i.e., along only the top or bottom edge), and individual fracture events are bending dominated, the crack path aligns with these modes [Figs. 1(d)–1(f)], enabling directional control of damage. As individual elements fracture, constraints are removed, necessarily resulting in the removal of SSS. However, bridging elements, which deform by bending [Figs. 1(d) and 1(e)], remain along the crack path. These bridging elements provide an opportunity to tune the toughening. Here, we

*Contact author: mdias@ed.ac.uk

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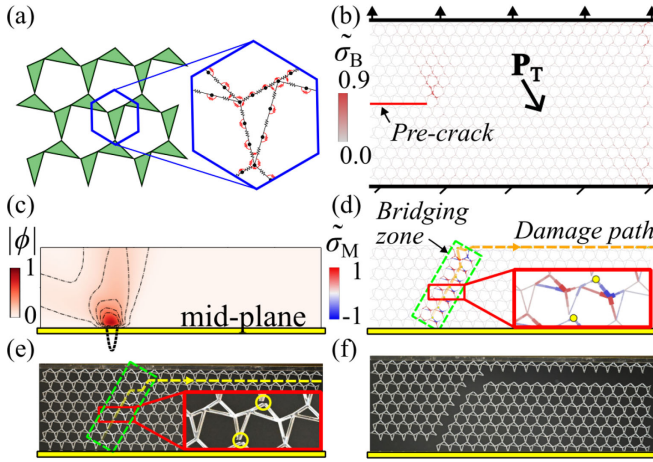


FIG. 1. (a) RDP1 geometry with discretization overview. (b) Normalized bending stress ($\tilde{\sigma}_B = |\sigma_B/\sigma_B^u|$) preceding the first failure event with boundary condition and polarization vector ($\mathbf{P}_T$) illustrated. (c) Mode visualization from a localized disturbance at the crack tip with magnitude $|\phi| = 1$. (d) Numerical and (e) experimental initial damage paths with bridging elements highlighted (green box). In (d), $\tilde{\sigma}_M = \sigma_A/\sigma_A^u + \text{sgn}(\sigma_A)|\sigma_B/\sigma_B^u|$ plots each elements maximum normalized stress; yellow circles represent locations where the node has split. (f) Complete fracture of sample. Refer to Supplemental Material, movie S1 for videos of this damage process [27].

combine numerical and experimental analysis to demonstrate how this phenomenon can be exploited to control damage and robustly increase fracture energy.

The role of the microstructure on topologically protected edge modes—Periodic Maxwell lattices are on the verge of mechanical stability, containing an equal number of FM and SSS. In practice, a finite lattice specimen will introduce or remove constraints at its boundary, resulting in an imbalance (loss or gain) between FM and SSS. For example, Maxwell lattices with pinned boundaries (i.e., fixed displacements at the boundary) and a *gapped* dispersion relation will possess extra SSS that exponentially localize around bulk discontinuities [19]. Importantly, these edge modes can exhibit a topologically protected directional preference, known as a topological polarization $\mathbf{P}_T$ [Fig. 1(b)], which arises from and depends on the unit cell’s geometry. $\mathbf{P}_T$ encodes a winding number across the Brillouin zone that links bulk geometry to boundary mode localization. Intuitively, it identifies the boundaries that host excess FM (see Supplemental Material for a detailed definition [27]).

We will focus on the response of distorted kagome lattices that can be parametrized in terms of the *regular kagome* (RK) lattice using four parameters $\{x_1, x_2, x_3, z_s\}$,

$$\mathbf{r}_\mu = \mathbf{r}_\mu^0 - \sqrt{3}x_\mu \mathbf{p}_\mu + x_{\mu-1} \mathbf{a}_{\mu+1} + \frac{z_s}{\sqrt{3}} \mathbf{p}_{\mu-1}, \quad (1)$$

where $\mathbf{r}_\mu^0$ are the basis vectors for the sites of the RK, $\mathbf{p}_\mu$ are the vectors normal to the bonds, $\mathbf{a}_\mu$ are the primitive lattice

vectors, and μ is evaluated mod 3 [28,29]. This parametrization introduces distortions along each of the three principal bond directions via the x_μ , while z_s controls the relative scale of adjacent triangles. A description of the lattice using the unit cell defined by Eq. (1) allows the topological polarization to be fully captured by a simple expression only depending on the sign of x_μ [19] when $\mathbf{a}_\mu$ is fixed:

$$\mathbf{P}_T = \frac{1}{2} \sum_{\mu=1}^3 \mathbf{a}_\mu \text{sgn}(x_\mu), \quad (2)$$

providing a unique definition of topologically distinct lattices through $\text{sgn}(x_1, x_2, x_3)$, distinguishing between the different equivalences classes.

While $\mathbf{P}_T$ remains identical within the same equivalence class, the form (i.e., direction and decay) of the nontrivial modes can be tuned. Applying the zero-energy condition, expressed in Fourier space as

$$E(q_1, q_2) = \det \mathbf{C}(q_1, q_2) = \det \mathbf{Q}(q_1, q_2) = 0, \quad (3)$$

allows the form of these modes in the bulk to be determined in terms of their form on an edge; where $\mathbf{C}$ is the compatibility matrix, $\mathbf{Q}$ is the equilibrium matrix and $q_{1,2}$ are wave numbers in the 1 and 2 directions. In other words, this energy constraint amounts to fixing one wave number component in terms of the other, effectively reducing the relevant degrees of freedom for the description of these evanescent modes—this gives rise to what is known as *bulk-boundary correspondence*. In essence, an evanescent mode that is described by a real wave number q_1 parallel to a boundary will be associated with a wave number $q_2 \equiv q_2(q_1)$ orthogonal to it [19]. By leveraging the bulk-boundary correspondence, mode amplitudes in the bulk are visualized by first reconstructing the boundary-specific response from a localized disturbance at the crack tip, and then computing the dependent orthogonal components [19,24]. Together, these determine the full amplitude distribution of FM in real space, $|\phi|$, as illustrated in Fig. 1(c). If the dispersion relation is gapped, the solution for the orthogonal component will be complex, $q_2 = k_2 + i\kappa_2$, where k_2 is the real part of the wave number and κ_2 , its imaginary part, which represents an inverse decay length in the 2 direction. Therefore, a sinusoidal disturbance decays into the bulk with respect to the edge’s inward facing normal at an angle given by

$$\theta = \arctan\left(\frac{k_2}{q_1}\right), \quad (4)$$

with clockwise rotation for a negative angle. As $\mathbf{C} = \mathbf{Q}^\dagger$, the q_2 component of the SSS will be the complex conjugate of the FM resulting in $\kappa^{\text{SSS}} = -\kappa^{\text{FM}}$ and $\theta^{\text{SSS}} = \theta^{\text{FM}} = \theta$. While the sign of κ is dependent on $\mathbf{P}_T$, $|\kappa|$, and θ are

dictated by the underlying cell geometry, allowing lattices even from the same equivalence class to have disparate forms for these modes.

Although the underlying theory assumes idealized lattices (zero-thickness hinges), it has been shown that at sufficiently high SR (> 10), the bending energy remains low enough for these topological modes to govern the mechanical response [24]. Crucially, the direction of the fracture path can be controlled by engineering the lattice $\mathbf{P}_T$ such that modes localize asymmetrically on one edge of the precrack. We therefore focus on lattices from the $(+, -, -)$ and $(-, +, -)$ equivalence classes, which we refer to as *left-up polarized* (LUP) and *right-down polarized* (RDP), respectively. In these cases, the edge modes localize exclusively on the bottom (LUP) or top (RDP) of the pre-crack [Fig. 1(c)]. This leads to asymmetric bending stress concentration around the crack tip [Fig. 1(b)], biasing fracture in these directions [Figs. 1(d)–1(f)]. In contrast, other equivalence classes exhibit symmetric mode localization, leading to bending stresses above and below the precrack and horizontal crack propagation under the applied boundary conditions. Additionally, to ensure that this phenomenon occurs, all lattices studied in this Letter

will have a constant $SR = |\mathbf{a}_1|/t = 15$. Where $t = 0.8$ mm is the bond thickness and $|\mathbf{a}_1| = 12$ mm is the magnitude of the primitive lattice vector in the 1 direction. The out-of-plane depth $b = 2.12$ mm.

Numerical model—Lattices are modeled with fixed boundaries using mass-spring networks incorporating axial and torsional springs [Figs. 1(a) and 1(b)]. Each bond is discretized into three elements to capture bending, and equilibrium is found via the FIRE 2.0 algorithm [30,31]. The fracture process is modeled by splitting a node at the end of the element(s) when the local failure condition,

$$\left| \frac{\sigma_A}{\sigma_A^u} \right| + \left| \frac{\sigma_B}{\sigma_B^u} \right| \geq 1, \quad (5)$$

is reached. Where σ_A and σ_B are the maximum axial and bending stresses in the element, and σ_A^u and σ_B^u are the material failure stress values in axial and bending, respectively. The material is modeled as linear-elastic until failure with elastic modulus, σ_A^u and σ_B^u set to 3120, 45, and 85 MPa. The elastic modulus is taken from the manufacturer's data sheet, while the other properties are found from material tests on 2.12 mm thick PMMA.

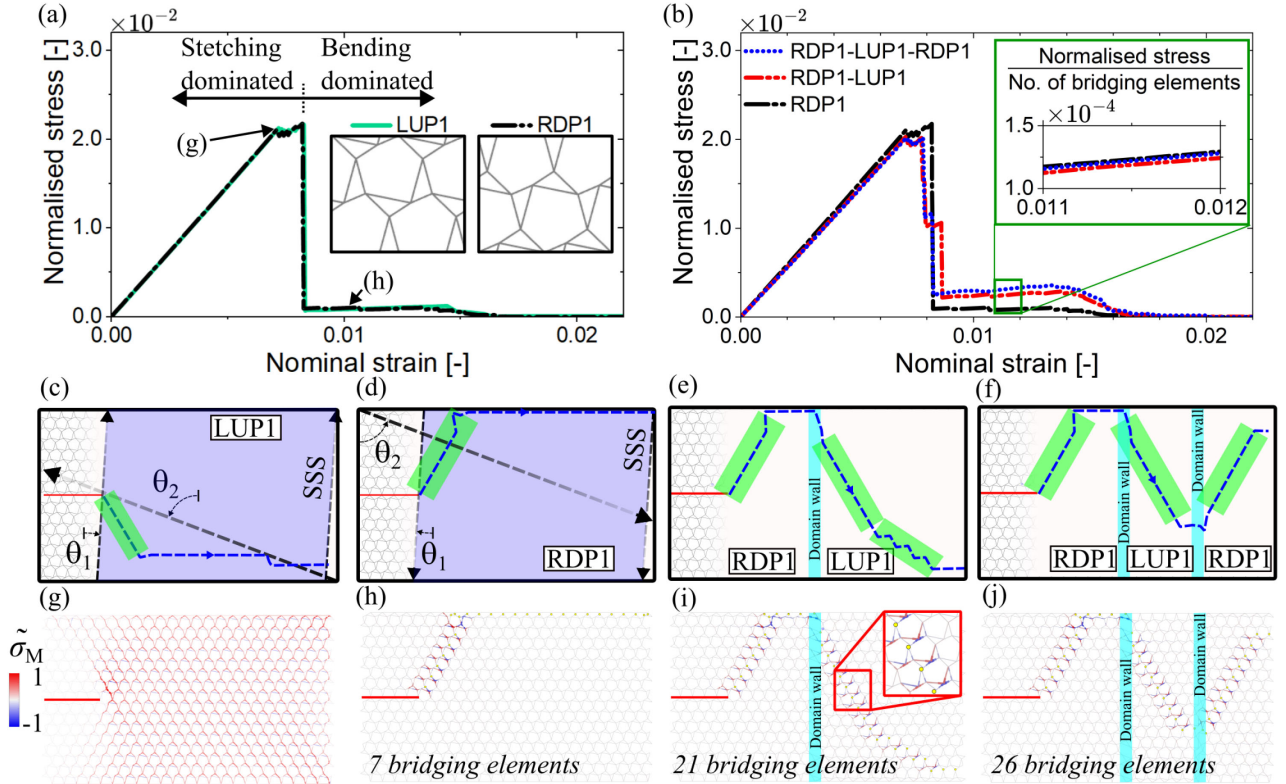


FIG. 2. Normalized stress vs nominal strain for (a) LUP1 and RDP1; and, (b) their combinations. Inset in (b) normalizes stress by the number of bridging elements [see (h)–(j)]. (c)–(f) Damage paths with bridging zones (green) for LUP1, RDP1, RDP1-LUP1, and RDP1-LUP1-RDP1. The long-wavelength propagation angles $\theta_{1,2}$ of the SSS are also indicated in (c),(d). Normalized stress in elements (g) preceding the first failure event for RDP1 and, following the initial stretching dominated damage for (h) RDP1; (i) RDP1-LUP1; and, (j) RDP1-LUP1-RDP1. In (g)–(j), $\tilde{\sigma}_M = \sigma_A/\sigma_A^u + \text{sgn}(\sigma_A)|\sigma_B/\sigma_B^u|$ plots each elements maximum normalized stress; yellow circles represent locations where the node has split. Refer to Supplemental Material, movies S1–S3 and Fig. 4 for experimental comparison [27].

Further details of the model are provided in the Supplemental Material [27]. When reporting results, the stress applied to the edge of the lattice is normalized with σ_A^u [i.e., $\sigma_n = F/(A_{\text{edge}}\sigma_A^u)$ where F is the force applied to the sites and A_{edge} is the projected area of the edge]. The nominal strain is $\epsilon_n = \delta/H$, where δ is the applied displacement and H is the initial height of the lattice.

Numerical results—RDP1 and LUP1 unit cells are rotationally symmetric with $(|x_1|, |x_2|, |x_3|) = (0.08, 0.08, 0.1)$, $z = 0$. Both lattices have a similar response [Fig. 2(a)] and damage process. The damage path initially propagates diagonally downwards or upwards before continuing along a horizontal path upon nearing the boundary, shown with a blue dashed line in Figs. 2(c), 2(d) respectively. Around the regions of diagonal damage propagation, bridging elements remain [Figs. 2(h), 1(d), and 1(e)], highlighted in green zones in Figs. 2(c) and 2(d). Along the horizontal damage propagation, the two surfaces are completely disconnected. Refer to Supplemental Material, movies S1 and S4 for the damage propagation observed in the simulations [27]. The increase in degrees of freedom, related to the initial damage propagation, results in the dominant behavior switching from stretching to bending. These two regions are indicated in Fig. 2(a). In the initial stretching dominated phase, the stress in the lattice [Fig. 2(g)] maps onto the SSS form [Fig. 2(g)]. As the initial damage propagates, these SSS are removed until only the bridging elements, dominated by bending, remain [Fig. 2(h)]. In both phases there is an opportunity to increase the fracture energy by adjusting the microstructure.

Increasing the number of bridging elements provides a methodology to increase the energy absorbed within the second, bending dominated phase. The number of bridging elements is proportional to the length of the diagonal damage path. To increase the diagonal path length, we can combine RDP1 and LUP1 unit cells, separated by domain walls, to direct the crack away from the boundary in a zigzag type propagation [Figs. 2(e), 2(f), 2(i), and 2(j)]. The initial stretching dominated phase remains similar when adding domain walls, provided they are not positioned to significantly alter the stress profile about the precrack, resulting in a similar peak stress and stiffening response [Fig. 2(b)]. However, the stiffness increases linearly with the number of bridging elements remaining within the second phase. The small insert within Fig. 2(b) normalizes the stress-strain response with respect to the number of bridging elements remaining within the second phase, highlighting this scaling.

Within the first phase, alignment of the SSS provides a methodology to increase the fracture energy. Figures 2(c) and 2(d) indicates the long-wavelength propagation directions ($\theta_{1,2}$) of the two modes. At the long-wavelength limit, only SSS for θ_1 can develop for the domain size simulated [Fig. 2(d)]. RDP2 (0.12, -0.04, 0.08) microstructure is adjusted so θ_1 remains similar but θ_2 is reduced

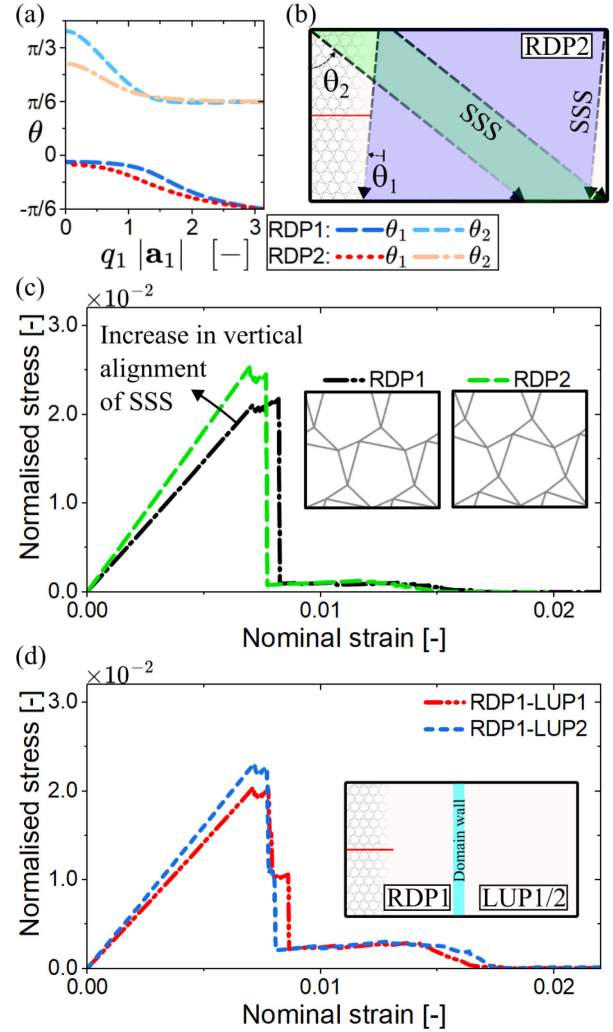


FIG. 3. (a) Direction of modes for RDP1-2. (b) Long-wavelength SSS along $\theta_{1,2}$ for RDP2. Effect of SSS alignment on normalized stress vs nominal strain (c) without domain walls for RDP1-2 and (d) with domain walls for RDP1-LUP1/2. See Supplemental Material, movie S4 for damage videos [27].

[Fig. 3(a)], allowing for the SSS associated with this mode to develop within the lattice [Fig. 3(b)]. This increases the initial stiffness [Fig. 3(c)]. However, the initial fracture occurs at a lower nominal strain, reducing the possible increase in fracture energy. By leveraging domain walls, it is possible to increase the stiffness while maintaining a similar nominal strain for fracture initiation. This is shown in Fig. 3(d), which compares the response for two lattices with a single domain walls separating RDP1 and LUP1/2 microstructure. Because of the more vertical alignment of LUP2 when compared to LUP1, the initial stiffness is larger for RDP1-LUP2, and, as RDP1 is located at the precrack in both cases, the nominal strain where fracture initiates is identical. The microstructure can be adjusted to reduce or increase the vertical alignment of θ_1 for a similar effect.

Experimental observation—Experiments were conducted on samples laser cut from 2.12 mm thick PMMA. These

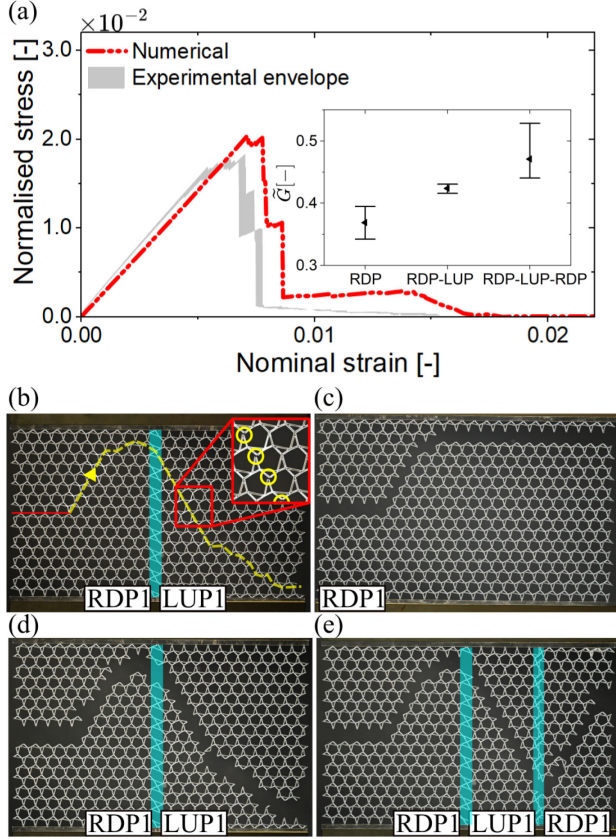


FIG. 4. Numerical vs experimental. (a) Normalized stress vs nominal strain with inset showing the normalized fracture energy $\tilde{G} = G/(\sigma_A^u |a_1| b t)$. G is the energy required to separate the lattice. (b) Initial damage path with bridging elements intact for RDP1-LUP1; and, complete fracture for (c) RDP1, (d) RDP1-LUP1, (e) RDP1-LUP1-RDP1. The numerical prediction of the damage path is provided in Figs. 2(d)–2(f) and 2(h)–2(j). Refer to Supplemental Material, movies S1–S3 for videos of the experiments [27].

samples were annealed for 2 h at 85 °C to reduce residual stresses. Three samples were tested for each lattice under type I loading with fixed boundaries. A comparison with the simulation for RDP1-LUP1 is provided in Fig. 4. See Supplemental Material Fig. S1 [27] for the comparison of other lattices and movie S1–S3 for recordings of the experiments. While the numerical model slightly overpredicts the peak stress, the general behavior is in close agreement with the two distinct phases being captured [Fig. 4(a)]. In the experiments, bridging elements are observed following the initial damage propagation [Fig. 4(b)]. These sequentially fracture, leading to the complete separation of the sample [Fig. 4(d)]. In all cases, the damage process predicted numerically was observed experimentally [Figs. 4(c) and 4(e) and Supplemental Material, movies S1–S3 [27]]. The good agreement with experiments suggests that this phenomenology is robust against manufacturing imperfections (this is also supported by a sensitivity study presented in [27]). However, while the initial magnitude of the stress within the

bending dominated phase is similar to that predicted numerically, it reduces below the prediction as loading continues [Fig. 4(a)]. Notwithstanding, the experiments indicate that increasing the damage path results in an increase in the fracture energy [Fig. 4(a)].

Conclusions—This Letter establishes a robust framework for functionalizing damage in Maxwell lattices by leveraging topologically protected edge modes. We show that by tuning the mode form through the microstructure design, one can control stress distributions and direct damage propagation at high SR. Two distinct failure regimes are identified, each offering a route to tailor fracture energy: in the initial, stretching-dominated stage, SSS are progressively removed as damage advances, with the vertical alignment of these SSS modulating the lattice’s effective stiffness and offering a key opportunity to enhance fracture energy; in the subsequent, bending-dominated stage, bridging elements persist along the damage path, where the absorbed energy scales with the fracture length. Combining lattice geometries to extend this path thus provides a reliable means to increase energy absorption during failure. Numerical predictions and experimental results converge, demonstrating the robustness and practicality of this approach. Beyond advancing the fundamental understanding of topological mechanics, our findings provide a general design strategy for engineering fracture behavior in metamaterials. This Letter highlights topology as a powerful tool for guiding damage in architected structures, opening new directions in the design of mechanically robust systems.

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Data availability—The data that support the findings of this article are openly available [32].

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