

Adaptive, Symmetry-Informed Bayesian Metrology for Precise Quantum Technology Measurements

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High precision measurements are essential to solve major scientific and technological challenges, from gravitational wave detection to healthcare diagnostics. Quantum sensing delivers greater precision, but an in-depth optimization of measurement procedures has been overlooked. Here, we present a systematic strategy for parameter estimation in the low-data limit that integrates experimental control parameters and natural symmetries. The method is guided by a Bayesian quantifier of precision gain, enabling adaptive optimization tailored to the experiment. We provide general expressions for optimal estimators for any parameter. The strategy's power is demonstrated in a quantum technology experiment, in which ultracold caesium atoms are confined in a micromachined hole in an optical fiber. We find a fivefold reduction in the fractional variance of the estimated parameter, compared to the standard measurement procedure. Equivalently, our strategy achieves a target precision with a third of the data points previously required. Such enhanced device performance and accelerated data collection will be essential for applications in quantum computing, communication, metrology, and the wider quantum technology sector.

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From advances in gravitational wave detection [1,2] to applications such as brain pattern imaging [3], magnetic sensing [4,5], or inertial navigation [6], quantum technologies are revolutionizing fundamental and applied research. Rapid estimation of platform parameters, as well as precise state preparation and control, underpins this success, enabling, e.g., efficient interferometric measurements [7,8] and high-fidelity readout within a limited coherence time in quantum computing [9,10]. Estimation efficiency [11,12] is critical for advancing quantum technologies [4,13] and for testing the fundamental laws of nature [14–18].

Local estimation theory is the standard for benchmarking measurement processes [19–22]. Here, the rate of information acquisition is increased by optimizing the (quantum) Fisher information [23–26], and optimal sensitivity is achieved when the Cramér-Rao bound is saturated. This often requires an asymptotically large number of repetitions [19,21], necessitating long acquisition times and possibly

leading to information loss at the data processing stage [27–32]. In contrast, Bayesian estimation theory [23,33] goes beyond by maximizing information acquisition for *any sample size* [30]. Its effectiveness has been demonstrated within a limited number of atomic, molecular, and optical experiments, primarily for phase estimation [34–46]. Despite the clear advantages and well-established foundations of the Bayesian approach [21,23,33,47–52], experimental adoption for quantum sensing applications remains limited.

Here, we present a systematic strategy for experimental sensing accelerating information acquisition, or equivalently, enabling higher precision. This is achieved through Bayesian optimization of experimental control parameters y in order to estimate an unknown parameter Θ , as illustrated in Fig. 1(a). Unlike standard adaptive strategies [53–56], our approach is guided by a precision gain quantifier that incorporates the problem's natural symmetries, ensuring both consistency and maximal information extraction. Our strategy applies to experimental parameters Θ that are not phases, such as atom number [57], decay rates [58,59], or relative weights [23,52], and significantly expands beyond conventional phase estimation. This transferable strategy can be readily adopted to boost performance across the quantum technology sector, and will provide significant benefit when probing challenging regimes, such as in searches for new fundamental

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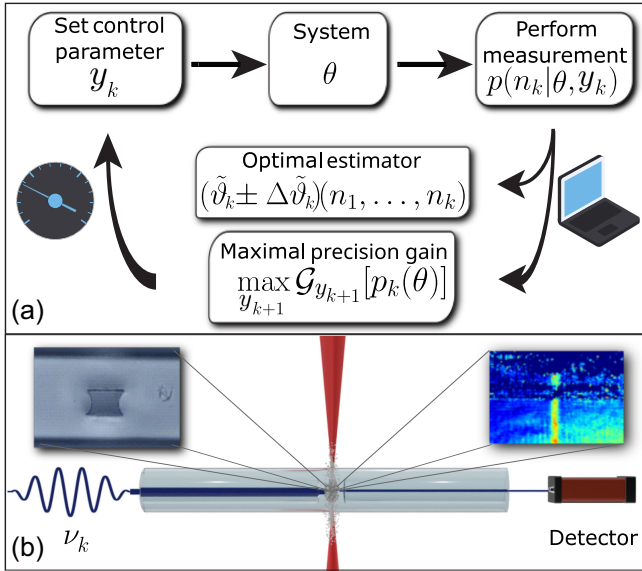


FIG. 1. (a) General adaptive, symmetry-informed, Bayesian estimation strategy. Information gained about system parameter Θ from a detected signal n_k determines a new optimal control parameter y_k that maximizes information gain in the next measurement shot, $k + 1$. (b) Application to an ensemble of cold Cs atoms confined in an optical dipole trap (red cones) within a microscopic hole (left inset) that intersects the core of an optical fiber. Atom number is probed via photon-absorption detection of light traveling along the fiber. The light's frequency ν_k is the control parameter to be optimized. The final estimate of atom number is sequentially improved as photon counts are recorded. The right inset shows an absorption image of the atoms passing through the optical fiber.

physics with wide *a priori* bounds for unknown parameters [2,60,61].

We further demonstrate this strategy's efficacy in an example experiment, where a cold atomic cloud is trapped inside an optical fiber [62,63] [see Fig. 1(b)], and the atom number is the system parameter to be estimated. Controlling the probing laser frequency according to our adaptive, symmetry-informed Bayesian protocol significantly enhances information acquisition speed and precision.

Adaptive, symmetry-informed estimation—Efficient estimation amounts to solving an optimization problem [23,47,55,64]. We present a general adaptive, symmetry-informed Bayesian strategy for parameter estimation in experiments where measurement precision depends on one or more control parameters and estimated parameters are not necessarily phases.

Let $\theta \in [\theta_{\min}, \theta_{\max}]$ be a *hypothesis* for an unknown parameter Θ to be determined indirectly through a measurement with outcome n . This outcome is related with θ and possibly a control parameter y through a likelihood function $p(n|\theta, y)$. For k outcomes $\mathbf{n} = (n_1, \dots, n_k)$ measured using control parameters $\mathbf{y} = (y_1, \dots, y_k)$, the likelihood is $p(\mathbf{n}|\theta, \mathbf{y}) = \prod_{i=1}^k p(n_i|\theta, y_i)$. Based on this, the

default choice for an estimator for Θ is the maximum likelihood estimator (MLE) $\tilde{\theta}_y(\mathbf{n}) = \text{argmax}_{\theta} [p(\mathbf{n}|\theta, \mathbf{y})]$. However, the MLE is known to be efficient only in the regime of large sample sizes [19,21].

This motivates adopting a Bayesian approach. Here, a prior probability $p(\theta)$, which encodes the initial information, is *updated* via Bayes's theorem as

$$p(\theta) \mapsto p(\theta|\mathbf{n}, \mathbf{y}) \propto p(\theta)p(\mathbf{n}|\theta, \mathbf{y}) \quad (1)$$

upon recording $\mathbf{n}$, where

$$p(\theta|\mathbf{n}, \mathbf{y}) \propto p(\theta) \prod_{i=1}^k p(n_i|\theta, y_i) \quad (2)$$

is the posterior probability. This probability contains all the information available about Θ [23,39].

In addition to using Bayes's theorem, one must respect the symmetry of the problem. For example, while phases obey circular symmetry, $\theta' = \theta + 2\pi$ [65], temperature usually appears in ratios of energies, which are invariant under transformations $\theta' = \gamma\theta$, for positive γ [30]. A symmetry function f that weights the posterior probabilities (2) can incorporate such symmetries into the measurement strategy, thereby enforcing the desired symmetry in the optimal estimation. For a given f , measurement outcomes $\mathbf{n}$, and control parameters $\mathbf{y}$, the symmetry-informed estimator for Θ is

$$\tilde{\theta}_{y,f}(\mathbf{n}) = f^{-1} \left[\int d\theta p(\theta|\mathbf{n}, \mathbf{y}) f(\theta) \right]. \quad (3)$$

This estimator is *optimal* under quadratic losses of the form $[f(\tilde{\theta}) - f(\theta)]^2$ [52]. When $f(\theta) = \theta$, the estimator is simply the mean of the posterior probability, suitable for location parameters with translation symmetry $\theta' = \theta + \gamma$, with arbitrary γ . Other symmetries necessitate different estimator functions. Accounting for the symmetries of individual applications is thus an essential step in generalizing this estimation strategy. Furthermore, the empirical error associated with this estimator is

$$\Delta \tilde{\theta}_{y,f}(\mathbf{n}) = \frac{\sqrt{\mathcal{L}_{y,f}(\mathbf{n})}}{|f'[\tilde{\theta}_{y,f}(\mathbf{n})]|}, \quad (4)$$

where the prime denotes a derivative and

$$\mathcal{L}_{y,f}(\mathbf{n}) = \int d\theta p(\theta|\mathbf{n}, \mathbf{y}) f(\theta)^2 - f[\tilde{\theta}_{y,f}(\mathbf{n})]^2 \quad (5)$$

is the loss incurred by processing $(\mathbf{n}, \mathbf{y})$ via Eq. (3).

Equations (3) and (4) provide maximum information for any parameter range or sample size and enable experimental symmetry-informed estimation through appropriate

choice of f . For example, the likelihood may be invariant under a symmetry group [66], or we may enforce invariance under all reparametrizations using information geometry [33,67]. These symmetries can be shown to determine maximum ignorance (MI) priors of the form [52]

$$p_{\text{MI}}(\theta) \propto f'(\theta), \quad (6)$$

where $p_{\text{MI}}(\theta)$ follows from the invariance condition [33]

$$p_{\text{MI}}(\theta')d\theta' = p_{\text{MI}}(\theta)d\theta \quad (7)$$

with θ' and θ related by an arbitrary symmetry. The function f entering Eqs. (3) and (4) is obtained from Eqs. (6) and (7). This rationale is applied later, in the derivation of Eqs. (14) and (15).

The information loss (5) can now be *adaptively* reduced by selecting y_k for the k th measurement to maximize the precision gain [39,52]

$$\mathcal{G}_{y_k, f} = \sum_n p(n|y_k) f[\tilde{\theta}_{y_k, f}(n)]^2. \quad (8)$$

Here, $p(n|y_k) = \int d\theta p_{k-1}(\theta) p(n|\theta, y_k)$, with

$$p_{k-1}(\theta) := \begin{cases} p(\theta) & \text{if } k = 1 \\ p(\theta) \prod_{i=1}^{k-1} p(n_i|\theta, y_i) & \text{for } k > 1, \end{cases} \quad (9)$$

and $\tilde{\theta}_{y_k, f}(n)$ is defined as in Eq. (3) using $p(\theta|n, y_k) \propto p_{k-1}(\theta) p(n|\theta, y_k)$. By construction, $\mathcal{G}_{y_k, f}$ guarantees adaptive optimization that respects the symmetry of the problem through f . Equations (3), (4), and (8) constitute our first result and are derived in the Supplemental Material [68].

Application to experimental atom number metrology—We now apply the equations derived above to a specific experiment to demonstrate their power. The example is a cold atom setup [62], where Cs atoms are trapped within a hole cut into an optical fiber; see Fig. 1(b). This setup is a promising platform [62,63] for the integration of cold-atom-based sensors, quantum memories and multiqubit gates within existing photonic circuit architectures [80]. The task is to determine the number of atoms N trapped within the fiber. Previously this required 30–100 shots per estimate and was the main rate-limiting step. Requiring many repetitions, or long measurement times, is typical, e.g., for atom-based systems such as atom-nanofiber experiments [81,82], precision magnetometers [83], and atom interferometry [84]. Here, we address this problem with an adaptive strategy based on Eq. (8) and an optimal atom number estimator $\tilde{N}_\nu(\mathbf{n}) \pm \Delta\tilde{N}_\nu(\mathbf{n})$ built upon Eqs. (3) and (4).

Near-resonant light propagating through the fiber is absorbed by the atoms in the intersection at a rate proportional to [62]

$$\zeta_\nu = \frac{\Gamma^2}{\Gamma^2 + 4(\nu - \nu_r)^2}, \quad (10)$$

where ν is the laser frequency, and ν_r and Γ are the resonant frequency and natural linewidth of the atomic transition, respectively. The light intensity is kept low compared to the atomic saturation intensity, leading to an expected photon count of

$$\bar{n}_\nu(\Phi, \Theta) = \Phi e^{-\zeta_\nu \Theta}. \quad (11)$$

Here, Φ is the expected photon number with no atoms present, $\exp(-\zeta_\nu \Theta)$ is the transmittance of the medium, and Θ its on-resonance optical depth. Both Φ and Θ are initially unknown. The atom number N we seek to determine is proportional to the optical depth Θ , i.e., $N = \kappa\Theta$ [62], with constant κ determined by the mode area of the waveguide and the on-resonance atomic absorption cross section. The standard method for estimating N , employed in most setups using optical absorption to detect or image cold atoms [85–89], is to count photons with and without atoms loaded into the hole ($n = n_a$ and $n = n_b$, respectively), and use the MLE $\tilde{\theta}_\nu(\mathbf{n}) = (\kappa/\zeta_\nu) \log(\langle n_a \rangle / \langle n_b \rangle)$, with error propagated from this formula. Here, ν is set on resonance with ν_r (i.e., $\zeta_\nu = 1$). This estimator results from the likelihood associated with photon count statistics,

$$p(n|\Phi, \Theta, \nu) = P(n|\bar{n}_\nu(\Phi, \Theta)), \quad (12)$$

where $P(n|z) = z^n e^{-z} / n!$, which arises because ordinary (unsqueezed) light exhibits a Poissonian distribution.

Instead, we will now apply the new protocol with the laser frequency ν as the control parameter y in Eq. (3). Symmetry-informed estimation is applied directly to the optical depth Θ , with Φ , the photon number without atoms, featuring as an undetermined nuisance parameter.

Let $\theta \in [\theta_{\min}, \theta_{\max}]$ and $\varphi \in [\varphi_{\min}, \varphi_{\max}]$ be the hypotheses for the real Θ and Φ , respectively. When two observers run the experiment at frequency ν , their average photon counts are comparable, but their initial hypotheses can differ due to prior ignorance. That is, $\bar{n}_\nu(\varphi', \theta') = \bar{n}_\nu(\varphi, \theta)$, so that $\varphi'/\varphi = \exp[\zeta_\nu(\theta' - \theta)]$. Equating both sides to a positive γ reveals transformations,

$$\varphi' = \gamma\varphi, \quad \zeta_\nu\theta' = \zeta_\nu\theta + \log(\gamma), \quad (13)$$

under which Eq. (12) is invariant. These imply that the expected number of photons is a scale parameter, while the optical depth is a location parameter [23]. Then, if both observers are equally ignorant, and given that ν does not inform Φ and Θ , their priors must respect the aforementioned invariance as $p_{\text{MI}}(\varphi', \theta') d\varphi' d\theta' = p_{\text{MI}}(\varphi, \theta) d\varphi d\theta$. As shown in the Supplemental Material [68], this leads to the multiparameter prior

$$p_{\text{MI}}(\varphi, \theta) = \left[(\theta_{\text{max}} - \theta_{\text{min}}) \log \left(\frac{\varphi_{\text{max}}}{\varphi_{\text{min}}} \right) \varphi \right]^{-1}. \quad (14)$$

Despite having no *a priori* knowledge about the unknown parameters beyond the initial ranges, this prior is *not* flat in φ . This is a consequence of the scale symmetry in Eq. (13) [23,66].

Given that Eq. (14) is a two-parameter prior, it yields two different symmetry functions. Marginalizing over θ or φ to obtain $f(\varphi)$ or $f(\theta)$ from Eq. (6), respectively, one finds

$$f(\varphi) = c_1 \log \left(\frac{\varphi}{c_2} \right) \quad \text{and} \quad f(\theta) = c'_1 \theta + c'_2, \quad (15)$$

where c_1 , c_2 , c'_1 , and c'_2 are free constants. Since our focus is θ , and φ is a nuisance parameter, only the linear function is required. Note that in other contexts, such as quantum thermometry, the logarithmic function has been important [30,39,54]. In general, the function f can take many forms; for example, this methodology could be applied equally effectively to experiments measuring a conversion efficiency such as Ref. [90], which is a weight parameter and would thus require $f(z) = 2\text{arctanh}(2z - 1)$ [52].

Applying the symmetry function $f(\theta)$ from Eq. (15) and with $N \propto \Theta$, Eqs. (3) and (4) deliver the optimal atom number estimator and its uncertainty $\tilde{N}_\nu(\mathbf{n}) \pm \Delta\tilde{N}_\nu(\mathbf{n})$ as the mean and variance of the marginalized posterior $p(\theta|\mathbf{n}, \nu)$, which is given by

$$p(\theta|\mathbf{n}, \nu) \propto \int_{\varphi_{\text{min}}}^{\varphi_{\text{max}}} \frac{d\varphi}{\varphi} \prod_{i=1}^k p(n_i|\varphi, \theta, \nu_i), \quad (16)$$

where $c'_1 = 1$ and $c'_2 = 0$ without loss of generality. Equation (16) respects the location symmetry associated with the optical depth θ and the scale symmetry associated with φ through the ignorance prior (6). This is our second result.

Absorption measurements at atom-optical interface—To employ the protocol experimentally, we measure the outcomes $\mathbf{n} = (n_{a,1}, n_{b,1}, \dots, n_{a,k}, n_{b,k})$, with $1 \leq k \leq 30$, alternating photon counts with and without atoms and setting $p(n_i|\varphi, 0, \nu_i)$ in Eq. (16) for the latter. The laser frequency is set to maximize the precision gain (8) with $\tilde{\theta}_{\nu_k}(\mathbf{n}) \propto \tilde{N}_{\nu_k}(\mathbf{n})$, thus finding the frequencies $\nu = (\nu_{1a}, \nu_{1b}, \dots, \nu_{ka}, \nu_{kb})$; see Fig. S1 in the Supplemental Material [68]. We choose prior ranges $5 \leq \varphi \leq 20$ and $0 \leq \theta \leq 8$. For the Cs D₂ $F = 4 \rightarrow F' = 5$ transition, the physical values are $\nu_r = 351.721961$ THz, $\Gamma = 2\pi \cdot 5.234$ MHz, and $\kappa = 84.9$ [62,91]. All experimental parameters other than ν are kept constant. Equations (8) and (16) are rapidly evaluated between shots by caching function results, managing tolerances, and adaptively truncating probabilities.

The effectiveness of the adaptive, symmetry-informed Bayesian strategy is quantified in comparison to four alternative methods. The first (“on-resonance”) is the standard method [62] discussed above. The second method

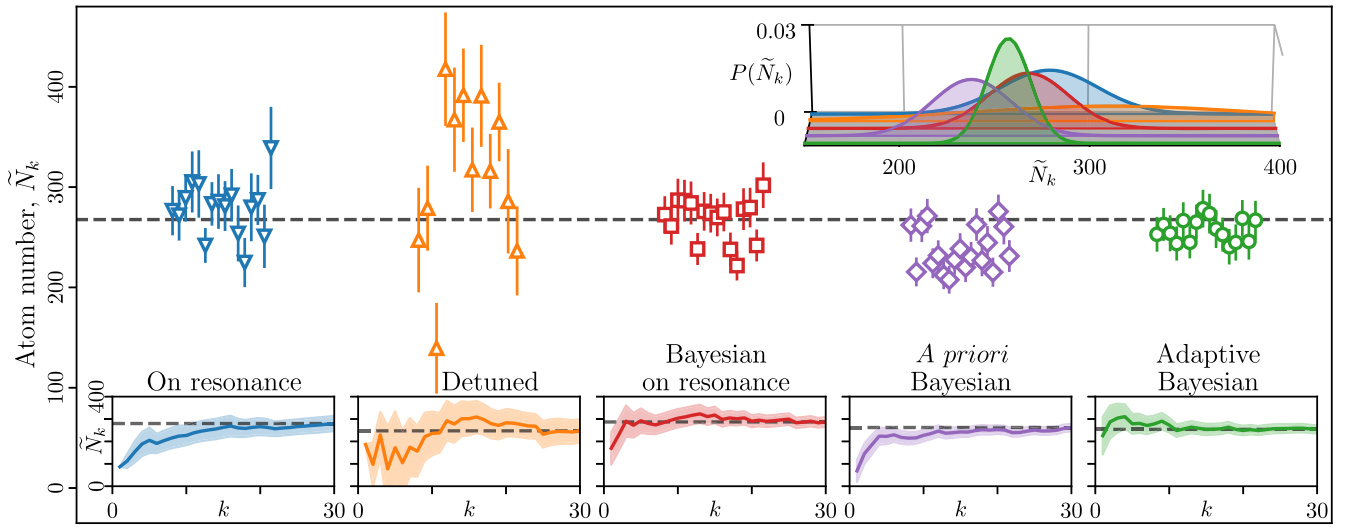


FIG. 2. Atom number estimates $\tilde{N}_k$ resulting from measurements using the standard (blue and orange) and Bayesian estimation strategies (red, purple, and green) described in the text. Each estimate employs $k = 30$ measurement shots. Error bars are given by the standard error for each 30-shot measurement in the on-resonance and detuned cases and by the variance of Eq. (16) otherwise. The lower insets show the progression of the estimate with increasing k , for a single measurement run, in each case. The shading denotes the aforementioned errors. The small differences in the mean value obtained with each method are consistent with normal experimental drifts in loading efficiency. The dashed lines indicate final estimates in the insets and overall average across all methods in the main figure. The inset in the top right corner gives a visual Gaussian representation, $P(\tilde{N}_k)$, of the mean and the standard deviation of the data presented in the main body of the figure for each method.

TABLE I. Comparison of atom number estimation strategies in terms of noise-to-signal ratio for $k = 30$ measurement shots, as well as mean minimal number of shots $k_{\min}$ after which the estimate stays within 10% of the final estimate, from the data shown in Fig. 2.

	On Resonance	Detuned	On-reso. Bayesian	<i>A priori</i> Bayesian	Adaptive Bayesian
NSR in %	0.89	6.10	0.62	0.77	0.19
$k_{\min}$	13	17	12	12	8

(“detuned”) is analogous, but with the laser frequency detuned below the atomic resonance, to $\nu_r - \nu = 5$ MHz. The third method (“on-resonance Bayesian”) uses resonant light, $\nu = \nu_r$, but adopts the Bayesian estimator from Eq. (16). In the fourth method (“*a priori* Bayesian”), the laser frequency is chosen to maximize the precision gain (8) for $k = 1$. This *a priori* optimization involves no outcomes [27,39], and here results in a detuning of $\nu_r - \nu = 1.95$ MHz.

Figure 2 shows atom number estimates yielded by the on-resonance (blue inverted triangles), detuned (orange triangles), on-resonance Bayesian (red squares), *a priori* Bayesian (purple rhombuses), and adaptive Bayesian (green circles) methods. We repeat each method m times and quantify its performance by the empirical noise-to-signal ratio (NSR) $\text{Var}(\tilde{N}_k)/\langle \tilde{N}_k \rangle^2$ [39], where $\langle \tilde{N}_k \rangle = (1/m) \sum_{j=1}^m \tilde{N}_{k,j}$ and $\tilde{N}_{k,j}$ denotes the k th estimate during the j th repetition. This figure of merit is appropriate as it is agnostic to the specific methodology employed. The

results, given in Table I, clearly show marked differences in performance between the strategies. While the two nonadaptive Bayesian protocols and the on-resonance method yield comparable NSRs of just under 1%, and all three outperform the detuned method by an order of magnitude, the adaptive protocol surpasses them all with an NSR of $< 0.2\%$. Notably, the latter outperforms the on-resonance method—which is the current standard for cold atom platforms—by a factor of 4.68. This approximately fivefold enhancement constitutes our third result.

Figure 3 shows that these conclusions hold even for a low shot number. There, the NSR is represented as a function of k . A crucial implication is that the NSR achieved by the standard technique (blue inverted triangles) using $k = 30$ shots is achieved by the adaptive protocol (green circles) with only $k = 9$ shots, i.e., about one third of the resources. This is our fourth result.

The adaptive protocol was also found to converge faster. When recording the mean number of measurement steps $k_{\min}$ needed before the current estimate settles to within $\pm 10\%$ of the final estimate based on 30 shots, we obtain the results listed in the last row of Table I. The adaptive protocol achieves the specified fractional precision with approximately 40% fewer shots (8 instead of 13) than the standard on-resonance method.

Discussion—We have developed a general symmetry-informed estimation strategy that will enable accelerated data collection across the quantum technology sector. We have demonstrated its application to a test platform: atom number estimation in a fiber experiment. The optimized protocol yielded a substantial enhancement in estimation

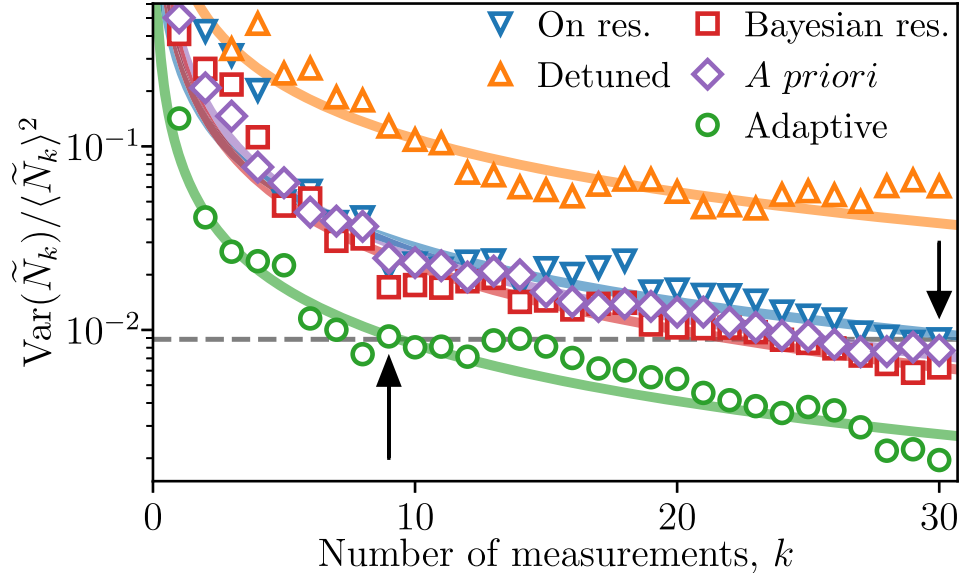


FIG. 3. Noise-to-signal ratio versus shot number k for the on-resonance (blue inverted triangles), detuned (orange triangles), on-resonance Bayesian (red squares), *a priori* Bayesian (purple rhombuses), and adaptive Bayesian (green circles) strategies. The data points are obtained empirically from the distribution of m estimates after each shot number k . The dashed line indicates the NSR for on-resonance method with 30 shots (downward facing arrow), which is achieved by our adaptive protocol with only nine shots (upward facing arrow).

precision (fivefold increase), or alternatively, increased acquisition speed (a third of data points), compared to current standards. These gains make our approach a powerful tool for quantum sensing, metrology, and computation in data-limited regimes. We have further shown that real-time adaptive optimization is feasible with standard computational resources, even in multiparameter setups [28,39]. In general, use of Eqs. (3), (4), and (8) expands beyond the application range of Bayesian phase estimation [65] to optimal Bayesian estimation of parameters with noncyclical symmetry, such as temperature, atom number, and state occupation. Given these generality and performance advantages, the presented strategy should be adopted in experiments ranging from precision spectroscopy and magnetometry to quantum state preparation and fundamental physics tests.

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Data availability—The data that support the findings of this article are openly available [92].

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