

Programmable Lattices for Non-Abelian Topological Photonics and Braiding

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Non-Abelian physics, originating from noncommutative sequences of operations, unveils novel topological degrees of freedom. In photonics, significant efforts have been devoted to developing reconfigurable non-Abelian platforms, serving both as classical testbeds for non-Abelian quantum phenomena and as programmable systems that harness topological complexities. Here, we establish topological spinor lattices for non-Abelian programmable photonics. We design a building block for reconfigurable unitary coupling between pseudospin resonances, achieving a universal set of rotation gates through the coupling loop. The lattice assembly of the building blocks enables the emulation of the extended quantum Hall family. We reveal the emergence of a non-Abelian interface even when the bulks are Abelian, which allows the topologically trivial engineering of topologically protected edge states. We also define the braid group for pseudospin observables, demonstrating non-Abelian braiding and the Yang-Baxter relations. Our results pave the way for realizing a reconfigurable emulator for Abelian and non-Abelian topological phenomena.

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A versatile building block for reconfigurable unitary operations lies at the heart of classical [1,2] and quantum [3] photonics. In programmable photonic circuits (PPCs) [2,4], the building block is a reconfigurable SU(2) gate. Systematic assembly [1] of these gates enables higher-dimensional unitary operations for wave manipulations [5], matrix calculations [6], and quantum computations [7].

Unitary operations play a crucial role also in non-Abelian physics [8–13]. Because symmetries and their gauge invariances are characterized by unitary operators, the noncommutative nature of internal symmetries requires matrix-valued gauge fields within noncommutative unitary groups $U(N > 1)$. In photonics, the implementation of non-Abelian gauge fields has been realized through static [14–23] and tunable platforms [24–26]. Although noninteracting photonic platforms cannot achieve fault-tolerant quantum computing [10], the implementation of non-Abelian gauge fields has enabled novel topological

phenomena as well as the emulation of anyonic systems [8]. When evaluating PPCs as reconfigurable unitary operators [2,4], the next challenge lies in devising a building block for reconfigurable, universal, and lattice-compatible non-Abelian matrix-valued gauge fields.

Here, we propose programmable lattices for exploring Abelian and non-Abelian photonics and their hybrids. By devising a versatile building block, we develop a reconfigurable spinor lattice for emulating topological phenomena. We implement a family of isospectral systems—distinct systems sharing identical spectral responses [27]—exhibiting Abelian topological phenomena: the quantum Hall effect (QHE), and quantum spin Hall effects (QSHEs) for different eigenbases. We also introduce the concept of a non-Abelian interface, revealing the hybridization of topologically nontrivial edge states and the consequent band-gap reopening. Programmable non-Abelian braiding for emulating anyonic systems is also demonstrated.

Programmable spinor lattices—We investigate a lattice composed of traveling-wave resonators [28] [Figs. 1(a) and 1(b)]. Near the operating frequency, the m th resonator supports degenerate counterclockwise (ψ_m^+) and clockwise (ψ_m^-) resonances, constituting a pseudospinor state $\Psi_m = [\psi_m^+, \psi_m^-]^T$ in an expanded Hilbert space for matrix-valued gauge fields [8]. We devise the lattice to develop a programmable U(2) gauge field. The core component is a waveguide loop coupler evanescently coupled to the

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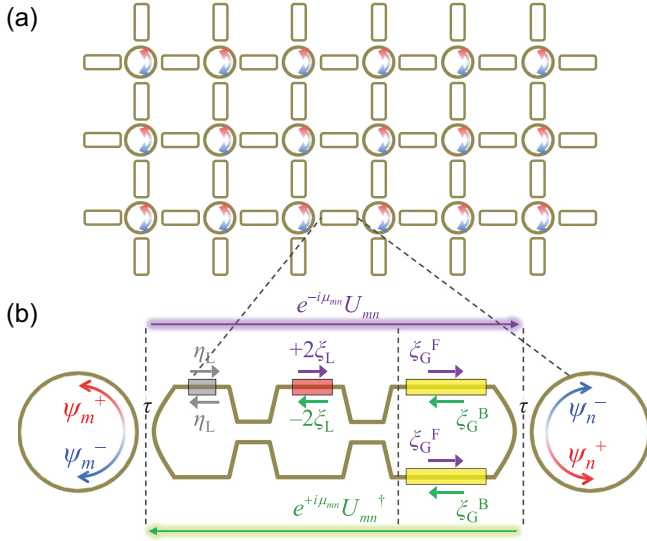


FIG. 1. Programmable spinor lattices. (a) A lattice and (b) its building block. Gray, red, and yellow boxes represent local-reciprocal, local-nonreciprocal, and global-nonreciprocal phase shifters, respectively.

resonators with a decay rate $1/\tau$. The coupler consists of two nonreciprocal parts: an SU(2) gate and a global phase shifter. The first part mainly follows the SU(2) gate design [2,4]; however, one phase shifter introduces a nonreciprocal phase shift (NRPS) $\pm 2\xi_L$, while the other, η_L , remains reciprocal. The second part introduces the NRPSs, ξ_G^F and ξ_G^B , globally in both arms. Based on the transfer-matrix method deriving photonic analog of quantum mechanical Hamiltonians [29–31], wave phenomena in the lattice are governed by the tight-binding Hamiltonian (Note S1 [32]),

$$H = -\frac{1}{2\tau} \sum_{\langle m,n \rangle} \begin{pmatrix} \hat{a}_{\uparrow m}^\dagger & \hat{a}_{\downarrow m}^\dagger \end{pmatrix} e^{i\mu_{mn}} U_{mn} \begin{pmatrix} \hat{a}_{\uparrow n} \\ \hat{a}_{\downarrow n} \end{pmatrix} + \text{H.c.}, \quad (1)$$

where $(\hat{a}_{\uparrow m}^\dagger, \hat{a}_{\uparrow m})$ and $(\hat{a}_{\downarrow m}^\dagger, \hat{a}_{\downarrow m})$ are the creation and annihilation operator pairs for the counterclockwise and clockwise pseudospin resonances, respectively, $\langle m, n \rangle$ denotes a pair of nearest-neighbor indices, μ_{mn} is the U(1) gauge field, and U_{mn} is the following SU(2) link variable (End Matter “Properties of U(1) and U(2) gauge fields” and “Practical implementation” and Notes S2–S4 of [32] for an extended discussion):

$$U_{mn} = \begin{bmatrix} e^{-\frac{i}{2}(\eta_L + \pi)} \sin \xi_L & e^{+\frac{i}{2}(\eta_L + \pi)} \cos \xi_L \\ e^{-\frac{i}{2}(\eta_L - \pi)} \cos \xi_L & e^{+\frac{i}{2}(\eta_L + \pi)} \sin \xi_L \end{bmatrix}. \quad (2)$$

U_{mn} can be represented as an α rotation about the $\mathbf{n}$ axis on the Bloch sphere, $U_{mn} = \exp(-i\alpha \mathbf{n} \cdot \boldsymbol{\sigma}/2)$, with the Pauli vector $\boldsymbol{\sigma} = \sigma_x \mathbf{x} + \sigma_y \mathbf{y} + \sigma_z \mathbf{z}$. Because U_{mn} allows for

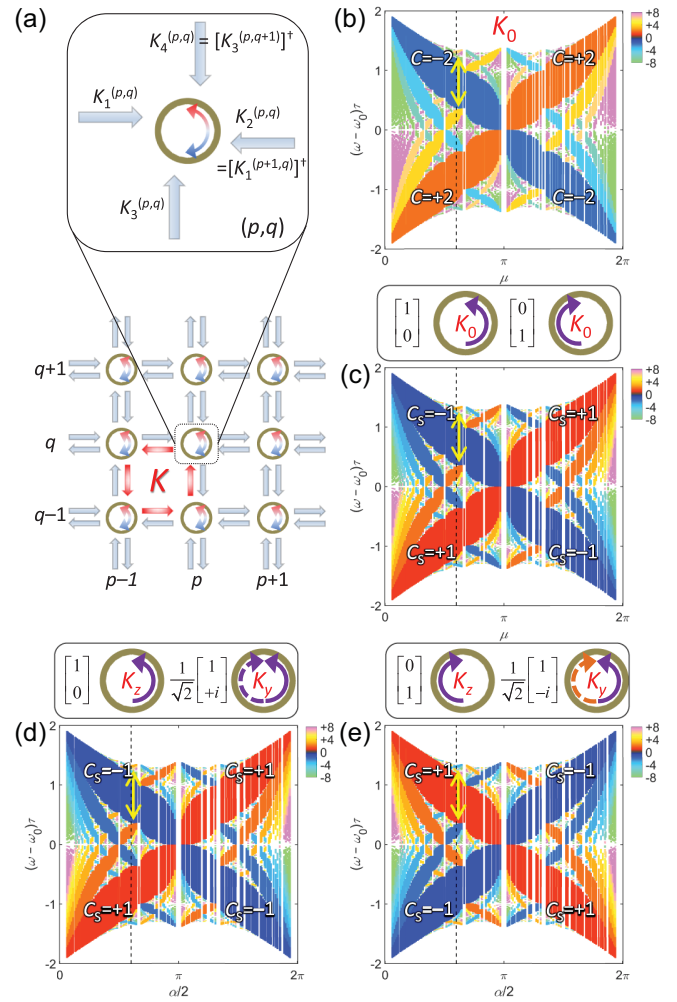


FIG. 2. Isospectral Abelian topological lattices. (a) Schematic for $K_\gamma^{(p,q)}$, where γ indicates the gauge field direction. (b),(c) QHE Hofstadter butterflies for C (b) and C_S (c) for K_0 . (d),(e) QSHE Hofstadter butterflies for C_S for K_y and K_z . The color bars in panels (b)–(e) indicate the value of C in (b) and of C_S in (c)–(e). Butterflies are obtained by solving the eigenvalue problem within a $\Lambda_x \times \Lambda_y$ supercell in the first Brillouin zone [42], which yields rational values of $\mu/(2\pi)$ and $\alpha/(4\pi)$, and, consequently, discrete vertical lines (End Matter “Butterfly calculation” and [44]).

complete rotations around the y and z axes (End Matter “System parameters for rotation operations,” and Note S5 [32]), we adopt these two rotations as fundamental operation modes.

Isospectral Abelian lattices—As the first example of non-Abelian phenomena, we develop a lattice for the U(2)-flux generalization of the Harper-Hofstadter (HH) model [42,43] using our building blocks. The topological nature of the lattice is governed by the loop operator of each plaquette: $K = P\Pi \exp(-i\mu) \exp(-i\alpha \mathbf{n} \cdot \boldsymbol{\sigma}/2)$, where $P\Pi$ denotes the path-ordered product of matrix-valued link variables around a plaquette. For the generalized HH model, the Hermiticity of the Hamiltonian leads to (End Matter “Butterfly calculation”)

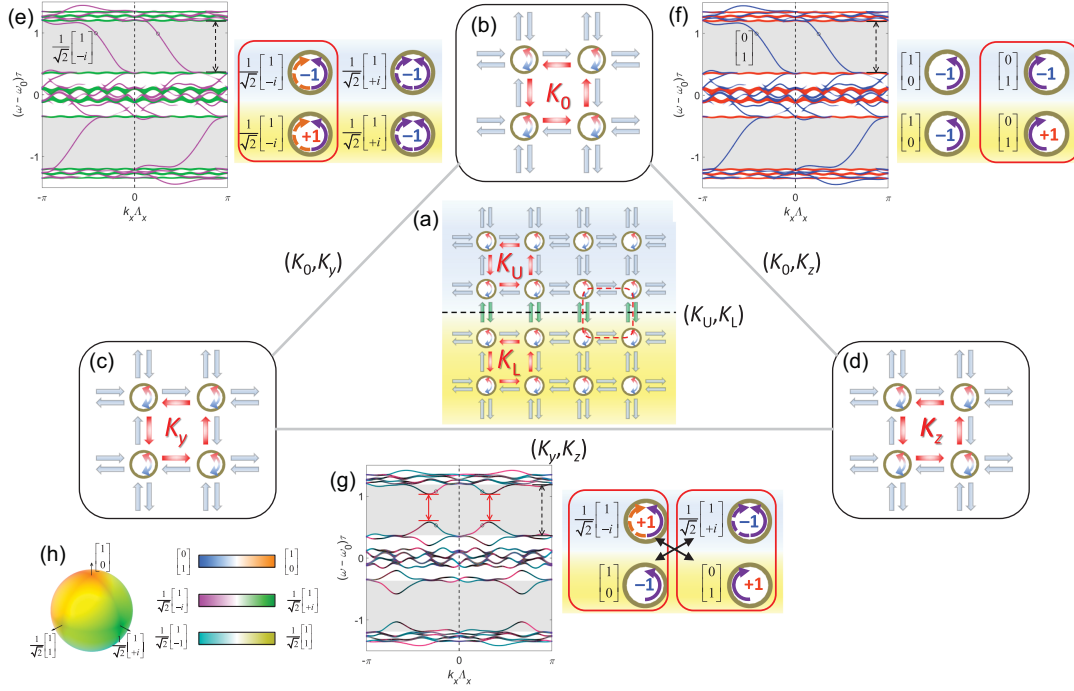


FIG. 3. Topological interfaces. (a) Schematic for the interface between the lattices. Green arrows indicate the coupling with the zero gauge field. Red dashed box denotes the interface loop. (b)–(d) Topological lattices with K_0 (b), K_y (c), and K_z (d). (e)–(g) Band diagrams projected onto the k_x axis [44], and the corresponding interface configurations for (K_0, K_y) (e), (K_0, K_z) (f), and (K_y, K_z) (g). The circles with colored arrows represent the eigenspinors and the corresponding C_S . (h) Color map applied to the bands of (e)–(g). Black dashed arrows in (e)–(g) depict the band gap of the bulk. Red solid arrows in (g) highlight the band gap reopening. The fluxes applied to each lattice correspond to black dashed lines in Figs. 2(b)–2(f), characterized by the supercells of $\Lambda_x = 1$ and $\Lambda_y = 10$ with $\mu = 0.6\pi$ and $\alpha/2 = 0.6\pi$.

$$K = \left[K_3^{(p,q+1)} \right]^\dagger \left[K_1^{(p+1,q+1)} \right]^\dagger K_3^{(p+1,q+1)} K_1^{(p+1,q)}, \quad (3)$$

where $K_\gamma^{(p,q)} = \exp(-i\mu_\gamma^{(p,q)}) U_\gamma^{(p,q)}$ denotes the γ -directional incident link variable to the (p, q) th resonator [Fig. 2(a)].

The loop operator K fully encompasses the $U(2)$ group due to the universality of fundamental operation modes. Consequently, the spinor lattice allows for the programmable emulation of both universal Abelian and non-Abelian topological phenomena on a platform. We focus on the spinor lattices generalizing the HH model within an Abelian group, which is characterized by μ and $\alpha/2$, and a single type of $\mathbf{n}$.

Firstly, we develop the standard HH model with $K = K_0 = \exp(-i\mu)\sigma_0$, where σ_0 is the 2×2 identity matrix. Figures 2(b) and 2(c) show the Hofstadter butterflies for this model, plotting the gaps colored by the gap Chern number C [Fig. 2(b)] and the spin gap Chern number C_S [Fig. 2(c)] following the convention of [45]. Notably, the time-reversal symmetry (TRS) of the spin-1 system is broken with nonzero NRPSs in each coupler. When $\mu \neq q\pi$ (integer q), the TRS of the effective spin-1/2 system is also broken with the same sign of C_S for both pseudospins—the signature of the QHE.

In the same platform, we can program the lattice to achieve the isospectral partner of the HH model with the

transformed eigenbasis. By tailoring the phase shifters, the standard QSHE with $K = K_z = \exp(-i\alpha\sigma_z/2)$ is achieved, leading to nonzero C_S [Figs. 2(d) and 2(e)] with the zero C . The unique form of the QSHE is also achieved with $K = K_y = \exp(-i\alpha\sigma_y/2)$. This Abelian $SU(2)$ gauge field derives the coupling between pseudospin modes, imposing the same topological characteristics of the QSHE on the system in the transformed basis: $[1, +i]^T/2^{1/2}$ and $[1, -i]^T/2^{1/2}$. Both QSHEs achieve the preserved TRS in the effective spin-1/2 system despite NRPSs—the broken TRS of a spin-1 physical system—as evidenced by the zero C . Therefore, the proposed platform allows for constituting an isospectral family of topological lattices, engineering the basis and TRS.

Non-Abelian topological interfaces—Utilizing the developed isospectral topological lattices, we explore unique non-Abelian phenomena observed at the interface between Abelian bulks, which we refer to as a non-Abelian interface (End Matter “Supercell analysis for interface bands”). Figure 3(a) describes the interface between the upper and lower lattices, which have the loop operators K_U and K_L , respectively. We assign K_0 , K_y , and K_z for $K_{U,L}$ [Figs. 3(b)–3(d)], comprising three interfaces: $(K_U, K_L) = (K_0, K_y)$, (K_0, K_z) , and (K_y, K_z) . In illustrating the eigenspinors and the corresponding band structures

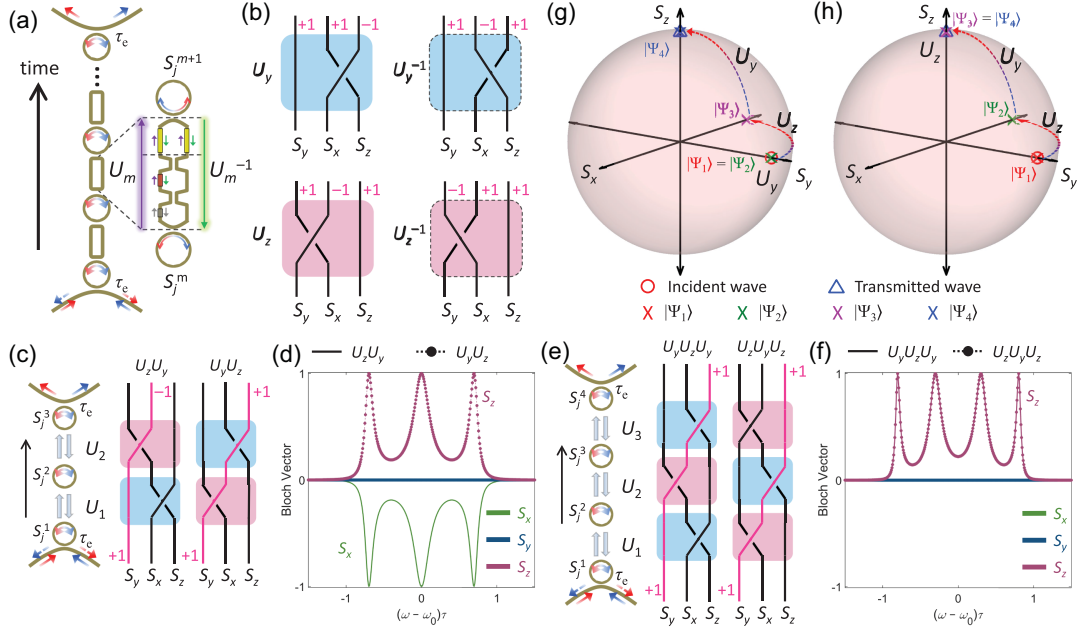


FIG. 4. Programmable non-Abelian braiding. (a) Schematic of braiding. The external waveguides are coupled to the boundary resonators with a decay rate $1/\tau_c$. (b) Generators for B_3 . The number ± 1 denotes the state after braiding when $S_j = 1$ is excited. (c), (d) Non-Abelian condition, and (e), (f) Yang-Baxter relation. (c) and (e) show the 1D lattices and the corresponding braid operations. (d) and (f) show the transmission spectra of the Bloch vector components in (c) and (e), respectively [44]. $\tau_c = 4\tau$. Black solid arrows in (a), (c), (e) denote the arrow of time. (g), (h) Bloch-sphere representation of the Yang-Baxter relation at $\omega = \omega_0$: $U_y U_z U_y$ (g) and $U_z U_y U_z$ (h). $|\Psi_{1,2}\rangle$ and $|\Psi_{3,4}\rangle$ overlap each other in (g) and (h), respectively. All the other parameters are the same as those in Fig. 3.

[Figs. 3(e)–3(g)], we utilize the mixed color map for spinor bases [Fig. 3(h) and End Matter “Band coloring”].

The interfaces, (K_0, K_y) and (K_0, K_z) , are classified as Abelian ones because K_0 commutes with any $SU(2)$ link variable. Figures 2(c)–2(e) show that the nontrivial topological interfaces in these Abelian configurations are achieved only for one spinor: $[1, -i]^T/2^{1/2}$ in Fig. 3(e) and $[0, 1]^T$ in Fig. 3(f). The consequent emergence of a topologically protected edge state for a spinor highlights the uniqueness of our Abelian topological interfaces (K_0, K_y) and (K_0, K_z) , in contrast to the codirectional chiral edge states in the QHE with (K_0, K'_0) and counterdirectional helical edge states in the QSHE with (K_z, K'_z) [43].

In comparison, we explore (K_y, K_z) , which introduces non-Abelian interface physics [Fig. 3(g)]. The configuration satisfies the non-Abelian condition only at the interface while maintaining Abelian bulks, because K_y and K_z lead to noncommutative loop products around the interface. In this configuration, the interface imposes nontrivial topological conditions—unequal C_S —on different pairs of the spinors from distinct bases: the pair of $[1, -i]^T/2^{1/2}$ and $[1, 0]^T$, and the pair of $[1, +i]^T/2^{1/2}$ and $[0, 1]^T$ [red boxes in Fig. 3(g)]. Because of an opposite spatial arrangement of C_S for these pairs, the induced helical edge states have opposite signs of group velocity, consistent with the preserved TRS in effective spin-1/2 systems. Furthermore, different basis representations for eigenspinors in the K_y and K_z regions, lacking orthogonality, lead to coupling within the pairs of

states that have the same C_S [black solid arrows in Fig. 3(g)]. This topologically trivial coupling results in the hybridization of helical edge states and the band anticrossing, restoring band gaps, which is the unique phenomenon of non-Abelian topological interfaces. Considering the relationship between topological phase transitions involving gap closures and bulk-boundary correspondence [17], the observed gap reopening implies the absence of a topological phase transition, which is consistent with the continuous deformation between K_y and K_z (Note S6 for edge-state profiles [32]).

Non-Abelian resonant braiding—To demonstrate the universality of our design as a building block for non-Abelian physics, we emulate non-Abelian braiding [8,9,18,21,46,47], which describes the exchange of anyons [10]. We construct a finite one-dimensional (1D) lattice of coupled resonators where the lattice axis represents the pseudotime axis [Fig. 4(a)]. We model particle states with half-spin observables— $S_j^m = \langle \Psi_m | \sigma_j | \Psi_m \rangle$ ($j = x, y$, and z), in the m th resonator—that inherit a twofold degeneracy essential for non-Abelian braiding [10,21]. The transfer function between each unit in the lattice emulates non-Abelian operation on the spin state that can be plotted on a Bloch sphere. We initiate the operation by coupling external waveguides to the resonators at the end and excite only one of them. By limiting the possible set of the transfer functions, the spin state can be visualized as three strands, where the j th strand S_j is a series of S_j^m . The lattice in this perspective

describes the temporal exchange of three particles as it braids S_x , S_y , and S_z , emulating the operations of the non-Abelian braid group B_3 . We sequence the strands as S_y , S_x , and S_z .

The design in Fig. 1(b) enables arbitrary rotation operations connecting spin observables to one another. Among them, we employ $U_y = \exp(-i\alpha_y\sigma_y/2)$ and $U_z = \exp(-i\alpha_z\sigma_z/2)$, and their inverses U_y^{-1} and U_z^{-1} [Fig. 4(b)] as the generators of B_3 describing the particle exchanges. We set $U_{y,z}$ and $U_{y,z}^{-1}$ to counterclockwise and clockwise exchanges, respectively. A rotation around the y (or z) axis induces a unitary interaction between the z - and x -spin observables (or the x - and y -spin observables), which is visualized by the braiding of the corresponding strands. Although the interaction can be freely adjusted by α_y and α_z , the allowed operations for B_3 are restricted by the criteria of the braid group—far commutativity and the Yang-Baxter relation [48]. While the far commutativity is automatically satisfied within B_3 , the Yang-Baxter relation requires $U_y U_z U_y = U_z U_y U_z$. Moreover, non-Abelian braiding enforces $U_y U_z \neq U_z U_y$. When restricting our discussion to $0 \leq \alpha_{y,z} < 2\pi$, the allowed rotations are achieved solely with $(\alpha_y, \alpha_z) = (\pi/2, \pi/2)$ and $(3\pi/2, 3\pi/2)$. These conditions impose an additional identity on the braiding operations—eightfold periodicity of U_y and U_z [49]—arising from the double-covering relationship between $SU(2)$ and $SO(3)$ groups.

Figures 4(c)–4(f) demonstrates B_3 realized with our design at $(\alpha_y, \alpha_z) = (\pi/2, \pi/2)$. Because a unitary coupling between two resonators corresponds to a generator, the non-Abelian and the Yang-Baxter relations are demonstrated using three [Figs. 4(c) and 4(d)] and four [Figs. 4(e) and 4(f)] coupled resonators, respectively. Figures 4(d) and 4(f) show the observation of the braid-group criteria in the transmission spectra for the incidence $S_y = +1$ (Note S7 [32]). The results indicate that the criteria are satisfied throughout the entire spectra. For example, the non-Abelian nature accounts for the distinction between $U_z U_y: S_y = +1 \rightarrow S_x = -1$ and $U_y U_z: S_y = +1 \rightarrow S_z = +1$. Similarly, the identity of $U_y U_z U_y = U_z U_y U_z$ is confirmed by their perfect overlay in the transmitted spin observables, also as illustrated in the Bloch-sphere representation [Figs. 4(g) and 4(h); Note S8 [32] for the non-Abelian case]. Despite the valid criteria in the entire spectra, complete conservation of the strands is achieved at the resonant-tunneling frequencies with perfect transmission. The proposed braiding geometry of the strands is programmable by controlling phase shifts in the loop couplers.

Utilizing the well-defined generators of B_3 , we can develop diverse sequences of braiding operators—namely, braid words [50]—that are distinguished by the Yang-Baxter relation and eightfold periodicity. When braid words differ under these constraints, they acquire distinct geometric phases even when the spin observables are identical,

enabling geometrical phenomena such as the Aharonov-Bohm effect (Note S9 [32]).

We have developed a universal and reconfigurable building block applicable to diverse non-Abelian phenomena in photonics. The reconfigurability of this building block allows for realizing isospectral quantum Hall family and constructing the interfaces between them. The non-Abelian interface exhibits unique characteristics, demonstrating the coexistence of topologically trivial and nontrivial effects. The 1D lattice of our building blocks enables the realization of non-Abelian braid group. Realizing matrix-valued gauge fields compatible with conventional PPCs represents a distinct novelty in spatial mode manipulation compared to recent cornerstone works in frequency-synthetic dimensions [25,26]. Our findings provide versatile platforms for exploring both Abelian and non-Abelian physics, as discussed in End Matter “Application examples.”

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Data availability—The data that support the findings of this article are openly available [44].

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End Matter

Properties of $U(1)$ and $U(2)$ gauge fields—The critical parameter for non-Abelian $U(2)$ gauges is the NRPS ξ_L , which couples counter-clockwise and clockwise resonances. The Abelian $U(1)$ gauge field is affected by both local and global phase shifts, as $\mu_{mn} = \xi_G^F + \xi_L + \eta_L/2$, while the necessary ξ_G^B becomes $\xi_G^B = -\eta_L - \xi_G^F - \pi(\text{modulo } 2\pi)$. We break the TRS of photons by setting $\xi_G^F \neq \xi_G^B$ or $\xi_L \neq 0$, but for emulation of QSHE

we can preserve the fermionlike pseudo TRS by tuning μ_{mn} [51] (Note S2 [32]).

Practical implementation—In practical implementation, the critical challenge lies in achieving a sufficient magnitude of NRPS within integrated platforms. In Note S3 of [32], we provide a design example of NRPS using the Tidy3D software [52]. Assuming the use of a cerium-substituted yttrium iron garnet (Ce:YIG) silicon

waveguide [53], the designed component achieves an NRPS of 1.4 rad/mm for a Faraday rotation of 6000 deg/cm at a wavelength of 1550 nm. The results show that a waveguide length of 9.0 mm is sufficient to achieve a 4π NRPS, which is linearly tunable with the external magnetic field. Within this NRPS scheme, we also conduct numerical analyses of the remaining components of our building block (Note S4 in [32]), which yield a prototype platform with a 0.62 mm² footprint and a total loss of 1.35 dB along the entire loop coupler. The resulting geometry also allows for employing the independent control of NRPSs in each element by utilizing integrated electromagnets [54].

Considering the feature size of unit cells in conventional photonic circuits under weak coupling conditions [29], and examining the loss and footprint of our design, achieving stronger NRPS or implementing efficient loss compensation will be crucial challenges for experimental demonstration. In terms of the proof of concept, demonstrations in the radio-frequency regime could be carried out first [55]. Considering the difficulty in achieving integration of magneto-optical structures, we can envisage nonmagnetic alternatives, such as time-varying systems for pseudomagnetic fields [56]. This approach also allows for the independent control of NRPSs through the electrical drive [57].

System parameters for rotation operations—By comparing Eq. (2) with $U_{mn} = \exp(-i\alpha \mathbf{n} \cdot \boldsymbol{\sigma}/2)$, which represents the rotation by angle α about the $\mathbf{n}$ axis, the required local phase shifts ξ_L and η_L can be determined from the given α and $\mathbf{n}$, and vice versa. In this end matter section, we specifically address rotations about $\mathbf{n} = \pm \mathbf{y}$ and $\pm \mathbf{z}$, while more general cases are explored in Note S5 of [32]. It is well known that the Taylor expansion of $\exp(-i\alpha \mathbf{n} \cdot \boldsymbol{\sigma}/2)$ leads to the following expression [58]:

$$U_{mn} = e^{-i\frac{\alpha}{2}\mathbf{n}\cdot\boldsymbol{\sigma}} = \sigma_0 \cos \frac{\alpha}{2} - i\mathbf{n} \cdot \boldsymbol{\sigma} \sin \frac{\alpha}{2}, \quad (\text{C1})$$

where σ_0 is the 2×2 identity matrix. Equation (2) can also be expressed using the Pauli vector $\boldsymbol{\sigma}$,

$$U_{mn} = -\left(\sin \frac{\eta_L}{2} \sin \xi_L\right) \sigma_0 - i\left(-\cos \frac{\eta_L}{2} \cos \xi_L\right) \sigma_x \\ - i\left(\sin \frac{\eta_L}{2} \cos \xi_L\right) \sigma_y - i\left(\cos \frac{\eta_L}{2} \sin \xi_L\right) \sigma_z. \quad (\text{C2})$$

For $\mathbf{n} = \pm \mathbf{z}$, the coefficients of the Pauli x and y matrices should be zero, leading to

$$\xi_L = \frac{\pi}{2}, \quad \eta_L = \pm \alpha - \pi + 4l_\eta \pi, \quad (\text{C3})$$

where l_η is an integer number. Similarly, for $\mathbf{n} = \pm \mathbf{y}$, the coefficients of the Pauli x and z matrices should be

zero, leading to

$$\xi_L = \pm \frac{\alpha}{2} - \frac{\pi}{2} + 2l_\xi \pi, \quad \eta_L = \pi, \quad (\text{C4})$$

where l_ξ is an integer number. We set $l_\eta = l_\xi = 0$, which determines the explicit ranges of local phase shifts ξ_L and η_L for the complete rotations about the $\pm y$ and $\pm z$ axes. The necessary phase shifts for each coupler operation mode are shown in Note S5 of [32], requiring $0 \leq \xi_L \leq \pi$ and $0 \leq \eta_L \leq 2\pi$ to achieve a universal U(2) flux across a plaquette.

Butterfly calculation—In calculating the Hofstadter butterflies in Figs. 2(b)–2(f), the loop operator K along the path $(p, q) \rightarrow (p+1, q) \rightarrow (p+1, q+1) \rightarrow (p, q+1)$ in Fig. 2(a) is defined by

$$K = K_4^{(p,q)} K_2^{(p,q+1)} K_3^{(p+1,q+1)} K_1^{(p+1,q)}. \quad (\text{D1})$$

To satisfy the Hermitian condition of the Hamiltonian, it is necessary that $K_2^{(p,q+1)} = [K_1^{(p+1,q+1)}]^\dagger$ and $K_4^{(p,q)} = [K_3^{(p,q+1)}]^\dagger$, which lead to Eq. (3). In Fig. 2, we consider three types of Abelian loop operators, $K_0 = \exp(-i\mu)$, $K_z = \exp(-i\alpha\sigma_z/2)$, and $K_y = \exp(-i\alpha\sigma_y/2)$, which can be collectively expressed as $K_d = \exp(-iv\sigma_d)$ ($v = \mu$ for $d = 0$, and $v = \alpha/2$ for $d = y$ and z). Similar to the Landau gauge [59], we set $K_1^{(p,q)} = \exp(iq\lambda_1\sigma_d)$ and $K_3^{(p,q)} = \exp(-ip\lambda_3\sigma_d)$, resulting in the homogeneous loop operator $K_d = \exp[-i(\lambda_1 + \lambda_3)\sigma_d]$ around the entire lattice.

To satisfy the Bloch boundary conditions for the band calculation, λ_1 and λ_3 are set to be rational numbers. We calculate the band necessary for obtaining the butterflies when both λ_1 and λ_3 are irreducible fractions with denominators no greater than the natural number $B = 33$, and the sum of λ_1 and λ_3 lies between 0 and 2π . In this configuration, μ and $\alpha/2$ are discretized into 163 nonuniform points over the range 0 to 2π . The gap Chern number and spin gap Chern number are calculated according to their conventional definitions [60,61], while the spinor bases for these quantities are defined differently for K_y and K_z , as $\{[1, 1]^T, [0, 1]^T\}$ for K_z , and $\{[1, +i]^T/2^{1/2}, [1, -i]^T/2^{1/2}\}$ for K_y .

Supercell analysis for interface bands—The results in Fig. 3 for interface physics are obtained from the lattice configurations using, $K_0 = P\Pi \exp(-i\mu_{mn}) = \exp(-i\mu)$, $K_y = P\Pi \exp(-i\alpha_{mn}\sigma_y/2) = \exp(-i\alpha\sigma_y/2)$ and, $K_z = P\Pi \exp(-i\alpha_{mn}\sigma_z/2) = \exp(-i\alpha\sigma_z/2)$ where $\mu = \alpha/2 = 0.6\pi$. These flux values are implemented in the coupled-resonator supercells with $\Lambda_x = 1$ and $\Lambda_y = 10$. Consequently, for the calculation of the band structures shown in Figs. 3(e)–3(g), we apply the supercells of $\Lambda_x = 1$ and $\Lambda_y = 20$, placing the interface at the center

of the cell. We solve the eigenvalue problem across the first Brillouin zone, defined by $-\pi \leq k_x \Lambda_x \leq +\pi$ and $-\pi \leq k_y \Lambda_y \leq +\pi$, where k_x and k_y are the x and y components of the reciprocal lattice vector, respectively. To focus on the interface modes, we filter out the modes near the upper and lower boundaries by applying a proper weighting to the optical intensity of eigenmodes. The remaining bands overlap for different values of k_y . It is worth noting that the edge states are almost invariant with respect to k_y .

Band coloring—Because the fundamental unit of our platform is a two-level system—traveling-wave resonator—comprising pseudospin resonances, the state of an eigenspinor at a specific resonator can be represented on the Bloch sphere. For a spinor state $|\Psi\rangle$, where $\langle m|\Psi\rangle = \Psi_m = [\psi_m^+, \psi_m^-]^T$ for the resonator lattice site operator $|m\rangle$, the state on the Bloch sphere is defined by three scalar degrees of freedom in Cartesian coordinates: $S_x = \langle \Psi|\sigma_x|\Psi\rangle$, $S_y = \langle \Psi|\sigma_y|\Psi\rangle$, and $S_z = \langle \Psi|\sigma_z|\Psi\rangle$. These scalars represent the x -, y -, and z -axis components of the position on the Bloch sphere, respectively, which characterize three pairs of eigenspinor bases. We apply corresponding color maps to these bases as shown in Fig. 3(h). The state of an eigenspinor at a specific band point (k_x, k_y, ω) , which will be located on the Bloch sphere, is visualized through additive color mixing, as depicted in the colored Bloch sphere of Fig. 3(h).

Application examples—Because of the programmability of the proposed platform, abundant design freedom in TRS, bulk-edge configuration, and braiding operations can be utilized for reconfigurable light manipulation. For example, the demonstrated isospectral quantum Hall family allows for the control of TRS across the effective spin-1/2 system, while the corresponding physical spin-1 system exhibits broken TRS. Abelian and non-Abelian interface states can be dynamically configured through

the manipulation of a set of phase shifters within the subregion of the system. Reconfigurable braiding operations enable topologically nontrivial transitions between different knots and links. Such dynamical nature suggests the extension of the recently emerging field of time-varying photonics [62] into non-Abelian topological phenomena. We also note that time-varying modulations allow for magnetic-free realizations of our building blocks using synthetic magnetic fields [56] or acousto-optic modulation [63].

Substantial issues still remain from both fundamental and application perspectives. Reviewing studies on different dimensional defects in photonic crystals [64], extending research from the interface to point non-Abelian configurations could emerge as a future topic. The interfaces between non-Abelian topological lattices will offer substantial degrees of freedom due to their complex mothlike spectra [25,65,66]. Additionally, given the robustness of topological protection against external influences, the potential impact of non-Abelian interface physics could lie in enabling gate operations for topologically nontrivial states, as demonstrated in our gap reopening example. We also note that our braid group characterized by resonant, discretized, spin-observable, and spectral realizations enables novel forms of knots and links of photonic states, which will be in sharp contrast to previous approaches based on propagating, adiabatic, and spatial-mode realizations using waveguide arrays [18,21].

Our Letter focuses on non-Abelian physics in 1D and 2D systems, specifically braiding and 2D topological lattices, respectively. To access the richer topological phenomena available in higher dimensions [67], we can envisage the generalization of non-Abelian physics to three-dimensional systems. Although our design possesses geometrical constraints, this extension could be realized by exploiting the remaining degrees of freedom—multimode resonances—that enable synthetic-dimensional physics [26,68].