

Forced 3D Reconnection in an Exponentially Separating Magnetic Field

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We present a solvable scenario for 3D reconnection in a sheared magnetic field. We consider a localized external force that is applied slowly to a flux tube and then maintained, generating an Alfvénic perturbation that spreads along the field lines. Separation of the sheared field lines reduces the scale of the perturbation across the field, enhancing magnetic diffusion. For a fusion-motivated equilibrium with exponential field-line separation, we find a reconnection timescale proportional to $S/\ln S$ under magnetohydrodynamics (MHD) and to $S^{1/3}$ for semicollisional electron-only reconnection, where S is the Lundquist number of the perturbed flux tube. We generalize these results to arbitrary magnetic geometries, showing that the semicollisional case is geometry independent. Interestingly, we find that slower field-line separation yields an increased reconnection rate in MHD.

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Introduction—Magnetic reconnection—the topological reconfiguration of a magnetic field in a conducting medium—is a fundamental process of key importance in both laboratory and astrophysical plasmas. In fusion devices, reconnection can cause sawtooth crashes [1,2] and act as both a trigger [3] and saturation mechanism [4] for edge-localized modes (ELMs). In astrophysical and space plasmas, reconnection mediates and drives turbulence [5,6], accelerates particles [7], and enables solar flares [8]. While reconnection in 2D is enabled by the tearing instability [9,10] (particularly in its nonlinear [11] and/or kinetic [12] regimes), it has been suggested that alternative 3D mechanisms may facilitate faster reconnection [13–15]. In this Letter, we present a solvable case of 3D reconnection occurring via a 3D analog of resistive tearing. The process we consider occurs when a force is applied locally to a sheared flux tube and maintained. This induces a resistively damped Alfvén-wave packet that thins as it propagates along separating field lines. Because the packet is associated with a local change in the direction of the magnetic field, its resistive diffusion modifies the connectivity of the global field.

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Although we generalize to arbitrary equilibria at the end of the Letter, we develop and illustrate our results for a particular fusion-motivated equilibrium [Eq. (2)] with field lines that separate exponentially with the distance ℓ along them [Fig. 1(a)]. It has been suggested, and is perhaps intuitively compelling, that the rate of reconnection of exponentially separating field lines is fast, i.e., at most logarithmic in the Lundquist number S (the ratio of the ideal to nominal resistive timescales) [13,14]. It seems intuitive that this would be the case for the scenario described above: a diffusing Alfvén wave would be sheared to resistive scales in $\ln S$ Alfvén timescales. However, we find that the reconnection timescale remains proportional to a finite power of S (see abstract) because the field-line separation arrests propagation of the wave. Indeed, we find *faster* reconnection for field lines that separate more slowly (provided that they separate faster than $\ell^{1/2}$).

Clebsch coordinates—We represent the equilibrium magnetic field $\mathbf{B}_0$ with Clebsch coordinates ψ and α , i.e., $\mathbf{B}_0 = \nabla\psi \times \nabla\alpha$. We take ψ to have units of magnetic flux and α to be dimensionless. The Clebsch coordinates are constant along unperturbed field lines and can therefore be used to label them; the arc length ℓ along the field line completes the coordinate set [16]. We take the applied force $\mathbf{f}$ to be localized to, and directed along, a particular α surface (i.e., $\mathbf{f} \propto \mathbf{B}_0 \times \nabla\alpha$). The distance perpendicular to $\mathbf{B}_0$ between two field lines, (ψ, α) and $(\psi + d\psi, \alpha)$, in an α surface is

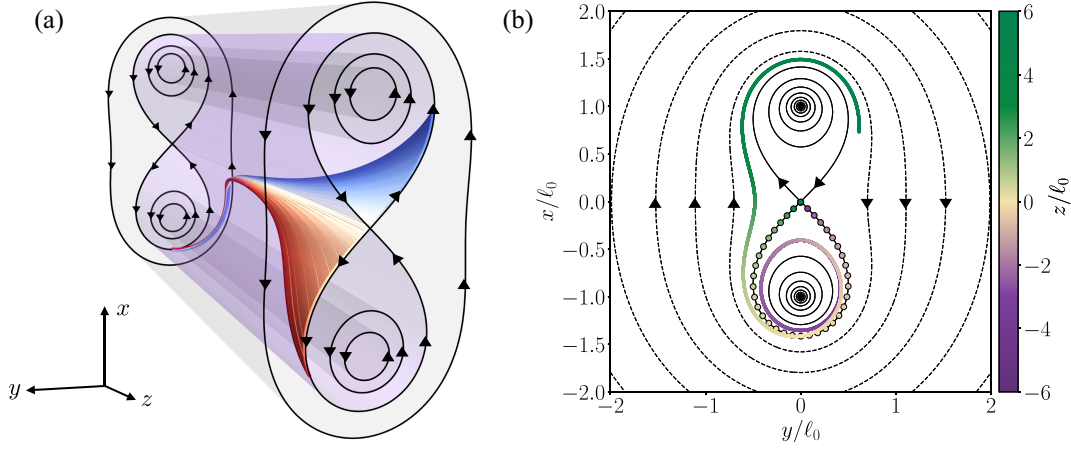


FIG. 1. Panel (a): The magnetic geometry considered in this Letter [Eq. (2)]. The α surface along which we consider displacing a field line is shown with ψ increasing from red to blue. Arrows show the projection of the magnetic-field direction onto the xy plane. Panel (b): Projection onto the xy plane of the field line through $(-\sqrt{2}a, 0, 0)$ at (i) $\tilde{t} = 0$ (dots with black outlines) and (ii) $\tilde{t} \gg \tilde{\tau}_{\text{rec}}$ (purple-green line). Note that the orientation of the y -axis in panel (b) is opposite to that in panel (a).

$$dl_{\perp} = \frac{|\nabla\alpha|}{B_0} d\psi, \quad (1)$$

$$|\nabla\alpha|^2 = \frac{\ell_0^2}{2a^4} \cosh\left(\frac{2z}{\ell_0}\right) + \frac{2a^2}{\ell_0^2} + \frac{36(z/\ell_0)^2}{\cosh(2z/\ell_0)} \quad (3)$$

so the rate of separation of the field lines is encoded in the dependence of $|\nabla\alpha|$ on ℓ . This dependence determines the reconnection rate in our theory. (Note that the coordinate α is not uniquely defined for given B_0 . Different definitions correspond to different applied forces, which may be associated with different reconnection rates.)

Model equilibrium—We consider as a model magnetic geometry the field of two wires, located at $x = \pm a$ and $y = 0$, each carrying current I_0 in the z direction. We embed this configuration in a uniform external field $B_{\text{ext}}\hat{z}$ so that, with $r^2 = x^2 + y^2$ and c the speed of light,

$$B_0 = B_{\text{ext}}\hat{z} + \hat{z} \times \frac{\nabla\psi}{a}, \quad \psi = \frac{aI_0}{c} \ln[(r^2 + a^2)^2 - 4a^2x^2]. \quad (2)$$

The equilibrium in Eq. (2) is visualized in Fig. 1(a). Toroidicity notwithstanding, each lobe resembles the magnetic field of a tokamak in a divertor configuration (with the lobes taken together, it resembles a “doublet” [17]). We expect the results of our Letter to be applicable to reconnection in such a device, forced, for example, by an ELM-control coil (whose effect on the topology of edge fields and transport may be crucial for confinement [18–20]).

We consider an odd-parity force that acts in the vicinity of the point $(-\sqrt{2}a, 0, 0)$ on the separatrix, inducing the reconnection of field lines passing through this region. As we explain in Sec. 1 of the End Matter, we may define α such that

on this field line, where $\ell(z)$ is the solution of

$$\frac{d\ell}{dz} = \frac{B_0}{B_{\text{ext}}} = \sqrt{1 + \frac{2a^2}{\ell_0^2 \cosh(2z/\ell_0)}}, \quad (4)$$

$\ell_0 = a^2 B_{\text{ext}}/4I_0$ is the magnetic-shear length (we take $\ell_0 = a$ in all numerical results presented in this Letter), and $B_0(\ell) = |B_0|$. This choice corresponds to a force in the $\hat{x}$ direction. According to Eqs. (1) and (3), field lines separate exponentially with ℓ as $\ell \rightarrow \infty$ [see Fig. 1(a)].

Dynamical model—We take the equilibrium magnetic field to be strong, and therefore, consider a perturbation that is extended along the field line. We consider two dynamical regimes: the *MHD case*, in which the perpendicular scale of the perturbation is much larger than the ion gyroradius ρ_i , and the *kinetic case*, in which it is much smaller than ρ_i . We employ the equations of low- β reduced MHD [21] to describe the MHD case and the semicollisional limit of the kinetic reduced electron heating model (KREHM) of Ref. [22] to describe the kinetic case (see Sec. 2 of the End Matter). The latter regime is interesting *a priori* because the parallel group velocity of *kinetic* Alfvén waves is an increasing function of their perpendicular wave-number [23]: the equilibrium shear causes them to accelerate and thus reach diffusive scales more quickly [24].

We assume eikonal forms for the velocity streamfunction Φ and perturbed magnetic-flux function Ψ , viz., $\Phi = \bar{\Phi}(\psi, \alpha, \ell, t)e^{ina}$ and $\Psi = \bar{\Psi}(\psi, \alpha, \ell, t)e^{ina}$, where $n \gg 1$ and $\bar{\Phi}$ and $\bar{\Psi}$ are slowly varying (i.e., n -independent) functions (cf. Refs. [25–28]). As we explain in Sec. 2 of the End Matter, this yields

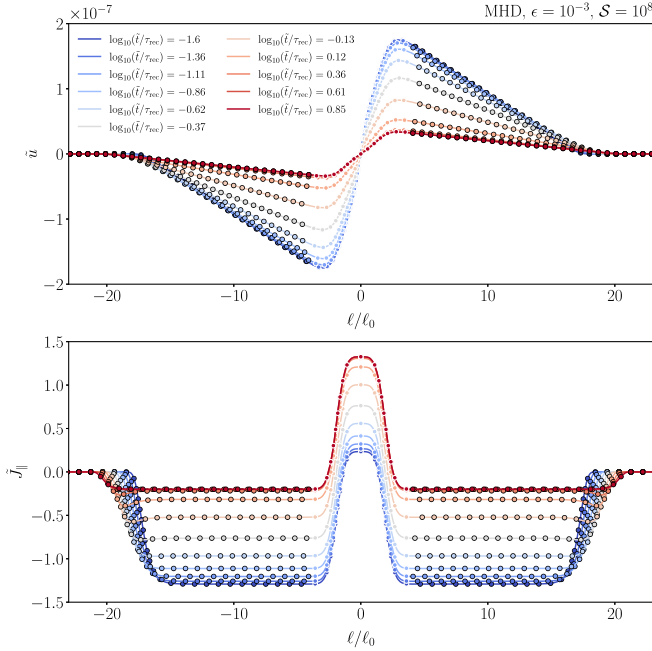


FIG. 2. Evolution of $\tilde{u}$ and $\tilde{J}_{\parallel}$ for $\tilde{t} \sim \tilde{\tau}_{\text{rec}}$ in the MHD case. Solid lines show numerical solutions of Eqs. (5) and (6) obtained with a finite-difference scheme. Dots show the theoretical prediction derived in the main text: dots with black outlines correspond to the outer solution [Eqs. (19) and (20) (corrected for the resistive diffusion of the inner solution, as described in Sec. 2 of the Supplemental Material [29])], dots with white outlines correspond to the inner solution [Eqs. (9) and (10)].

$$\left(\begin{cases} |\tilde{\nabla}\alpha|^2 & \text{(MHD)} \\ 1 & \text{(kinetic)} \end{cases} \right) \times \frac{1}{\tilde{B}_0(\tilde{\ell})^2} \frac{\partial \tilde{u}}{\partial \tilde{t}} = \frac{\partial \tilde{J}_{\parallel}}{\partial \tilde{\ell}} + \tilde{F}(\tilde{\ell}, \tilde{t}), \quad (5)$$

and

$$\frac{1}{|\tilde{\nabla}\alpha|^2} \frac{\partial \tilde{J}_{\parallel}}{\partial \tilde{t}} = \frac{1}{\tilde{B}_0(\tilde{\ell})} \frac{\partial [\tilde{B}_0(\tilde{\ell}) \tilde{u}]}{\partial \tilde{\ell}} - \frac{\tilde{J}_{\parallel}}{S}. \quad (6)$$

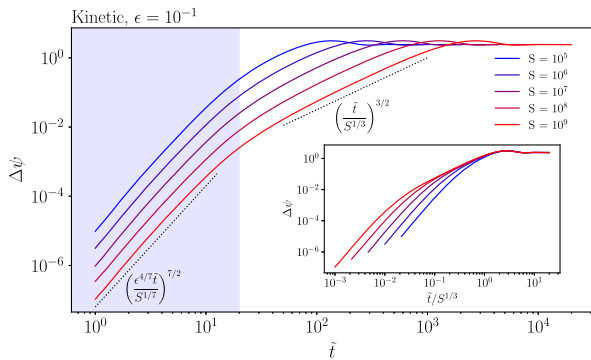
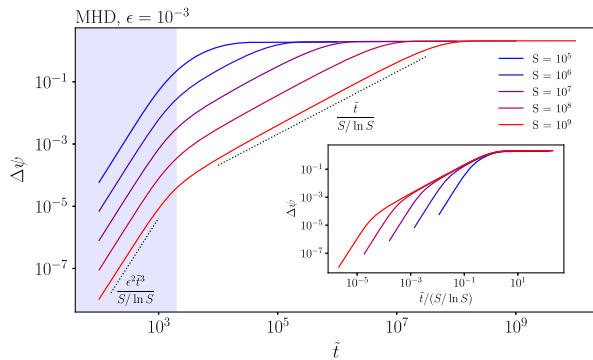


FIG. 3. Evolution of $\Delta\psi$ [Eq. (8)] as a function of time in the MHD (left) and kinetic (right) cases in the numerical solution of Eqs. (5) and (6). At $\tilde{t} \gg 1/\epsilon$, the evolution is as predicted by Eqs. (22) and (23) (see insets, which show time rescaled by the appropriate function of S). Blue regions indicate $\tilde{t} \lesssim \epsilon^{-1}$ when the force $\tilde{F}$ is increasing in size. The power laws for $\Delta\psi$ in these regions are derived in Sec. 3 of the Supplemental Material [29].

The variables that appear in Eqs. (5) and (6) are as follows. The velocity, directed parallel to the α surface, is $\delta u_{\perp} = \mathbf{b}_0 \times \nabla \Phi = u(\psi, \alpha, \ell, t) e^{i n \alpha} \mathbf{b}_0 \times \nabla \alpha$; $\delta B_{\perp} = \mathbf{b}_0 \times \nabla \Psi = \delta B_{\perp}(\psi, \alpha, \ell, t) B_0(\ell) e^{i n \alpha} \mathbf{b}_0 \times \nabla \alpha$ is the magnetic-field perturbation in the same surface; $\mathbf{b}_0(\ell) = \mathbf{B}_0/B_0$; and $J_{\parallel} = |\nabla \alpha|^2 \delta B_{\perp}$ is the perturbed parallel-current density. Tildes signify normalization to the dimensionally appropriate combination of the shear length ℓ_0 and the ideal timescale τ_A . The latter is the shear-length crossing time of, in the MHD case, an Alfvén wave (i.e., $\tau_A = \ell_0/v_{A,\text{ext}}$, with $v_{A,\text{ext}}$ the Alfvén speed associated with B_{ext}) or, in the kinetic case, a kinetic Alfvén wave (i.e., $\tau_A = \sqrt{2} \ell_0^2 / n \rho_i v_{A,\text{ext}}$). Similarly normalized, $\tilde{F}(\tilde{\ell}, \tilde{t})$ represents a force with the same eikonal dependence on α as Φ and Ψ , which pushes the field line along the α surface. We take $\tilde{F}$ to have odd parity in $\tilde{\ell}$ and to turn on smoothly on the (normalized) timescale $\epsilon^{-1} \gg 1$, reaching a constant value at $\tilde{t} \gg \epsilon^{-1}$. In the numerical results shown in Figs. 1(b), 2, and 3, we choose $\tilde{B}_0(\tilde{\ell})^2 \tilde{F}(\tilde{\ell}, \tilde{t}) = \tilde{\ell}^3 e^{-\tilde{\ell}^2} (1 - e^{-\epsilon^2 \tilde{t}^2})$, absorbing the amplitude of $\tilde{F}$ into the definitions of $\tilde{u}$, $\delta \tilde{B}_{\perp}$, and $\tilde{J}_{\parallel}$. We define the Lundquist number for the perturbed flux tube to be $S = \tau_A/\tau_{\eta}$, where $\tau_{\eta} = \ell_0^2/n^2\eta$ is the resistive timescale at the perpendicular scale ℓ_0/n . Thus,

$$S = \begin{cases} S/n^2 \sim 10^4 (100/n)^2 & \text{(MHD)} \\ \rho_{\star} S / \sqrt{2} n \sim 10^4 (100/n) & \text{(kinetic)}, \end{cases} \quad (7)$$

where $S = v_{A,\text{ext}} \ell_0 / \eta$ is the Lundquist number of the global equilibrium, η is the magnetic diffusivity, and $\rho_{\star} \equiv \rho_i / \ell_0$ is the normalized ion gyroradius. The numerical values quoted in Eq. (7) correspond to $S \sim 10^8$ and $\rho_{\star} \sim 10^{-2}$, which are typical for a tokamak.

Equations (5) and (6) are linear in $\tilde{u}$ and $\tilde{J}_{\parallel}$ even though we have not required their amplitudes to be small compared with their perpendicular wavelengths. In this sense, the solutions we derive are valid nonlinearly [30].

Reconnected flux—In the absence of the resistive final term, Eq. (6) conserves

$$\Delta\psi = \int_{-\infty}^{\infty} d\tilde{\ell} \frac{\tilde{B}_0(\tilde{\ell})}{|\tilde{\nabla}\alpha|^2} \tilde{J}_{\parallel} = \int_{-\infty}^{\infty} d\tilde{\ell} \tilde{B}_0(\tilde{\ell}) \delta\tilde{B}_{\perp}. \quad (8)$$

$\Delta\psi$ is the difference (“step”) in the coordinate ψ of the perturbed field line between $\tilde{\ell} = -\infty$ and $\tilde{\ell} = +\infty$: $\Delta\psi \neq 0$ implies reconnection of field lines from inside the separatrix with those outside it [see Fig. 1(b)]. In a fusion device, this would cause the loss of confinement of hot plasma from inside the separatrix by streaming. The total flux reconnected through the separatrix [see Fig. 1(b)] is $\int d\alpha d\psi \sim \Delta\psi/n$. Our goal is to determine the evolution of $\Delta\psi$ for finite $S \gg 1$.

Late-time diffusive solutions—We consider the late-time ($\epsilon\tilde{t} \gg 1$) evolution of a diffusively spreading pulse induced by the force $\tilde{F}$. We plot numerical solutions of Eqs. (5) and (6) in Fig. 2. In what follows, we determine these solutions and their reconnection rates (Fig. 3) for $S \gg 1$ analytically, using matched asymptotic expansions. There are two relevant solution domains: an inner region, $\tilde{\ell} \ll \ln S$, where inertia [left-hand side of Eq. (5)] is negligible, and an outer region, $\tilde{\ell} \gg \ln(\ln S)$, where induction [first term in Eq. (6)] is. We justify neglecting these terms in the above-stated domains *a posteriori*—see Sec. 1 of the Supplemental Material [29].

MHD case: (i) Inner region ($\tilde{\ell} \ll \ln S$): Neglecting inertia, Eq. (5) yields

$$\tilde{J}_{\parallel\text{in}}(\tilde{\ell}, \tilde{t}) = - \int_{-\infty}^{\tilde{\ell}} d\tilde{\ell}' \tilde{F} + \tilde{J}_{\parallel 0}(\tilde{t}); \quad (9)$$

substituting this into Eq. (6) yields, for $\epsilon\tilde{t} \gg 1$,

$$\tilde{u}_{\text{in}} = \frac{1}{\tilde{B}_0} \int_0^{\tilde{\ell}} d\tilde{\ell}' \frac{\tilde{B}_0}{S} \left(\frac{S}{|\tilde{\nabla}\alpha|^2} \frac{d\tilde{J}_{\parallel 0}}{d\tilde{t}} + \tilde{J}_{\parallel 0} - \int_{-\infty}^{\tilde{\ell}'} d\tilde{\ell}'' \tilde{F} \right). \quad (10)$$

The inner solution depends on a single unknown function of time, $\tilde{J}_{\parallel 0}(\tilde{t})$, which we shall determine by matching to the outer solution. We can deduce the value of $\tilde{J}_{\parallel 0}(\tilde{t})$ at early times by assuming that the forcing-ramp-up time ϵ^{-1} is small compared with the normalized reconnection time-scale $\tilde{\tau}_{\text{rec}}$, whatever the latter turns out to be (we show how to relax this restriction in Sec. 3 of the Supplemental Material [29]). In that case, $\Delta\psi$ is unchanged between $\tilde{t} = 0$ and $\epsilon^{-1} \ll \tilde{t} \ll \tilde{\tau}_{\text{rec}}$. At such times, Eqs. (8) and (9) require that

$$\tilde{J}_{\parallel 0}(\tilde{t}) = \mathcal{J}_0 \equiv |\Delta'| \int_{-\infty}^{\infty} d\tilde{\ell} \frac{\tilde{B}_0(\tilde{\ell})}{|\tilde{\nabla}\alpha|^2} \int_{-\infty}^{\tilde{\ell}} d\tilde{\ell}' \lim_{\tilde{t} \rightarrow \infty} \tilde{F}, \quad (11)$$

where $1/|\Delta'| \equiv \int_0^{\infty} d\tilde{\ell} \tilde{B}_0(\tilde{\ell})/|\tilde{\nabla}\alpha|^2$.

MHD case: (ii) Outer region [$\tilde{\ell} \gg \ln(\ln S)$]: Dropping $\tilde{F}$ in Eq. (5) (as $\tilde{\ell} \gg 1$) and neglecting induction in Eq. (6), $\tilde{u}_{\text{out}}$ satisfies the diffusion equation

$$\frac{\partial \tilde{u}_{\text{out}}}{\partial \tilde{t}} = \frac{S}{|\tilde{\nabla}\alpha|^2} \frac{\partial^2 \tilde{u}_{\text{out}}}{\partial \tilde{\ell}^2}. \quad (12)$$

We now specialize to the equilibrium [Eq. (2)]. For $\tilde{\ell} \gg 1$, Eq. (3) reads $|\tilde{\nabla}\alpha|^2 = \Lambda e^{2\tilde{\ell}}$, where $\Lambda = \exp[2\lim_{\tilde{\ell} \rightarrow \infty} (\tilde{\ell}(\tilde{z}) - \tilde{z})]/4$ is obtained from Eq. (4). Equation (12) then admits a similarity variable, $\xi = \Lambda e^{2\tilde{\ell}}/4S\tilde{t}$. Changing variables from $\tilde{\ell}$ to ξ , Eq. (12) becomes

$$\tilde{t} \frac{\partial \tilde{u}_{\text{out}}(\xi, \tilde{t})}{\partial \tilde{t}} = \xi \frac{\partial^2 \tilde{u}_{\text{out}}(\xi, \tilde{t})}{\partial \xi^2} + (1 + \xi) \frac{\partial \tilde{u}_{\text{out}}(\xi, \tilde{t})}{\partial \xi}, \quad (13)$$

of which the series

$$\tilde{u}_{\text{out}}(\xi, \tilde{t}) = \sum_n \tilde{t}^{p_n} f_n(\xi) \quad (14)$$

is a solution if $f_n(\xi)$ satisfies

$$\xi \frac{d^2 f_n}{d\xi^2} + (1 + \xi) \frac{df_n}{d\xi} - p_n f_n = 0, \quad \forall n. \quad (15)$$

The solutions of Eq. (15) that vanish at $\xi \rightarrow \infty$ are $f_n(\xi) = C_n e^{-\xi} U(1 + p_n, 1, \xi)$, where C_n is a constant and U is Tricomi’s confluent hypergeometric function.

MHD case: (iii) Matching solutions: Both the inner and outer solutions are valid for $1 \ll \ln(\ln S) \ll \tilde{\ell} \ll \ln S$ and must, therefore, match there. In this range, $\tilde{u}$ decreases linearly with $\tilde{\ell}$, i.e., $\tilde{u} \rightarrow c_1(\tilde{t}) - c_2(\tilde{t})\tilde{\ell}$ (Fig. 2). Apart from at very late times $\tilde{t} \gtrsim S$ [see “MHD case (v)”], only the first term in Eq. (10) for $\tilde{u}_{\text{in}}$ contributes to c_1 ; the other two (resistive) terms determine c_2 . From the outer solution, $c_1 = \lim_{\tilde{\ell} \rightarrow 0} \tilde{u}_{\text{out}}$. Equating the two expressions for c_1 , the matching condition is

$$\lim_{\tilde{\ell} \rightarrow 0} \tilde{u}_{\text{out}} = \frac{S}{|\Delta'|} \frac{d}{d\tilde{t}} \lim_{\tilde{\ell} \rightarrow 0} \frac{\partial \tilde{u}_{\text{out}}}{\partial \tilde{\ell}}, \quad (16)$$

where we have used $\tilde{J}_{\parallel 0} = S \partial \tilde{u}_{\text{out}} / \partial \tilde{\ell}$ in the matching region. Using the above-stated solutions of Eq. (15) and standard results for limits of Tricomi’s function, Eq. (14) implies that

$$\lim_{\xi \rightarrow 0} \tilde{u}_{\text{out}} = - \sum_n \frac{C_n \tilde{t}^{p_n} \ln(\Theta_n \xi)}{\Gamma(1 + p_n)} + O(\xi), \quad (17)$$

where $\Theta_n \equiv e^{2\gamma_E + F(1+n)}$, γ_E is the Euler-Mascheroni constant, and F is the digamma function. For $\tilde{t} \ll S$, the

logarithm in Eq. (17) is constant and approximately equal to $\ln(\Lambda/4S)$. Equations (16) and (17) together require that $p_{n+1} = p_n + 1$, $C_{n+1} = -C_n/\tilde{\tau}_{\text{rec}}$, where

$$\tilde{\tau}_{\text{rec}} = \frac{2S}{|\Delta'| \ln(4S/\Lambda)}. \quad (18)$$

Finally, Eq. (11) requires $p_0 = 0$, $C_0 = -\mathcal{J}_0/2S$, whence

$$\tilde{u}_{\text{out}} = -\frac{\mathcal{J}_0}{2S} \sum_{n=0}^{\infty} (-1)^n e^{-\xi} U(1+n, 1, \xi) \left(\frac{\tilde{t}}{\tilde{\tau}_{\text{rec}}} \right)^n, \quad (19)$$

$$\tilde{J}_{\parallel \text{out}} = \mathcal{J}_0 \sum_{n=0}^{\infty} (-1)^n \xi e^{-\xi} U(1+n, 2, \xi) \left(\frac{\tilde{t}}{\tilde{\tau}_{\text{rec}}} \right)^n. \quad (20)$$

The inner solution is given by Eqs. (9) and (10), with $\tilde{J}_{\parallel 0}$ the $\xi \rightarrow 0$ limit of Eq. (20), i.e.,

$$\tilde{J}_{\parallel 0} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \left(\frac{\tilde{t}}{\tilde{\tau}_{\text{rec}}} \right)^n = \mathcal{J}_0 \exp \left(-\frac{\tilde{t}}{\tilde{\tau}_{\text{rec}}} \right). \quad (21)$$

Equations (9), (10), (19), and (20) are as we obtain from the numerical solution of Eqs. (5) and (6) (see Fig. 2). Qualitatively, the solution for $\tilde{t} \ll \tilde{\tau}_{\text{rec}}$ [the $n = 0$ terms in Eqs. (19) and (20)] is a self-similar spreading with similarity variable ξ . The solution is diffusive, but it corresponds to negligible reconnection [Eq. (22)] until the current starts to decay at $\tilde{t} \sim \tilde{\tau}_{\text{rec}}$, which is when the propagating front reaches $\tilde{\ell} \sim \ln S$ (corresponding to $\xi \sim 1$).

MHD case: (iv) Reconnection rate: It follows from Eqs. (8), (9), and (21) that

$$\Delta\psi = \frac{1}{|\Delta'|} (\tilde{J}_{\parallel 0} - \mathcal{J}_0) = -\frac{\mathcal{J}_0}{|\Delta'|} \left[1 - \exp \left(-\frac{\tilde{t}}{\tilde{\tau}_{\text{rec}}} \right) \right], \quad (22)$$

which confirms that $\tilde{\tau}_{\text{rec}}$ [Eq. (18)] is the reconnection timescale [in units of τ_A , defined below Eq. (6)]. Equation (22) matches our numerical solution, as shown in Fig. 3.

MHD case: (v) Matching solutions, $\tilde{t} \gtrsim S$: Combining Eqs. (10) and (21), we find that the contribution of the final term (involving $\tilde{F}$) in Eq. (10) to $c_1(\tilde{t})$ is negligible only for $\tilde{t} \ll S$ —at later times, this term must also be included in the matching condition, Eq. (16). Physically, at $\tilde{t} \sim S$, resistivity can no longer be neglected at the location of the force, and the flux tube begins to “slip” diffusively there. We derive the corresponding solution in Sec. 2 of the Supplemental Material [29] and [31]. However, because $\tilde{\tau}_{\text{rec}} \ll S$ as $S \rightarrow \infty$, the solution in the preceding sections is the one of primary interest.

Kinetic case: The derivation of the reconnection rate in the kinetic case of Eq. (5) is analogous to the MHD case; we present it in Sec. 3 of the End Matter. The chief difference is that, unlike Eq. (12), the diffusion equation for

the outer region in the kinetic case [Eq. (C1)] is independent of $|\nabla\alpha|$, so the diffusion coefficient is not suppressed by field-line separation. We find [cf. Eq. (22)]

$$\Delta\psi = -\frac{\mathcal{J}_0}{|\Delta'|} \left[1 - E_{3/2} \left(-\left(\frac{\tilde{t}}{\tilde{\tau}_{\text{rec}}} \right)^{3/2} \right) \right], \quad (23)$$

where $\tilde{\tau}_{\text{rec}} = (S/|\Delta'|^2)^{1/3}$ and $E_{3/2}(x)$ is the Mittag-Leffler function, a generalization of the exponential function defined by $E_\alpha(x) = \sum_{k=0}^{\infty} x^k / \Gamma(1 + \alpha k)$.

Generalization to other geometries—As explained above, the outer-region diffusion equation is independent of $|\nabla\alpha|(\ell)$ in the kinetic case. The solution, Eq. (23), is therefore valid for *any* magnetic geometry, not just the one described by Eq. (2). By contrast, $|\nabla\alpha|$ does appear in Eq. (13) (MHD case). For an equilibrium with $|\nabla\alpha| \propto \tilde{\ell}^\delta$ for $\tilde{\ell} \gg 1$, it is readily verified that Eq. (13) has a similarity variable $\xi = \tilde{\ell}/(S\tilde{t})^{1/2(\delta+1)}$. It follows that [32]

$$\Delta\psi \rightarrow \left(\frac{\tilde{t}}{\tilde{\tau}_{\text{rec}}} \right)^{\frac{\delta+3/2}{\delta+1}}, \quad \tilde{\tau}_{\text{rec}} \propto \frac{S^{(2\delta+1)/(2\delta+3)}}{|\Delta'|^{2(\delta+1)/(2\delta+3)}} \quad (24)$$

as $\tilde{t}/\tilde{\tau}_{\text{rec}} \rightarrow 0$. Equation (24) implies that MHD reconnection becomes *slower* as the rate of field-line separation increases (i.e., as δ increases). In the case of linear shear, i.e., $\delta = 1$ (as for field lines *not* on the separatrix), $\tilde{\tau}_{\text{rec}} \propto S^{3/5}$ [33]. Faster reconnection occurs for field lines that separate more slowly (decreasing δ), but only up to $\delta = 1/2$, as then, $|\Delta'|$ diverges [see its definition below Eq. (11)]. Physically, $\delta = 1/2$ is the smallest value of δ for which the diffusive solution can be causally connected to the origin at $t \sim \tilde{\tau}_{\text{rec}}$ [34]. The shortest possible MHD-reconnection timescale in our theory as $S \rightarrow \infty$ is, therefore, $\tilde{\tau}_{\text{rec}} \sim S^{1/2}/|\Delta'|^{3/4}$, corresponding to $\delta \rightarrow 1/2^+$.

Conclusion—We have derived nonlinear solutions for 3D MHD and electron-only reconnection in a sheared magnetic field. In our solutions, reconnection is induced by diffusing Alfvén- and semicollisional kinetic-Alfvén-wave packets generated by an external force applied gradually to exponentially separating field lines. Their diffusion changes the connectivity of the global field in timescales $\propto S/\ln S$ and $\propto S^{1/3}$, respectively. We expect our results to be quantitatively applicable to reconnection at separatrices, as in tokamak-edge-field stochasticization by an ELM-control coil, although we note that the anisotropic ordering $k_{\parallel} \gg k_{\perp}$ necessary for a tractable theory is not rigorously applicable in that case. Qualitatively, our Letter provides a solvable case of 3D magnetic reconnection, with the interesting and unexpected property that the reconnection rate is greater for field lines that diverge more slowly [Eq. (24)] for MHD dynamics and is geometry independent in the case of semicollisional electron-only reconnection [22].

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- [29] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/rd5d-9317> for *a posteriori* justification of terms neglected in our asymptotic analysis (Sec. 1); late-time corrections to the MHD reconnection rate arising from resistive slippage at the origin (Sec. 2); and the early-time evolution of the reconnected flux before the applied force reaches its final, maintained value (Sec. 3).
- [30] We require amplitudes to be small compared with the equilibrium scales in order for Eq. (3) to be valid for the perturbed magnetic field.
- [31] Qualitatively, the effect of the modification is that decay of $\tilde{u}_{\text{out}}$ and $\tilde{J}_{\parallel \text{out}}$ is arrested at a nonzero value.
- [32] In deriving these expressions, we have used the relations $\tilde{J}_{\parallel 0} = \text{Slim}_{\ell \rightarrow 0} \partial \tilde{u}_{\text{out}} / \partial \tilde{\ell}$, $d\tilde{J}_{\parallel 0} / d\tilde{t} = \lim_{\ell \rightarrow 0} \tilde{u}_{\text{out}}$, and

$\lim_{\tilde{t} \rightarrow 0} \tilde{u}_{\text{out}} \propto \tilde{t}^{p_0} f_0(\xi)$ [Eq. (14)], which together imply that $d \ln J_{\parallel 0} / d\tilde{t} \propto \mathcal{S}^{-(1+2\delta)/2(1+\delta)} \tilde{t}^{1/2(\delta+1)}$ as $\tilde{t} \rightarrow 0$.

- [33] This dependence was inevitable as, in the case of linear shear, Eqs. (5) and (6) are isomorphic to the 2D tearing-mode equations [25].
- [34] In the dimensionless units employed in the Letter, the distance that the diffusive pulse travels before reconnection

occurs is $(\mathcal{S}\tilde{\tau}_{\text{rec}})^{1/2(\delta+1)} \sim \mathcal{S}^{2/(2\delta+3)}$ (obtained by setting $\xi \sim 1$ at $t = \tilde{\tau}_{\text{rec}}$, with $|\Delta'| \sim 1$), while the distance that an Alfvén wave propagates in the same time, i.e., the causal distance, is $\tilde{\tau}_{\text{rec}} \sim \mathcal{S}^{(2\delta+1)/(2\delta+3)}$. The reconnecting solution cannot be established if the former distance is greater than the latter, which occurs for $\delta < 1/2$.

End Matter

Two-wire magnetic-field geometry—In this section, we derive Eqs. (3) and (4) from the main text, which give the dependence of $|\nabla\alpha|^2$ and B_0 on the field-line distance ℓ for the case of the field [Eq. (2)]. The ℓ dependence of $|\nabla\alpha|^2$ determines the reconnection rate in our theory [see Eq. (1)].

From the vector product of $\mathbf{B}_0 = \nabla\psi \times \nabla\alpha$ with $\nabla\psi$,

$$\nabla\alpha = \frac{\mathbf{B}_0 \times \nabla\psi}{|\nabla\psi|^2} + \lambda \nabla\psi, \quad \lambda = - \int_L^\ell \frac{d\ell'}{B_0} \mathcal{I}(\psi, \alpha, \ell'), \quad (\text{A1})$$

where the lower limit $L = L(\alpha, \psi)$ is arbitrary and

$$\mathcal{I}(\psi, \alpha, \ell) = \frac{\mathbf{B}_0 \times \nabla\psi}{|\nabla\psi|^2} \cdot \nabla \times \frac{\mathbf{B}_0 \times \nabla\psi}{|\nabla\psi|^2} \quad (\text{A2})$$

is the local shear. It follows from Eq. (A1) that

$$|\nabla\alpha|^2 = \frac{B_0^2}{|\nabla\psi|^2} + \lambda^2 |\nabla\psi|^2. \quad (\text{A3})$$

We now evaluate Eq. (A3) on the separatrix of the magnetic field [Eq. (2)]. Substituting Eq. (2) into Eq. (A2), we obtain

$$\mathcal{I} = - \frac{B_{\text{ext}} \nabla\psi \cdot \nabla |\nabla\psi|^2}{a |\nabla\psi|^4} = - \frac{\ell_0}{a^4} \left(3 + \frac{(a^4 - \chi)}{r^4} \right), \quad (\text{A4})$$

where $\chi \equiv \exp(c\psi/aI_0) = (r^2 + a^2)^2 - 4a^2x^2$, and we have used the fact that $|\nabla\chi|^2 = 16r^2\chi$. On the separatrix, where $\chi = a^4$, $\mathcal{I} = -3\ell_0/a^4$, whence

$$\frac{|\nabla\psi|^2}{B_{\text{ext}}^2} = \frac{a^2 r^2}{\ell_0^2}, \quad \frac{B_0^2}{B_{\text{ext}}^2} = 1 + \frac{r^2}{\ell_0^2}, \quad \lambda = \frac{3\ell_0 z}{B_{\text{ext}} a^4}. \quad (\text{A5})$$

It remains to determine $r(\ell)$. Because χ is constant along field lines, the x and y components of field-line trajectories are $x(r, \chi) = \pm [1/(2a)] \{ \sqrt{[(r^2 + a^2)^2 - \chi]} \}$ and $y(r, \chi) = \pm [1/(2a)] \{ \sqrt{[\chi - (r^2 - a^2)^2]} \}$, respectively. We determine the field-line trajectory in z by integrating $(\partial z / \partial r)_{\psi, \alpha} = (\mathbf{B}_0 \cdot \nabla z) / (\mathbf{B}_0 \cdot \nabla r)$ to yield (in the quadrant $y > 0$, $x < 0$)

$$\frac{z - a\alpha}{2I_0 a^2} = \frac{\chi}{a^4} \int_{r_0}^r \frac{r dr}{\sqrt{[(r^2 + a^2)^2 - \chi]} [\chi - (r^2 - a^2)^2]}. \quad (\text{A6})$$

We have fixed the constant of integration in Eq. (A6) by choosing $\alpha = z/a$ on the semi-infinite surface $y = 0$, $x < -a$ [see Fig. 1(b)]. In general, Eq. (2) requires $a\alpha = z + g(\chi, r) + f(\chi)$, where $g(\chi, r)$ is the function that generates the constant field $B_{\text{ext}} \hat{z}$ and $f(\chi)$ is arbitrary—our choice corresponds to $f(\chi) = -g(\chi, r_0(\chi))$ with $r_0(\chi) = \sqrt{a^2 + \sqrt{\chi}}$. Evaluating the integral [Eq. (A6)] with $\chi = a^4$ and (without loss of generality) $\alpha = 0$, we find

$$r^2 = \frac{2a^2}{\cosh(2z/\ell_0)}. \quad (\text{A7})$$

Equations (3) and (4) in the main text follow from substitution of Eq. (A7) into Eqs. (A5) and (A3).

Dynamical model—In this section, we derive Eqs. (5) and (6) from the main text, starting from the semicollisional kinetic reduced electron heating model (KREHM), a subsidiary limit of gyrokinetics derived by Ref. [22]. Semicollisional KREHM describes low-frequency (compared with the ion gyrofrequency), anisotropic ($k_{\parallel}/k_{\perp} \ll 1$, with $k_{\parallel}$ and $k_{\perp}$ the wavenumber along and across the equilibrium magnetic field, respectively) dynamics of a semicollisional plasma (collisional electrons but collisionless ions) at small plasma β and large ion-to-electron temperature ratio. In such a plasma, ions have no mean flow, and the magnetic field is advected by the $\mathbf{E} \times \mathbf{B}$ motion of electrons. The equations of semicollisional KREHM are [22]

$$\frac{\partial}{\partial t} \hat{G}\Phi + \{\Phi, \hat{G}\Phi\} = v_A(\ell) \frac{\partial}{\partial \ell} \nabla_{\perp}^2 \Psi + \{\Psi, \nabla_{\perp}^2 \Psi\} \quad (\text{B1})$$

$$\frac{\partial}{\partial t} \Psi + \{\Phi, \Psi\} = \frac{\partial}{\partial \ell} [v_A(\ell) \Phi] + \eta \nabla_{\perp}^2 \Psi, \quad (\text{B2})$$

where Φ is the streamfunction for the velocity field perpendicular to the local unperturbed magnetic field, $\mathbf{u}_{\perp} = \mathbf{b}_0 \times \nabla_{\perp} \Phi$; Ψ is the flux function for the magnetic perturbation, $\delta \mathbf{B}_{\perp} = \sqrt{4\pi\rho_0} \mathbf{b}_0 \times \nabla_{\perp} \Psi$; $v_A(\ell) = B_0(\ell) / \sqrt{4\pi\rho_0}$ is the local Alfvén speed associated with the unperturbed field strength $B_0(\ell)$ and density ρ_0 ; the Poisson bracket is $\{\Phi, \Psi\} \equiv \mathbf{b}_0 \cdot (\nabla_{\perp} \Phi \times \nabla_{\perp} \Psi)$; $\mathbf{b}_0$ is the unit vector in the direction of $\mathbf{B}_0$ [Eq. (B5)];

$\hat{G} = 2(\hat{\Gamma}_0 - 1)/\rho_i^2$, where ρ_i is the ion gyroradius; $\hat{\Gamma}_0$ is the operator whose representation in Fourier space is as multiplication by $\Gamma_0(\alpha_i) \equiv I_0(\alpha_i)e^{-\alpha_i}$, where $\alpha_i \equiv k_\perp^2 \rho_i^2/2$; and I_0 is the modified Bessel function of the first kind.

In this Letter, we focus on the limits of small and large $k_\perp \rho_i$, i.e., dynamics above and below the ion gyroscale, respectively. In the former case, $\hat{\Gamma}_0 = k_\perp^2 \rho_i^2/2$. It follows that $\hat{G} = \nabla_\perp^2$, whence Eqs. (B1) and (B2) become the equations of reduced MHD [21]. We refer to this limit as the MHD case in the main text. In the opposite limit $k_\perp \rho_i \gg 1$ (the “kinetic” case in the main text), Γ_0 is exponentially small, and $\hat{G} = -2/\rho_i^2$ becomes a multiplication by a constant.

We make the ansatz

$$\Phi = \bar{\Phi}(\psi, \alpha, \ell, t)e^{ina}, \quad \Psi = \bar{\Psi}(\psi, \alpha, \ell, t)e^{ina} \quad (\text{B3})$$

for the solution of Eqs. (B1) and (B2), where $n \gg 1$ enforces narrow localization in the $\nabla\alpha$ direction, while $\bar{\Phi}$ and $\bar{\Psi}$ encode slow (n -independent) variation of the perturbation with ψ and α . Equation (B3) fixes the polarization of the wave to be in the $\mathbf{b}_0 \times \nabla\alpha$ direction; it can be changed by redefining α to incorporate a shift by a function of ψ (which leaves $\mathbf{B}_0$ unchanged).

The Poisson brackets in Eqs. (B1) and (B2) vanish after substitution of Eq. (B3), because the perpendicular gradients of Φ and Ψ are both along $\nabla\alpha$. Thus, any solution in the form of Eq. (B3) is a nonlinear solution of Eqs. (B1) and (B2). Substituting Eq. (B3) into Eqs. (B1) and (B2), and taking $\mathbf{b}_0 \times \nabla_\perp$ of the result, yields

$$G\left(\frac{n^2|\nabla\alpha|^2\rho_i^2}{2}\right)\frac{\partial u_\perp}{\partial t} = -n^2v_A^2\frac{\partial}{\partial\ell}(|\nabla\alpha|^2\delta B_\perp), \quad (\text{B4})$$

$$\frac{\partial\delta B_\perp}{\partial t} + \eta n^2|\nabla\alpha|^2\delta B_\perp = \frac{1}{B_0}\frac{\partial(B_0 u_\perp)}{\partial\ell}, \quad (\text{B5})$$

where we have defined $u_\perp$ and $\delta B_\perp$ such that

$$\delta\mathbf{u}_\perp = u_\perp(\psi, \alpha, \ell, t)e^{ina}\mathbf{b}_0 \times \nabla\alpha, \quad (\text{B6})$$

$$\delta\mathbf{B}_\perp = \delta B_\perp(\psi, \alpha, \ell, t)B_0e^{ina}\mathbf{b}_0 \times \nabla\alpha, \quad (\text{B7})$$

and G is the function satisfying $\hat{G}\Phi = G\Phi$, given by

$$G\left(\frac{n^2|\nabla\alpha|^2\rho_i^2}{2}\right) = \frac{2}{\rho_i^2}\left[\Gamma_0\left(\frac{n^2|\nabla\alpha|^2\rho_i^2}{2}\right) - 1\right] \\ \rightarrow \begin{cases} -n^2|\nabla\alpha|^2 & \text{as } n^2|\nabla\alpha|^2\rho_i^2/2 \rightarrow 0, \\ -2/\rho_i^2 & \text{as } n^2|\nabla\alpha|^2\rho_i^2/2 \rightarrow \infty. \end{cases} \quad (\text{B8})$$

Equations (B4) and (B5) are identical to Eqs. (5) and (6) after normalizing all variables as described in the main text and appending to them the forcing term in Eq. (5).

Kinetic case—Here, we present the derivation of the reconnection rate for the kinetic case, Eq. (23). The methodology is the same as for the MHD case considered in the main text.

Kinetic case: (i) Inner region ($\tilde{\ell} \ll \mathcal{S}^{2/3}$): We neglect inertia in the inner region (justified *a posteriori* in Sec. 1 of the Supplemental Material [29]). This being the only term in which the kinetic and MHD cases of Eq. (5) differ, Eqs. (9) and (10) remain valid for the kinetic case.

Kinetic case: (ii) Outer region ($\tilde{\ell} \gg \ln \mathcal{S}$): As justified *a posteriori* in Sec. 1 of the Supplemental Material [29], we neglect the forcing term in Eq. (5) and the induction term in Eq. (6). Then, $\tilde{u}_{\text{out}}$ obeys the diffusion equation [cf. Eq. (12) for the MHD case]:

$$\frac{\partial \tilde{u}_{\text{out}}}{\partial \tilde{t}} = \mathcal{S} \frac{\partial^2 \tilde{u}_{\text{out}}}{\partial \tilde{\ell}^2}. \quad (\text{C1})$$

We recast Eq. (C1) in a form similar to Eq. (13) by changing variables from $\tilde{\ell}$ to a new similarity variable, $\xi = \tilde{\ell}/(4\mathcal{S}\tilde{t})^{1/2}$, which yields

$$\tilde{t} \frac{\partial \tilde{u}_{\text{out}}}{\partial \tilde{t}} = \frac{1}{4} \frac{\partial^2 \tilde{u}_{\text{out}}}{\partial \xi^2} + \frac{1}{2} \xi \frac{\partial \tilde{u}_{\text{out}}}{\partial \xi}. \quad (\text{C2})$$

The solutions of the ordinary differential equations that result from substituting Eq. (14) into Eq. (C2) are $f_n(\xi) = C_n e^{-\xi^2} U((1+2p_n)/2, 1/2, \xi^2)$.

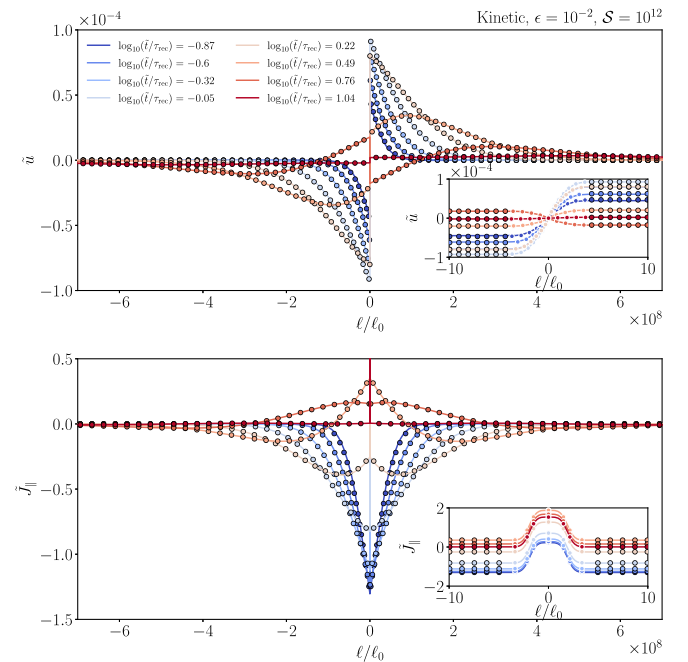


FIG. 4. As Fig. 2, but for the kinetic case.

Kinetic case: (iii) Matching solutions: Because Eq. (6) is the same in the MHD and kinetic cases, Eq. (16) is the matching condition for both [see below Eq. (C7) for justification of the neglect of the second two terms in Eq. (10) in forming the matching condition]. From the small-argument limit of Tricomi's function, we have

$$\lim_{\xi \rightarrow 0} \tilde{u}_{\text{out}} = \sqrt{\pi} \sum_n C_n \tilde{t}^{p_n} \left[\frac{1}{\Gamma(1+p_n)} - \frac{2\xi}{\Gamma(1/2+p_n)} \right] \quad (\text{C3})$$

up to $O(\xi^2)$. Equation (16) then requires that $p_{n+1} = p_n + 3/2$ and gives the recursion relation $C_{n+1} = -|\Delta'|/S^{1/2} C_n$, so $C_n = (-|\Delta'|/S^{1/2})^n C_0$. Matching to the early-time solution, Eq. (11), gives $p_0 = 1/2$, $C_0 = -\mathcal{J}/\sqrt{\pi} S^{1/2}$, whence,

$$\tilde{u}_{\text{out}} = -\mathcal{J}_0 \sqrt{\frac{\tilde{t}}{\pi S}} \sum_{n=0}^{\infty} (-1)^n e^{-\xi^2} U_{n,1/2}(\xi) \left(\frac{\tilde{t}}{\tilde{\tau}_{\text{rec}}} \right)^{3n/2}, \quad (\text{C4})$$

$$\tilde{J}_{\parallel \text{out}} = \frac{\mathcal{J}_0}{\sqrt{\pi}} \sum_{n=0}^{\infty} (-1)^n e^{-\xi^2} \xi U_{n,3/2}(\xi) \left(\frac{\tilde{t}}{\tilde{\tau}_{\text{rec}}} \right)^{3n/2}, \quad (\text{C5})$$

where

$$\tilde{\tau}_{\text{rec}} = \left(\frac{S}{|\Delta'|^2} \right)^{1/3}, \quad U_{n,\alpha} \equiv U(1+3n/2, \alpha, \xi^2). \quad (\text{C6})$$

These solutions are as we find numerically (Fig. 4). Qualitatively, both $\tilde{u}_{\text{out}}$ and $\tilde{J}_{\parallel \text{out}}$ spread self-similarly for $\tilde{t} \ll \tilde{\tau}_{\text{rec}}$, with $\tilde{u}_{\text{out}}$ also growing like $\tilde{t}^{1/2}$. Reconnection occurs when the current begins to decay at $\tilde{t} \sim \tilde{\tau}_{\text{rec}}$, which is when the propagating front reaches $\tilde{\ell} \sim S^{2/3}$. The $\xi \rightarrow 0$ limit of Eq. (C5) yields

$$\tilde{J}_{\parallel 0} = \mathcal{J}_0 E_{3/2} \left[-(\tilde{t}/\tilde{\tau}_{\text{rec}})^{3/2} \right], \quad (\text{C7})$$

where $E_{3/2}(x)$ is the Mittag-Leffler function. Combining Eqs. (8) and (C7) yields Eq. (23) for $\Delta\psi$.