

Dynamical Complexity of Non-Gaussian Many-Body Systems with Dissipation

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We characterize the dynamical state of many-body bosonic and fermionic many-body models with intersite Gaussian couplings, on-site non-Gaussian interactions, and local dissipation comprising incoherent particle loss, particle gain, and dephasing. We first establish that, for fermionic systems, if the dephasing noise is larger than the non-Gaussian interactions, irrespective of the Gaussian coupling strength, the system state is a convex combination of Gaussian states at all times. Furthermore, for bosonic systems, we show that if the particle loss and particle gain rates are larger than the Gaussian intersite couplings, the system remains in a separable state at all times. Building on this characterization, we establish that at noise rates above a threshold, there exists a classical algorithm that can efficiently sample from the system state of both the fermionic and bosonic models. Finally, we show that, unlike fermionic systems, bosonic systems can evolve into states that are not convex Gaussian even when the dissipation is much higher than the on-site non-Gaussianity. Similarly, unlike bosonic systems, fermionic systems can generate entanglement even with noise rates much larger than the intersite couplings.

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Introduction—Whether many-body quantum systems evolve into classically nontrivial states under decoherence is of fundamental interest to the theory of open quantum systems and also has implications for the quantum advantage in quantum computers and simulators [1–6]. Traditionally, this has been mostly studied for many-body spin models, including extensive recent activity in both the discrete-time setting (i.e., quantum circuits interspersed with noise) and in the continuous-time setting (modeled by a many-body Lindblad master equation [7,8]). For discrete-time models, early results showed that sufficiently high noise suppresses entanglement thus enabling classical simulation [9]. Recent results have shown classical simulability, even with a small amount of depolarizing noise, for both sampling or computing local observables in both random [10–19] and structured models [20–22]. These results have partly been generalized to the geometrically local continuous-time setting, which more accurately models analog quantum simulators, to show that the system remains classically simulable when the noise rate is larger than the interaction terms in the Hamiltonian [8].

Quantum simulators based on platforms such as ultra-cold atoms in optical lattices [23–27], superconducting circuits [28,29], or nonlinear photonics [30–32], are often described by a family of Hamiltonians that, only in certain regimes, reduce to spin systems. They are modeled by a fermionic or bosonic lattice with two kinds of terms: (i) Gaussian coupling terms which are linear or quadratic in creation-annihilation operators (e.g., particle hopping or pair production); and (ii) non-Gaussian interaction terms that typically act on particles only on one site. Consequently, there are two relevant frequency scales: the strength of the Gaussian couplings J and that of the on-site (non-Gaussian) interactions U . If $J = 0$ or $U = 0$, this model is classically simulable. When $J = 0$, the Hamiltonian is a sum of single-site terms which maps product states to product states which can be classically simulated. When $U = 0$, the dynamics is Gaussian and thus local observables can be efficiently computed and, for fermions, even sampling is efficient [33–36]. However, when both $J, U \neq 0$ and the system is noiseless, this model is universal for quantum computation [37,38] and thus worst-case hard to simulate classically. While the presence of dissipation should make the model classically simulable, the amount and type of local dissipation needed remains unclear.

For fermionic systems, the impact of noise on non-Gaussianity was studied in a circuit model with Gaussian gates and non-Gaussian ancillas, which showed that the state remains a convex combination of Gaussian states above a noise threshold [39–41]. Studies analyzing

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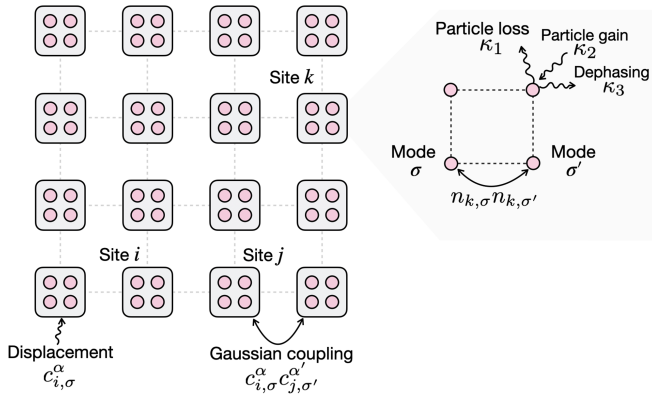


FIG. 1. Sketch of the model, with n sites on a lattice, where each site contains L modes. There are Gaussian couplings between the different sites, while the non-Gaussian interactions are only onsite. Nonlocal couplings are allowed. In the bosonic case, interactions of the form $n_{i,\sigma}^2$ are also allowed.

continuous-time dynamics have focused on the non-Gaussianity introduced via two-body dissipation [7]. For bosonic systems, previous studies have either focused on understanding their complexity as a function of evolution time in the absence of noise [42–44], or for the specific task of boson sampling in the presence of noise [45–49]. However, the classical simulability of the noisy continuous-time model motivated above remains unresolved.

In this Letter, we rigorously address this question—we consider fermionic and bosonic systems with n sites, each containing locally L modes (Fig. 1). The annihilation operators corresponding to the σ th mode at the i th site, where $\sigma \in \{1, 2, \dots, L\}$ and $i \in \{1, 2, \dots, n\}$, is given by $a_{i,\sigma}$. We will use the Hermitian operators $c_{i,\sigma}^\alpha$, with $\alpha \in \{1, 2\}$, defined as $c_{i,\sigma}^1 = (a_{i,\sigma} + a_{i,\sigma}^\dagger)/\sqrt{2}$, $c_{i,\sigma}^2 = -i(a_{i,\sigma} - a_{i,\sigma}^\dagger)/\sqrt{2}$, which represent either Majorana operators (for fermions) or position and momentum quadrature operators (for bosons). The noisy dynamics is described by the Lindblad master equation.

$$\frac{d\rho(t)}{dt} = -i[H(t), \rho(t)] + \sum_{i,\sigma} \mathcal{L}_{i,\sigma} \rho(t), \quad (1)$$

where $H(t)$ is a (possibly time dependent) Hamiltonian describing the system. $\mathcal{L}_{i,\sigma}$ captures the noise, which is assumed to act locally on every mode (i, σ) , and is modeled by

$$\mathcal{L}_{i,\sigma}(\cdot) = \sum_{l=1}^3 \kappa_l \left(L_{i,\sigma}^{(l)}(\cdot) L_{i,\sigma}^{(l)\dagger} - \frac{1}{2} \{ L_{i,\sigma}^{(l)\dagger} L_{i,\sigma}^{(l)}, (\cdot) \} \right), \quad (2)$$

with jump operators $L_{i,\sigma}^{(1)} = a_{i,\sigma}$, $L_{i,\sigma}^{(2)} = a_{i,\sigma}^\dagger$, $L_{i,\sigma}^{(3)} = a_{i,\sigma}^\dagger a_{i,\sigma}$ and decay rates $\kappa_1, \kappa_2, \kappa_3$ respectively. The jump operator $a_{i,\sigma}$ models particle loss, $a_{i,\sigma}^\dagger$ models incoherent particle

gain, and $a_{i,\sigma}^\dagger a_{i,\sigma}$ models dephasing. We remark that our conclusions also hold for other physically relevant dissipators such as $L_{i,\sigma}^{(1)} = c_{i,\sigma}^1$, $L_{i,\sigma}^{(2)} = c_{i,\sigma}^2$ which, in the bosonic case, would correspond to white noise fluctuations in the quadratures. We use the same noise Lindbladian for both bosons and fermions—for the bosonic case, we will additionally assume that the particle loss occurs at a rate strictly higher than (both coherent and incoherent) particle gain (see Supplemental Material [50] for the exact assumption) so as to avoid an unbounded growth of the number of particles with t which would be unphysical in an actual experiment.

We will assume that the Hamiltonian can be written as $H(t) = H_g(t) + H_{ng}(t)$, where $H_g(t)$ contains Gaussian general intersite terms:

$$H_g(t) = \sum_{i,j} \sum_{\alpha,\alpha'} J_{i,\sigma;j,\sigma'}^{\alpha,\alpha'}(t) c_{i,\sigma}^\alpha c_{j,\sigma'}^{\alpha'} + \sum_{i,\alpha,\sigma} \Omega_{i,\sigma}^\alpha(t) c_{i,\sigma}^\alpha, \quad (3a)$$

and $H_{ng}(t)$ contains on-site non-Gaussian terms which account for particle-particle repulsion and attraction between different modes at the same site:

$$H_{ng}(t) = \sum_{i,\sigma,\sigma'} U_{i,\sigma;i,\sigma'}(t) n_{i,\sigma} n_{i,\sigma'}. \quad (3b)$$

Note that in the fermionic case $\Omega_{i,\sigma}^\alpha(t) = 0$, since physical Hamiltonians must preserve fermionic parity, while in the bosonic case $\Omega_{i,\sigma}^\alpha(t)$ can be a nonzero real scalar. For fermions, we can assume that $J_{i,\sigma;j,\sigma'}^{\alpha,\alpha'}(t)$ is purely imaginary and antisymmetric, i.e.,

$$J_{i,\sigma;j,\sigma'}^{\alpha,\alpha'}(t) = [J_{j,\sigma';i,\sigma}^{\alpha',\alpha}(t)]^* = -J_{j,\sigma';i,\sigma}^{\alpha',\alpha}(t), \quad (4)$$

while for bosons it can be assumed to be purely real and symmetric:

$$J_{i,\sigma;j,\sigma'}^{\alpha,\alpha'}(t) = J_{j,\sigma';i,\sigma}^{\alpha',\alpha}(t). \quad (5)$$

The non-Gaussian onsite interactions $U_{i,\sigma;i,\sigma'}(t)$ can be assumed to be real and symmetric for both fermions and bosons.

We also define the parameters J, Ω, U as the smallest constants such that for every mode (i, σ) and all times t

$$\sum_{j \neq i, \sigma'} \sum_{\alpha, \alpha'} |J_{i,\sigma;j,\sigma'}^{\alpha,\alpha'}(t)| \leq J, \quad \sum_{\alpha} |\Omega_{i,\sigma}^\alpha(t)| \leq \Omega \quad \text{and} \quad \sum_{\sigma'} |U_{i,\sigma;i,\sigma'}(t)| \leq U. \quad (6)$$

The parameter J captures the Gaussian coupling strength between a mode and the modes at all other sites, U captures

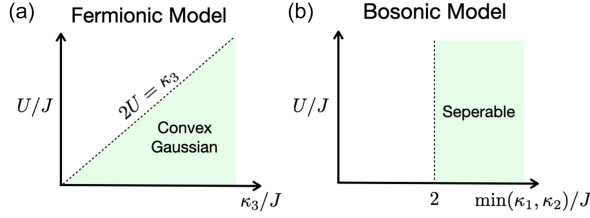


FIG. 2. Phase diagram for both bosonic and fermionic systems in the presence of generic noise. (a) For the fermionic system the state remains convex Gaussian at all times for error rates $\kappa_3 \geq 2U$. (b) In bosonic systems the state remains separable at all times for error rates $\kappa_1, \kappa_2 \geq 2J$.

the on-site non-Gaussian interaction strength, and Ω captures the coherent drive at each site. We will also assume that J , U , and Ω are $O(1)$ constants, which is true in most physical models. Finally, we remark that we do not need to assume geometrical locality of the model—our results will apply to geometrically local and nonlocal models.

The initial state $\rho(0)$ is either a product state (when analyzing entanglement), a Gaussian state (when analyzing non-Gaussianity), or both (e.g., the vacuum state). In the bosonic case, additionally, $\rho(0)$ will be assumed to satisfy $\text{Tr}[n_{i,\sigma}^k \rho(0)] \leq C_0^k k^{\alpha_0 k + \beta_0}$, $\forall (i, \sigma)$, for $k \in \{1, 2, \dots\}$, and for some $C_0, \alpha_0, \beta_0 > 0$: this condition guarantees that the probability of finding $\geq k$ particles in a mode decreases superpolynomially with k , as would be expected in a physically preparable bosonic state [62].

Results—Our results, depicted in Fig. 2, show the simulability of the fermionic and bosonic models as a function of J, U, κ_i . We first establish that when the noise rate is larger than the on-site non-Gaussian interaction strength U , the fermionic model remains convex Gaussian

at all times, and can therefore be classically efficiently sampled from.

Theorem 1—For an initial Gaussian state, if $\kappa_3 \geq 2U$, then the state of the fermionic model at time t , $\rho(t)$, is a convex combination of Gaussian states for all $t \geq 0$. Furthermore, $\rho(t)$ can be classically sampled in the Fock state basis to an ϵ total variation error in $\text{poly}(n, t, 1/\epsilon)$ time.

Physically, Theorem 1 suggests that when $\kappa_3 \geq 2U$, dephasing noise destroys non-Gaussianity faster than $H_{\text{ng}}(t)$ creates it, so that $\rho(t)$ always remains convex Gaussian. Since $H_g(t)$ preserves convex Gaussianity, the noise threshold in Theorem 1 is independent of J . Notably, it is dephasing that results in this convex Gaussianity. With only incoherent particle loss-gain, $\rho(t)$ could evolve into a non-Gaussian state at short times even with large dissipation. This arises from the fermionic parity structure—the density matrix of the fermionic model has the form $\rho(t) = \rho_+(t) + \rho_-(t)$, where $\rho_{\pm}(t)$ is supported only on even-odd parity states. Because of this structure, convex Gaussianity in $\rho(t)$ requires both $\rho_{\pm}(t)$ to be convex Gaussian [40]. $H_{\text{ng}}(t)$ generates non-Gaussianity individually in both $\rho_{\pm}(t)$. However, the loss-gain dissipators, to first order, switch the parity of the state and do not act within the two parity subspaces. Consequently, they cannot immediately counter the non-convex-Gaussianity created by $H_{\text{ng}}(t)$.

We provide a complete proof of Theorem 1 in the supplement: The starting point is a trotterization of the Lindbladian in Eq. (1)—in each trotter step, we express the evolution as (a) a Gaussian unitary corresponding to $H_g(t)$, particle loss and gain followed by (b) the single-site channels generated by $H_{\text{ng}}(t)$ and dephasing (Fig. 3). Analyzing the single-site channel, we show that for $\kappa_3 \geq 2U$, this channel maps an input convex-Gaussian

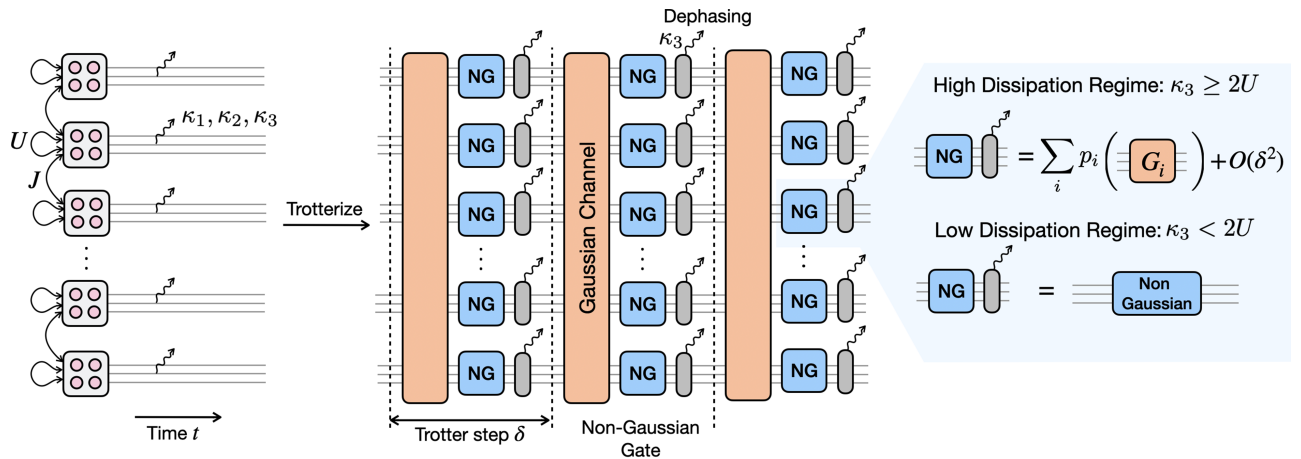


FIG. 3. Schematic depiction of the trotterization schemes in the proof of Theorem 1 (fermionic systems with weak non-Gaussianity). For simplicity, we only depict a 1D setting, with each site containing three modes ($L = 3$). A single trotter step consists of a Gaussian channel (orange rectangles) that includes the combined effect of $H_g(t)$ and the particle gain and loss dissipators, followed by non-Gaussian gates (blue rectangles) interspersed with the dephasing dissipator (gray curved rectangles). Crucially, a non-Gaussian gate followed by sufficiently strong dephasing can be written as a convex combination of Gaussian channels.

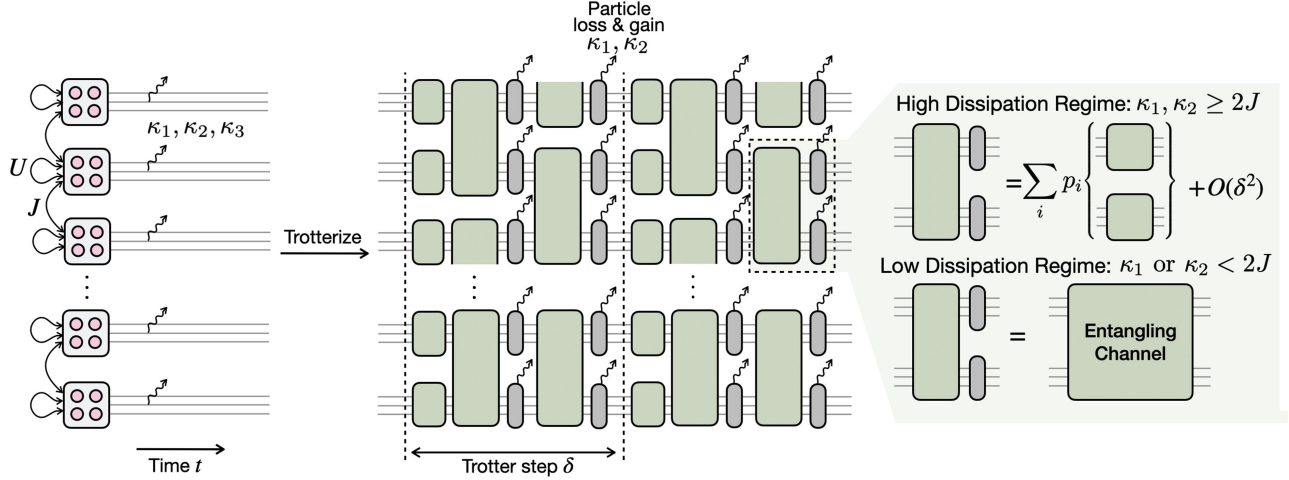


FIG. 4. Schematic depiction of the trotterization schemes in the proof of Theorem 2 (bosonic systems with weak intersite couplings). For simplicity, we only depict a 1D setting, with each site containing three modes ($L = 3$). A single trotter step consists of a layer of single-site channels which include the Hamiltonian terms acting on that site and the dephasing dissipator, followed by two-site gates interspersed with particle gain and loss dissipators (in gray circles). Crucially, a two-site gate followed by sufficiently strong noise can be written as a convex combination of single-site channels.

state to a convex-Gaussian state. Furthermore, we explicitly construct the output convex-Gaussian state, which allows us to sample from it [35,63].

Next, we consider the bosonic model and establish that when the noise rate is larger than the intersite Gaussian coupling, $\rho(t)$ remains separable at all times, and can therefore be classically efficiently sampled from in the Fock state basis.

Theorem 2—Suppose $\rho(t)$ is the state obtained after evolving the bosonic model for time t with an initial product state, then for $\kappa_1, \kappa_2 \geq 2J$ the state $\rho(t)$ is separable for all $t \geq 0$. Furthermore, there is a randomized classical algorithm that can sample $\rho(t)$ in the Fock state basis to ϵ total variation error in $\text{poly}(n, t, 1/\epsilon)$ time.

Our result formalizes the intuition that, when noise exceeds the intersite coupling strength, a buildup of entanglement is prohibited and no classically nontrivial state is generated. Notably, the noise threshold is determined by particle loss and gain noise. This arises from the dephasing dissipator being diagonal in the Fock basis, while evolution under $H_g(t)$ creates entanglement through off-diagonal elements (i.e., coherences). Thus, to first order, dephasing cannot counter this entanglement generation. Particle gain or loss dissipators, however, are not diagonal in the Fock basis and can prevent it. In the Supplemental Material [50], we also extend Theorem 2 to quantum spin models with single spin noise and bosonic models with intersite non-Gaussian couplings [50]. Unlike previous percolation-based arguments limiting entanglement to $O(\log n)$ qubit clusters for sufficiently strong noise [8,9], we show that the state is entirely separable, and provide an explicit construction that can be efficiently sampled from.

A detailed proof of Theorem 2 is provided in Supplemental Material [50]. Similar to Theorem 1,

we begin by a first-order trotterization of the model but with a different decomposition of the Lindbladian: we express it as a product of (a) single site gates, which contain the unitary generated by $H_{ng}(t)$, single-site terms in $H_g(t)$ and the dephasing dissipator, and (b) two-site channels which contain the unitary generated by the intersite terms in $H_g(t)$ paired together with the particle gain and loss dissipators (Fig. 4). We denote the channel acting between modes (i, σ) and (j, σ') at the trotter step τ as $\Phi_{\tau\delta, (\tau-1)\delta}^{i, \sigma; j, \sigma'}$, where δ is the size of the trotter step. Importantly, the trotterization is performed in such a way that the channel $\Phi_{\tau\delta, (\tau-1)\delta}^{i, \sigma; j, \sigma'}$ can be understood as a time evolution of the intersite Gaussian couplings between modes (i, σ) and (j, σ') , followed by noise on both modes. This effectively redistributes the single-site noise into “gate-based” noise on the intersite gates. Analyzing $\Phi_{\tau\delta, (\tau-1)\delta}^{i, \sigma; j, \sigma'}$, we show that for $\kappa_1, \kappa_2 \geq 2J$, it is separability preserving and thus the state remains separable at all times. Furthermore, we also explicitly construct an $O(\delta^2)$ approximation to the separable state after each time step and obtain an explicit algorithm to sample from $\rho(t)$.

Tightness of Theorems 1 and 2—We can now ask if a version of Theorem 1 holds for bosonic systems, i.e., is there a noise threshold $\kappa_{th}(U)$ dependent only on the non-Gaussian strength U and uniform in J, Ω that guarantees a convex Gaussian state at all times? For dephasing noise, we provide numerical evidence to the contrary: even for $\kappa_3 \gg U$, single-mode dynamics can yield states with negative Wigner function, and thus not convex-Gaussian states can be generated at timescales $\sim 1/U$ [50,64]. While negativity of the Wigner function suggests classical simulation hardness [64,65], we do not rule out the

existence of an efficient classical algorithm. When $\kappa_3 = 0$, in Supplemental Material [50] we show that no matter how small U is relative to κ_1, κ_2 , computing expected local particle numbers is bounded-error quantum polynomial time (BQP) hard if J, Ω can be arbitrarily large but $O(1)$ [50]. This builds upon Refs. [66,67] which perform a universal gate set on a single bosonic mode with an arbitrarily small effective gate-error rate by engineering the displacement and squeezing. We extend this technique to also implement an entangling gate between two oscillators thus yielding a high-fidelity universal multimode gate-set. Together with results from Ref. [68], this suggests that when the noise is nonunitary (i.e., $\kappa_1 \neq \kappa_2$), by using sufficiently large J and Ω , a fault-tolerant quantum computation can be encoded into the model [69–71]. Thus, it is unlikely to be able to classically compute even local observables in this setting unless BQP = BPP.

Finally, we consider if a version of Theorem 2 holds for fermions, i.e., do noise rates larger than the Gaussian intersite couplings result in separability at all times. We answer this question in the negative: in Supplemental Material [50] we show by analyzing few-mode fermionic models that, in contrast with bosons, no matter how high κ_2, κ_3 are, the system does not remain separable at all times and can exhibit entanglement at timescales $\sim \min(\kappa_2^{-1}, \kappa_3^{-1})$. In fact, this short-time nonseparability holds not only if we consider separability with respect to all observables [72], but also if we consider a weaker notion of separability with respect to only parity-conserving observables [73]. However, this result does not rule out separability at longer times or other routes to classical simulation.

Conclusion and outlook—We have characterized the classical complexity of simulating the continuous-time evolution of fermionic and bosonic systems as a function of the noise, Gaussian and non-Gaussian interaction strengths. Future theoretical directions include the study extending our results to non-Markovian models of dissipation.

The models considered in this Letter can be experimentally realized in several platforms. The bosonic model can be implemented in superconducting systems where the Gaussian Hamiltonian can be controlled by designing capacitive couplings between different qubits and the single-site non-Gaussianity by the nonlinear Josephson potential in the qubit [28,29]. We can also use cold bosonic atoms in optical lattices where the strength of both the Gaussian and the non-Gaussian Hamiltonians can be controlled by tuning the optical lattice potential [26,27]. The fermionic model can be implemented either with cold atoms in optical lattices by using a fermionic species of atoms [23–25], or in solid-state systems such as moiré superlattices hosting triions [74–76]. Since all of these systems will have intrinsic particle loss, gain, and dephasing, tuning the parameters in the Gaussian and non-Gaussian Hamiltonians could allow us to access the parameter regimes in Theorems 1 and 2. A major challenge

in experimentally verifying the threshold behavior predicted by Theorems 1 and 2 would be verifying the presence (or absence) of entanglement and non-Gaussianity in $\rho(t)$. While this could be hard to do for large systems, we remark that the difference in the threshold behavior in fermionic and bosonic models that we described can be understood even with systems with few (≤ 4) fermionic or bosonic modes, which is well within the regime where a full state tomography can already be performed.

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Data availability—The data that support the findings of this article are openly available [77].

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