

Solubility Enhanced Surfactant-Induced Flow in Air-Liquid-Air Sheets

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Liquid interfaces appear throughout nature and engineering and are typically contaminated by surface active agents (surfactants), which are characterized by a wide range of solubility. We demonstrate that solubility enhances by an order of magnitude surfactant-induced flow in air-liquid-air films, in contrast to previously studied geometries where solubility dampens the flow. The enhancement is described by a single parameter comparing the depletion length to the film thickness. Our experiments are well described by an asymptotic theory of the Navier-Stokes equations, including surfactant kinetics.

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Interfaces influence dynamics in the environment, health, and industrial processes. For example, sea spray aerosols from bursting bubbles strongly affect ocean-atmosphere mass exchange [1,2]. Exhaled aerosols transmit pathogens [3], while thin liquid films line the lungs [4] and eyes [5]. In industry, emulsions, coatings, and foams exhibit intricate interfacial flow dynamics.

Furthermore, most liquid interfaces are contaminated. A common type is surface active agents or surfactants: molecules of both biological and anthropogenic origins that adsorb to the interface and lower surface tension. These diverse origins yield surfactants with distinct chemical properties governing their physicochemical behavior [6,7]. For example, the solubility (adsorption equilibrium constants) of biological surfactants, including polysaccharides, lipids, proteins, and humics, in the ocean can vary by several orders of magnitude [8]. The formation of sea spray aerosols involves several surfactant-dependent processes, including wave breaking [9,10], bubble scavenging during flotation [11], turbulent bubble breakup [12,13], bubble lifetime [14], and bursting dynamics [15,16]. These processes can be modulated by surfactant properties, of which solubility is an important example, contributing to the variation in relative abundance of compounds between ocean surface water and sea spray aerosols [8]. Such

complexity remains a major source of uncertainty in environmental models.

Surfactant concentration gradients produce surface tension gradients, which drive so-called *Marangoni* flow. A commonly studied configuration is the localized deposition of surfactant onto a clean air-liquid interface, generating outward spreading flow. This configuration has been studied for close to two centuries [17,18], primarily on a deep liquid subphase or on thin air-liquid-solid films. In both configurations, surfactant solubility, a chemical property, reduces the spreading rate [19–22], a flow property, as surface surfactants desorb into the bulk, leading to a weaker Marangoni stress due to a lower surface surfactant gradient [20–22].

In contrast, we focus on surfactant deposition on an air-liquid-air film [23–28], which is less studied despite its ubiquity. For example, localized Marangoni stress is a proposed mechanism for rupture of a bubble cap (an air-liquid-air film) [14,24], thereby influencing bubble lifetime, and in turn, oceanic aerosol production. More generally, air-liquid-air films arise in foam and soap films [29,30], bubble coalescence [31], and the breakup of rain drops [32]. Understanding when and how such films thin and rupture is therefore a recurring question.

Specifically, we consider a surfactant-laden drop deposited onto a Savart sheet [24,33,34] [Fig. 1(a)]. The deposition leads to a localized region of low surface tension, driving an outward radial flow and sheet thinning; Fig. 1(b) shows spreading patterns with front radius $r_f(t)$.

Surfactant deposition on thin air-liquid-air films of low viscosity liquids (such as water) is characterized by the deposition length scale (along the film) $\mathcal{L}$, the initial uniform film thickness h_i , and a timescale $\mathcal{T}$ obtained by balancing inertia $\rho h_i \mathcal{L} \mathcal{T}^{-2}$ (where ρ is the liquid density) and Marangoni stresses $\Delta \Sigma_0 \mathcal{L}^{-1}$ arising from

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the surfactant-induced surface tension deficit $\Delta\Sigma_0: \mathcal{T} = [\rho h_i \mathcal{L}^2 / (\Delta\Sigma_0)]^{1/2}$ [24,27,28].

Taking typical values of $\mathcal{L} = O(100 \mu\text{m})$, $\Delta\Sigma_0 = O(1 \text{ mN/m})$, and $h_i = O(10 \mu\text{m})$, the magnitude of $\mathcal{T}$ is $O(100 \mu\text{s})$. In the late-time spreading limit where time $t \gg \mathcal{T}$, experiments [24] with ethanol drops showed that the surfactant front $r_f(t)$ propagates with $r_f(t)/\mathcal{L} \sim (t/\mathcal{T})^{1/2}$. In the thin-film limit ($h_i \ll \mathcal{L}$) with insoluble surfactants, the Navier-Stokes equations simplify, and the late-time $t^{1/2}$ propagation was derived theoretically [27,28], accounting for inertia, Marangoni, capillary, and viscous extensional stresses with surfactant transport.

The insoluble surfactant dynamics are described by two nondimensional numbers (see End Matter): the Marangoni number $\mathcal{M}$ (ratio of Marangoni to capillary stresses), and Reynolds number Re (ratio of inertia to viscous extensional stresses). For typical values of $\mathcal{L}$ and $\mathcal{T}$ as above, diffusion along the sheet is not considered: the Péclet number $\mathcal{L}^2/(D\mathcal{T}) = O(10^6) \gg 1$ since the typical diffusion coefficient $D = O(10^{-10} \text{ m}^2/\text{s})$.

Here, we show experimentally and theoretically that surfactant solubility strongly enhances the front propagation, in contrast to the reduction observed on air-liquid-solid films and deep liquid subphases. We show that the $t^{1/2}$ scaling for $r_f(t)$ still holds,

$$r_f(t) = \eta_f \mathcal{L} \left(\frac{t}{\mathcal{T}} \right)^{\frac{1}{2}} = \eta_f \left(\frac{\Delta\Sigma_0 \mathcal{L}^2}{\rho h_i} \right)^{\frac{1}{4}} t^{\frac{1}{2}}, \quad (1)$$

demonstrate experimentally that solubility increases the prefactor η_f , and derive a theoretical prediction for it. The overlaid curves on Fig. 1(b) are the theoretical predictions (to be derived) without fitting parameters.

The enhancement arises since in an air-liquid-air film, the surfactant-containing region thins as the front $r_f(t)$ propagates, which pushes bulk surfactant to adsorb onto the surface, strengthening the Marangoni stress [Figs. 1(c) and 1(d)]. This push can be understood by noting that incompressible stretching of a material element increases its surface-area-to-volume ratio: stretching acts to dilute the surface surfactants, while acting to retain bulk surfactant concentration, and surfactants therefore adsorb onto the surface. Thus, the bulk fluid acts as a reservoir that replenishes the surface with surfactants. Bulk replenishment of the surface is central to many surfactant phenomena [7,36–39]. Equivalently, the interface acts as a sink for bulk surfactants.

The late-time propagation for finitely soluble surfactants has the same spatial structure [Fig. 1(c)] as for insoluble surfactants, which consists of a propagating surfactant front (region II), a thin region with spatially uniform surfactant concentration behind the surfactant front (region I), and a region ahead of the surfactant front without surfactants (region III). This structure is physically expected since inertial surfactant flow for air-liquid-air films mathematically maps onto the compressible gas equations upon

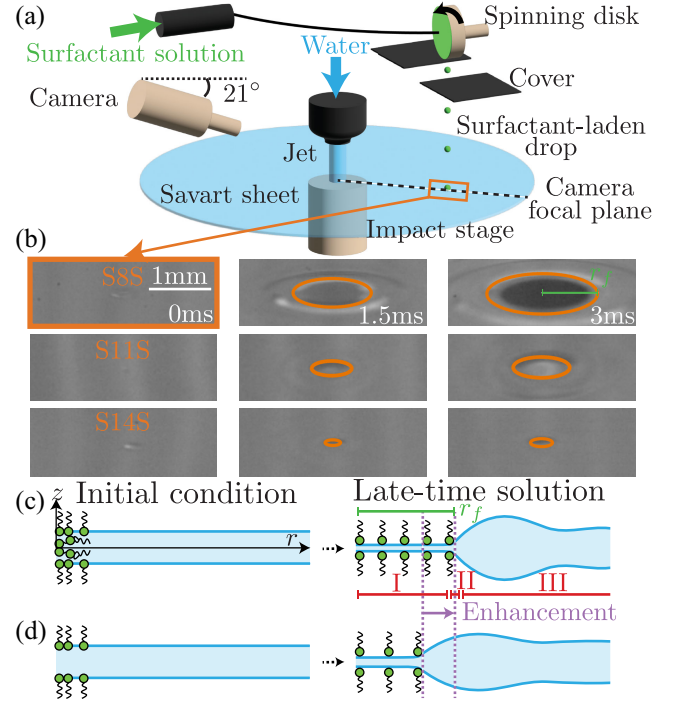


FIG. 1. Summary of the surfactant deposition experiment and solubility-driven enhancement of front propagation and film thinning. (a) Experimental setup for surfactant deposition onto an air-liquid-air (Savart) sheet, produced by a jet impacting a stage. Monodisperse surfactant-laden drops are produced by a spinning disk atomizer. (b) Sample dynamics of surfactant-induced front propagation with radial distance r_f . Frames are centered on the deposition pattern. Rows are surfactants of decreasing solubility (top to bottom): S8S, S11S, S14S, at the half concentration case (see Table I), corresponding to static surface tension deficit $\Delta\Sigma_0 \approx 2\text{--}3 \text{ mN/m}$. Columns show time evolution. Axisymmetric patterns appear elliptic due to the camera angle of $\approx 21^\circ$ to the horizontal. The orange curves overlaid on the experimental frames are the theoretical predictions without any fitting parameters [ratio of minor to major axes of the overlaid ellipse is set as $\sin(21^\circ)$]. See Supplemental Material [35] for video versions. (c) Schematic film evolution. (d) Analogous to (c) but without bulk surfactants. The purple arrow shows the enhancement of surfactant front propagation and hence the thinning. Diagrams in (a), (c), (d) are not to scale.

identifying surface surfactant concentration with (negative) gas pressure and film thickness with gas density [28,40,41] (see End Matter). The surfactant front propagation therefore has analogies to classical compressible gas shock propagation.

To quantify enhancement via solubility, we account for surfactant physicochemistry. We consider low bulk and surface surfactant concentrations where adsorption and desorption fluxes are linear. Then, the surface surfactant concentration Γ_0 at kinetic equilibrium with the bulk surfactant concentration c_0 satisfies

$$k_d \Gamma_0 = k_a c_0, \quad (2)$$

TABLE I. Depletion lengths $k_a k_d^{-1}$ for sodium alkyl sulfates obtained by fitting the curve $k_d \Gamma_0 = k_a c_0$ to Fig. 4(b) of [42]. We allow for $\pm 30\%$ uncertainty range of $k_a k_d^{-1}$. The base case concentration c_0 of the sodium alkyl sulfate solutions with measured surface tension of 67 ± 1 mN/m is shown. Experiments are also performed at double and half the base case concentration.

	S8S	S9S	S10S	S11S	S12S	S13S	S14S
$k_a k_d^{-1}$ [μm]	0.07	0.15	0.3	0.5	1.0	2.0	3.5
c_0 [mM]	13	6.0	2.6	1.5	0.84	0.44	0.29

where k_a and k_d are constants. Similarly, at dilute concentrations, surface tension $\sigma(\Gamma)$ varies linearly with surface surfactant concentration Γ . Hence the static surface tension deficit $\Delta\Sigma_0$ for c_0 is

$$\Delta\Sigma_0 = -\left.\frac{d\sigma}{d\Gamma}\right|_{\Gamma=0} k_a k_d^{-1} c_0. \quad (3)$$

A natural length scale $k_a k_d^{-1}$ appears, which is commonly referred to as the depletion length [7] or equilibrium adsorption constant. Estimates for $k_a k_d^{-1}$ are available from published equilibrium measurements (Table I). Smaller $k_a k_d^{-1}$ means that a higher c_0 is required to obtain the same Γ_0 . The nondimensional parameter comparing surface and bulk concentrations at equilibrium is

$$\Lambda_d = \frac{k_a}{k_d h_i} = \frac{\Gamma_0}{c_0 h_i}, \quad (4)$$

where h_i is the initial thickness of the liquid sheet onto which surfactants are deposited.

Our experimental setup is as follows. We use sodium alkyl sulfates $[\text{CH}_3(\text{CH}_2)_{n-1}\text{OSO}_3\text{Na}]$, which are well-documented anionic surfactants [42,43] with a hydrophobic hydrocarbon tail and hydrophilic sodium sulfate head, henceforth labeled SnS s. The choice allows us to systematically explore surfactant solubility since a longer hydrocarbon tail length n leads to lower solubility. We use SnS s for $n = 8, \dots, 14$ as purchased (Thermo Fisher, Fisher Scientific, Neta Scientific), which are used in many cleaning and personal care products. Furthermore, SnS s are often used as model surfactants, e.g., as proxies for lipids found in the ocean [8]. In particular, S12S is known as SDS (sodium dodecyl sulfate). We use solutions of SnS s dissolved in deionized (DI) water. As the base case, the bulk surfactant concentration c_0 of each SnS solution is set (Table I) so that the static surface tension of the solution is 67 ± 1 mN/m (Wilhelmy plate method), giving a deficit $\Delta\Sigma_0 \approx 5$ mN/m relative to pure DI water (72 mN/m). The concentrations c_0 are $\approx 10\%$ of the critical micelle concentration for each SnS . Experiments at double and half

concentrations are also performed to check sensitivity to c_0 and the associated $\Delta\Sigma_0$.

The surfactant-laden drops are produced by placing the surfactant solution on a fast spinning disk enabling controlled production of small monodisperse drops in all directions parallel to the disk plane [44,45]; a cover selects the deposition location. Such control is necessary for quantitative theoretical comparisons. By separately imaging the disk, we estimate the mean drop radius to be $r_d \approx 22 \pm 2$ μm for a disk of radius 1 cm rotating at angular frequency ≈ 2000 rad s^{-1} . Since the operating conditions are held constant, this mean drop radius is taken to apply across all experiments.

The air-liquid-air sheet, referred to as a Savart sheet, is formed by a (tap) water jet of diameter $d_j \approx 3.3$ mm with velocity $v_j \approx 4$ ms^{-1} striking a horizontal impact stage, where an adjustable sleeve allows the sheet to be produced at a right angle to the jet [33,34]. A nozzle with a large contraction ratio keeps the jet, and hence the sheet, laminar at high velocities. The camera field of view, onto which the surfactant deposition is aimed, is located at a radial distance $r_s \approx 5$ –8 cm from the jet. The Savart sheet thickness formula [33,34] gives $0.13 d_j^2 / r_s = 20$ –30 μm , which we checked by interferometry.

By imaging at 22 000 fps (Phantom v2012 paired with a Nikon Micro 200 and 36 mm extension tube, giving ≈ 20 $\mu\text{m}/\text{pixel}$ resolution), we track the evolution of the front position $r_f(t)$, which appears in the image as the major radius of an ellipse due to the imaging angle. We saved instances where the drop deposition was visible in the frame, recording until either rupture or advection out of the frame. Sample images of the experiment are shown in Fig. 1(b). Here, for S8S in Fig. 1(b), the black regions are not holes but extremely thin films. Once a hole forms, Taylor-Culick-type retraction [39,46–49] is rapid and readily visible.

The temporal evolution of the front radius for the base concentration case (≈ 5 mN/m surface tension deficit) is shown in Fig. 2(a). Each case is repeated 30 times. The mean and standard error are shown for $t > 1.5$ ms. At earlier times, the spreading front is often not clearly visible; our focus is on the late-time propagation. The solid lines show linear least-square fits to a $t^{1/2}$ power law, in good agreement with the scaling predicted for insoluble surfactant deposition. Treating the exponent as free parameter shows values close to $1/2$ [e.g., 0.5 ± 0.15 for the data shown in Fig. 2(a)]. We do not analyze these deviations further, attributing them to experimental uncertainties and transient effects since the $t^{1/2}$ scaling only holds at late times $t \gg T$ while our observational window is $t/T = O(10$ –100) for $T = O(100)$ μs .

The fitted prefactor η_f is defined from (1) by setting

$$\mathcal{L} = \sqrt{\frac{4r_d^3}{3h_i}}, \quad (5)$$

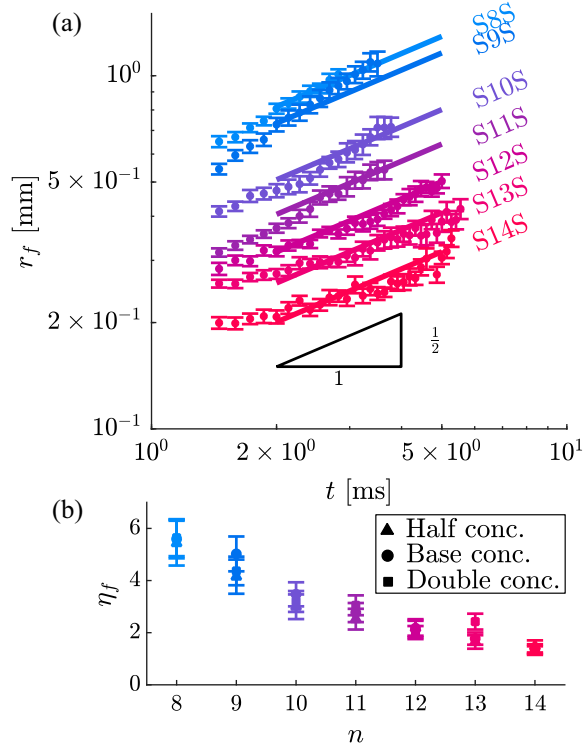


FIG. 2. Experimental data of the enhancement of front propagation via surfactant solubility. (a) Surfactant front radius r_f against time t for each surfactant (S8S to S14S shown as blue to red) for the base case concentrations shown in Table I where surface tension deficit ≈ 5 mN/m. The error bars show the mean ± 1 standard error. The solid lines show the best linear least-square fits to a $t^{1/2}$ power law. (b) The nondimensional prefactor η_f (1) and (5) against n , the number of carbons in SnS . Here, $r_d = 22$ μm is the drop radius, $\rho = 997$ kg m^{-3} the water density, $h_i = 25$ μm the initial thickness of the liquid sheet, and $\Delta\Sigma_0 = 2.5, 5, 10$ mN/m the static surface tension deficit of the surfactant solution for half (triangle), base (dot), and double (square) concentrations respectively. The error bars are from the uncertainty of the values $\Delta\Sigma_0$ (± 1 mN/m), r_d (± 2 μm), and h_i (± 5 μm).

where $\mathcal{L}$ is obtained by equating the characteristic sheet volume $\pi\mathcal{L}^2 h_i$ to the drop volume $(4/3)\pi r_d^3$. Substituting (5) into (1), we have $r_f = \eta_f [(4\Delta\Sigma_0 r_d^3)/(3\rho h_i^2)]^{1/4} t^{1/2}$. The plot of η_f against n is shown in Fig. 2(b). Two additional concentrations are shown, where the concentration is doubled and halved respectively, i.e., $\Delta\Sigma_0 \approx 10$ mN/m and $\Delta\Sigma_0 \approx 2.5$ mN/m, where data points are obtained as in Fig. 2(a). Figure 2(b) shows that for fixed $\Delta\Sigma_0$, η_f increases as n decreases, corresponding to higher solubility; the same enhancement is also seen in Fig. 1(b). For fixed n , all η_f collapse across $\Delta\Sigma_0$, meaning that larger surface tension deficits result in faster front propagation, consistent with (1). Overall, varying n over the range considered changes the front growth by factor ≈ 4 , suggesting that solubility is a key parameter in characterizing the dynamics.

Now, we propose a theoretical model for our experimental measurements. First, we consider the sheet. Since the field of view is small, the sheet is locally flat and uniformly translating; background stretching is weak (see End Matter), consistent with the axisymmetric patterns experimentally observed in Fig. 1(b). Additionally, forming a Savart sheet creates large surface area and hence the surface of the sheet is devoid of additional or parasitic surface active compounds [24,33]. Next, the drop retains the original solution concentration c_0 and hence the total surfactant in the deposited drop is $(4/3)\pi r_d^3 c_0$. Since the deposited drops are small enough that coalescence occurs rapidly relative to our experimental measurement timescale (see End Matter), the drop impact and coalescence dynamics can be neglected. Thus, we consider the model problem of axisymmetric surfactant deposition onto a clean flat film of uniform thickness h_i otherwise at rest.

At late times, the region containing the surfactants is thin and the surfactants are pushed onto the surface due to increased surface-area-to-volume ratio. Since the surfactants are mostly on the surface, the late-time similarity solution that describes the film dynamics here is the same as that of insoluble surfactants with the same total amount of surfactants. Then, the theoretical prediction for the enhanced soluble surfactant front growth is derived by modifying the late-time similarity solution for insoluble surfactants deposited onto a clean flat film otherwise at rest [27,28]. Since the characteristic surface surfactant concentration is $h_i c_0/2$ (distributing $h_i c_0$ between top and bottom surfaces), there is an enhancement factor $h_i c_0/(2\Gamma_0) = (2\Lambda_d)^{-1}$ over Γ_0 . This gives (see End Matter) an effective surface tension deficit $\Delta\Sigma_{\text{eff}}$,

$$\Delta\Sigma_{\text{eff}} = \frac{\Delta\Sigma_0}{2\Lambda_d}, \quad (6)$$

and

$$r_f = \eta_f^{\text{insol}} \left(\frac{4\Delta\Sigma_{\text{eff}} r_d^3}{3\rho h_i^2} \right)^{1/4} t^{1/2}, \quad (7)$$

where $\eta_f^{\text{insol}} = \eta_f^{\text{insol}}(\mathcal{M}, \text{Re})$ is a function of $\mathcal{M}$ and Re that weakly depends on Λ_d and is obtained numerically from the similarity solution to the ordinary differential equations for insoluble surfactant deposition. Then, comparing (6) and (7) with (1) and (5) gives

$$\eta_f = \eta_f^{\text{insol}} (2\Lambda_d)^{-1/4}. \quad (8)$$

In particular, it can be shown analytically that for $\Lambda_d \ll 1$, the function η_f^{insol} tends to a constant: $\lim_{\Lambda_d \rightarrow 0} \eta_f^{\text{insol}}(\mathcal{M}, \text{Re}) = 2$ [50]. Thus, for $\Lambda_d \ll 1$,

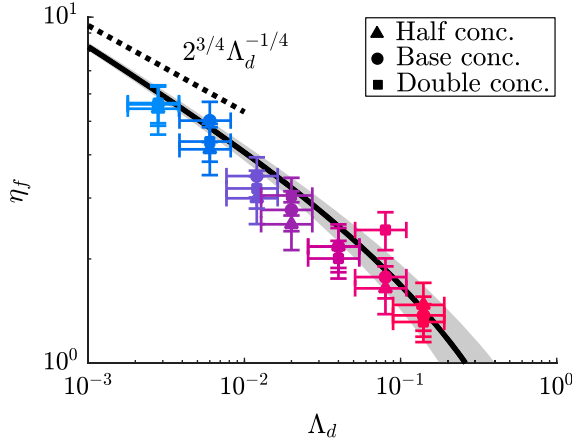


FIG. 3. Comparison of the experimental data and theoretical prediction of the effect of the nondimensional depletion length Λ_d on the nondimensional prefactor η_f . Experimental Λ_d (4) follows from Table I and the experimental η_f is as in Fig. 2(b). The error bars are from the uncertainty of the physical values $k_a k_d^{-1}$ ($\pm 30\%$), $\Delta\Sigma_0$ (± 1 mN/m), r_d (± 2 μ m), and h_i (± 5 μ m). The theoretical prediction (8) for $\Delta\Sigma_0 = 5$ mN/m is shown as a solid curve. The shaded region is the range of theoretical predictions accounting for the range $\Delta\Sigma_0 = 2.5$ –10 mN/m. The theory uses the experimental values $(r_d, h_i, \Sigma, \mu, \rho) = (22$ μ m, 25 μ m, 72 mN/m, 0.9 mPas, 997 kg m $^{-3}$). The dotted line shows the theoretical asymptote (9) for $\Lambda_d \ll 1$. There are no fitting parameters.

$$\eta_f = 2^{\frac{3}{4}} \Lambda_d^{-\frac{1}{4}}, \quad (9)$$

which is a scaling law for Λ_d with an analytical prefactor.

Comparisons between the theoretical prediction (8) and the experimental data are shown in Fig. 3, where the front prefactor is plotted against Λ_d . There are no fitting parameters; all are from experimental measurements. The experimental data are given by symbols and the theoretical prediction is given by a solid curve. The dotted curve shows the asymptote of the theory (9) for $\Lambda_d \ll 1$. There is excellent agreement, and the $\Lambda_d^{-1/4}$ scaling for the front growth is visible both experimentally and theoretically.

In conclusion, we have shown experimentally and theoretically that surfactant solubility enhances surfactant-induced film thinning due to the replenishment of surface surfactants from the bulk. The dynamics are dictated by a single nondimensional parameter Λ_d , which enhances the effective surface tension deficit (6) with a $\Lambda_d^{-1/4}$ front-propagation scaling for $\Lambda_d \ll 1$. Theoretically, such scaling implies that the minimum thickness is proportional to $\Lambda_d^{1/2}$ (see End Matter), which explains the thin region seen for S8S in Fig. 1(b). Our results suggest the front-propagation velocity differs by roughly 4 between S8S and S14S, implying a factor $4^2 = 16$ difference in the minimum thickness thinning rate. The order-of-magnitude difference in the thinning rate will significantly impact

whether surfactant-induced thinning ultimately leads to rupture, such as by van der Waals forces [51–53], background flows [54,55], or entrained bubbles [56–58]. This is visible without a camera: spraying the Savart sheet with S8S, the sheet shows visibly greater disruption for small n , reflecting the likelihood of hole formation before the deposition pattern reaches the sheet edge.

The essence of the enhancement mechanism is as follows. The region behind the surfactant front is, as a whole, the thinnest region of the sheet and thins over time [$O(t^{-1})$; see End Matter], pushing (due to increased surface-area-to-volume ratio) surfactants onto the surface, and making finitely soluble surfactants effectively insoluble at late times. The important parameter is the depletion length $k_a k_d^{-1}$ relative to the film thickness, i.e., Λ_d (4). By contrast, the surfactant-containing region does not thin over time as a whole for air-liquid-solid films or a deep liquid subphase. For the former, the film thickness approaches a steady profile in self-similar spatial coordinates, i.e., remaining $O(t^0)$ [22,59], since the no-slip condition at the solid surface resists motion more than in air-liquid-air films.

Future work could investigate the influence of the enhancement via solubility on the stability of liquid sheets in more complex systems, e.g., predicting surface bubble lifetimes. It will also be interesting to consider multiple-component surfactant-laden solutions. Technologically, observing the surfactant front growth gives information about the solution composition, with the advantage that only picoliters of the solution (the deposited drop) are required. Finally, the problem of soluble surfactant deposition allows for more analysis both numerically and theoretically, which will be published elsewhere [50].

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Data availability—Experimental data taken for this study (seven surfactants at three concentrations deposited 30 times leading to 630 instances) is available at [60].

- [1] F. Veron, Ocean spray, *Annu. Rev. Fluid Mech.* **47**, 507 (2015).
- [2] L. Deike, Mass transfer at the ocean–atmosphere interface: The role of wave breaking, droplets, and bubbles, *Annu. Rev. Fluid Mech.* **54**, 191 (2022).

- [3] L. Bourouiba, The fluid dynamics of disease transmission, *Annu. Rev. Fluid Mech.* **53**, 473 (2021).
- [4] F. Possmayer, Y. Y. Zuo, R. A. W. Veldhuizen, and N. O. Petersen, Pulmonary surfactant: A mighty thin film, *Chem. Rev.* **123**, 13209 (2023).
- [5] R. J. Braun, Dynamics of the tear film, *Annu. Rev. Fluid Mech.* **44**, 267 (2012).
- [6] G. G. Fuller and J. Vermant, Complex fluid-fluid interfaces: Rheology and structure, *Annu. Rev. Chem. Biomol. Eng.* **3**, 519 (2012).
- [7] H. Manikantan and T. M. Squires, Surfactant dynamics: Hidden variables controlling fluid flows, *J. Fluid Mech.* **892**, P1 (2020).
- [8] S. M. Burrows, O. Ogunro, A. A. Frossard, L. M. Russell, P. J. Rasch, and S. M. Elliott, A physically based framework for modeling the organic fractionation of sea spray aerosol from bubble film Langmuir equilibria, *Atmos. Chem. Phys.* **14**, 13601 (2014).
- [9] X. Liu and J. H. Duncan, The effects of surfactants on spilling breaking waves, *Nature (London)* **421**, 520 (2003).
- [10] M. A. Erinin, C. Liu, X. Liu, W. Mostert, L. Deike, and J. H. Duncan, The effects of surfactants on plunging breakers, *J. Fluid Mech.* **972**, R5 (2023).
- [11] D. C. Blanchard, The ejection of drops from the sea and their enrichment with bacteria and other materials: A review, *Estuaries* **12**, 127 (1989).
- [12] G. Soligo, A. Roccon, and A. Soldati, Breakage, coalescence and size distribution of surfactant-laden droplets in turbulent flow, *J. Fluid Mech.* **881**, 244 (2019).
- [13] Z. Wu, T. Aurégan, and L. Deike, Fast and slow surfactants in turbulent bubble breakup, *arXiv:2601.03157* [*J. Fluid Mech.* (to be published)].
- [14] S. Poulain, E. Villermaux, and L. Bourouiba, Ageing and burst of surface bubbles, *J. Fluid Mech.* **851**, 636 (2018).
- [15] C. R. Constante-Amores, L. Kahouadji, A. Batchvarov, S. Shin, J. Chergui, D. Juric, and O. K. Matar, Dynamics of a surfactant-laden bubble bursting through an interface, *J. Fluid Mech.* **911**, A57 (2021).
- [16] C. R. Constante-Amores, J. Chergui, S. Shin, D. Juric, J. R. Castrejón-Pita, and A. A. Castrejón-Pita, Role of surfactant-induced Marangoni stresses in retracting liquid sheets, *J. Fluid Mech.* **949**, A32 (2022).
- [17] J. Thomson, Xlii. on certain curious motions observable at the surfaces of wine and other alcoholic liquors, *Philos. Mag.* **10**, 330 (1855).
- [18] O. K. Matar and R. V. Craster, Dynamics of surfactant-assisted spreading, *Soft Matter* **5**, 3801 (2009).
- [19] P. Joos and J. Van Hunsel, Spreading of aqueous surfactant solutions on organic liquids, *J. Colloid Interface Sci.* **106**, 161 (1985).
- [20] R. Baños, F. Méndez, O. Bautista, and J. Arcos, Spreading of a diffusive soluble surfactant in a deep layer with inertial effects, *Phys. Rev. E* **111**, 055501 (2025).
- [21] D. Halpern and J. Grotberg, Dynamics and transport of a localized soluble surfactant on a thin film, *J. Fluid Mech.* **237**, 1 (1992).
- [22] O. E. Jensen and J. B. Grotberg, The spreading of heat or soluble surfactant along a thin liquid film, *Phys. Fluids A* **5**, 58 (1993).
- [23] M. Bowen and B. S. Tilley, On self-similar thermal rupture of thin liquid sheets, *Phys. Fluids* **25** (2013).
- [24] B. Néel and E. Villermaux, The spontaneous puncture of thick liquid films, *J. Fluid Mech.* **838**, 192 (2018).
- [25] G. Kitavtsev, M. A. Fontelos, and J. Eggers, Thermal rupture of a free liquid sheet, *J. Fluid Mech.* **840**, 555 (2018).
- [26] M. Motaghian, R. Shirsavar, M. Erfanifam, M. Sabouhi, E. Van Der Linden, H. A. Stone, D. Bonn, and M. Habibi, Rapid spreading of a droplet on a thin soap film, *Langmuir* **35**, 14855 (2019).
- [27] J. Eshima, H. A. Stone, and L. Deike, Precursors of thin film rupture: Similarity solution of surfactant-driven, inertial capillary waves, *Phys. Rev. Lett.* **134**, 214002 (2025).
- [28] J. Eshima, L. Deike, and H. A. Stone, Similarity solutions and regularisation of inertial surfactant dynamics, *J. Fluid Mech.* **1023**, A15 (2025).
- [29] D. Langevin, *Emulsions, Microemulsions and Foams* (Springer, New York, 2020).
- [30] I. Cantat, S. Pertant, C. Raufaste, and E. Rio, Capillary drainage in horizontal soap films: Theoretical review and experimental illustrations, *Annu. Rev. Fluid Mech.* **58**, 111 (2026).
- [31] D. Y. C. Chan, E. Klaseboer, and R. Manica, Film drainage and coalescence between deformable drops and bubbles, *Soft Matter* **7**, 2235 (2011).
- [32] E. Villermaux and B. Bossa, Single-drop fragmentation determines size distribution of raindrops, *Nat. Phys.* **5**, 697 (2009).
- [33] P. Marmottant, E. Villermaux, and C. Clanet, Transient surface tension of an expanding liquid sheet, *J. Colloid Interface Sci.* **230**, 29 (2000).
- [34] C. Clanet and E. Villermaux, Life of a smooth liquid sheet, *J. Fluid Mech.* **462**, 307 (2002).
- [35] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/rbyz-h78n> for sample videos from the experiment of the surfactant deposition.
- [36] A. Bussonnière and I. Cantat, Local origin of the viscoelasticity of a millimetric elementary foam, *J. Fluid Mech.* **922**, A25 (2021).
- [37] K. J. Stebe, S.-Y. Lin, and C. Maldarelli, Remobilizing surfactant retarded fluid particle interfaces. I. Stress-free conditions at the interfaces of micellar solutions of surfactants with fast sorption kinetics, *Phys. Fluids A* **3**, 3 (1991).
- [38] D. Fernández-Martínez, E. J. Vega, J. M. Montanero, U. J. Gutiérrez-Hernández, and D. Fernández Rivas, Influence of surfactant kinetics on rapid interface creation via microjet impact on liquid pools, *J. Colloid Interface Sci.* **717**, 140335 (2026).
- [39] A. Guillemot, J. Pierre, and A. Bussonnière, Bursting of large soap films: Steady soluble aureole, *J. Fluid Mech.* **1030**, A20 (2026).
- [40] J.-M. Chomaz, The dynamics of a viscous soap film with soluble surfactant, *J. Fluid Mech.* **442**, 387 (2001).
- [41] I. Kim and S. Mandre, Marangoni elasticity of flowing soap films, *Phys. Rev. Fluids* **2**, 082001(R) (2017).
- [42] I. Varga, R. Mészáros, and T. Gilányi, Adsorption of sodium alkyl sulfate homologues at the air/solution interface, *J. Phys. Chem. B* **111**, 7160 (2007).
- [43] C.-H. Chang and E. I. Franses, Adsorption dynamics of surfactants at the air/water interface: A critical review of mathematical models, data, and mechanisms, *Colloids Surf. A* **100**, 1 (1995).

- [44] C. N. Davies and P. K. P. Cheah, Spinning generators of homogeneous aerosols, *J. Aerosol Sci.* **15**, 719 (1984).
- [45] H. Bocanegra Evans, N. Dam, D. van der Voort, G. Bertens, and W. van de Water, Measuring droplet size distributions from overlapping interferometric particle images, *Rev. Sci. Instrum.* **86** (2015).
- [46] G. I. Taylor, The dynamics of thin sheets of fluid. III. Disintegration of fluid sheets, *Proc. R. Soc. A* **253**, 313 (1959).
- [47] F. E. C. Culick, Comments on a ruptured soap film, *J. Appl. Phys.* **31**, 1128 (1960).
- [48] W. R. McEntee and K. J. Mysels, Bursting of soap films. I. An experimental study, *J. Phys. Chem.* **73**, 3018 (1969).
- [49] N. Savva and J. Bush, Viscous sheet retraction, *J. Fluid Mech.* **626**, 211 (2009).
- [50] J. Eshima, L. Deike, and H. A. Stone, The effects of surfactant solubility on inertial Marangoni flow: Theory and numerics (to be published).
- [51] D. Vaynblat, J. R. Lister, and T. P. Witelski, Rupture of thin viscous films by van der Waals forces: Evolution and self-similarity, *Phys. Fluids* **13**, 1130 (2001).
- [52] H. Wee, B. W. Wagoner, and O. A. Basaran, Spontaneous rupture of surfactant-covered thin liquid sheets, *Phys. Rev. Fluids* **7**, 094005 (2022).
- [53] H. Wee, A. H. Kumar, X. Liu, and O. A. Basaran, Escape from pinch-off during contraction of liquid sheets and two-dimensional drops of low-viscosity fluids, *Phys. Rev. Fluids* **9**, 103601 (2024).
- [54] J. C. Burton and P. Taborek, Two-dimensional inviscid pinch-off: An example of self-similarity of the second kind, *Phys. Fluids* **19** (2007).
- [55] M. A. Fontelos, G. Kitavtsev, and R. M. Taranets, Asymptotic decay and non-rupture of viscous sheets, *Z. Angew. Math. Phys.* **69**, 1 (2018).
- [56] H. Lhuissier and E. Villermaux, “Effervescent” atomization in two dimensions, *J. Fluid Mech.* **714**, 361 (2013).
- [57] D. Lohse and E. Villermaux, Double threshold behavior for breakup of liquid sheets, *Proc. Natl. Acad. Sci. U.S.A.* **117**, 18912 (2020).
- [58] A. K. Dixit, C. Zhao, S. Zaleski, D. Lohse, and V. Sanjay, Holes in sheets: Double-threshold rupture of draining liquid films, *Phys. Rev. Lett.* **136**, 084001 (2026).
- [59] O. E. Jensen and J. B. Grothberg, Insoluble surfactant spreading on a thin viscous film: Shock evolution and film rupture, *J. Fluid Mech.* **240**, 259 (1992).
- [60] J. Eshima, T. Aurégan, E. Villermaux, H. A. Stone, and L. Deike, Data set for “Solubility enhanced surfactant-induced flow in air-liquid-air sheets”, [10.34770/pk7y-9v87](https://doi.org/10.34770/pk7y-9v87) (2026).
- [61] H. P. Kavehpoor, Coalescence of drops, *Annu. Rev. Fluid Mech.* **47**, 245 (2015).

End Matter

Here, we expand on the details of the experimental modeling assumptions and show the derivation of (7) from the similarity solution of insoluble surfactant deposition [27,28].

Experimental modeling assumption details: First, we explain why stretching due to the background flow of the Savart sheet is assumed weak relative to the flow induced by the Marangoni stress. The background flow is directed outward from the jet and is nearly uniform [33] with velocity $v_j \approx 4 \text{ ms}^{-1}$, so a patch of radius r_f located at distance r_s from the jet center is stretched azimuthally (relative to the jet center) at velocity $\approx v_j r_f / r_s$. Integrating over time with r_f proportional to $t^{1/2}$ gives deformation $\approx (2/3) v_j r_f t / r_s \lesssim 0.2 r_f$ for $t < 5 \text{ ms}$ (taking $r_s \approx 6.5 \text{ cm}$); the stretching is thus weak and the patterns seen in Fig. 1(b) are approximately axisymmetric. Furthermore, we measure r_f along the ray from the jet center to minimize background azimuthal stretching effects on our measurements. Specifically, since the focal plane of the camera lies along the ray from the jet center [see Fig. 1(a)], r_f is taken as half the horizontal chord intersecting the ellipse in the frame [see Fig. 1(b)].

Second, we explain why the drop coalescence dynamics can be neglected. The coalescence is estimated to occur on the inertio-capillary timescale $\sqrt{\rho r_d^3 / \Sigma} \approx 0.01 \text{ ms}$ [61] (where Σ is the characteristic surface tension), consistent with experiments where coalescence is seen to complete within $\approx 0.1 \text{ ms}$,

far shorter than the timescale of the surfactant front growth (in our experiments, observable up to $\approx 5 \text{ ms}$). In other words, r_d in the experiment is small enough that the details of the drops do not need to be considered.

Derivation of the similarity solution: For insoluble surfactant deposition with initial surfactant concentration Γ_i (top-bottom symmetric) onto a flat sheet of thickness h_i otherwise at rest, letting $N_\Gamma = 2\pi \int_0^\infty \Gamma_i r dr$ and $\Gamma_m = \max \Gamma_i$, the characteristic radial length scale $\mathcal{L}$ and timescale $\mathcal{T}$ are given by

$$\mathcal{L} := \left(\frac{N_\Gamma}{\pi \Gamma_m} \right)^{\frac{1}{2}}, \quad (\text{B1a})$$

$$\mathcal{T} := \left(\frac{\rho h_i \mathcal{L}^2}{\Delta \Sigma} \right)^{\frac{1}{2}}, \quad (\text{B1b})$$

where the characteristic surface tension deficit $\Delta \Sigma := -\Gamma_m (d\sigma/d\Gamma)|_{\Gamma=0}$. Let μ and Σ be the dynamic viscosity and surface tension of pure DI water respectively. There are two nondimensional numbers,

$$\mathcal{M} := \frac{\Delta \Sigma \mathcal{L}^2}{h_i^2 \Sigma} = \frac{N_\Gamma \left(\left. \frac{-d\sigma}{d\Gamma} \right|_{\Gamma=0} \right)}{\pi h_i^2 \Sigma}, \quad (\text{B2a})$$

$$\text{Re} := \left(\frac{\rho \Delta \Sigma \mathcal{L}^2}{h_i \mu^2} \right)^{\frac{1}{2}} = \left(\frac{\rho N_\Gamma \left(\left. \frac{-d\sigma}{d\Gamma} \right|_{\Gamma=0} \right)}{h_i \mu^2 \pi} \right)^{\frac{1}{2}}, \quad (\text{B2b})$$

where the Marangoni number $\mathcal{M}$ is the ratio of Marangoni to capillary stress and the Reynolds number Re is the ratio of inertia to viscous extensional stress.

Then, assuming a thin-film limit $h_i/\mathcal{L} \ll 1$, the following nondimensional equations are found for radial velocity $\tilde{u}(\tilde{r}, \tilde{t})$, thickness $\tilde{h}(\tilde{r}, \tilde{t})$, and surface surfactant concentration $\tilde{\Gamma}(\tilde{r}, \tilde{t})$, with nondimensional variables defined by $\tilde{r} = r/\mathcal{L}$, $\tilde{t} = t/T$, $\tilde{u} = uT/\mathcal{L}$, $\tilde{h} = h/h_i$, $\tilde{\Gamma} = \Gamma/\Gamma_m$,

$$\frac{\partial \tilde{u}}{\partial \tilde{t}} + \tilde{u} \frac{\partial \tilde{u}}{\partial \tilde{r}} = -\frac{2}{\tilde{h}} \frac{\partial \tilde{\Gamma}}{\partial \tilde{r}} + \frac{1}{2\mathcal{M}} \frac{\partial}{\partial \tilde{r}} \left[\frac{1}{\tilde{r}} \frac{\partial}{\partial \tilde{r}} \left(\tilde{r} \frac{\partial \tilde{h}}{\partial \tilde{r}} \right) \right] + \frac{4}{\text{Re}} \frac{1}{\tilde{h}} \left[\frac{\partial}{\partial \tilde{r}} \left(\frac{\tilde{h}}{\tilde{r}} \frac{\partial}{\partial \tilde{r}} (\tilde{r} \tilde{u}) \right) - \frac{1}{2} \frac{\tilde{u}}{\tilde{r}} \frac{\partial \tilde{h}}{\partial \tilde{r}} \right], \quad (\text{B3a})$$

$$\frac{\partial \tilde{h}}{\partial \tilde{t}} = -\frac{1}{\tilde{r}} \frac{\partial}{\partial \tilde{r}} (\tilde{r} \tilde{u} \tilde{h}), \quad (\text{B3b})$$

$$\frac{\partial \tilde{\Gamma}}{\partial \tilde{t}} = -\frac{1}{\tilde{r}} \frac{\partial}{\partial \tilde{r}} (\tilde{r} \tilde{u} \tilde{\Gamma}). \quad (\text{B3c})$$

When $\mathcal{M}^{-1} = \text{Re}^{-1} = 0$, system (B3) is mathematically the 2D axisymmetric compressible gas equations upon replacing $-2\tilde{\Gamma}$ with pressure, and $\tilde{h}$ with the density (and pressure/density $^\gamma = \text{constant}$ for $\gamma = 1$) [40,41].

The late-time $\tilde{t} = t/T \gg 1$ similarity solutions are found by matching the three regions as shown in Fig. 1(c).

Below, we briefly summarize scaling arguments in [27,28]. Subscripts denote the region under discussion. Let $\Delta \tilde{r}$ be the characteristic width of a given region. Since the film is undisturbed at far field, $\tilde{h}_{\text{III}} \sim 1$. Since $\tilde{u}_{\text{III}} \sim \Delta \tilde{r}_{\text{III}} \tilde{t}^{-1}$, balancing inertia $\tilde{u}_{\text{III}} \tilde{t}^{-1}$ with capillary $\tilde{h}_{\text{III}} (\Delta \tilde{r}_{\text{III}})^{-3}$ and/or viscous extensional stress $\tilde{u}_{\text{III}} (\Delta \tilde{r}_{\text{III}})^{-2}$ gives $(\tilde{u}_{\text{III}}, \tilde{h}_{\text{III}}, \Delta \tilde{r}_{\text{III}}) \sim (\tilde{t}^{-1/2}, \tilde{t}^0, \tilde{t}^{1/2})$. Upon integrating across region II where there is a balance between Marangoni and capillary stress, which can be thought of as a jump region, $\tilde{\Gamma}_I \sim \tilde{h}_{\text{III}}^2 (\Delta \tilde{r}_{\text{III}})^{-2} \sim \tilde{t}^{-1}$. Then by global conservation of surfactants, $\Delta \tilde{r}_I \sim \tilde{t}^{1/2}$. Hence, $\tilde{u}_I \sim \tilde{t}^{-1/2}$ and using that $\tilde{\Gamma}_I/\tilde{h}_I$ is conserved in Lagrangian coordinates [40], it follows that $\tilde{h}_I \sim \tilde{t}^{-1}$. Thus, $(\tilde{u}_I, \tilde{h}_I, \tilde{\Gamma}_I, \Delta \tilde{r}_I) \sim (\tilde{t}^{-1/2}, \tilde{t}^{-1}, \tilde{t}^{-1}, \tilde{t}^{1/2})$.

In particular, the surfactant front and minimum thickness at late times $\tilde{t} = t/T \gg 1$ are given respectively by [27,28]

$$\tilde{r}_f = \eta_f^{\text{insol}} \tilde{t}^{\frac{1}{2}}, \quad \tilde{h}_{\text{min}} = (\eta_f^{\text{insol}})^{-2} \tilde{t}^{-1}, \quad (\text{B4})$$

where $\tilde{r}_f = r_f/\mathcal{L}$, $\tilde{h}_{\text{min}} = h_{\text{min}}/h_i$, and $\eta_f^{\text{insol}} = \eta_f^{\text{insol}}(\mathcal{M}, \text{Re})$ is a prefactor, set by $\mathcal{M}$ and Re that can be found from a

shooting method for the similarity solution ordinary differential equations [27,28].

In order to compare against experiments tracking the surfactant front, we wish to convert (B4) back to dimensional variables. Such a step involves the factor

$$\mathcal{L} T^{-\frac{1}{2}} = \left(\frac{\Delta \Sigma \mathcal{L}^2}{\rho h_i} \right)^{\frac{1}{4}} = \left(\frac{N_\Gamma \left(\frac{d\sigma}{d\Gamma} \right)_{\Gamma=0}}{\pi \rho h_i} \right)^{\frac{1}{4}}. \quad (\text{B5})$$

As discussed in the main text the total amount of surfactants deposited in the experiment is $(4/3)\pi r_d^3 c_0$. Then, the top-bottom symmetric insoluble surfactant deposition with the same amount of surfactants satisfies

$$2N_\Gamma = \frac{4}{3} \pi r_d^3 c_0. \quad (\text{B6})$$

Substituting (B6) and (3) into (B2), (B4), and (B5) gives the expression

$$r_f = \eta_f^{\text{insol}} (2\Lambda_d)^{-\frac{1}{4}} \left(\frac{4r_d^3 \Delta \Sigma_0}{3\rho h_i^2} \right)^{\frac{1}{4}} \tilde{t}^{\frac{1}{2}}, \quad (\text{B7})$$

as desired, where $\eta_f^{\text{insol}} = \eta_f^{\text{insol}}(\mathcal{M}, \text{Re})$ with $\mathcal{M}$ and Re given by

$$\mathcal{M} = (2\Lambda_d)^{-1} \frac{4r_d^3 \Delta \Sigma_0}{3h_i^3 \Sigma} \quad (\text{B8a})$$

$$\text{Re} = (2\Lambda_d)^{-\frac{1}{2}} \left(\frac{4\rho \Delta \Sigma_0 r_d^3}{3h_i^2 \mu^2} \right)^{\frac{1}{2}}. \quad (\text{B8b})$$

It is also observed that a factor $\Lambda_d^{1/2}$ for h_{min} arises since $h_{\text{min}} = (\eta_f^{\text{insol}})^{-2} (2\Lambda_d)^{1/2} [\rho h_i^4 \mathcal{L}^4 / ([4/3] r_d^3 \Delta \Sigma_0)]^{1/2} \tilde{t}^{-1}$. Note that the predictions [Eqs. (B7) and (B8)] vanish in the insoluble limit $\Lambda_d \rightarrow \infty$ since the total amount of surfactants deposited $(4/3)\pi r_d^3 c_0$ vanishes as $c_0 \rightarrow 0$ [for fixed $\Delta \Sigma_0$, larger $k_a k_d^{-1}$ leads to smaller c_0 ; see Eq. (3)].

A technical remark is that a particular definition of length scale $\mathcal{L} = [(4/3)r_d^3 h_i^{-1}]^{1/2}$ was chosen to nondimensionalize the experimental data and to define η_f [Eqs. (1) and (5)], which assumes that $\Gamma_m = h_i c_0/2$ [see the definition (B1a)]. However, the theoretical prediction [Eqs. (B7) and (B8)] is independent of Γ_m and therefore does not require assumptions about the exact value of Γ_m .