

# Nanoscale Mirrorless Superradiant Lasing

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We predict collective free-space lasing in a dense nanoscopic emitter arrangement where dipole-dipole coupled atomic emitters synchronize their emission and exhibit lasing behavior without the need for an optical resonator. At the example of a subwavelength-spaced linear emitter chain with varying fractions of pumped and unpumped emitters, we present a comprehensive study of this mirrorless lasing phenomenon. The total radiated power transitions from subradiant suppression under weak pumping to superradiant enhancement at stronger pumping, while exhibiting directional emission confined to a narrow spatial angle. At the same time multiple independent spectral emission lines below the lasing threshold merge toward a single narrow spectral line at high pump power. The most substantial enhancement and line narrowing occur when a fraction of unpumped atoms is present. We show that this leads to superradiant lasing near the bare atomic frequency, making the system a promising candidate for a minimalist active optical frequency reference.

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**Introduction**—More than half a century after their first realization, lasers have proven to be a ubiquitous and versatile tool in wide areas of science and technology [1–4]. Generically, lasers share an optical resonator and a gain medium amplifying light via stimulated emission. Superradiant lasers are a special subclass of so-called single-mode lasers, where the spectral bandwidth of the optical gain medium is much smaller than the linewidth of the optical resonator modes. Here, the emission spectrum is locked around the gain maximum [5–8]. The most promising route to realizing a superradiant laser to date relies on a laser-cooled gas of individual atoms with a very narrow optical transition, often called a clock transition, providing for light amplification [3]. Theoretical predictions suggest an unprecedented precision and accuracy of the emitted radiation at the bare atomic transition frequency with minimal sensitivity to technical noise [9,10], in close analogy to hydrogen maser technology.

Independent of these active clock developments over the years there have been several observations of narrow-band and directional coherent light emission from dense laser-driven atomic gases [11–15]. While often there are nonlinear optical mixing processes or collective Raman scattering identified as the most likely origin of these

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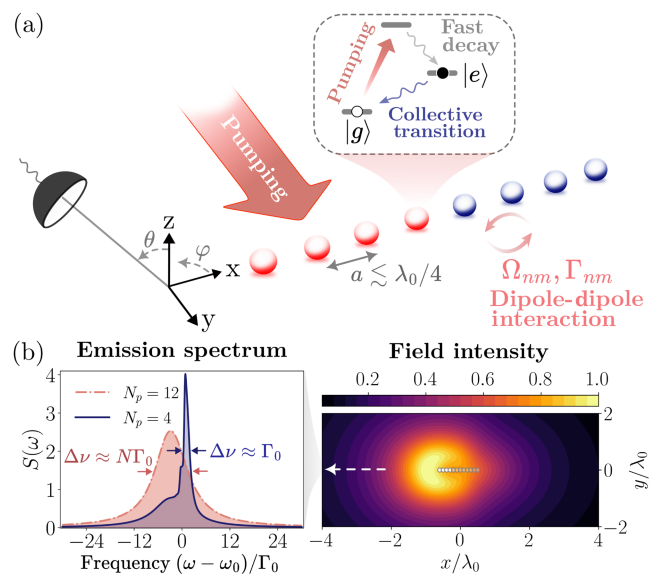


FIG. 1. (a) Schematics: a linear chain of quantum emitters in free space with a spacing  $a$  smaller than the  $|g\rangle \leftrightarrow |e\rangle$  transition wavelength  $\lambda_0$ . The incoherent pumping of  $N_p < N$  emitters in the chain creates a population in the excited state  $|e\rangle$  from where the emitters decay at rate  $\Gamma_0$  and interact collectively among each other through long-range dipole-dipole interactions. (b) Right panel: directional steady-state superradiant emission from a chain of  $N = 12$  emitters with  $N_p = 4$  being pumped. The normalized electric field intensity distribution is plotted from the  $z$  direction ( $z/\lambda_0 = 1$ ) with the emitters linearly polarized perpendicular to the chain axis and spaced by  $a/\lambda_0 = 0.1$ . Left panel: emission spectrum [Eq. (8)] in the direction of maximal emission  $(\varphi, \theta) = (\pi, \pi/2)$  (dashed white line) exhibits a narrowing linewidth  $\Delta\nu$  for partial pumping compared to a fully pumped ensemble. The incoherent pumping rate  $R = 10\Gamma_0$ .

phenomena, in some cases population inversion and lasing in driven ensembles appear to be the most plausible source of this radiation even without the presence of an optical resonator or external feedback [16–19]. A detailed experimental observation and analysis of such a phenomenon was recently presented as an extremely bad-cavity limit in the context of a bad-cavity laser [20]. There appears to be a connection to the phenomenon of random lasing eventually appearing in driven disordered media [21,22] or air lasing [23], which is sometimes tied to amplified spontaneous emission.

For atomic dipole emitters localized at distances much smaller than the transition wavelength, common interaction with the radiation field results in collective super- and subradiance [13–15,24,25]. In the case of initial population inversion, one can observe superradiant emission bursts followed by subradiance periods with excitation energy storage much longer than the natural lifetime of the emitters [26,27]. However, when the emitters are continuously repumped to their upper state and placed within an optical resonator with a large linewidth, superradiant stable steady-state emission of narrow-band radiation can occur, potentially creating an active optical frequency reference. We predict atomic dipole synchronization for dense nanoscopic emitter arrangements, exhibiting analogous behavior without any optical resonator. As a motivating result to study mirrorless lasing in more detail, our earlier theoretical work showed that a ring-shaped nanoarray of ground-state quantum emitters can serve as an optical resonator for a gain emitter placed in the center of the ring [28], unexpectedly indicating a laserlike behavior for suitable operating conditions.

In this Letter, we investigate generic examples of regular subwavelength arrays of quantum emitters, where a certain fraction is incoherently driven to a nonthermal population inverted state. We focus on the spontaneous buildup of coherence between the individual quantum emitters, as well as the power, directional, and spectral properties of the collectively emitted radiation emerging from the nonlinear coupled dipole dynamics. Key questions are to identify the conditions for which continuous superradiance emerges in the system as well as its relation to conventional lasing.

*Theoretical model*—We consider  $N$  identical two-level emitters with spontaneous decay rate  $\Gamma_0 = \omega_0^3 \mu_0^2 / (3\pi c^3 \epsilon_0 \hbar)$  tightly trapped in a regular spatial configuration at subwavelength distances and coupled via free-space dipole-dipole interaction. Although we assume subwavelength distances between the emitters, they are still large enough so that we can neglect molecular interactions and use pairwise free-space dipole-dipole interaction, including the radiative long-range contributions, which decrease as  $1/r$  in the far field ( $r$  being the separation between the emitters), leading to collective radiation properties via interference.

Our open system can thus be described by the following master equation in the Born-Markov approximation:  $\dot{\rho} = -i[\mathcal{H}, \rho] + \mathcal{L}_{\text{decay}}[\rho] + \mathcal{L}_{\text{pump}}[\rho]$  [29,30], where  $\hbar = 1$ ,  $\rho$  is the emitter density matrix, and the Hamiltonian in the rotating frame of the emitter frequency  $\omega_0$  reads

$$\mathcal{H} = \sum_{n,m \neq n}^N \Omega_{nm} \sigma_n^+ \sigma_m^- \quad (1)$$

The incoherent part describing the collective spontaneous emission is accounted for by the Lindblad term

$$\mathcal{L}_{\text{decay}}[\rho] = \frac{1}{2} \sum_{n,m}^N \Gamma_{nm} (2\sigma_n^- \rho \sigma_m^+ - \sigma_n^+ \sigma_m^- \rho - \rho \sigma_n^+ \sigma_m^-). \quad (2)$$

The incoherent pumping process can be consistently modeled as inverse spontaneous emission, which populates the excited quantum emitter state without resonant coherent driving. Hence we add an extra Lindblad superoperator for the pump,

$$\mathcal{L}_{\text{pump}}[\rho] = \frac{1}{2} \sum_{n=1}^N R_n (2\sigma_n^+ \rho \sigma_n^- - \sigma_n^- \sigma_n^+ \rho - \rho \sigma_n^- \sigma_n^+), \quad (3)$$

where the pump rate  $R_n$  can vary as illustrated in Fig. 1(a), such that only a fraction of the emitters is pumped.

Dipole-dipole interaction is represented by the coherent and dissipative part,

$$\Omega_{nm} - \frac{i\Gamma_{nm}}{2} = -\mu_0 \omega_0^2 \mathbf{d}_n^* \cdot \mathbf{G}(\mathbf{r}_n - \mathbf{r}_m, \omega_0) \cdot \mathbf{d}_m, \quad (4)$$

of the free-space electromagnetic Green's function with  $\mathbf{r}_n$  and  $\mathbf{d}_n$  being the position and dipole polarization of the  $n$ th emitter, which reads [31]

$$\mathbf{G}(\mathbf{r}, \omega_0) = \frac{e^{ik_0 r}}{4\pi k_0^2 r^3} \left[ (k_0^2 r^2 + ik_0 r - 1) \mathbb{1} - (k_0^2 r^2 + 3ik_0 r - 3) \frac{\mathbf{r} \otimes \mathbf{r}}{r^2} \right], \quad (5)$$

where  $r = |\mathbf{r}|$  and  $k_0 = 2\pi/\lambda_0$  is the wave number corresponding to the emitter transition frequency. For a single emitter, Eq. (4) yields the vacuum emission rate  $\Gamma_{nn} = \Gamma_0$ , while the energy shift  $\Omega_{nn}$  arising from  $\mathbf{G}$  function leads to a divergence and will be set to zero in the following, as it would lead to a renormalized resonance frequency  $\omega_0$ . We will assume throughout this Letter that all emitters are linearly polarized in the  $z$  direction  $\mathbf{d} = (0, 0, 1)^T$ . However, qualitatively similar conclusions are reached for other polarizations.

The total rate of the ensemble photon emission is then given by [32]

$$I(t) = \sum_{n,m=1}^N \Gamma_{nm} \langle \sigma_n^+ \sigma_m^- \rangle. \quad (6)$$

Note that besides the individual diagonal terms in the final double sum, the relative phase coherence of emitter pairs in the off-diagonal terms can strongly modify the total emission power. We will see later that these off-diagonal terms can be destructive (subradiance) or constructive (superradiance), depending on the chosen operating conditions, and vanish in dilute emitter ensembles. The electric field intensity  $\langle \mathbf{E}^-(\mathbf{r}) \mathbf{E}^+(\mathbf{r}) \rangle$  at any position  $\mathbf{r}$  can be directly calculated from the electric field generated by the dipole operators via

$$\mathbf{E}^+(\mathbf{r}) = \mu_0 \omega_0^2 \sum_{n=1}^N \mathbf{G}(\mathbf{r} - \mathbf{r}_n, \omega_0) \cdot \mathbf{d}_n \sigma_n^-, \quad (7)$$

where we assume the initial field to be in the vacuum state, neglecting any input noise [29,30].

To analyze the steady-state spectral properties of the system and particularly the linewidth  $\Delta\nu$  of the emitted field, we calculate the directional spectrum  $S(\omega, \theta, \varphi)$  via the Fourier transform of the electric field intensity in the far field ( $r \gg \lambda_0$ ). Here, the angles  $(\theta, \varphi)$  parametrize the direction of detection, shown in Fig. 1(a). The directional far-field spectrum for an ensemble of atoms with identical dipole polarizations  $\mathbf{d}$  and positioned along the  $x$  direction is then given by [32–37]

$$S(\omega, \varphi, \theta) = \frac{3\Gamma_0}{8\pi} \left( 1 - (\mathbf{R}(\theta, \varphi) \cdot \mathbf{d})^2 \right) \times 2\Re \left\{ \int_0^\infty d\tau e^{-i\omega\tau} \sum_{n,m=1}^N e^{ik_0(x_m - x_n) \cos \varphi \sin \theta} \langle \sigma_n^+(\tau) \sigma_m^-(0) \rangle \right\}, \quad (8)$$

with  $\sigma_m^-(0)$  referring to the stationary state of  $\sigma_m^-$  under the time evolution, and the unit vector  $\mathbf{R}(\theta, \varphi) = [\cos(\varphi) \sin(\theta), \sin(\varphi) \sin(\theta), \cos(\theta)]$  describing the direction of detection; see Supplemental Material (SM) for details [38]. In Fig. 1(b), we show the steady-state emission properties for a chain of  $N = 12$  emitters, where a fraction of  $N_p$  emitters is incoherently pumped with the rate  $R = 10\Gamma_0$ . The dipole moment orientation is chosen perpendicular to the chain direction, and the emitter spacing is  $a/\lambda_0 = 0.1$ . The electric field intensity distribution in the stationary state and the emission spectrum in the direction of maximum emission  $(\varphi, \theta) = (\pi, \pi/2)$  are plotted. We observe the emergence of a narrow spectral peak close to the bare atomic transition frequency  $\omega_0$  when only a fraction (in this case  $N_p = 4$ ) of emitters is pumped compared to all of them being pumped. We will study the superradiance and spectral properties in more detail in the following sections.

*Gain and superradiance of a partially pumped array*—In contrast to the collectively pumped array studied here, in a conventional laser the optical resonator and the gain medium are separate physical entities. In this section, we will use this analogy, dividing our emitter ensemble into an antenna (resonator) and a gain subgroup, where only the latter is incoherently pumped. At the most basic example of a chain of  $N$  quantum emitters with the first  $N_p < N$  being pumped, we will demonstrate that this can lead to superior power emission at diminishing frequency shifts as desired for an active clock.

Figure 2 shows how for  $N$  emitters the total photon emission in Eq. (6) transitions from subradiance to superradiance compared to the case of independent emitters. The steady-state emission of  $N_p$  independent emitters with pumping rate  $R$  can be readily obtained and reads,

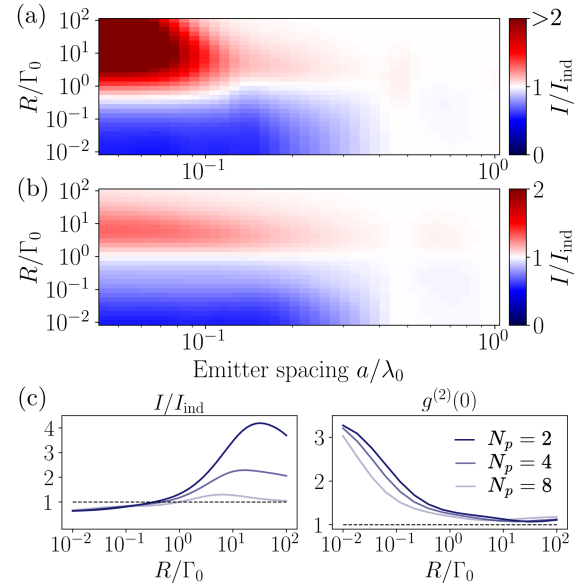


FIG. 2. Collective enhancement of photon emission: steady-state emission intensity  $I/I_{\text{ind}}$  normalized to the independent emitter case ( $a \gg \lambda_0$ ) as a function of the emitter spacing  $a$  and pumping rate  $R$ , where in (a) the first  $N_p = 4$  emitters are pumped, whereas in (b)  $N_p = N = 8$  all emitters are pumped. The white region indicates the crossover from the steady-state subradiant (blue region) to superradiant emission (red region). (c) Parameter cut as a function of the pumping rate for  $a = 0.05\lambda_0$  for various pumped fractions as well as the second-order correlation function. Collective superradiant emission is enhanced in the presence of unpumped emitters due to dipole-dipole exchange interaction. Simulations are based on the full quantum master equation.

$I_{\text{ind}} = N_p \Gamma_0 R / (R + \Gamma_0)$ . One can see the onset of superradiant emission starting from  $a \approx 0.5\lambda_0$ , which becomes appreciable at  $a \lesssim 0.25\lambda_0$  as the coherent dipole-dipole interaction increases. To evaluate the statistical properties of the emitted light one typically calculates the second-order correlation function at zero time delay. In Fig. 2(c), we observe that  $g^{(2)}(0)$  steady-state values are close to unity above the pumping threshold, in particular at spacings  $a \lesssim 0.1\lambda_0$ , where the emission is superradiant; see SM for details on the second-order correlation function calculations [38].

Furthermore, Figs. 2(a) and 2(c) show that a lower fraction  $N_p/N < 1$  of pumped emitters leads to a larger collective gain of the total emitted power. This enhancement compared to independent emitters follows from the presence of coherent dipole-dipole exchange between all  $N$  emitters in the ensemble, enabling excitation transfer to the unpumped emitters. Since this unpumped emitter fraction is not broadened by the incoherent pumping rate or power broadening, it acts as an effective high-Q resonator, which induces enhanced narrowband emission analogous to a Purcell enhancement. The pumped fraction then creates a broadband gain filtered and amplified by the unpumped emitters.

**Emission power and spectral linewidth using cumulant expansion**—When one goes to larger systems beyond ten emitters, a numerically exact, full quantum simulation of the master equation gets soon out of reach of numerical capabilities. As we are centrally interested in gain and inversion, a common cutoff method toward the few excitation manifolds cannot be used. Typically, a simple mean-field approach will not work as well, as no absolute phase will build up in the system, thus resulting in zero mean-field averages. In the following, we will study the system dynamics using a second-order cumulant expansion [38–40] including all pair correlations. Notably, in related physical models, this approximation turned out to capture the dynamics and steady-state properties remarkably well. For smaller systems, we benchmark the results against full quantum solutions of the master equation; see SM [38].

Here, we present the results for the steady-state total photon emission and spectral linewidth in the direction of maximal emission as a function of the total emitter number and a fixed pumped emitter number  $N_p$ . We obtain the direction of maximal emission  $(\varphi, \theta)$  by calculating the electric field intensity distribution in the far field via Eq. (7) and subsequently calculate the emission spectrum using Eq. (8). The linewidth is defined as the full width at half maximum of the maximum peak in the spectrum, as the system can exhibit multiple peaks in any spatial direction, as shown in Fig. 4. Surprisingly, Fig. 3(b) shows a sharp transition of the linewidth from  $\Delta\nu \approx R + N_p\Gamma_0$  to  $\Delta\nu \approx \Gamma_0$  as the total number of emitters  $N$  is increased while keeping  $N_p$  fixed. This is quite reminiscent of an ultracold cloud of atoms coupled to a single-mode cavity, where the system

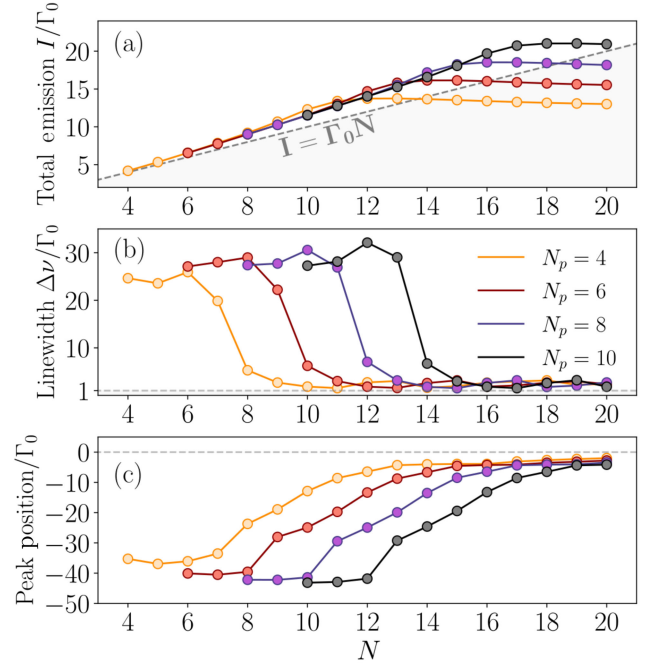


FIG. 3. Narrow linewidth superradiance: radiation from a linear chain of emitters spaced by  $a = 0.05\lambda_0$  with increasing total emitter number  $N$  for a fixed number of pumped emitters  $N_p$ . (a) The steady-state emission power [Eq. (6)] shows superradiance ( $> N\Gamma_0$ ) and reaches saturation for large  $N$ . (b) The spectral linewidth  $\Delta\nu$ , evaluated in the direction of maximal emission [Eq. (8)] exhibits a sudden narrowing with increasing  $N - N_p$  when  $N$  reaches  $N/N_p \approx 3/2$ . (c) The spectral peak position  $\omega - \omega_0$  approaches the atomic resonance even in the presence of strong dipole-dipole frequency shifts. In all plots,  $R = 20\Gamma_0$ .

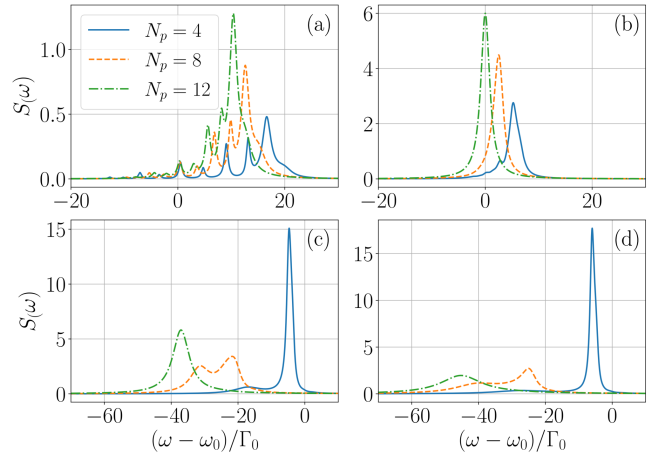


FIG. 4. Steady-state light emission spectra in the direction of maximal emission for a chain of  $N = 12$  emitters spaced by  $a = 0.05\lambda_0$  for various pumped fractions  $N_p = 4, 8, 12$  and pump powers (a)  $R = 0.2\Gamma_0$ , (b)  $R = 1\Gamma_0$ , (c)  $R = 10\Gamma_0$ , (d)  $R = 20\Gamma_0$ . We observe a transition from multiple independent lines to synchronized emission around a single frequency. Note that, for a pumped fraction of less than half, a powerful narrow line emission persists even for a very strong pump. The area under the spectral curves corresponds to the total emitted power.

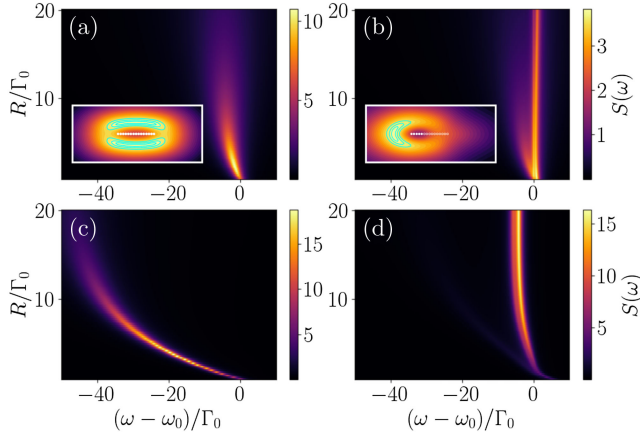


FIG. 5. Pump power dependence of emission spectra in the direction of maximal emission for a chain of  $N = 15$  particles. The results are shown for emitter spacings of  $a = 0.1\lambda_0$  with (a)  $N_p/N = 1$  and (b)  $N_p/N = 1/3$  pumped fraction, and for  $a = 0.05\lambda_0$  with (c)  $N_p/N = 1$  and (d)  $N_p/N = 1/3$ . The presence of nonpumped emitters leads to the appearance of a single narrow emission line close to the atomic resonance. Insets: electric field intensity in the  $xy$  plane for  $R = 10\Gamma_0$ .

transitions to collective superradiant lasing above a certain pumping threshold [9,41]. In the present case, the pumping rate  $R = 20\Gamma_0$  is chosen such that the system operates in the superradiant regime (colored as red in Fig. 2). As presented in Fig. 3(c), the spectral peak position approaches the atomic resonance frequency  $\omega_0$  as the number of unpumped emitters  $N - N_p$  increases, despite the initial strong dipole-dipole frequency shifts observed when all emitters are pumped.

*Light emission directionality and spectra*—Although the array’s spatial extent considered thus far is only of the order of the optical wavelength, the emitted radiation pattern differs from the single-emitter dipolar pattern and exhibits directionality. Simultaneously, the emitted light can exhibit a narrow spectral peak close to the bare emitter transition frequency  $\omega_0$ , if pumped above a certain threshold power, as shown in Fig. 4 for  $a = 0.05\lambda_0$ . This is of particular interest, as the system demonstrates superradiant emission with a narrow spectral linewidth without the need for optical elements or nearby resonators, unlike superradiant lasing in optical cavities [9,56,57]. Furthermore, Fig. 5 shows the pump power dependence of the emission spectra for a partially (right) and fully pumped (left) array of  $N = 15$  emitters. In case of partial pumping, one can see a striking persistence of a single narrow spectral peak close to the bare transition frequency, which spectral intensity becomes particularly apparent at smaller spacings as can be seen in Fig. 5(d). The insets show the electric field intensity distribution in the  $xy$  plane.

The numerical simulations were performed with the open-source frameworks `QuantumCumulants.jl` [40] and `QuantumOptics.jl` [58].

*Conclusions*—Our results point toward a narrow-linewidth superradiant directional light source solely composed of a dense regular array of two-level dipole emitters under partial incoherent pumping. The collective light emission transitions from sub- to superradiance as the pump rate exceeds the individual spontaneous decay rate. While strong pumping provides population inversion, a particularly large emission gain is observed when the emitters are spaced at subwavelength distances, where the strong and directional narrow-band emission resembling lasing originates solely from resonant free-space dipole-dipole interaction. The emission power and spectral purity, meaning a single spectral peak, are significantly enhanced when only a fraction of the atomic emitters are pumped and the remaining free atoms act as narrow-band outcoupling antennas.

A laserlike action in the absence of an optical resonator has been previously experimentally observed in various systems and was termed “mirrorless” lasing [16,20,59]. For a uniformly pumped ensemble, the superradiant power can reach twice the power of independent atomic emission. However, when additional unpumped emitters are present, strong pumping can lead to much larger gain factors only limited by the total number  $N$  of unpumped and pumped atoms. The spectrum then splits into a single narrow peak close to the atomic resonance and a weak broad, strongly shifted peak directly from the pumped atoms. In the extreme case, only a small fraction of gain atoms can thus provide for strong narrow-band emission close to the bare atomic frequency, which is particularly favorable for an active optical frequency standard [3,8,56].

While we focused our model on the generic case of one-dimensional arrays, other geometries such as ring-shaped or planar arrays could prove to be effective [28]. Geometric control over the atomic emitter ensemble can be expected to lead to improved spatial and spectral properties of the emitted light. We anticipate superior performance from much larger ensembles of emitters, which are currently out of reach of our numerical capabilities but potentially available in experiments. To relax the requirements on tight atomic positioning, operations on narrow transitions in the (far-)infrared region could present a promising experimental approach.

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*Data availability*—The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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