

Next-to-Leading-Order QCD Corrections to Nucleon Dirac Form Factors

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The leading-order perturbative QCD (pQCD) predictions for nucleon electromagnetic form factors were first made in late 1970s. In this Letter, for the first time, we accomplish the calculation of the next-to-leading-order (NLO) QCD corrections to nucleon Dirac form factors at large momentum transfer to leading-twist accuracy in the collinear factorization approach, specifically within the Krankl and Manashov renormalization scheme. The effect of NLO perturbative corrections turns out to be positive and substantial. Taking the nucleon leading-twist light-cone distribution amplitudes determined from the recent lattice simulations as input, we find that the state-of-the-art pQCD predictions significantly underestimate the available nucleon Dirac form factors in both spacelike and timelike domains. This nuisance indicates that some additional soft nonfactorizable contribution might be called for to account for the measured nucleon electromagnetic form factor data up to $Q^2 \approx 30 \text{ GeV}^2$.

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Introduction—Composite particles made of strongly coupled, relativistic quarks and gluons, protons, and neutrons are the building blocks of atomic nuclei, and consequently, of our planet and all its inhabitants, and even more, of our visible Universe. Understanding the internal structure of nucleons from quantum chromodynamics (QCD), the underlying theory of strong interaction, is the fundamental challenge faced by contemporary hadron and nuclear physics [1].

Nucleon electromagnetic form factors (EMFFs), which gauge the distributions of electric charge and magnetization inside the nucleon, are the fundamental probes into the internal structure of nucleons [1]. Nucleon EMFFs can be directly accessed from ep elastic scattering experiments, which are encoded in the following nucleon transition matrix elements:

$$\langle P' | j_{\text{em}}^\mu | P \rangle = \bar{N} \left[F_1(Q^2) \gamma^\mu - F_2(Q^2) \frac{i\sigma^{\mu\nu} q_\nu}{2m_N} \right] N, \quad (1)$$

with the electromagnetic current $j_{\text{em}}^\mu = \sum_f e_f \bar{\psi}_f \gamma^\mu \psi_f$. N and m_N denote the Dirac spinor and mass of the nucleon, respectively. $q^\mu \equiv P^\mu - P'^\mu$, and $Q^2 \equiv -q^2 \geq 0$ characterizes the squared momentum transfer. The scalar functions F_1 , F_2 are referred to as the (helicity-conserving) Dirac form factor and the (helicity-flipped) Pauli form factor, respectively.

After Hofstadter's pioneering work in the mid-1950s [2,3], intensive experimental efforts have been continuously devoted to measuring the nucleon EMFFs $F_{1,2}^{p,n}$ over the past seven decades (for comprehensive experimental review, see Refs. [4–7]). In the early 1990s, SLAC measured the spacelike proton EMFFs at $Q^2 = 31.3 \text{ GeV}^2$ [8]. In the prospective electron-ion colliders EIC and EicC, the momentum transfer for nucleon EMFFs may be further extended to $Q^2 \sim 40\text{--}50 \text{ GeV}^2$ [9]. Over the past two decades, the high-luminosity e^+e^- colliders have also started to explore nucleon EMFFs in the timelike region [10–16].

On the theoretical side, enormous intellectual endeavors have also been invested in unveiling the QCD dynamics beneath the nucleon EMFFs. At small and moderate Q^2 , the measured nucleon EMFFs have been confronted with the

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theoretical predictions made from various nonperturbative approaches—e.g., the vector meson dominance model [17–22], constituent quark model [23–25], cloudy bag model [26–28], Dyson-Schwinger equation [29–31], light-front Hamiltonian [32], chiral perturbation theory [33,34], light-cone sum rule [35,36], and lattice QCD [37–42], as well as dispersive analysis [43,44].

Thanks to asymptotic freedom, the nucleon EMFFs with large momentum transfer are generally believed to be adequately accounted for by perturbative QCD (pQCD). Within the collinear factorization formalism [45–52], they can be expressed as the perturbatively calculable hard-scattering kernel convoluted with nonperturbative light-cone distribution amplitudes (LCDAs) of the nucleon. The helicity selection rule implies that, at asymptotically large Q^2 , F_1 scales with $1/Q^4$, and F_2 scales with $1/Q^6$ [53], modulo logarithms of Q^2 [54]. Hence, the nucleon Dirac form factor dominates the Pauli form factor in such a limit.

The lowest-order (LO) hard-scattering kernel for F_1 was known in the late 1970s, shortly after the advent of QCD [46,47,52,55]. Curiously, notwithstanding that nearly half a century has elapsed, the next-to-leading-order (NLO) perturbative correction to this fundamental observable of the nucleon remains elusive today, due to severe technical obstacles. [Note that the status of pQCD predictions for nucleon EMFFs lags far behind that for pion EMFFs, where the one-loop QCD corrections were known in the early 1980s [56–59], and even the two-loop corrections have recently become available [60].] The theoretical stagnation for such a basic observable over such a long time is rather unusual, in contrast with ep deep inelastic scattering, where the three-loop QCD corrections became available about two decades ago [61].

It is the very goal of this Letter to fill this overdue gap by accomplishing the calculation of the NLO perturbative correction to the nucleon Dirac form factors and conducting a comprehensive phenomenological investigation.

Collinear factorization for the nucleon Dirac form factor—At large momentum transfer, the F_1 can be factorized as the following convolution integrals [It is worth mentioning that this factorization formula is anticipated to be subject to modification starting at the two-loop order, due to emergence of the new leading region contribution [51,62].]:

$$Q^4 F_1(Q^2) = \Phi_3^*(y) \otimes_y T_H(x, y, Q^2) \otimes_x \Phi_3(x) + \mathcal{O}\left(\frac{1}{Q^2}\right), \quad (2)$$

where $\otimes$ is a shorthand for the convolution integration over x [$dx \equiv dx_1 dx_2 dx_3 \delta(1 - \sum_i x_i)$]. $T_H(x, y)$ signifies the hard-scattering kernel, and $\Phi_3(x)$ represents the twist-3 nucleon LCDA. This factorized structure is illustrated in Fig. 1.

For definiteness, let us specialize to the proton. The leading-twist-3 LCDA of the proton assumes the following operator definition [63,64]:

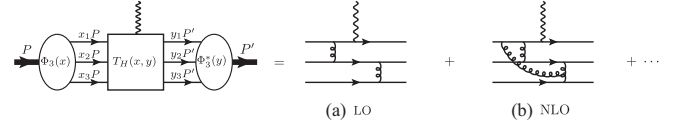


FIG. 1. Pictorial illustration of the nucleon EMFF in collinear factorization. For concreteness, we show typical LO and NLO Feynman diagrams for partonic reactions.

$$\begin{aligned} & \langle 0 | \epsilon_{ijk} (u_+^T(a_1 n) C \not{n} u_-^j(a_2 n)) \not{n} d_+^k(a_3 n) | p^\uparrow(P) \rangle \\ &= -\frac{1}{2} P \cdot n \not{n} u_p^\uparrow(P) \int [dx] e^{-iP \cdot n \sum x_i a_i} \Phi_3(x), \end{aligned} \quad (3)$$

where i, j, k denote the color indices; $q_\pm \equiv \frac{1}{2}(1 \pm \gamma_5)q$ represent the right- and left-handed components of the massless quark field; and $u_p^\uparrow(P)$ signifies the Dirac spinor of the $h = \frac{1}{2}$ proton in relativistic normalization. C represents the charge conjugation matrix, and n^μ is a lightlike vector. For simplicity, we have suppressed those lightlike gauge links whose role is to ensure gauge invariance. Renormalizing the nonlocal baryonic operator in Eq. (3), one recovers the celebrated Efremov-Radyushkin-Brodsky-Lepage (ERBL) evolution equation [47,49]: $d\Phi_3(x, \mu_F)/d \ln \mu_F^2 = V(x, y) \otimes_y \Phi_3(y, \mu_F)$.

Perturbative matching in the KM scheme—We first sketch how to achieve the desired factorization formula (2). With $Q^2 \gg \Lambda_{\text{QCD}}^2$, asymptotic freedom indicates that the transition matrix element in Eq. (1) in d -dimensional spacetime can be factorized as

$$\begin{aligned} \langle \Pi^\mu(Q^2) \rangle &\equiv \langle P' | j_{\text{em}}^\mu | P \rangle = \langle P' | [\bar{Q}_{\alpha\beta\gamma}(y)]^{\overline{\text{MS}}} | 0 \rangle \\ &\otimes_y \mathcal{T}_{\alpha\beta\gamma; \alpha'\beta'\gamma'}(x, y) \otimes_x \langle 0 | [\mathcal{Q}_{\alpha'\beta'\gamma'}(x)]^{\overline{\text{MS}}} | P \rangle, \end{aligned} \quad (4)$$

where $\alpha, \beta, \gamma, \dots$ denote the spinor indices affiliated with each quark line, and $\mathcal{T}$ signifies the perturbative coefficient function. The three-quark operator $\mathcal{Q}_{\alpha'\beta'\gamma'}(x)$ is defined via the Fourier transform of the nonlocal three-quark operators with lightlike separations $\epsilon^{ijk} u_\alpha^i(a_1 n) u_\beta^j(a_2 n) d_\gamma^k(a_3 n)$, where the lightlike gauge links are tacitly inserted [65].

It is advantageous to adopt the renormalization scheme originally proposed by Krankl and Manashov [66] (the KM scheme henceforth). [The KM scheme has several remarkable virtues. It warrants the vanishing of evanescent operators [67] in $d = 4$ dimensions, so that one only needs to work on the four-dimensional operator basis. The renormalization procedure in the KM scheme preserves Fierz identities between the renormalized operators [65]. The self-consistency of the KM scheme has been checked through three-loop accuracy [68]. It is also worth stressing that the KM scheme has also been routinely utilized by the lattice QCD practitioners to calculate the baryon LCDAs [69–71].] The $\overline{\text{MS}}$ -renormalized three-quark operators in

Eq. (4) are connected with the bare one through

$$[\mathcal{Q}_{\alpha\beta\gamma}(x)]^{\overline{\text{MS}}} = \mathcal{Z}_{\alpha\beta\gamma}^{\alpha'\beta'\gamma'}(x, x') \otimes \mathcal{Q}_{\alpha'\beta'\gamma'}(x'). \quad (5)$$

For the reader's convenience, the one-loop expression of $\mathcal{Z}$ in the KM scheme is explicitly given in the Supplemental Material [72].

The perturbative coefficient function $\mathcal{T}$ in Eq. (4) can be decomposed into

$$\mathcal{T}_{\alpha\beta\gamma, \alpha'\beta'\gamma'}(x, y) = T_{lmn}(x, y) \gamma_{\alpha\alpha'}^{(l)} \otimes \gamma_{\beta\beta'}^{(m)} \otimes \gamma_{\gamma\gamma'}^{(n)}. \quad (6)$$

Note that T_{lmn} is a scalar function, and the Lorentz index μ is affiliated with one of the $\gamma^{(l)}$, $\gamma^{(m)}$, and $\gamma^{(n)}$ matrices with $\gamma_{\mu_1 \dots \mu_n}^{(n)} \equiv \gamma^{[\mu_1} \gamma^{\mu_2} \dots \gamma^{\mu_n]}$ [65,66].

The central task now is to deduce the coefficient function $T_{lmn}(x, y)$. Since it is insensitive to long-distance physics, one can replace the incoming and outgoing nucleons with “fictitious” nucleons, viz. the free, collinear three-quark states $|q_1(u_1 P) q_2(u_2 P) q_3(u_3 P)\rangle$ and $|q_1(v_1 P') q_2(v_2 P') q_3(v_3 P')\rangle$. A virtue of the KM scheme is that the quark flavor structure becomes irrelevant in determining $\mathcal{Z}$ and $\mathcal{T}$ [66].

For a fictitious nucleon, the left side of Eq. (4) can be computed in perturbation theory:

$$\langle \Pi^i(Q^2) \rangle = \mathcal{A}_{lmn}(u, v) \bar{u}_{\alpha\beta\gamma} \left[\gamma_{\alpha\alpha'}^{(l)} \otimes \gamma_{\beta\beta'}^{(m)} \otimes \gamma_{\gamma\gamma'}^{(n)} \right] u_{\alpha'\beta'\gamma'}, \quad (7)$$

where $u_{\alpha'\beta'\gamma'} \equiv u_{\alpha'}(u_1 P) u_{\beta'}(u_2 P) u_{\gamma'}(u_3 P)$.

Analogously, we also evaluate the nonlocal matrix elements on the right side of Eq. (4) perturbatively. After dropping all the scaleless integrals, we obtain

$$\langle 0 | [\mathcal{Q}_{\alpha\beta\gamma}(x)]^{\overline{\text{MS}}} | P \rangle = \mathcal{Z}_{\alpha\beta\gamma}^{\alpha'\beta'\gamma'}(x, u) u_{\alpha'\beta'\gamma'}. \quad (8)$$

Practically, we utilize the following matching equation [Note that one has to reexpand the products of γ matrices in the right-hand side of Eq. (9), and then cast them in the basis of the totally antisymmetrized products of γ matrices.] to deduce $\mathcal{T}$:

$$\begin{aligned} \mathcal{A}_{lmn}(u, v) \gamma_{\alpha\alpha'}^{(l)} \otimes \gamma_{\beta\beta'}^{(m)} \otimes \gamma_{\gamma\gamma'}^{(n)} &= \tilde{\mathcal{Z}}_{\alpha\beta\gamma}^{\alpha_2\beta_2\gamma_2}(y, v) \\ &\otimes \left[T_{l'm'n'}(x, y) \gamma_{\alpha_2\alpha_1}^{(l')} \otimes \gamma_{\beta_2\beta_1}^{(m')} \otimes \gamma_{\gamma_2\gamma_1}^{(n')} \right] \otimes \mathcal{Z}_{\alpha'\beta'\gamma'}^{\alpha_1\beta_1\gamma_1}(x, u). \end{aligned} \quad (9)$$

Sketch of the calculation—We start with the partonic reaction $\gamma^* + q_1 q_2 q_3 \rightarrow q_1 q_2 q_3$. We use the packages HepLib [73] and FeynArts [74] to generate the Feynman diagrams and the corresponding amplitudes. There are about 48 tree-level diagrams, and 2040 one-loop diagrams.

As is evident in Fig. 1, the amplitude in each diagram bears the form $\bar{u}_{\alpha\beta\gamma} [\Gamma_{\alpha\alpha'}^{(1)} \otimes \Gamma_{\beta\beta'}^{(2)} \otimes \Gamma_{\gamma\gamma'}^{(3)}] u_{\alpha'\beta'\gamma'}$. Moving $\not{p}$ to

the rightmost and $\not{p}'$ to the leftmost repeatedly using the anticommunication relation, [Note that ℓ also appears ubiquitously in the intermediate stage. By Lorentz covariance, one can trade the loop momentum ℓ for the metric tensor or the external momenta P and P' .] and employing the Dirac equation, each $\Gamma^{(i)}$ can be expressed as the product of a string of Dirac γ matrices, which can be further cast into the form prescribed in Eq. (7).

The aforementioned operations can be automated by HepLib [73]. We then use Apart [75] and FIRE [76] for partial fraction and IBP reduction. We end up with about 100 one-loop two-point and three-point master integrals, which are computed using the differential equation method [77,78]. Upon renormalizing the QCD coupling in the $\overline{\text{MS}}$ scheme, we obtain the UV-finite expression of the NLO coefficients $\mathcal{A}_{lmn}$ in Eq. (7), which, however, still possess an uncanceled single infrared pole. Validated by the matching Eq. (9), we find that, hearteningly, the IR poles in $\mathcal{A}_{lmn}$ are exactly canceled by the corresponding poles in $\mathcal{Z}$, so that the short-distance coefficients $T_{lmn}(x, y)$ are indeed IR finite in a pointwise manner. Therefore, we verify that the collinear factorization does hold at one-loop order for the nucleon Dirac form factor.

To recover the desired structure in Eq. (2), we substitute the expression [65,79]

$$\begin{aligned} \langle 0 | [\mathcal{Q}_{\alpha\beta\gamma}(x)]^{\overline{\text{MS}}} | P \rangle &= \frac{1}{4} \left\{ \mathcal{V}(\not{p}C)_{\alpha\beta} (\gamma_5 N)_\gamma + \mathcal{A}(\not{p}\gamma_5 C)_{\alpha\beta} N_\gamma \right. \\ &\quad \left. + \mathcal{T}(i\sigma_{\mu\nu} P^\nu C)_{\alpha\beta} (\gamma_\mu \gamma_5 N)_\gamma \right\} \end{aligned} \quad (10)$$

into Eq. (4). Using the relations among $\mathcal{V}(x)$, $\mathcal{A}(x)$, $\mathcal{T}(x)$, and $\Phi_3(x)$ [65,79], we obtain $Q^4 \langle \Pi^i(Q^2) \rangle = \Phi_3(x) \otimes_x T_H(x, y) \otimes_y \Phi_3^*(y) \bar{N} \gamma^\mu N$. The intended Dirac form factor through NLO accuracy can then be predicted in line with Eq. (2).

Master formula for the nucleon Dirac form factor at NLO—The leading-twist nucleon LCDA is conveniently expanded in the orthogonal polynomial basis [80]:

$$\Phi_3(x, \mu_F) = f_N(\mu_F) \sum_{0 \leq i \leq j} a_{ij}(\mu_F) \phi_{ij}(x), \quad (11)$$

where $\phi_{ij}(x) = 120 x_1 x_2 x_3 \mathcal{P}_{ij}(x)$, and f_N denotes the nucleon decay constant. The first few orthogonal polynomials $\mathcal{P}_{ij}(x)$ have been tabulated in [80,81] [e.g., Eq. (12) in Ref. [81]]. All the nonperturbative dynamics is encoded in the expansion coefficients $a_{ij}(\mu_F)$.

For phenomenological usage, it is advantageous to recast Eq. (2) from the convolutional form into an algebraic one:

$$Q^4 F_1(Q^2) = f_N^2(\mu_F) \sum_{ijkl} a_{ij}(\mu_F) \mathcal{H}^{ijkl} a_{kl}(\mu_F), \quad (12)$$

with $\mathcal{H}^{ijkl} \equiv \phi_{ij}(x) \otimes_x T_H(x, y) \otimes_y \phi_{kl}(y)$.

The short-distance coefficients $\mathcal{H}^{ijkl}$ can be organized in powers of α_s . Through NLO accuracy, they can be parametrized as

$$\mathcal{H}^{ijkl} = \frac{1600\pi^2\alpha_s^2}{3} \left\{ e_u c_0^{ijkl} \left[1 + \frac{\alpha_s}{\pi} \left(c_1^{ijkl} L_\mu + c_2^{ijkl} \right) \right] + e_d d_0^{ijkl} \left[1 + \frac{\alpha_s}{\pi} (d_1^{ijkl} L_\mu + d_2^{ijkl}) \right] + \mathcal{O}(\alpha_s^2) \right\}, \quad (13)$$

with $L_\mu \equiv \ln \mu^2/Q^2$. For simplicity, we have set $\mu_R = \mu_F = \mu$. Note that $\mathcal{H}^{ijkl} = \mathcal{H}^{klij}$ and $c_1^{ijkl} = d_1^{ijkl}$. For a given $ijkl$, the coefficients $c_{0,1,2}^{ijkl}$ and $d_{0,1,2}^{ijkl}$ can be analytically inferred, whose explicit expressions are given in the Supplemental Material [72].

We conduct the fourfold convolution integration following two different routes: one is to utilize the numerical integrator Cubature [82] by implementing a multiple-precision floating point around 100 digits, and the other is to carry out the integration analytically with the assistance of the package PolyLogTools [83]. A perfect agreement is found between the analytic and numerical approaches. In Table I, we enumerate the values for c_i and d_i associated with some low-lying $(ijkl)$. [We notice that a PhD thesis by Knödlseider in 2015 was dedicated to calculating the NLO QCD correction to the nucleon Dirac form factor at leading twist [84]. Though the analytic result of the one-loop amplitude was obtained in Ref. [84], the IR-finite hard-scattering kernel $T^{(1)}(x, y)$ has not been given in a pointwise manner. More importantly, since the author failed to accomplish the fourfold convolution integrations in a numerically stable fashion, no attempt at confronting the NLO pQCD predictions with the measured nucleon form factor data was made in Ref. [84].]

Equipped with Table I, Eq. (12) constitutes the master formula of rendering predictions for the proton Dirac form factor through one-loop accuracy. In the isospin symmetric limit, the neutron Dirac form factor F_1^n can be obtained from Eq. (12) by making the replacement $e_u \leftrightarrow e_d$ in (13). It is also straightforward to adapt this master formula from the spacelike domain to the timelike one, provided that one makes the replacement $L_\mu \rightarrow L_\mu + i\pi$ in Eq. (13), with Q^2 now indicating the squared invariant mass of the nucleon-antinucleon pair.

Phenomenological exploration—It is now the time to confront the state-of-the-art pQCD predictions with the available large- Q^2 data. We take the data points for the spacelike proton Dirac form factor from Refs. [8,85], [Note that it is the magnetic Sachs form factor of the nucleon, $G_M(Q^2)$, that is directly accessible in ep scattering

TABLE I. Numerical values of c_i and d_i affiliated with various $\mathcal{H}^{ijkl}$ with $0 \leq \{ijkl\} \leq 2$.

$ijkl$	c_0^{ijkl}	c_2^{ijkl}	d_0^{ijkl}	d_2^{ijkl}	$c_1^{ijkl} = d_1^{ijkl}$
0000	1.0000	21.334	2.0000	17.491	4.8333
1000	8.1667	24.347	-8.1667	24.347	5.3889
1010	168.78	28.623	174.22	28.251	5.9444
1100	3.5000	28.699	-3.5000	18.233	5.5000
1110	38.111	29.655	-38.111	29.655	6.0556
1111	59.889	27.016	21.778	26.056	6.1667
2000	21.350	26.260	8.0500	24.454	5.7222
2010	89.833	31.367	-89.833	31.367	6.2778
2011	93.100	32.158	9.8000	37.615	6.3889
2020	234.71	34.390	54.880	33.494	6.6111
2100	8.7500	27.639	-8.7500	27.639	5.9444
2110	159.25	32.756	159.25	32.904	6.5000
2111	44.917	32.687	-44.917	32.687	6.6111
2120	84.525	35.895	-84.525	35.895	6.8333
2121	159.25	37.626	159.25	37.603	7.0556
2200	3.2000	27.381	0.10000	45.283	6.0000
2210	12.250	31.624	-12.250	31.624	6.5556
2211	7.1167	39.349	2.6833	35.342	6.6667
2220	30.345	35.977	4.9350	36.841	6.8889
2221	7.1750	38.780	-7.1750	38.780	7.1111
2222	7.7900	34.975	1.7200	34.974	7.1667

experiments. At large Q^2 , it nevertheless becomes legitimate to assume $F_1(Q^2) \approx G_M(Q^2)$, and the data points for the timelike proton Dirac form factor from Refs. [11–15,86,87]. We discard many irrelevant data at small momentum transfer. In contrast, the quality of data and the covered range of Q^2 for neutron EMFFs is much less competitive with respect to that for protons. The data points for the spacelike neutron Dirac form factor are taken from Refs. [88,89], while those for the timelike neutron Dirac form factor are taken from Refs. [10,16].

As the key nonperturbative input, our present knowledge about the nucleon LCDAs is far from confirmative. During the past few decades, the low-lying expansion coefficients of nucleon LCDAs have been extensively investigated by different approaches [65,70,81,90–94]. In our numerical analysis, we have taken eight different ansätze of nucleon LCDAs as input. Six of them are obtained from the QCD sum rules, referred to as CZ [90], COZ [95], KS [92], SB [93], BLW [94], and ABO [65]. Two of them are provided by the recent lattice QCD simulations, abbreviated as LAT14 [81] and RQCD [70].

The predicted values of the expansion coefficients of the nucleon are usually given at the initial scale $\mu_0 = 1$ or 2 GeV, and they are scattered in a wide range in literature. The one-loop renormalization group equations are invoked to evolve f_N and $a_{ij}(\mu)$ from μ_0 to another scale, viz. $f_N(\mu) = f_N(\mu_0) [\alpha_s(\mu)/\alpha_s(\mu_0)]^{2/(3\beta_0)}$ and $a_{ij}(\mu) = a_{ij}(\mu_0) [\alpha_s(\mu)/\alpha_s(\mu_0)]^{\gamma_{ij}/\beta_0}$. $\beta_0 = 11 - \frac{2}{3}n_f$ with $n_f = 3$ signifies the one-loop coefficient of the QCD β function,

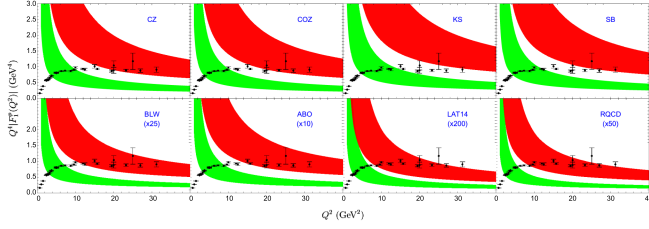


FIG. 2. pQCD predictions vs data for $Q^4 F_1^p(Q^2)$ in the spacelike region. The values of the expansion coefficients of proton LCDA are taken with eight different ansätze. The green and red bands correspond to the LO and NLO pQCD predictions, which are obtained by sliding $\mu_R = \mu_F = \mu$ from $Q/2$ to Q . Since the pQCD predictions in the lower row are much smaller than the data, we have multiplied them by a magnifying factor (as specified inside the parenthesis in each plot) to render the theoretical bands visible.

while γ_{ij} is the anomalous dimension associated with each expansion coefficient, whose explicit form can be found in Ref. [81]. The package FAPT [96] is utilized to evaluate the running QCD coupling constant to two-loop accuracy.

From Figs. 2–5, we confront our state-of-the-art pQCD predictions with the available large- Q^2 data for proton and neutron Dirac form factors, in both spacelike and timelike cases. The theoretical errors are estimated by varying $\mu_R = \mu_F = \mu$ from $Q/2$ to Q .

It has been widely known that the LO predictions in pQCD for the proton Dirac form factor are generally much smaller than the large- Q^2 data, for virtually all the popular models of nucleon LCDAs [97]. [To resolve this discrepancy, historically it has been proposed [90,93,98–101] that the strong coupling constants should be evaluated at the exact gluon virtualities in each diagram, so that the LO prediction can be significantly enhanced to account for the data. The α_s in this recipe might be associated with a scale of order Λ_{QCD} , evaluation of which requires some *ad hoc* treatment. Moreover, it is difficult to adapt this recipe to the NLO case.] As is evident from Figs. 2–5, the magnitudes of the NLO QCD correction are positive and significant. Once we adopt the CZ [90], COZ [95], KS [92], and SB [93] parametrizations of nucleon LCDAs, the pQCD predictions at NLO accuracy appear to be in magnitudes compatible with the large- Q^2 data for

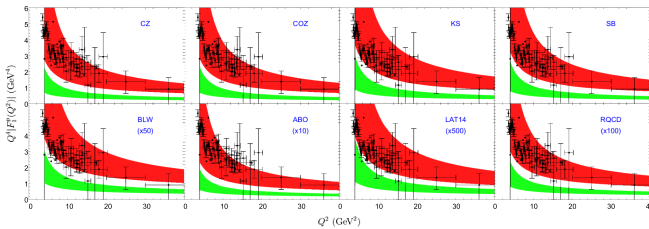


FIG. 3. Same as Fig. 2, except switching from the spacelike to the timelike region.

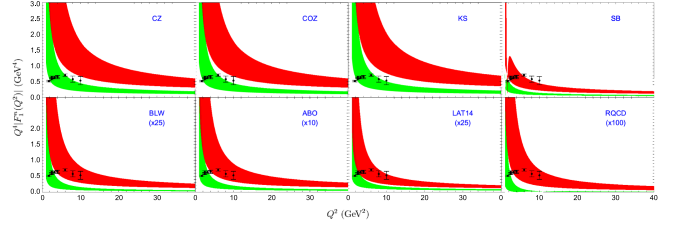


FIG. 4. Similar to Fig. 2, but with the proton replaced with the neutron. Here, pQCD predictions are confronted with the data for $Q^4 F_1^n(Q^2)$ in the spacelike region.

both proton and neutron Dirac form factors, in both spacelike and timelike regions. Nevertheless, we must stress that the monotonically decreasing tendency of the NLO predictions in variation with Q^2 is clearly at odds with that exhibited in the actual data.

In sharp contrast, due to quite small expansion coefficients predicted by BLW [94], ABO [65], LAT14 [81], and RQCD [70], the resulting pQCD predictions fall far below the experimental data. From Fig. 2–5, one clearly sees that, even after including NLO corrections, the pQCD predictions based on these parametrizations are still an order of magnitude smaller than the measured Dirac form factor of protons and neutrons.

Since lattice QCD is universally believed to be the only model-independent, nonperturbative approach to unravel the hadronic structure, the sheer failure of the parametrized nucleon LCDAs provided by LAT14 and RQCD when confronted with the data is rather disquieting. To settle this alarming discrepancy, it seems imperative to call for another independent lattice study based on a different algorithm. The large momentum effective theory (LaMET) [102,103] is a viable option, which allows one to ascertain the hadron's parton distributions directly in the x space on a Euclidean lattice. Some theoretical exploration to infer light octet baryons' LCDAs in the LaMET approach has recently been launched [104,105]. Had the prospective LaMET predictions for nucleon LCDAs turned out to be compatible with the RQCD predictions, one would then be inevitably led to the conclusion that the hard-scattering mechanism alone in Eq. (2) would be inadequate even at $Q^2 \approx 30 \text{ GeV}^2$, and some additional soft nonfactorizable contributions are expected to play an important role.

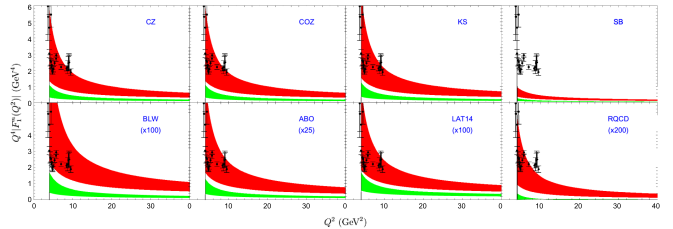


FIG. 5. Same as Fig. 4, except switching from the spacelike to the timelike region.

Summary and outlook—In this Letter, we report the first complete calculation of the NLO QCD corrections to the nucleon Dirac form factors. The collinear factorization formalism has been explicitly verified to hold true at one-loop order for hard-exclusive processes involving baryons.

It has long been known that the LO predictions in pQCD severely underestimate the measured nucleon Dirac form factor data. Historically, some *ad hoc* frozen strong coupling recipes were advocated to reconcile this discrepancy. Remarkably, the effect of NLO perturbative corrections has turned out to be positive and significant. Nevertheless, even after including the sizable NLO QCD corrections, the pQCD predictions based on the recent lattice QCD inputs of nucleon LCDAs are still considerably smaller than the measured nucleon form factors. This symptom may indicate that the hard-scattering mechanism alone might be inadequate even at Q^2 values accessible in future EIC and EicC experiments, and other soft, non-factorizable contributions need be supplemented to account for the data.

In a sense, this Letter may herald the coming of a new phase of perturbative QCD, where NLO QCD corrections start to be systematically explored for a class of important hard-exclusive reactions in the baryon sector, exemplified by the nucleon Pauli (axial, gravitational) form factors, nucleon Compton scattering, $\gamma\gamma \rightarrow N\bar{N}$, quarkonium decay into $N\bar{N}$, and so on. It is conceivable that the effect of NLO QCD corrections could be significant in many occasions, including some which will thereby facilitate a critical comparison between pQCD prediction and data.

Note added—Recently, a similar work has appeared by Huang *et al.* [106], who have also computed the NLO QCD correction to the nucleon Dirac form factor in the spacelike domain, yet within a specific evanescent-to-physical operator mixing scheme. After conversion into the KM scheme, their results in the Supplemental Material in the published version of Ref. [106] are compatible with ours.

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