

Ising Machine by Dimensional Collapse of Nonlinear Polarization Oscillators

Salvatore Chivazzo,¹ Marcello Calvanese Strinati,² Claudio Conti³,^{*} and Davide Pierangeli^{3,1,*}

¹*Institute for Complex Systems, National Research Council (ISC-CNR), 00185 Rome, Italy*

²*Enrico Fermi Research Center (CREF), 00184 Rome, Italy*

³*Department of Physics, Sapienza University of Rome, 00185 Rome, Italy*



(Received 4 April 2025; revised 22 June 2025; accepted 9 July 2025; published 4 August 2025)

Ising machines show promise as ultrafast hardware for optimizations encoded in Ising Hamiltonians but fall short in terms of success rate and performance scaling. Here, we propose a novel Ising machine that exploits the three-dimensional nature of nonlinear polarization oscillators to counteract these limitations. Based on the evolution of the optical polarization in third-order nonlinear media, the high-dimensional machine reaches the Ising ground state by the mechanism of “dimensional collapse”: the dynamics on the Poincaré sphere undergoes a self-induced collapse into polarization fixed points mapping an Ising spin. Collapse from a spherical to a binary spin occurs when the polarization oscillator experiences iterative loss and anisotropic feedback. The photonic setup consists of polarization modulated pulses in a $\chi^{(3)}$ crystal subject to measurement and feedback. We numerically demonstrate the polarization machine achieves enhanced success probability on benchmark graphs and an exponential improvement in performance scaling with respect to coherent Ising machines due to its high-dimensional operation. The proposed Ising machine paves the way for a new class of Ising solvers with enhanced computing capabilities.

DOI: 10.1103/qs29-2xqc

Physical systems that implement spin Hamiltonians are attracting vast attention as unconventional computing paradigms to overcome the time and energy limitations of digital hardware in combinatorial optimization and machine learning. Ising machines (IMs) [1] can impact a myriad of applications by accelerating the ground state (GS) search of the Ising model encoding the nondeterministic-polynomial-hard problem [2]. Realizations range from classical and quantum annealers made by optical devices [3–12], superconducting circuits [13], magnetic junctions [14], and memristors [15] to dynamical-system solvers made by networks of electronic [16–23], acoustic [24], and optical oscillators [25,26], lasers [27,28], and polaritons [29], whose computing principles have led to novel heuristics [30–33]. Photonic IMs [34] such as coherent IMs (CIMs) based on degenerate optical parametric oscillators [35–41] and optoelectronic oscillators [42–44] are especially promising for achieving ultrafast and large-scale solutions by virtue of optical bandwidth and parallelism.

A critical limitation of IMs is their trapping in local minima due to the approximate embedding of the Ising Hamiltonian within the dynamics of the analog platform.

This leads to suboptimal solutions and restricts the functioning to narrow conditions, which results in a reduced success rate and poor scaling. Intensive research is aimed at improving IM performance [45–48]. Recent works have proposed the use of multidimensional spins to leverage the additional dimensions as an escaping mechanism leading to the GS [49–51]. These approaches stimulate the idea of building superior IMs by using high-dimensional ($D \geq 3$) instead of low-dimensional oscillators. However, a system of high-dimensional oscillators mapping Ising spins has yet to be identified.

In this Letter, we propose a high-dimensional IM based on a novel type of optical oscillator: nonlinear polarization oscillators (NPOs) in third-order nonlinear media. We exploit NPOs as three-dimensional computing units represented by the Poincaré sphere. Unlike current IMs that encode spins in phase or amplitude, our approach maps spherical spins in the polarization of the optical field. To pass from 3D to 1D spins encoded in polarization, we introduce a novel high-dimensional computing mechanism that we call “dimensional collapse.” A designed feedback forces the multidimensional dynamics to settle into the fixed points of a target Ising model. The phenomenon occurs when NPOs evolve iteratively under the combined effect of loss and anisotropic feedback. Dimensional collapse can be realized in practice by a system of time-multiplexed pulses modulated in polarization that evolve iteratively in a $\chi^{(3)}$ crystal subject to measurement and feedback. This photonic setup implements the polarization IM (PIM), a network of NPOs coupled by feedback.

*Contact author: davide.pierangeli@roma1.infn.it

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI.

The PIM operates through a transition on the Poincaré sphere with NPOs that spontaneously collapse into a binary steady-state configuration mapping the Ising GS. Extensive simulations demonstrate enhanced success probability and exponentially improved scaling of performance metrics on the benchmark maximum cut (Max-Cut) and complete graphs of hundreds of spins. Exploiting the extra dimensions to escape from local minima, the PIM solves problems for which CIMs hardly converge to the GS. The generality of the collapse phenomenon and the simplicity of the proposed setup suggest a promising route to realize high-dimensional IMs.

NPOs describe the dynamics of the optical polarization in nonlinear media [52–54]. We consider light at frequency ω propagating in a lossless crystal with anisotropic susceptibility tensor $\chi^{(3)}$ and no birefringence, where the polarization changes by nonlinear effect [Fig. 1(a)]. The z evolution of the Stokes vector $\mathbf{S} = (S_0, S_1, S_2, S_3)$ admits the motion invariants $S_1^2 + S_2^2/\eta = r^2$ and $S_1^2 + S_2^2 + S_3^2 = S_0^2$, being $\dot{S}_0 = dS_0/dz = 0$, where S_0 is the optical intensity, which results in the NPO equation [54]

$$\ddot{S}_i + \alpha_i S_i + \beta_i S_i^3 = 0, \quad (1)$$

with $i = 1, 2, 3$. The oscillator coefficients read as $\alpha_1 = \chi^2 \eta [S_0^2 + (1 - 2\eta)r^2]$ and $\beta_1 = 2\chi^2 \eta (\eta - 1)$, and similar for $i = 2, 3$, where χ is the nonlinear coefficient and η the anisotropy factor, as detailed in Supplemental Material (SM) [55]. The NPO of Eq. (1) has four stable fixed points (centers) at $S_1 = \pm S_0$ and $S_3 = \pm S_0$, and two saddle points at $S_2 = \pm S_0$. Trajectories on the Poincaré sphere for an input vector in proximity of these points are shown in Fig. 1(b) by numerical integration of Eq. (1) with $\chi = 1$ cm/GW, $\eta = 1.1$ (e.g., a $\text{KTa}_x\text{Nb}_{1-x}\text{O}_3$ crystal at visible wavelengths).

The stable points of the NPO offer robust ways to encode binary information. The mapping to an Ising spin requires a mechanism that breaks the symmetry between the couples of centers in Fig. 1(b) and leads the dynamics to converge

either on the S_1 or S_3 axis. We find that iterative loss and self-feedback provide this mechanism, giving rise to two attracting fixed points that have the same polarization as the original fixed points. The iterative dynamics of a single NPO with loss and feedback writes in general as

$$S_i^{k+1}[0] = a_i S_i^k[L] + b_i f_i^k(\mathbf{S}[L]), \quad (2)$$

where $S_i^k[0]$ ($S_i^k[L]$) is the i th Stokes component at the k iteration at the input (output) of the crystal of length L , being the intensity $S_0^k[0]^2 = S_1^k[0]^2 + S_2^k[0]^2 + S_3^k[0]^2$. In Eq. (2), the feedback $f_i^k(\mathbf{S})$ acts separately on each component with strength b_i and the parameters $0 < a_i < 1$ set the losses. In absence of feedback ($b_i = 0$), the optical field exponentially decays to zero for any a_i . All the trajectories collapse to the origin [Fig. 1(c)]. Fixed points different from zero emerge with feedback, which provides nonlinear gain and counterbalances the field decay. The new fixed points appear as a result of the balance between gain and loss. As illustrated in Fig. 1(d), the trajectories collapse on two stable points, which lie on a Poincaré sphere of smaller radius (see SM [55]). The feedback dynamics inherits two of the fixed polarizations of the NPO, since these are invariant in propagation in the $\chi^{(3)}$ crystal. Dimensional collapse on this binary state requires two conditions: (i) anisotropic and (ii) saturable feedback. We meet these conditions by using feedback that (i) acts on a single component S_i and (ii) depends on the polarization state. The feedback function is $f_i^k = S_i^k[L]/S_0^k[L]$. Fixed points on either the S_1 or S_3 axis emerge when the only nonzero strength is b_1 or b_3 , respectively. Figure 2 shows the dynamics of the collapse on S_1 . Other cases are detailed in SM [55].

We propose an experimental realization of the collapse and design an IM based on NPOs. The setup is an optoelectronic feedback system in which N optical pulses of variable polarization and intensity propagate iteratively in the crystal. The j th pulse is modulated according to measurement and feedback. The scheme, shown in

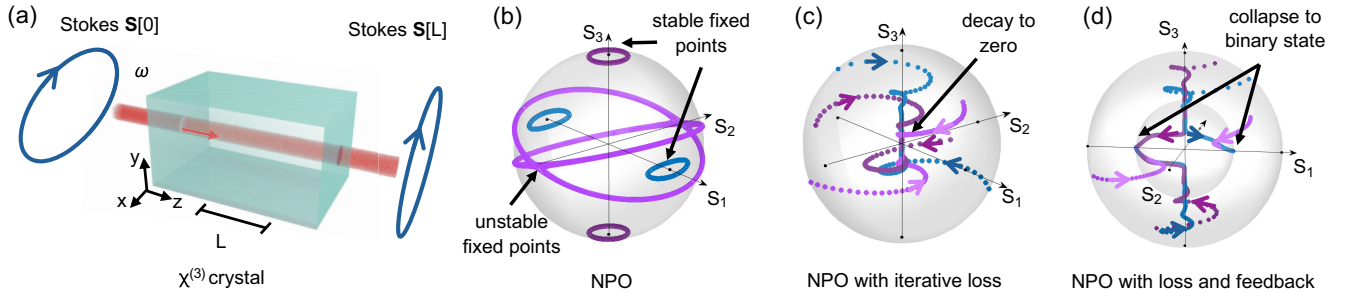


FIG. 1. Nonlinear polarization oscillator (NPO) and its dimensional collapse. (a) Polarization variation by nonlinear propagation in a third-order crystal. (b) Trajectories on the Poincaré sphere by solving Eq. (1) for input Stokes vectors near the stable (blue, purple) and unstable (magenta) fixed points. (c) Iterative evolution of the NPO with iterative loss. The field decays to zero, causing all trajectories to fall to the origin. (d) Self-feedback induces dimensional collapse: two of the stable points in (b) reemerge on a Poincaré sphere of smaller radius. All trajectories converge to a binary polarization state.

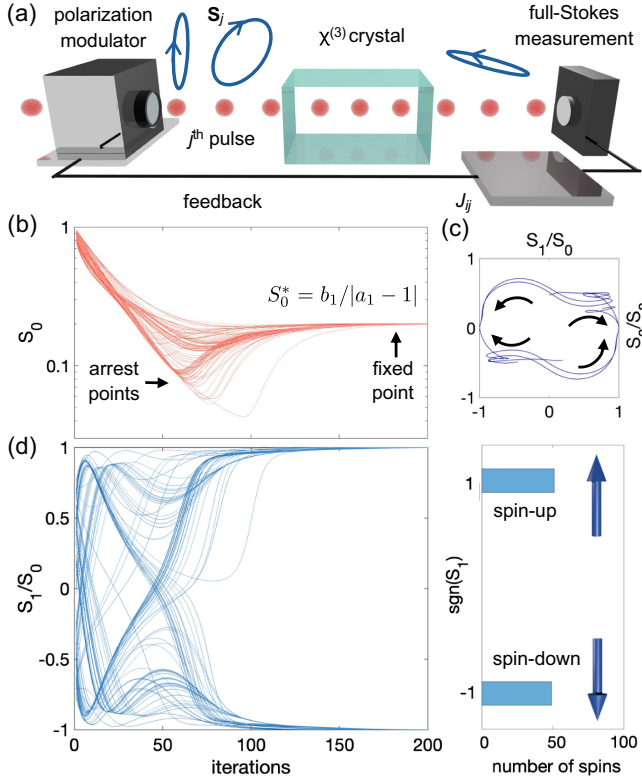


FIG. 2. Polarization Ising machine (PIM). (a) Scheme of the photonic setup. The Stokes vector of a train of pulses is modulated iteratively by polarization measurement and feedback. (b)–(d) Collapse dynamics of 100 uncoupled NPOs for $\chi = 0.3$ cm/GW, $\eta = 1.1$, $L = 1.2$ cm, $a_1 = 0.95$, $a_3 = 0.98$, $b_1 = 0.01$. Evolution of (b) S_0 , (d) S_1 , and (c) trajectories on the $S_1 - S_3$ plane. NPOs collapse to the stable point $S_1/S_0 = \pm 1$, which maps an Ising spin. Spin-up and spin-down have equal probability due to randomness in the initial S_1^0 and S_3^0 .

Fig. 2(a), includes a polarization modulator made by a series of electro-optic phase and amplitude modulators, and a full-Stokes polarimeter. A digital processor, such as a field programmable gate array, processes the measurements and programs the modulators, coupling the NPOs. Since we only modulate the laser at each iteration, the scheme does not require pulse synchronization and phase coherence. We modulate the j th input $\mathbf{S}_j[0]$ according the iterative map in Eq. (2) with $0 < b_1 < 1$, $b_2 = b_3 = 0$, $a_1 = a_2$, and $a_3 > a_1$, as explicit in SM [55]. The PIM operates via the feedback term

$$f_{1j}^k = \frac{1}{N} \sum_i^N J_{ij} S_{1i}^k[L] / S_{0i}^k[L], \quad (3)$$

which acts only on the S_{1j} component of the j th NPOs. The NPOs thus collapse to stable points on the S_1 axis. J_{ij} is the coupling matrix. Equation (3) provides each NPO a different saturable gain that depends on its interaction.

We first illustrate the collapse dynamics to $S_1/S_0 = \pm 1$ for $N = 100$ uncoupled NPOs subject to self-interaction $J_{ij} = 1$. As shown in Fig. 2(b), S_0 shows a decay until an arrest point, which occurs for $S_0^k \approx b_1$. After this intensity minimum, the growing feedback (see SM [55]) leads to the fixed point $\mathbf{S}^* = (S_0^*, \pm S_0^*, 0, 0)$. The steady-state intensity is $S_0^* = b_1/|a_1 - 1|$ for any initial $\mathbf{S}_j^0$. Figure 2(c) shows the projection of the trajectories on the $S_1 - S_3$ plane. The NPO evolves on the Poincaré sphere and collapses to horizontal or vertical polarization [Fig. 2(d)]. At the steady state, S_{1j} maps an Ising spin $\sigma_j = \pm 1$ defined by $\sigma_j = S_{1j}/S_{0j}$. The histogram in Fig. 2(d) indicates that spin-up and spin-down have equal probability, confirming that free NPOs under self-feedback behave as independent Ising spins at the steady state. When coupled via the feedback in Eq. (3), two interacting NPOs behave as two interacting Ising spins, settling into a stable ferromagnetic or antiferromagnetic configuration, as detailed in SM [55].

To study a large network of NPOs evolving by Eqs. (2) and (3), we discretize Eq. (1) and evaluate each $S_{ij}^k[L]$ as

$$S_{ij}[L] = S_{ij}[0] + L\dot{S}_{ij}[0] - L^2\alpha_{ij}S_{ij}[0] - L^2\beta_{ij}S_{ij}^3[0]. \quad (4)$$

The approximation is accurate for $L < 1/\sqrt{\alpha_1}$ [55] and allows us to efficiently simulate the PIM by avoiding the integration of N Eq. (1) at each iteration.

To demonstrate the PIM finds the Ising GS for a given J_{ij} , we run unweighted Max-Cut problems from the Binary Quadratic–Max-Cut (BiqMac) Library [56], which have known GS energy. The dynamics of the Stokes components during a single run on a 100-spin graph with 50% edge probability ($g05_{100,2}$) is shown in Figs. 3(a)–3(d) for $a_1 = 0.70$, $b_1 = 0.40$, $a_3 = 0.99$. Figure 3(a) shows how S_0 decays until the arrest points and then stabilizes at the steady state. The final S_{0j} values now differ as the feedback [Eq. (3)] varies on each NPO. This intensity heterogeneity is not detrimental due to the polarization encoding of the spin. S_1 goes to zero before undergoing a transition into the fixed points $S_1/S_0 = \pm 1$ [Fig. 3(a)]. Along S_3 , we observe an initial locking to circular polarizations ($S_3/S_0 = \pm 1$) followed by a transition toward zero [Fig. 3(d)]. S_2 remains small [Fig. 3(c)] but its sign plays a role. This high-dimensional dynamics can lead to the GS of the corresponding Ising Hamiltonian $H = -\sum_{ij} J_{ij} \sigma_i \sigma_j$. We map the polarization states to the spin configuration by $\sigma_j = \text{sgn}(S_{1j})$ and evaluate H at each iteration. The convergence of H to the GS is shown in Fig. 3(e) for different successful runs. The probability of finding the GS at the steady state defines the success probability (SP). The SP strongly depends on the machine parameters. In Fig. 3(f), we report the SP, averaged over 10 repetitions of 200 runs, when scanning the linear loss a_1 and feedback strength b_1 . We find a broad region of the parameter space where the PIM achieves a high SP. The SP increases by decreasing a_1

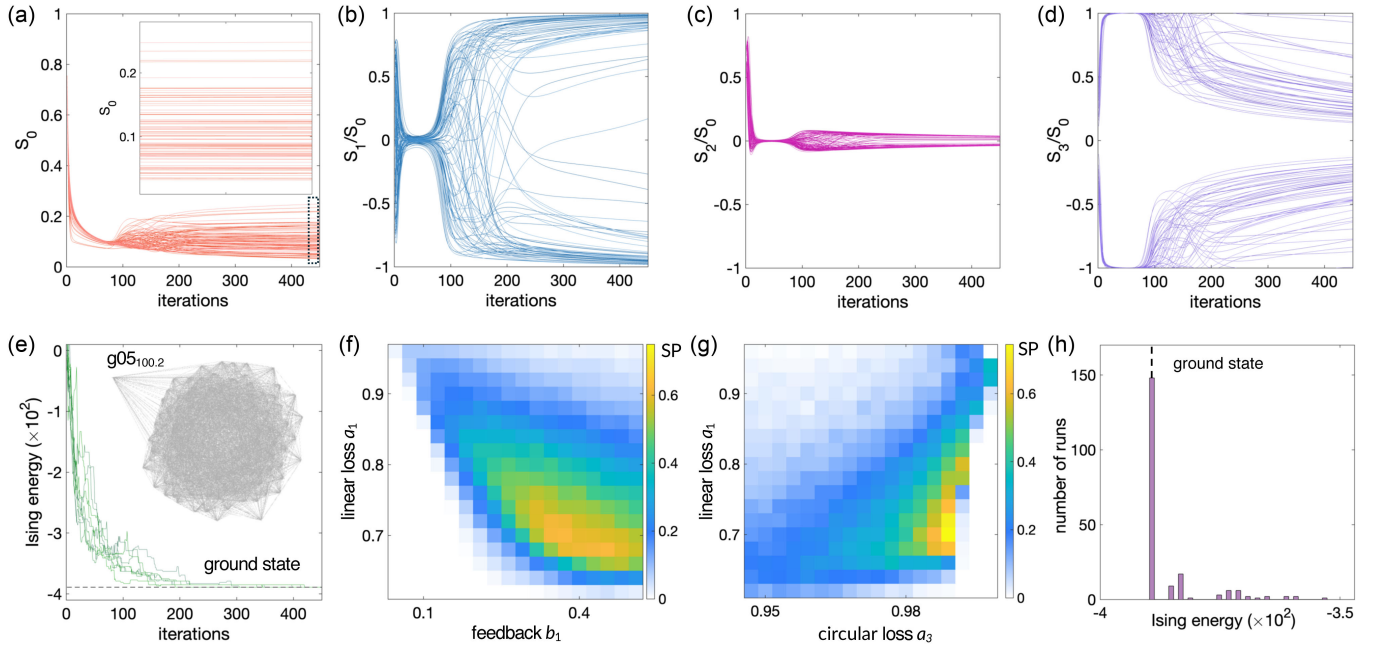


FIG. 3. PIM high-dimensional dynamics on a 100-spin Max-Cut problem. Evolution of (a) S_0 (the dotted region is enlarged), (b) S_1/S_0 , (c) S_2/S_0 , and (d) S_3/S_0 when solving the $g05_{100.2}$ benchmark graph (inset). (e) Ising energy H for various runs with random initial conditions. The dashed line indicates the GS energy. (f),(g) Success probability (SP) by scanning the feedback strength and losses. (h) Distribution of H at the steady state over 200 runs. Results are for $a_1 = 0.70$, $b_1 = 0.40$, $a_3 = 0.99$, $\chi = 0.3$ cm/GW, $\eta = 1$, $L = 1.2$ cm, $S_2^0/S_0^0 = 0.66$, $S_0^0 = 1$ GW/cm².

(larger loss), meaning that a fast initial decay of S_1 aids the GS search. Increasing b_1 accelerates the convergence (see SM [55]). The loss a_3 plays a key role in the SP. The map in Fig. 3(g) shows a narrow region of optimal operation. A slight decrease in a_3 causes the SP to drop. We attribute the effect to the initial growth experienced by the circular components. When decreasing a_3 , the transient states $S_3/S_0 = \pm 1$ are not reached or only last for a few iterations. Therefore, initial locking to $S_3/S_0 = \pm 1$ is crucial for the PIM operation. This evidence indicates the key role of the extra dimensions in leading to the solution. The GS is reached via a jump to the poles of the Poincaré sphere followed by the transition to the fixed points.

The PIM computing process is based on a self-induced dynamical transition from spherical to Ising spins. After the transition, the NPO network settles into a configuration that minimizes the power dissipation rate. In analogy with other dynamical IMs, the PIM embeds the Ising Hamiltonian by the principle of minimum loss [57]. As detailed in SM [55], the PIM dissipation rate maps H when NPOs collapse to fixed points. Therefore, at the steady state, the configuration of minimum dissipation can yield the Ising GS. Remarkably, the PIM embedding is robust to amplitude heterogeneity [45] thanks to the polarization encoding. This property contributes to enhancing the SP.

The PIM achieves a high SP on benchmark problems. On the $g05_{100.2}$ graph, tuning the parameters we find a maximum $SP_{\max} = 0.78 \pm 0.03$. We remark that SP is

evaluated by using only the outcome of the dynamics, without considering transient GSs, as this condition is relevant in hardware. The steady-state energy distribution over 200 runs is shown in Fig. 3(f). An SP considerably larger than simulated CIMs [58] is achieved also for other BiqMac graphs (see SM [55]). Remarkably, we find a finite SP even for problems classified as “Ising hard” for IMs (e.g., $g05_{100.3}$). For these problems, CIMs always converge to a local minimum in the absence of noise as the GS is not connected to the bifurcation branch [59]. In the PIM, these GSs become accessible through the extra dimensions. NPOs can be initialized in each run in different polarizations, and hence their collapse has multiple outcomes. Results varying the initial S_j^0 indicate the GS search can benefit from a larger input space (see SM [55]). Moreover, we observe that a minimum length L and nonlinearity χ are required to reach the GS, which confirms that nonlinear propagation is essential for the PIM computing capability. The larger SP as a function of η is found for $\eta = 1$ (isotropic crystal).

To study the scaling of the PIM computational performance, we run complete graphs with random $J_{ij} = \pm 1$ (K graphs) of increasing size up to $N = 500$. The SP is evaluated using the GS found via simulated annealing. Figure 4(a) reports $SP_{\max}$ when scanning a_1 , b_1 , and a_3 , averaged on 20 random graphs for each N . $SP_{\max}$ shows an exponential decay well fitted by the exponential function $p_s(N) \propto e^{-\gamma_{\text{PIM}} N}$ with $\gamma_{\text{PIM}} = 9 \pm 1 \times 10^{-3}$. We compare

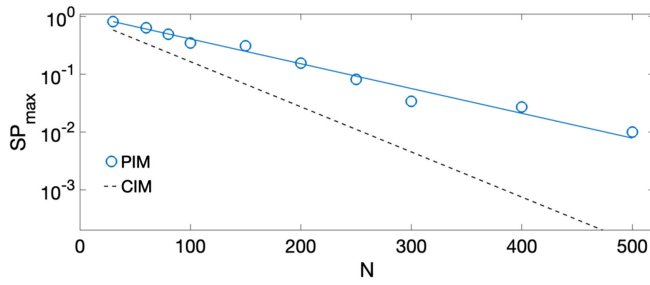


FIG. 4. Scaling of the PIM computational performance. $SP_{\max}$ on K graphs of size N . The results (dots) are fitted by an exponential (blue line) with decay rate $\gamma_{\text{PIM}} = 9 \pm 1 \times 10^{-3}$. The finite-size scaling for CIMs from [50] (dotted line) shows the PIM scaling advantage.

this decay with the exponential decay of CIMs on K graphs, for which a finite-size scaling analysis (up to $N = 100$) gives $\gamma_{\text{CIM}} \approx 18 \times 10^{-3}$ [50]. The PIM shows a decrease of $SP_{\max}$ that is exponentially slower, with an enhancement given by the decay rate ratio $\gamma_{\text{CIM}}/\gamma_{\text{PIM}} \approx 2$. We ascribe the exponential improvement in performance over CIMs to the high-dimensional operation of the PIM. A similar scaling advantage has been recently reported in the hyperspin machine with dimensional annealing [50], where the Ising model is embedded in a higher dimension by using parametric oscillators behaving as circular spins. However, while dimensional annealing adiabatically changes selected couplings to effectively reduce the dimensionality during the evolution, the PIM dimensional collapse from 3D to 1D spins is induced by the feedback and occurs spontaneously without an external annealing. The performance analysis is completed in SM [55] with additional figures of merit.

In conclusion, we have introduced a high-dimensional IM made by polarization oscillators. Leveraging the dynamics of the optical field on the Poincaré sphere, the PIM operates intrinsically in three dimensions. Dimensional collapse by self-feedback provides a novel mechanism to pass dynamically from 3D to 1D spins. The concept can trigger novel heuristic algorithms for optimization. The collapse mechanism is general, as the NPO is a fundamental example of a high-dimensional nonlinear oscillator. It can be applied to other oscillators and nonlinear optical setups, enabling these systems for IMs.

The computing process shares aspects with quantum annealing [60] but it is purely classical. The analogy is based on the correspondence between the Poincaré and the Bloch spheres, i.e., the Stokes representation of the qubit [61]. The feedback makes NPOs interact along S_1 in analogy with the interaction between quantum spins that occurs along the z axis. The losses initially align the NPOs along S_3 , mimicking the effect that a transverse magnetic field would have on classical spins. In quantum annealing, the magnetic field is also transverse (x axis) and time-dependent. The spontaneous growth of the feedback (interaction) during the dynamics mimics an annealing

process at a fixed temperature, favoring the Ising GS as quantum fluctuations do in quantum annealing. This formal analogy makes a link for further studies on the spin behavior of the optical polarization.

The proposed setup reveals how to realize high-dimensional IMs by simple photonic platforms. Extensive simulations demonstrate the PIM achieves a larger success rate and exponential improvement in scaling over CIMs. The performance can be further enhanced by annealing the machine parameters, introducing noise, or engineering the feedback signal. Our setup can be readily implemented in experiments, where it can operate with an iteration time of nanoseconds by using available GHz electro-optic modulators. NPOs can also form for few-photon pulses in systems that exhibit nonlinearity at the single-photon level [62]. The PIM opens the way to a new class of combinatorial optimization hardware that exploits the polarization of light.

Acknowledgments—We acknowledge funding from the European Union HORIZON-ERC-2024-STG LOOP “Optical polarization for ultrafast computing” No. 101162173, HORIZON-ERC-2023-ADG HYPERSPIM “Optoelectronic and all-optical hyperspin machines for large-scale computing” No. 101139828, and EUNextGenerationEU under the National Recovery and Resilience Plan (NRRP) by MUR PRIN 2022N738SA MetaComputing and PRIN 2022597 MBS PHERMIAC, and National Quantum Science and Technology Institute (NQSTI) MUR No. PE0000023-NQSTI.

Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the European Union or the European Research Council. Neither the European Union nor the granting authority can be held responsible for them.

Data availability—The data that support the findings of this Letter are not publicly available. The data are available from the authors upon reasonable request.

-
- [1] N. Mohseni, P. L. McMahon, and T. Byrnes, Ising machines as hardware solvers of combinatorial optimization problems, *Nat. Rev. Phys.* **4**, 363 (2022).
 - [2] A. Lucas, Ising formulations of many NP problems, *Front. Phys.* **2**, 5 (2014).
 - [3] D. Pierangeli, G. Marcucci, and C. Conti, Large-scale photonic Ising machine by spatial light modulation, *Phys. Rev. Lett.* **122**, 213902 (2019).
 - [4] M. Prabhu *et al.*, Accelerating recurrent Ising machines in photonic integrated circuits, *Optica* **7**, 551 (2020).
 - [5] D. Pierangeli, G. Marcucci, and C. Conti, Adiabatic evolution on a spatial-photonic Ising machine, *Optica* **7**, 1535 (2020).

- [6] D. Pierangeli, M. Rafayelyan, C. Conti, and S. Gigan, Scalable spin-glass optical simulator, *Phys. Rev. Appl.* **15**, 034087 (2021).
- [7] M. Leonetti, E. Hörmann, L. Leuzzi, G. Parisi, and G. Ruocco, Optical computation of a spin glass dynamics with tunable complexity, *Proc. Natl. Acad. Sci. U.S.A.* **118**, e2015207118 (2021).
- [8] H. Yamashita, K. I. Okubo, S. Shimomura, Y. Ogura, J. Tanida, and H. Suzuki, Low-rank combinatorial optimization and statistical learning by spatial photonic Ising machine, *Phys. Rev. Lett.* **131**, 063801 (2023).
- [9] L. Luo, Z. Mi, J. Huang, and Z. Ruan, Wavelength-division multiplexing optical Ising simulator enabling fully programmable spin couplings and external magnetic fields, *Sci. Adv.* **9**, eadg6238 (2023).
- [10] S. T. Yu, M. G. He, S. Fang, Y. Deng, and Z. S. Yuan, Spatial optical simulator for classical statistical models, *Phys. Rev. Lett.* **133**, 237101 (2024).
- [11] J. Sakellariou, A. Askitopoulos, G. Pastras, and S. I. Tsintzos, Encoding arbitrary Ising Hamiltonians on spatial photonic Ising machines, *Phys. Rev. Lett.* **134**, 203801 (2025).
- [12] D. Veraldi *et al.*, Fully programmable spatial photonic Ising machine by focal plane division, *Phys. Rev. Lett.* **134**, 063802 (2025).
- [13] S. Boixo, T. F. Rønnow, S. V. Isakov, Z. Wang, D. Wecker, D. A. Lidar, J. M. Martinis, and M. Troyer, Evidence for quantum annealing with more than one hundred qubits, *Nat. Phys.* **10**, 218 (2014).
- [14] W. A. Borders, A. Z. Pervaiz, S. Fukami, K. Y. Camsari, H. Ohno, and S. Datta, Integer factorization using stochastic magnetic tunnel junctions, *Nature (London)* **573**, 390 (2019).
- [15] F. Cai *et al.*, Power-efficient combinatorial optimization using intrinsic noise in memristor Hopfield neural networks, *Nat. Electron.* **3**, 409 (2020).
- [16] T. Wang and J. Roychowdhury, OIM: Oscillator-based Ising machines for solving combinatorial optimisation problems, in *International Conference on Unconventional Computation and Natural Computation* (Springer International Publishing, Cham, 2019), pp. 232–256.
- [17] J. Chou, S. Bramhavar, S. Ghosh, and W. Herzog, Analog coupled oscillator based weighted Ising machine, *Sci. Rep.* **9**, 14786 (2019).
- [18] L. Bello, M. Calvanese Strinati, E. G. Dalla Torre, and A. Pe'er, Persistent coherent beating in coupled parametric oscillators, *Phys. Rev. Lett.* **123**, 083901 (2019).
- [19] N. Casilli, T. Kaisar, L. Colombo, S. Ghosh, P. X. L. Feng, and C. Cassella, Parametric frequency divider based Ising machines, *Phys. Rev. Lett.* **132**, 147301 (2024).
- [20] P. Álvarez, D. Pittilini, F. Miserocchi, S. Raamamurthy, G. Margiani, O. Ameye, J. del Pino, O. Zilberberg, and A. Eichler, Biased Ising model using two coupled Kerr parametric oscillators with external force, *Phys. Rev. Lett.* **132**, 207401 (2024).
- [21] S. Dutta, A. Khanna, A. S. Assoa, H. Paik, D. G. Schlom, Z. Toroczkai, A. Raychowdhury, and S. Datta, An Ising Hamiltonian solver based on coupled stochastic phase-transition nano-oscillators, *Nat. Electron.* **4**, 502 (2021).
- [22] L. Q. English, A. V. Zampetaki, K. P. Kalinin, N. G. Berloff, and P. G. Kevrekidis, An Ising machine based on networks of subharmonic electrical resonators, *Commun. Phys.* **5**, 333 (2022).
- [23] H. Lo, W. Moy, H. Yu, S. Sapatnekar, and C. H. Kim, An Ising solver chip based on coupled ring oscillators with a 48-node all-to-all connected array architecture, *Nat. Electron.* **6**, 771 (2023).
- [24] A. Litvinenko, R. Khymyn, V. H. González, R. Ovcharov, A. A. Awad, V. Tyberkevych, A. Slavin, and J. Åkerman, A spinwave Ising machine, *Commun. Phys.* **6**, 227 (2023).
- [25] A. Marandi, Z. Wang, K. Takata, R. L. Byer, and Y. Yamamoto, Network of time-multiplexed optical parametric oscillators as a coherent Ising machine, *Nat. Photonics* **8**, 937 (2014).
- [26] T. Inagaki, K. Inaba, R. Hamerly, K. Inoue, Y. Yamamoto, and H. Takesue, Large-scale Ising spin network based on degenerate optical parametric oscillators, *Nat. Photonics* **10**, 415 (2016).
- [27] M. Babaieian *et al.*, A single shot coherent Ising machine based on a network of injection-locked multicore fiber lasers, *Nat. Commun.* **10**, 1 (2019).
- [28] V. Pal, S. Mahler, C. Tradonsky, A. A. Friesem, and N. Davidson, Rapid fair sampling of the XY spin Hamiltonian with a laser simulator, *Phys. Rev. Res.* **2**, 033008 (2020).
- [29] N. G. Berloff, M. Silva, K. Kalinin, A. Askitopoulos, J. D. Töpfer, P. Cilibrizzi, W. Langbein, and P. G. Lagoudakis, Realizing the classical XY Hamiltonian in polariton simulators, *Nat. Mater.* **16**, 1120 (2017).
- [30] H. Goto, K. Tatsumura, and A. R. Dixon, Combinatorial optimization by simulating adiabatic bifurcations in nonlinear Hamiltonian systems, *Sci. Adv.* **5**, eaav2372 (2019).
- [31] C. Roques-Carnes *et al.*, Heuristic recurrent algorithms for photonic Ising machines, *Nat. Commun.* **11**, 1 (2020).
- [32] K. P. Kalinin and N. G. Berloff, Global optimization of spin Hamiltonians with gain-dissipative systems, *Sci. Rep.* **8**, 1779 (2018).
- [33] T. Leleu, F. Khoiratee, T. Levi, R. Hamerly, T. Kohno, and K. Aihara, Scaling advantage of chaotic amplitude control for high-performance combinatorial optimization, *Commun. Phys.* **4**, 266 (2021).
- [34] Y. Gao, G. Chen, L. Qi, W. Fu, Z. Yuan, and A. J. Danner, Photonic Ising machines for combinatorial optimization problems, *Appl. Phys. Rev.* **11**, 041307 (2024).
- [35] Z. Wang, A. Marandi, K. Wen, R. L. Byer, and Y. Yamamoto, Coherent Ising machine based on degenerate optical parametric oscillators, *Phys. Rev. A* **88**, 063853 (2013).
- [36] P. L. McMahon *et al.*, A fully programmable 100-spin coherent Ising machine with all-to-all connections, *Science* **354**, 614 (2016).
- [37] T. Inagaki *et al.*, A coherent Ising machine for 2000-node optimization problems, *Science* **354**, 603 (2016).
- [38] R. Hamerly *et al.*, Experimental investigation of performance differences between coherent Ising machines and a quantum annealer, *Sci. Adv.* **5**, eaau0823 (2019).
- [39] T. Honjo *et al.*, 100,000-spin coherent Ising machine, *Sci. Adv.* **7**, eabh0952 (2021).
- [40] Y. Okawachi, M. Yu, J. K. Jang, X. Ji, Y. Zhao, B. Y. Kim, M. Lipson, and A. L. Gaeta, Demonstration of chip-based

- coupled degenerate optical parametric oscillators for realizing a nanophotonic spin-glass, *Nat. Commun.* **11**, 1 (2020).
- [41] M. C. Strinati, D. Pierangeli, and C. Conti, All-optical scalable spatial coherent Ising machine, *Phys. Rev. Appl.* **16**, 054022 (2021).
- [42] F. Böhm, G. Verschaffelt, and G. Van der Sande, A poor man's coherent Ising machine based on opto-electronic feedback systems for solving optimization problems, *Nat. Commun.* **10**, 1 (2019).
- [43] Q. Cen *et al.*, Large-scale coherent Ising machine based on optoelectronic parametric oscillator, *Light Sci. Appl.* **11**, 333 (2022).
- [44] Z. Li *et al.*, Scalable on-chip optoelectronic Ising machine utilizing thin-film lithium niobate photonics, *ACS Photonics* **11**, 1703 (2024).
- [45] T. Leleu, Y. Yamamoto, P. L. McMahon, and K. Aihara, Destabilization of local minima in analog spin systems by correction of amplitude heterogeneity, *Phys. Rev. Lett.* **122**, 040607 (2019).
- [46] M. Calvanese Strinati, L. Bello, E. G. Dalla Torre, and A. Pe'er, Can nonlinear parametric oscillators solve random Ising models?, *Phys. Rev. Lett.* **126**, 143901 (2021).
- [47] F. Böhm, T. V. Vaerenbergh, G. Verschaffelt, and G. Van der Sande, Order-of-magnitude differences in computational performance of analog Ising machines induced by the choice of nonlinearity, *Commun. Phys.* **4**, 149 (2021).
- [48] J. Wang, D. Ebler, K. M. Wong, D. S. W. Hui, and J. Sun, Bifurcation behaviors shape how continuous physical dynamics solves discrete Ising optimization, *Nat. Commun.* **14**, 2510 (2023).
- [49] M. Calvanese Strinati and C. Conti, Multidimensional hyperspin machine, *Nat. Commun.* **13**, 7248 (2022).
- [50] M. Calvanese Strinati and C. Conti, Hyperscaling in the coherent hyperspin machine, *Phys. Rev. Lett.* **132**, 017301 (2024).
- [51] J. S. Cummins and N. G. Berloff, Vector Ising spin annealer for minimizing Ising Hamiltonians, *Commun. Phys.* **8**, 1 (2025).
- [52] Y. P. Svirko and N. I. Zheludev, *Polarization of Light in Nonlinear Optics* (Wiley, New York, 2000), p. 240.
- [53] N. I. Zheludev, Polarization instability and multistability in nonlinear optics, *Sov. Phys. Usp.* **32**, 357 (1989).
- [54] G. Gregori and S. Wabnitz, New exact solutions and bifurcations in the spatial distribution of polarization in third-order nonlinear optical interactions, *Phys. Rev. Lett.* **56**, 600 (1986).
- [55] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/qs29-2xqc> for details on the NPO in $\chi^{(3)}$ crystals, the dimensional collapse of the single NPO and two interacting NPOs, the model of the PIM, the mapping of the Ising Hamiltonian, and additional results for various operating conditions and benchmark problems.
- [56] <https://biqmac.aau.at/biqmaclib.html>.
- [57] S. K. Vadlamani, T. P. Xiao, and E. Yablonovitch, Physics successfully implements Lagrange multiplier optimization, *Proc. Natl. Acad. Sci. U.S.A.* **117**, 26639 (2020).
- [58] T. Sevenants, G. Van der Sande, and G. Verschaffelt, Requirements on bit resolution in optical Ising machine implementations with analog spin variables, *Commun. Phys.* **8**, 11 (2025).
- [59] J. Lamers, G. Verschaffelt, and G. Van der Sande, Using continuation methods to analyse the difficulty of problems solved by Ising machines, *Commun. Phys.* **7**, 378 (2024).
- [60] T. Kadowaki and H. Nishimori, Quantum annealing in the transverse Ising model, *Phys. Rev. E* **58**, 5355 (1998).
- [61] J. B. Altepeter, E. R. Jeffrey, and P. G. Kwiat, Photonic state tomography, *Adv. At. Mol. Opt. Phys.* **52**, 105 (2005).
- [62] K. H. Nielsen *et al.*, Programmable nonlinear quantum photonic circuits, [arXiv:2405.17941](https://arxiv.org/abs/2405.17941).