

Gravitational Waves from Feebly Interacting Particles in a First Order Phase Transition

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First order phase transitions are well motivated and extensively studied sources of gravitational waves (GWs) from the early Universe. The vacuum energy released during such transitions is assumed to be transferred primarily either to the expanding bubble walls, whose collisions source GWs, or to the surrounding plasma, producing sound waves and turbulence, which source GWs. In this Letter, we study an alternative possibility that has not yet been considered: the released energy gets transferred primarily to feebly interacting particles that do not form a coherent interacting plasma but simply free-stream individually. We develop the formalism to study the production of GWs from such configurations and demonstrate that such GW signals have qualitatively distinct characteristics compared with conventional sources and are potentially observable with near-future GW detectors.

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Motivation—Gravitational waves (GWs) provide a unique probe of a variety of very early Universe phenomena. One of the most attractive targets for GW searches is a first order phase transition (FOPT) [1–7], where the metastable early Universe false vacuum decays through nucleation, expansion, and percolation of bubbles of true vacuum. The properties of GW signals generated by FOPTs have been extensively studied (see, e.g., Refs. [8–11] for reviews). FOPTs can generically occur in several beyond the standard model (BSM) scenarios, where the existence and breaking of additional symmetries in extended sectors (which could include dark sectors) is motivated by various shortcomings of the standard model (SM). Such dark sector FOPTs can be realized across a broader range of energy scales [12–25], offering detection prospects with various current and near future GW detectors such as LIGO-Virgo [26,27], LISA [28], DECIGO [29], Big Bang Observer [30], Einstein Telescope [31], and Cosmic Explorer [32].

FOPTs produce GWs in several ways. If the bubble walls carry most of the energy released in the transition, GWs are sourced by the scalar field energy in the bubble walls when the walls collide [4–7,33–39] or by particles produced from bubble collisions [40]. In the presence of significant interactions between the walls and the plasma, the released

energy is instead primarily transferred to the plasma, and GWs are produced by sound waves (SWs) [41–46] and turbulence [7,44,47–51]. These contributions have distinct spectral features determined by the behavior of the walls or SWs during percolation.

In this Letter, we study a new source of GWs from FOPTs that has so far not been considered: the energy released in the phase transition (PT) can be transferred primarily to feebly interacting particles (FIPs) that free-stream without interacting over the timescale of the PT. Such scenarios can readily occur in dark sectors, which can contain particles with feeble interactions in many realistic scenarios [52]. In such cases, the standard sources mentioned above carry negligible fractions of the total energy and cannot be efficient GW sources. This seemingly nightmare scenario for GW searches, where a FOPT does not lead to observable signals even with otherwise favorable parameters, deserves greater scrutiny. In this Letter, we develop the formalism to study the evolution of such FIPs during the PT, the subsequent production of GWs, and observation prospects with the next generation of GW detectors.

Framework—Consider a FOPT involving a dark sector scalar s obtaining a vacuum expectation value $\langle s \rangle$ at temperature T (in general, $T \lesssim \langle s \rangle$), producing bubbles of true vacuum (broken phase), whose walls expand into the false vacuum (symmetric phase) with velocity v_w (and Lorentz factor γ_w). For simplicity, we assume that the dark sector constitutes the dominant form of radiation and the energy in the SM bath is negligible [53]. We parametrize the energy density released during the PT as [54]

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$$\alpha = \frac{\Delta V}{\rho_{\text{rad}}} = \frac{\Delta V}{\frac{\pi^2}{30} g_*^D T^4}, \quad (1)$$

where ΔV is the difference in the potential energies of the two vacua, ρ_{rad} is the radiation energy density, and $g_*^D = g_{\text{bosons}} + (7/8)g_{\text{fermions}}$ represents the total number of degrees of freedom in the dark sector. β/H denotes the inverse timescale for the phase transition normalized by the Hubble time; for $v_w \approx 1$, β also represents the average bubble size at collision.

Consider a particle X in the bath that is massless in the false vacuum but obtains a mass m in the true vacuum due to its coupling to s . A massless X particle with energy E can cross into the bubble wall only if $\gamma_w E \gtrsim m$. The pressure on the bubble wall [57] due to a full thermal distribution of particles crossing into the bubble and becoming massive is $\mathcal{P}_{\text{max}} \approx (1/24)m^2 T^2$ [34]. For $m > \sqrt{g_*^D} \alpha T$ (we dropped an $\mathcal{O}(1)$ prefactor for simplicity), we have $\Delta V < \mathcal{P}_{\text{max}}$, and the released energy is entirely absorbed by an appropriate fraction of the X population crossing into the bubble and becoming massive, resulting in a steady-state, terminal bubble wall velocity.

For our numerical studies, we focus on three benchmark (BM) cases (which satisfy $\gamma T > m$):

$$(m/T, v_w) = \left\{ \begin{array}{ccc} \text{BM1} & \text{BM2} & \text{BM3} \\ (1, 0.7) & (2, 0.95) & (3, 0.99) \end{array} \right\}. \quad (2)$$

Larger values of m/T are also possible, with a smaller fraction of the X population entering the bubbles while the majority gets reflected; we focus on $m/T \sim 1$ purely for convenience as this does not require keeping track of the reflected population.

Because of the energy transfer from the bubble walls to the particles, the massive X particles in the broken phase gain momenta in the direction of wall propagation, forming extended shells that trail the walls and expand outward. Figure 1 shows the distribution of energy-momentum T_{ij} of particles within a bubble for the BM cases (see Supplemental Material for details of the

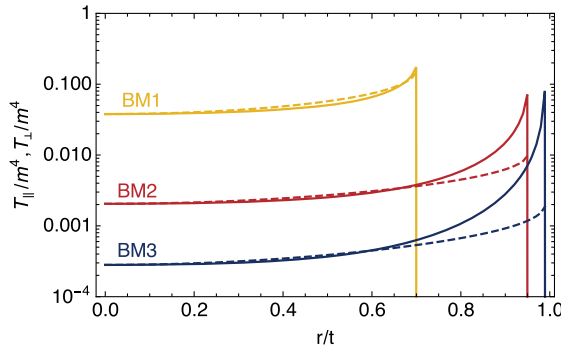


FIG. 1. Energy-momentum profile of particles inside an expanding bubble. The solid (dashed) curves denote $T_{\parallel}$ ($T_{\perp}$).

computation [62]), with $T_{\parallel} = T_{xx}$ and $T_{\perp} = T_{yy} = T_{zz}$ for wall motion in the x direction. The profiles are found to be self-similar (depending only on r/t , the time-dependent bubble radius divided by the time since nucleation), with distributions more sharply peaked for higher m/T and γ_w , as faster walls can drag particles along more strongly. The energy is mostly concentrated in extended shells with thickness comparable to the bubble radius, with a loose tail that extends inward.

We are interested in scenarios where this population of massive particles, or their decay products—we will denote the relevant particle by Y —only have feeble interactions (i.e., effectively do not interact) over the timescale of the phase transition. In the broken phase, Y could interact with other particles within a bubble during the expansion phase or with particles inside other bubbles after collision. In both cases, the condition for Y to be noninteracting during the phase transition is $n_Y \sigma R_* < 1$. Here n_Y is the average number density of Y particles ($\sim T^3$ assuming a full thermal distribution); σ is the relevant interaction cross section; and R_* , the average bubble size at collision, represents the timescale over which the PT completes. If $Y = X$, there are unavoidable X - s and (s -mediated) X self-scattering processes arising from the mass-generating coupling; nevertheless, the above condition can be satisfied with appropriate parameters. Alternatively, if X decays rapidly to other dark sector particles Y in the broken phase, such interactions are trivially avoided. We discuss details of the underlying particle physics model in Supplemental Material [62]. Since the massive FIPs Y could constitute a significant fraction of the total energy in the Universe, we assume that Y is metastable and decays into SM final states after the PT completes in order to avoid potential constraints from overclosure.

Gravitational wave signals—We now discuss the GW signal generated in the FIP scenario, drawing comparisons with GWs from the more familiar sound wave source. Gravitational waves h_{ij} are the transverse-traceless part of the Friedmann-Lemaître-Robertson-Walker metric $dx^2 = -dt^2 + a^2(t)(\delta_{ij} + h_{ij})dx^i dx^j$, sourced by the energy-momentum tensor $T_{\mu\nu}$ through the wave equation $\square h_{ij} = 16\pi G \Lambda_{ij,kl} T_{kl}$, where $\square$, G , and $\Lambda_{ij,kl}$ are the d'Alembertian, the Newtonian constant, and the tensor that projects out the transverse-traceless components, respectively. The GW spectrum at the time of production, Ω_{GW}^* , is calculated from the Fourier transform of $T_{\mu\nu}$:

$$\Omega_{\text{GW}}^*(k) \equiv \frac{1}{\rho_{\text{tot}}} \frac{d\rho_{\text{GW}}}{d \ln k} \propto \int \frac{d\hat{k}}{4\pi} \Lambda_{ij,kl}(\hat{k}) T_{ij}^*(\omega, \vec{k}) T_{kl}(\omega, \vec{k}) \Big|_{\omega=|\vec{k}|}, \quad (3)$$

where ρ_{tot} is the total energy density of the Universe.

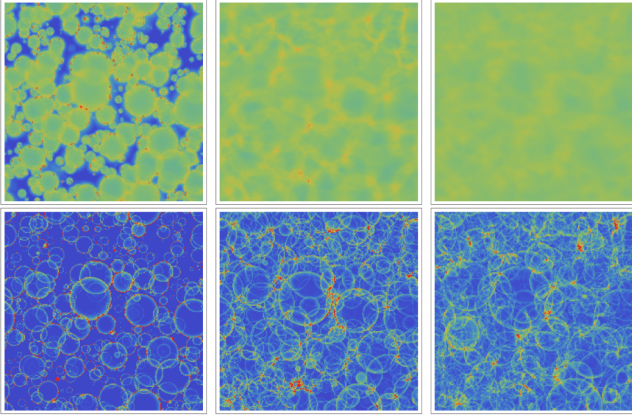


FIG. 2. Snapshots of time evolution of $(T_{ij}T_{ij})^{1/4}$ in the FIP scenario (BM1) (top) contrasted with an analogous simulation in the interacting scenario (SWs) [63] (bottom). These plots are for illustrative purposes only to highlight the qualitative differences between the two cases. Blue $\rightarrow$ green $\rightarrow$ yellow $\rightarrow$ red (normalized differently for the two cases) denotes increasing $(T_{ij}T_{ij})^{1/4}$; the red dots in the top row are numerical artifacts.

To calculate GW production, we develop a novel numerical scheme to calculate the energy-momentum tensor from the superposition of FIPs from multiple bubbles. We treat each spacetime point as a “sprinkler,” which emits particles with a definite spectrum when a bubble wall passes through it; the GW spectrum for a given bubble nucleation history is obtained by summing all sprinkler contributions. Further details of the calculations and simulations are provided in Supplemental Material [62]. We compute GW signals for a stable bosonic FIP particle X that is thermalized in the symmetric phase [92].

There are two main differences compared with the SW source. First, as clearly seen in Fig. 2, the FIP shells extend to the center of the bubble, whereas the sound shells have clear end points corresponding to the sound speed $c_s \simeq 1/\sqrt{3}$ determined by hydrodynamics [55,56,93,94]. The second concerns the behavior of T_{ij} when the shells cross after bubbles collide. For the FIPs case, since the particles free-stream without interacting, the total energy is obtained by simply adding the individual contributions from each bubble

$$T_{ij}(t, \vec{x}) = \sum_{I: \text{bubbles}} T_{ij}^{(I)}(t, \vec{x}). \quad (4)$$

Although the shells cross without interacting and superimpose trivially, the spherical symmetry of each shell nevertheless gets broken after collision (not visible in the figure). Particles that enter the bubble from the symmetric phase gain mass and get dragged along with the wall, but particles that enter from another bubble are already massive and therefore maintain their inward velocity, resulting in a nonisotropic distribution of particles in the shells.

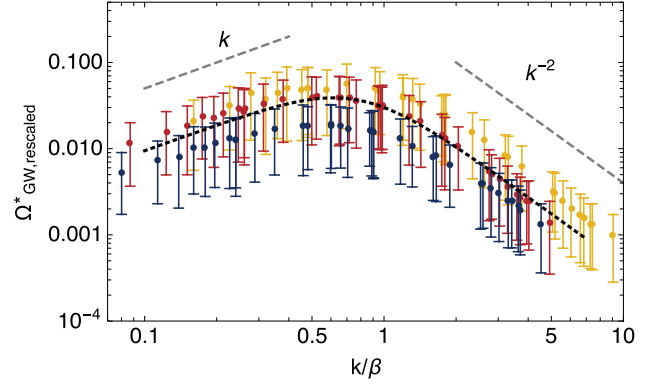


FIG. 3. Gravitational wave spectrum obtained from simulations with 50 nucleation histories, rescaled as $\Omega_{\text{GW}}^* / \{(\bar{K}^{(\text{GW})})^2 [(1/24)m^2 T^2 / \rho_{\text{tot}}]^2 (H/\beta)^2\}$. The dashed black curve shows the broken power law fit $s/(1+s^3)$. The error bars correspond to the variance associated with the average over propagation directions. We have checked that the variance due to the different nucleation histories is significantly smaller.

By contrast, for the interacting fluid (sound shells), the linearized fluid equation of motion $(\partial_t^2 - c_s^2 \nabla^2)v_i^{(\text{fluid})} = 0$ (neglecting vorticity) implies that the *fluid velocity field*, rather than the energy momentum, superimposes linearly:

$$v_i^{(\text{fluid})}(t, \vec{x}) = \sum_{I: \text{bubbles}} v_i^{(\text{fluid})(I)}(t, \vec{x}). \quad (5)$$

The GW source thus behaves nonlinearly in the superposition of the fluid shells,

$$T_{ij}^{(\text{fluid})}(t, \vec{x}) \sim w(t, \vec{x})v_i(t, \vec{x})v_j(t, \vec{x}) + (\delta_{ij} \text{ piece}), \quad (6)$$

where w is the fluid’s enthalpy. Consequently, new correlations get imprinted at the scale $k \sim (\text{shell thickness})^{-1}$, allowing for the accumulation of GWs at the same scale long after the collisions take place, enhancing the signal by a factor $\lesssim \beta/H$ [41–46]. For FIPs, as the particle shell thickness continues to expand as the FIPs propagate, the signal is instead imprinted over a larger range of wave numbers, resulting in a broader signal.

We evaluate Eq. (3) to obtain the GW signals from FIPs (see Supplemental Material [62] for details) for the three BM cases [Eq. (2)] and show the result in Fig. 3. The resulting signal (at production) can be parameterized as

$$\Omega_{\text{GW}}^*(k) \sim \left(\frac{H}{\beta}\right)^2 \left(\frac{\frac{1}{24}m^2 T^2}{\rho_{\text{tot}}}\right)^2 \left[(\bar{K}^{(\text{GW})})^2 \frac{2s}{1+s^3} \right], \quad (7)$$

with $s = 0.77 \times k/\beta$. Here, $(1/24)m^2 T^2 \approx \Delta V$; hence the factor $\{[(1/24)m^2 T^2 / \rho_{\text{tot}}]\}^2$ represents the characteristic scaling $[\alpha/(1+\alpha)]^2$ of GW signals, which is also observed for GW signals from SWs and bubble collisions. The part in the square parenthesis, obtained from a fit to the simulation

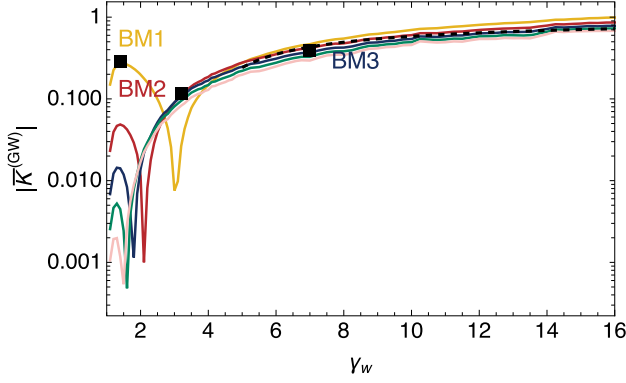


FIG. 4. Absolute value of $\bar{K}^{(\text{GW})}$ as a function of γ_w for $m/T = (1, 2, 3, 4, 5)$ (top to bottom). The black dashed curve is a numerical fit with formula $\bar{K}^{(\text{GW})} \sim 1 - 3/\gamma_w^{0.86}$. The black squares represent our BM points.

data, consists of two pieces. The spectral *shape* is approximately universal: the spectra peak is at $k \sim 0.77\beta$ and scales as $\sim k^1 (k^{-2})$ in the IR (UV). In the far IR, we expect the shape to scale as $\sim k^3$ as correlations are lost beyond a Hubble time; we do not recover this scaling in our simulations as we ignore the expansion of the Universe.

The function $\bar{K}^{(\text{GW})}$ encodes the details of the underlying process, i.e., the dependence on m/T and v_w (see Supplemental Material [62] for derivations). It quantifies the fraction of energy in the FIP distribution that is relevant for GW production [95]. The bar denotes an average over all propagation directions. We provide numerical values for various choices of $(m/T, \gamma_w)$ in Fig. 4. $\bar{K}^{(\text{GW})}$ approaches universal behavior at large γ_w . At small γ_w , $|\bar{K}^{(\text{GW})}|$ features a dip at a particular value of γ_w for each m/T ; this is because $\bar{K}^{(\text{GW})}$ roughly correlates with the average radial velocity of the particle distribution, and for each m/T there exists a γ_w for which the particle distribution is approximately static in the plasma frame, making GW production inefficient.

Rescaling Eq. (7) by the appropriate redshift factors gives the present day values

$$f = 1.2 \times 10^{-6} \text{ Hz} \frac{\beta}{H_*} \frac{T}{100 \text{ GeV}} \left(\frac{g_*^D}{5} \right)^{1/4} \left(\frac{106.75}{g_{\text{SM}}} \right)^{1/12} \frac{k}{\beta}, \quad (8)$$

and

$$\Omega_{\text{GW}}^0 = 5.0 \times 10^{-5} \frac{5}{g_*^D} \left(\frac{106.75}{g_{\text{SM}}} \right)^{1/3} \Omega_{\text{GW}}^*. \quad (9)$$

In Fig. 5, we plot the resulting GW signals for a few scenarios [96] corresponding to PTs at temperatures ranging from 5 TeV to 100 PeV against the power-law integrated sensitivities of various upcoming GW detectors.

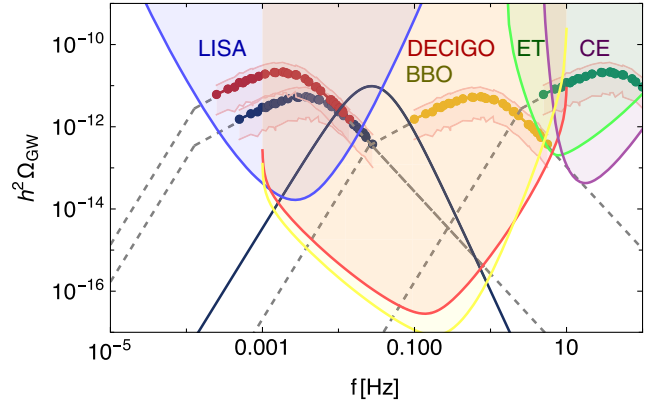


FIG. 5. Gravitational wave signals from FIPs with $m/T = 3$, $v_w = 0.99$, and $g_*^D = 5$ (which determine $\alpha = 0.23$). The different curves correspond to $T = 5 \text{ TeV}$, $\beta/H = 100$ (blue); $T = 5 \text{ TeV}$, $\beta/H = 50$ (red); $T = 1 \text{ PeV}$, $\beta/H = 100$ (yellow); and $T = 100 \text{ PeV}$, $\beta/H = 50$ (green). For comparison, we also show a sound wave signal (solid curve), using the same PT parameters as the blue curve, and $\alpha = 0.23$. The power-law integrated sensitivity curves for GW experiments [28–32] are for 1 year observation time, with signal-to-noise ratio = 1, obtained from Ref. [98].

The signal curves present the results of our simulations (colored dots) extrapolated in the UV with a k^{-2} -tail and a k^1 -tail in the IR, which breaks into a k^3 -tail at the Hubble scale at the time of the transition. If the FIPs decay soon after the PT completes, we expect the IR component to shut off exponentially, leaving another characteristic imprint on the signal. For comparison, we also include the GW signal predicted from a SW source (blue curve, made with PTPlot [10]), with the same PT parameters as the blue FIP curve.

The peak position of the SW signal is set by the size of the sound shells [43], whereas the peak of the FIP signal is set by the bubble size, explaining the order of magnitude difference in peak frequency. The amplitude of the GW signal from FIPs is comparable to the SW signal: despite the β/H enhancement of the latter due to the signal accumulating at the same scale, FIPs appear to be more efficient at sourcing GW signals as there is no energy loss inefficiency due to their noninteracting nature. The plot shows that various upcoming space- or ground-based GW detectors can be sensitive to FIP-GW signals produced from such PTs across a broad range of energy scales. Furthermore, the accumulation of the GW signal over a broader range of wave numbers also results in the FIP signal having a broader peak that scales as $\sim k^1$, which is a distinguishing characteristic of this scenario.

Discussion—We have discussed a novel source of gravitational waves from a FOPT: feebly interacting particles carrying the dominant fraction of the latent energy released in a phase transition in a dark sector. This provides an interesting and realistic alternative to conventionally

studied FOPT GW sources, and we have demonstrated that the GW spectra are qualitatively different: compared with a SW source with similar PT parameters, the FIP-induced signal has comparable amplitude but is broader and scales differently with frequency. The spectral shape is most similar to that from the bulk flow model [36,37], which models GW production from thin shells; the crucial difference with our setup is the extended FIP shell thickness, which can imprint distinguishable UV features in the signal. We have shown that these novel GW signals could be observable in next-generation GW experiments [99]. Such FIP configurations appear to be particularly efficient GW sources since their noninteracting nature eliminates energy loss inefficiencies and the GW signals are comparable in strength to those encountered with SW sources. We have provided an analytic formula in Eq. (7) that (together with Fig. 4) can be used to estimate the GW signal from FIPs in specific BSM models and study detection prospects without performing numerical simulations.

We studied a simplified, idealized scenario where the energy released in the phase transition goes entirely into FIPs, which act as the only source of GWs. Realistic scenarios likely involve additional features: for instance, a fraction of the particle distribution will get reflected, and some fraction of the FIPs may self-scatter or decay into interacting SM states on the same timescale as the phase transition. These can produce SWs in the plasma, which will produce additional GWs while suppressing the FIP signal. Even in such cases, the two contributions can likely be distinguished in an observed signal from their distinct features (see Fig. 5). Similarly, the case where FIPs produced from the decay of particles crossing into the bubble are significantly lighter than the parent particle will also change the signal; nevertheless, we expect the spectral features to remain the same. Such aspects merit further study in the future.

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Data availability—The data are not publicly available. The data are available from the authors upon reasonable request.

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