

Nature of $\chi_{c1}(3872)$ and $T_{cc}^+(3875)$

Nora Brambilla^{1,2,3,*} Abhishek Mohapatra^{1,†} Tommaso Scirpa^{1,‡} and Antonio Vairo^{1,§}

¹*Technical University of Munich, TUM School of Natural Sciences, Physics Department,
James-Frank-Straße 1, 85748 Garching, Germany*

²*Technical University of Munich, Institute for Advanced Study, Lichtenbergstraße 2 a, 85748 Garching, Germany*

³*Technical University of Munich, Munich Data Science Institute, Walther-von-Dyck-Straße 10, 85748 Garching, Germany*



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Two decades ago the $\chi_{c1}(3872)$ state was discovered in the hadron spectrum with two heavy quarks. The discovery fueled a surge in experimental research, uncovering dozens of so called XYZ exotics states lying outside the conventional quark model, as well as theoretical investigations into new forms of matter, such as quark-gluon hybrids, tetraquarks, and pentaquarks, with the potential of disclosing new information about the fundamental strong force. Among the XYZs, the $\chi_{c1}(3872)$ and $T_{cc}^+(3875)$ states stand out for their striking characteristics and unleashed many discussions about their nature. Here, we address this question using the Born-Oppenheimer effective field theory (BOEFT) and show how QCD settles the issue of their composition. Not only do we describe well the main features of the $\chi_{c1}(3872)$ and $T_{cc}^+(3875)$ states, but we also obtain predictions in the bottomonium sector. This opens the way to systematic applications of BOEFT to all XYZs.

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Introduction—The discovery [1] of the $\chi_{c1}(3872)$ [also known as $X(3872)$] state in the charmonium spectrum changed the field of the strong interactions, marking a significant breakthrough. A state with $J^{PC} = 1^{++}$ and isospin $I = 0$, it is remarkable for its mass being within 100 keV of the $D^{*0}\bar{D}^0$ threshold. Considering the central value, it is approximately 50 keV below the $D^{*0}\bar{D}^0$ threshold and has a very narrow width of 1.19 MeV [2]. Its properties suggest that it may be a tetraquark, consisting of four quarks bound together, $Q\bar{Q}q\bar{q}$. Additionally, a plethora of other exotic states, initially termed XYZs, emerged in the sector of the spectrum containing two heavy quarks ($Q\bar{Q}$ or QQ) at or above the strong decay thresholds, i.e., the energy above which these states can decay into a pair of heavy-light mesons $M\bar{M}$ [2,3]. They include states that are electrically charged, like the $Z_c^\pm$, $Z_b^\pm$, and pentaquarks P ($Q\bar{Q}qqq$). The $T_{cc}^+(3875)$ state, discovered recently by the LHCb Collaboration [4], stands out being charged, with a pronounced narrow peak (width of about 0.4 MeV), in the $D^0D^0\pi^+$ mass spectrum, about

360 keV below the D^+D^0 threshold, with $I = 0$ and $J^P = 1^+$ [4,5]. It is directly produced in hadroproduction and is the longest-lived exotic matter particle ever found. The XYZ states have been studied at the existing collider experiments and will be studied at the upcoming new experiments at FAIR [6] and EIC [7]. The $\chi_{c1}(3872)$ state has been observed with large (~ 30 nb) production cross sections both at CDF [8] and CMS [9] and in heavy ion collisions [10]. It is observed also in e^+e^- production and in B decays. Its decays to charm hidden states, some decay branching ratios, the production in $e^+e^- \rightarrow \omega\chi_{c1}(3872)$ [11], and the compositeness value [12,13], point to a charmonium component in its Fock space decomposition. The characteristics of these exotics reveal them as novel, strongly correlated systems with the potential of disclosing information about the strong force.

The existence of exotic hadron states with more than the minimal quark content was already proposed in the quark model [14–16] and in QCD [16,17] with the addition of states involving gluons in the binding, the hybrids. The search for exotic states started in the 1960s, but firm evidence came only when the XYZs were discovered. These new structures triggered a huge theoretical effort. The simplest system consisting of two quarks and two antiquarks (tetraquark) is already a very complicated one and it is unclear whether any kind of clustering occurs in it. To simplify the problem, models have been based on *ad hoc* choices of dominant configurations and interactions [3,16,18–30]. For the tetraquark, examples include hadronic molecules, which assume constituent color singlet mesons bound by residual nuclear forces [21,31,32], and

*Contact author: nora.brambilla@tum.de

†Contact author: abhishek.mohapatra@tum.de

‡Contact author: tommaso.scirpa@tum.de

§Contact author: antonio.vairo@tum.de

compact tetraquarks, which assume bound states between colored diquarks and antidiquarks [33–36]. For a long time, these two modelizations competed in asserting the nature of the XYZs, and the $\chi_{c1}(3872)$ state in particular [13,37].

QCD lattice calculations of some XYZ masses exist [38–62] but remain challenging as they require studying coupled channel scatterings on the lattice [63,64].

Born-Oppenheimer effective field theory (BOEFT)—The BOEFT is a nonrelativistic effective field theory obtained from QCD on the basis of symmetry and scales separation. It is suitable to describe bound states of two heavy quarks and any light degrees of freedom (LDF). Strongly coupled potential nonrelativistic QCD (pNRQCD) for quarkonium is the simplest realization of this theory [65–68]. The original BOEFT for hybrids was obtained in Ref. [69], with subsequent work [70–73] up to Ref. [74], where the equations for tetraquarks and pentaquarks were derived.

The first scale separation exploited by BOEFT is the scale separation between the mass m_Q of the heavy quarks and the energy scale of the LDF, the gluons or light quarks part of the binding, which is of the order of the non-perturbative hadronic scale Λ_{QCD} : $m_Q \gg \Lambda_{\text{QCD}}$. QCD static energies for all LDF excitations in the presence of two quarks separated by a distance r are classified in terms of the *BO quantum numbers*. Given $\mathbf{K}$, the total LDF angular momentum, we define $\Lambda = |\hat{\mathbf{r}} \cdot \mathbf{K}|$ ($\Lambda = 0, 1, 2, \dots$ are denoted by $\Sigma, \Pi, \Delta, \dots$). The BO quantum numbers Λ_η^σ identify the cylindrical $D_{\infty h}$ representations, where $\eta = g, u$ is the $[C]P$ eigenvalue ± 1 (for QQ only parity is defined), and $\sigma = \pm 1$ (only for Σ) is the eigenvalue of reflection through a plane containing the two sources. Excited states are indicated as $\Lambda_\eta^{\sigma'}, \Lambda_\eta^{\sigma''}, \dots$. We may also associate with the LDF the label $\kappa \equiv \{k^{P[C]}, f\}$, where $k(k+1)$ is the eigenvalue of $\mathbf{K}^2$ and f is the flavor. As $r \rightarrow 0$, the cylindrical symmetry reduces to a spherical symmetry labeled by κ , and some BO static energies become degenerate. The QCD static energies are written in terms of gauge invariant generalized Wilson loops characterized by the BO quantum numbers [74]. Some of these Wilson loops have been computed in lattice QCD [41,45,75–86].

The next scale separation is the one between the energy of the LDF and the binding energy of the heavy quarks: $\Lambda_{\text{QCD}} \gg E$. This scale separation allows us to define the BOEFT and obtain coupled Schrödinger equations to describe the exotics. The potentials appearing in these equations are the static energies introduced above. The potentials mix. The mixing may occur either because the potentials become degenerate as $r \rightarrow 0$ or because they carry the same BO quantum numbers and get close (degenerate) at some finite distance [74].

We present first-time predictions for the $\chi_{c1}(3872)$ state obtained from Schrödinger coupled equations derived from the BOEFT, and results for the $T_{cc}^+(3875)$ state obtained from a single-channel equation, whose T_{bb} analog has been studied previously in Refs. [41,44–47]. Moreover, in

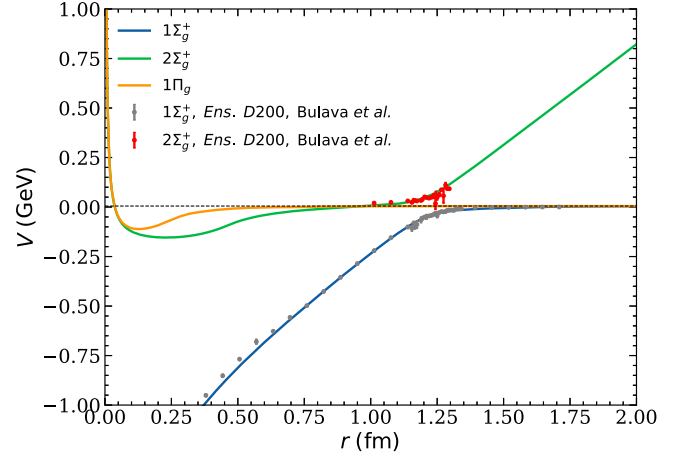


FIG. 1. Lowest isospin-0 adiabatic $Q\bar{Q}$ tetraquark BO potentials as functions of r [74], with $V_{2\Sigma_g^+}$ and $V_{1\Pi_g}$, corresponding to the 1^{--} adjoint meson at short distances (note the repulsive behavior of the color octet potential), approaching from below the $M\bar{M}$ threshold set at 0.005 GeV. The *adiabatic potentials* are the eigenvalues of the potential matrix defined in Eqs. (1)–(3) [see Eqs. (2)–(4) in Supplemental Material [87]] [74]. We show our potentials along with lattice data from the D200 ensemble [88] with $2\Sigma_g^+$ data limited to the avoided crossing region.

Ref. [74], we have shown that BOEFT constrains the potentials at short and large distances. At short distances r , we rely on the multipole expansion and express the potentials as a power series in r . The constant term in the series is the adjoint meson mass in the case of the $\chi_{c1}(3872)$ state and the triplet meson mass in the case of the $T_{cc}^+(3875)$ state; it depends only on κ . At large distances, due to BO quantum numbers conservation, the potentials evolve into the $M\bar{M}$ static energies. The adjoint or triplet meson masses are the key parameters for the existence of states close to the heavy-light meson thresholds [73].

$\chi_{c1}(3872)$ state: Coupled Schrödinger equations and potentials—In the BO picture, we take the state $\chi_{c1}(3872)$ as a bound state below the spin-isospin averaged $D\bar{D}$ threshold of the lowest $Q\bar{Q}$ tetraquark potential associated with the isospin-0 $k^{PC} = 1^{--}$ adjoint meson. Without spin interactions, different spin combinations, $\{S = 0, S = 1\}$ with S the heavy-quark-pair spin, form degenerate multiplets. So, the ground-state J^{PC} multiplet for $k^{PC} = 0^{++}$ is $\{0^{++}, 1^{+-}\}$ and for $k^{PC} = 1^{--}$ is $\{1^{+-}, (0, 1, 2)^{++}\}$. We identify the $\chi_{c1}(3872)$ state with the 1^{++} state in the ground-state multiplet corresponding to $k^{PC} = 1^{--}$ and described by the BO potentials $V_{\Sigma_g^+}$ and V_{Π_g} .

The potentials $V_{\Sigma_g^+}$ and V_{Π_g} mix at short distance, and with the quarkonium $V_{\Sigma_g^+}$ potential, due to avoided level crossing at around 1.2 fm, see Fig. 1, giving rise to coupled channel Schrödinger equations [Eq. (1) in Supplemental Material [87]] [74]. The equations depend on the angular momentum $\mathbf{L} = \mathbf{L}_Q + \mathbf{K}$, with $\mathbf{L}_Q$ the heavy quark pair orbital angular momentum; $l(l+1)$ are the eigenvalues of $\mathbf{L}^2$.

We use the lattice parametrization of Ref. [88] to model $V_{\Sigma_g^+}$ and $V_{\Sigma_g^{+'}}$ around the string breaking region. For $V_{\Sigma_g^{+'}}$ and V_{Π_g} , we model the short-distance behavior using the quenched BO-potential parametrization from Ref. [89], due to a lack of lattice data, and the long-distance behavior with a two-pion exchange potential [90]:

$$V_{\Sigma_g^+}(r) = V_0 + \frac{\gamma}{r} + \sigma r, \quad (1)$$

$$V_{\Lambda}(r) = \begin{cases} \kappa_8/r + \Lambda_{1^{--}} + A_{\Lambda}r^2 + B_{\Lambda}r^4 & r < R_{\Lambda} \\ F_{\Lambda}e^{-r/d}/r^2 + E_1 & r > R_{\Lambda} \end{cases}, \quad (2)$$

where $\Lambda \in \{\Sigma_g^{+'}, \Pi_g\}$, $\gamma = -0.434$, $\sigma = 0.198 \text{ GeV}^2$, $\kappa_8 = 0.037$, $A_{\Sigma_g^{+'}} = 0.0065 \text{ GeV}^3$, $B_{\Sigma_g^{+'}} = 0.0018 \text{ GeV}^5$, $A_{\Pi_g} = 0.0726 \text{ GeV}^3$, $B_{\Pi_g} = -0.0051 \text{ GeV}^5$, $d \sim 1/(2m_{\pi}) \sim 1/0.3 \text{ GeV}^{-1} \sim 0.66 \text{ fm}$, $E_1 = 0.005 \text{ GeV}$, $V_0 = -1.142 \text{ GeV}$, and the parameters F_{Λ} and R_{Λ} are determined by imposing continuity up to first derivatives. The mixing potential $V_{\Sigma_g^+ - \Sigma_g^{+'}}$ must vanish at $r \rightarrow 0$ based on pNRQCD [91] and it approaches zero asymptotically at $r \rightarrow \infty$, with a peak near the string-breaking region. Hence, we parameterize $V_{\Sigma_g^+ - \Sigma_g^{+'}}$ as

$$V_{\Sigma_g^+ - \Sigma_g^{+'}} = \begin{cases} gr/r_1 & r < r_1 \\ g & r_1 \leq r \leq r_2 \\ ge^{-(r-r_2)/r_0} & r > r_2 \end{cases}, \quad (3)$$

where $r_0 = 0.5 \text{ fm}$ is the Sommer scale, and $g = 0.05 \text{ GeV}$, $r_1 = 0.95 \text{ fm}$, and $r_2 = 1.51 \text{ fm}$ are fixed on the lattice data [88], and by demanding the continuity of the potential.

The charm and bottom quark masses entering the kinetic energy in the Schrödinger equations are set to be the spin-isospin averaged D and B meson masses, $m_c = 1.973 \text{ GeV}$ and $m_b = 5.313 \text{ GeV}$. The physical spin-isospin averaged meson-(antimeson) thresholds that we use in this Letter are $E_{D\bar{D}} = E_{DD} = 3.946 \text{ GeV}$, $E_{B\bar{B}} = E_{BB} = 10.627 \text{ GeV}$, and $E_{DB} = 7.287 \text{ GeV}$ [2]. We treat the 1^{--} adjoint meson mass $\Lambda_{1^{--}}$ as a free parameter to be fixed on the $\chi_{c1}(3872)$ mass.

Results for $\chi_{c1}(3872)$ and χ_{b1} states—The binding energy depends on the 1^{--} adjoint meson mass $\Lambda_{1^{--}}$. Solving the coupled Schrödinger equations with the potentials in Eqs. (1)–(3) and $\Lambda_{1^{--}}$ as a free parameter, we obtain a bound state around 90 keV below the $D\bar{D}$ threshold, which we identify with $\chi_{c1}(3872)$, for the value $\Lambda_{1^{--}}^* = 919 \text{ MeV}$. The scattering length a corresponding to the $E_b = 90 \text{ keV}$ binding energy is $a = \hbar/\sqrt{m_c E_b} = 14.81 \text{ fm}$. Additionally, we find a deeper bound state around 409 MeV below the $D\bar{D}$ threshold with mass 3537 MeV, which we identify with the spin-averaged $\chi_c(1P)$ state.

Our adjoint meson mass $\Lambda_{1^{--}}^*$ is consistent with the estimate from quenched lattice QCD with valence quarks [92] and changes only within 50 to 80 MeV depending on the one pion or two pions long-distance parameterization of $V_{\Sigma_g^{+'}}$ and V_{Π_g} . The $\chi_{c1}(3872)$ composition entails 8% of quarkonium and 92% of tetraquark, split into $P_{\Sigma'} = 38\%$ and $P_{\Pi} = 54\%$. These percentages show very little dependence on the specific parametrization of the long-distance part of the mixing potential in Eq. (3), e.g., whether it approaches zero with an exponential, power law, or Gaussian profile. Instead, increasing g up to 0.06 GeV and adjusting $\Lambda_{1^{--}}^*$ within 60 MeV to get again the $\chi_{c1}(3872)$ state raises the quarkonium fraction to 13%. Notably, for a state exactly at the threshold, the tetraquark probability dominates, $P_{\Sigma'} + P_{\Pi} \gtrsim 99\%$, with a negligible quarkonium probability, $P_{\Sigma} \lesssim 1\%$; the opposite happens for a deeper state like $\chi_c(1P)$.

The radiative decays of $\chi_{c1}(3872)$, $\chi_{c1}(3872) \rightarrow \gamma J/\psi$, and $\chi_{c1}(3872) \rightarrow \gamma \psi(2s)$, provide key insights into its internal structure and whether it has a significant charmonium component. In particular, the ratio $R_{\gamma\psi} = \Gamma_{\chi_{c1}(3872) \rightarrow \gamma \psi(2s)} / \Gamma_{\chi_{c1}(3872) \rightarrow \gamma J/\psi}$, has been the subject of significant theoretical [93–109] and experimental [110–114] interest in deciphering the nature of the $\chi_{c1}(3872)$ state. Theoretical predictions for $R_{\gamma\psi}$ from different models are given in Table 1 of Ref. [114]: $R_{\gamma\psi} \ll 1$ is expected for a pure $D\bar{D}^*$ molecule [93–95, 98–100], while $R_{\gamma\psi} \gtrsim 1$ is expected for a $\chi_{c1}(2P)$ charmonium-molecule mixture or a pure $\chi_{c1}(2P)$ charmonium state [95, 101–109]. Recently, based on Run 1 and Run 2 data, the LHCb Collaboration reported the updated value $R_{\gamma\psi} = 1.67 \pm 0.25$ [114]. As mentioned above, we obtain naturally a 8% $\chi_{c1}(2P)$ charmonium component in the $\chi_{c1}(3872)$ state from the mixing due to the avoided level crossing. Assuming the radiative decay is going only through the quarkonium component and considering the average of the several quark model results for the $\chi_{c1}(2P) \rightarrow \gamma J/\psi$ and $\chi_{c1}(2P) \rightarrow \gamma \psi(2s)$ widths given in Table 1 of Ref. [115], our rough estimates for the $\chi_{c1}(3872)$ state, based on a 8% charmonium component, are $\Gamma_{\chi_{c1}(3872) \rightarrow \gamma J/\psi} = 3.9 \pm 2.6 \text{ keV}$ and $\Gamma_{\chi_{c1}(3872) \rightarrow \gamma \psi(2s)} = 11.7 \pm 5.1 \text{ keV}$, which are consistent with the experimental results $\Gamma_{\chi_{c1}(3872) \rightarrow \gamma J/\psi}^{\text{PDG}} = 9.3 \pm 5.1 \text{ keV}$ and $\Gamma_{\chi_{c1}(3872) \rightarrow \gamma \psi(2s)}^{\text{PDG}} = 54 \pm 33 \text{ keV}$ [2]. Our prediction for the ratio is $R_{\gamma\psi} = 2.99 \pm 2.36$, which agrees with the recent LHCb value within errors. Moreover, our result indicating a $c\bar{c}$ component in the $\chi_{c1}(3872)$ state between 8% and 13% is close to the 5% estimate mentioned in some literature to explain the production rate of $\chi_{c1}(3872)$ in pp collisions [11, 116–118]. Additionally, using the Weinberg compositeness criterion [119, 120], which defines Z as the probability of finding a compact component in the bound state, the BESIII Collaboration reported $Z = 0.18^{+0.20}_{-0.23}$ based on the coupled channel

analysis of the $\chi_{c1}(3872)$ line shape [12]. Reference [13] reported $0.052 < Z < 0.14$ based on an analysis of high-statistics LHCb data for the $\chi_{c1}(3872)$ line shape [121]. Within error bars, these values for Z are consistent with our estimate of the $c\bar{c}$ component in $\chi_{c1}(3872)$. Finally, also the lattice studies in Refs. [42,43] suggest the existence of a quarkonium component. With $\Lambda_{1^{--}}^*$ tuned on the $\chi_{c1}(3872)$ mass and the potentials in Eqs. (1)–(3), we predict in the bottomonium sector a bound state about 15 MeV below the $B\bar{B}$ threshold, with quarkonium probability around 1.5%. Additionally, we find three deeper bound states around 744 MeV, 391 MeV, and 108 MeV below the $B\bar{B}$ threshold with masses 9.883 GeV, 10.236 GeV, and 10.519 GeV, respectively, which we identify with the spin-averaged $\chi_b(1P)$, $\chi_b(2P)$, and $\chi_b(3P)$ states.

Spin splitting—Considering the spin-dependent corrections to the potentials, we can determine the spin splittings. For the $k^{PC} = 1^{--}$ adjoint meson, the lowest $Q\bar{Q}$ tetraquark multiplet (with $l = 1$) includes a $S = 0$ state 1^{+-} and three $S = 1$ states $(0, 1, 2)^{++}$. The BOEFT predicts that spin-dependent interactions arise already at order $1/m_Q$ in tetraquarks and in hybrids, differently from quarkonium [71,122–124].

The spin-dependent potentials depend on nonperturbative correlators that for hybrids have been fixed [122,123] on a direct lattice calculation of the charmonium hybrid masses [38,39]. Currently, no lattice calculations for the spin splittings of the tetraquark multiplets exist. Therefore, we try to estimate the spin corrections for the lowest $c\bar{c}$ tetraquark by averaging the lattice results from Refs. [38,39] for the spin structure of the lowest $c\bar{c}$ hybrid multiplet. We also account for the spin splitting δ_Q , which scales like $1/m_Q$, between heavy mesons M^* and M : $\delta_c = 141$ MeV and $\delta_b = 45$ MeV [2]. The energies of M and M^* relative to their spin average are $-3\delta_Q/4$ and $\delta_Q/4$, respectively. This implies that the isospin averaged thresholds $D\bar{D}$ and $D^*\bar{D}$ (or $D\bar{D}^*$) are about 212 MeV and 70.5 MeV below, while $D^*\bar{D}^*$ is about 70.5 MeV above the spin-isospin averaged value. With the spin corrections from Refs. [38,39], the 1^{+-} and 2^{++} states are shifted above by 11(11) MeV and 58(14) MeV, while the 0^{++} and 1^{++} states are shifted below by 100(11) MeV and 74(14) MeV relative to the spin averaged (see Fig. 2). The values in parentheses represent the lattice statistical errors of Refs. [38,39] combined in quadrature. An alternative estimate [73] of these corrections due to $M\bar{M}$ threshold splittings is given in Supplemental Material [87] and agrees inside errors. The $\chi_{c1}(3872)$ state, identified with the 1^{++} state, is about 3.5(14) MeV below the isospin averaged $D^*\bar{D}$ threshold, which is consistent with the experimental fact that the state is 100 keV within $D^{*0}\bar{D}^0$ and 4 MeV below the isospin averaged $D^*\bar{D}$ threshold [2]. Given $E_{D\bar{D}} = 3.946$ GeV, the masses of the 1^{+-} , 0^{++} , 1^{++} , and 2^{++} states are 3.957(11) GeV, 3.846(11) GeV, 3.872(14) GeV, and 4.004(14) GeV, respectively. Moreover, the 1^{+-} state could

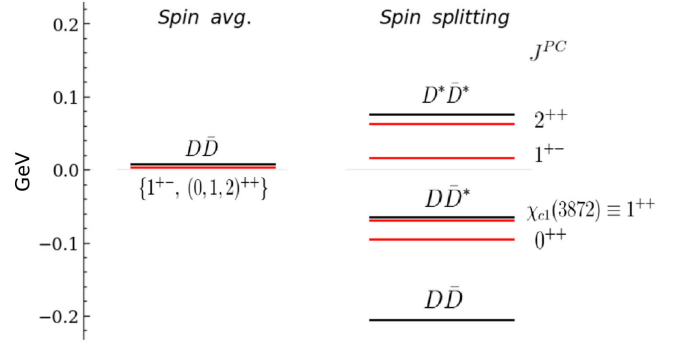


FIG. 2. Spectrum of the lowest $c\bar{c}$ tetraquark multiplet $\{1^{+-}, (0, 1, 2)^{++}\}$ corresponding to the 1^{--} adjoint meson relative to the heavy meson pair threshold. On the left, we display the spin-averaged case, and on the right, we display the spectrum with respect to the $D\bar{D}$, $D^*\bar{D}$ (or $D\bar{D}^*$), and $D^*\bar{D}^*$ thresholds after including spin corrections.

be identified with $X(3940)$ given that the masses are consistent within errors [125]. Spin splittings in the bottom sector are discussed in Supplemental Material [87].

$T_{cc}^+(3875)$ state: Schrödinger equations and potential—In the BOEFT, we take the state $T_{cc}^+(3875)$ as a bound state below the spin-isospin averaged DD threshold of the lowest QQ tetraquark potential associated with the isospin-0 $k^P = 0^+$ triplet meson. For $k^P = 0^+$, due to the Pauli exclusion principle, the ground-state multiplet only includes a heavy-quark-pair spin-triplet ($S = 1$) state 1^+ [74]. We identify the $T_{cc}^+(3875)$ state with this state in the ground-state multiplet (with $l = 0$) of the BO potential $V_{\Sigma_g^+}$ associated at $r \rightarrow 0$ with the 0^+ triplet meson. In this case, we have a single channel Schrödinger equation to solve [Eq. (5) in Supplemental Material [87]], with the potential $V_{\Sigma_g^+}$ based on the short-distance behavior of [89] and the long-distance behavior induced by a two-pion exchange potential [90]:

$$V_{\Sigma_g^+} = \begin{cases} \kappa_3/r + \Lambda_{0^+} + A_{\Sigma_g^+} r^2 & r < R_{\Sigma_g^+} \\ F_{\Sigma_g^+} e^{-r/d}/r^2 & r > R_{\Sigma_g^+} \end{cases}, \quad (4)$$

where $\kappa_3 = -0.120$ and $A_{\Sigma_g^+} = 0.197$ GeV³ [89], while the parameters $F_{\Sigma_g^+}$ and $R_{\Sigma_g^+}$ are determined by imposing continuity up to the first derivatives. In Fig. 3, we compare our parametrization of the potential $V_{\Sigma_g^+}(r)$ for isospin-0 QQ tetraquarks with lattice QCD data [90,126]. We treat the 0^+ triplet meson mass Λ_{0^+} as a free parameter to be fixed on the $T_{cc}^+(3875)$ mass.

Results for $T_{cc}^+(3875)$, T_{bb} , and T_{bc} states—The binding energy depends on the 0^+ triplet meson mass Λ_{0^+} (isospin $I = 0$). For $Q_1 Q_2$, the ground-state multiplet includes the states $\{0^+, 1^+\}$ with $\{S = 0, S = 1\}$, which are degenerate without spin interactions. Solving the Schrödinger equation gives $\Lambda_{0^+} = 664$ MeV, resulting in a bound state about

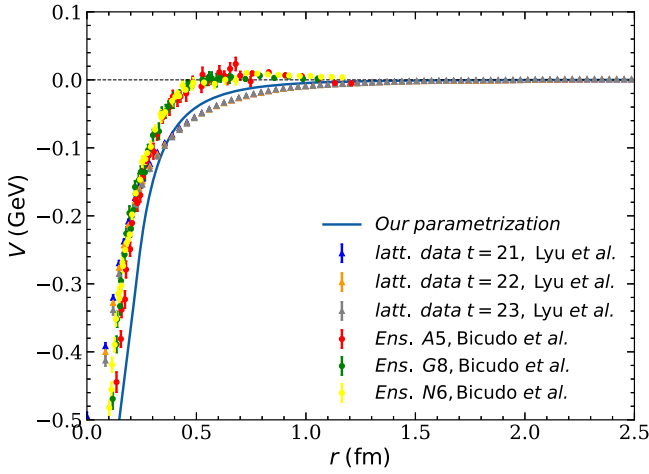


FIG. 3. Lowest isospin-0 QQ tetraquark BO potential as a function of r [74]. The BO potential $V_{\Sigma_j^+}$, corresponding to the 0^+ triplet meson at $r \rightarrow 0$, approaches at $r \rightarrow \infty$ the MM threshold, which is set to zero energy. Our potential is depicted alongside the lattice data from Refs. [90,126].

323 keV below the DD threshold, which we identify with the $T_{cc}^+(3875)$ state and a scattering length $a = 7.82$ fm, which is consistent with the LHCb result $a = 7.16 \pm 0.51$ fm [4]. Notice that the operator that defines the triplet (see Ref. [74]) coincides with a good diquark [127]. Having fixed $\Lambda_{0^+}^*$, we predict a deeply bound T_{bb} state ($J^P = 1^+$), 116 MeV below the BB threshold and a T_{bc} state ($J^P = \{0^+, 1^+\}$) 24 MeV below the DB threshold. Our T_{bb} binding energy result aligns well with *ab initio* lattice studies [48,51–55,57,59,60,128] and lattice based BO approximation studies [41,44–47]. Similarly, our T_{bc} binding energy agrees well with lattice QCD predictions [49,56,61,62]. A positive 30 MeV spin splitting correction for T_{bb} is reported in Ref. [47].

Summary—The BOEFT predicts a set of coupled channel Schrödinger equations to describe tetraquarks in QCD. In particular, the $\chi_{c1}(3872)$ state emerges as a bound state of coupled Schrödinger equations involving two tetraquark potentials and the quarkonium potential, while the $T_{cc}^+(3875)$ state emerges as a bound state of a single tetraquark potential. The value of the adjoint mass and the triplet mass together with the structure of the potential, which are constrained by the BOEFT, originate states with a very large radius, small binding energy, and other properties compatible with experiments. The states are neither simple molecules nor compact tetraquarks but result from a conspiracy between the short- and long-range behavior of potentials—constrained by symmetry and by lattice QCD near the string-breaking region. Thanks to the BOEFT factorization, fixing the parameters in the cc sector allows direct predictions in the bb and bc sectors. The existence of the $T_{cc}^+(3875)$ state below threshold may be related to the corresponding triplet meson being a good diquark [74].

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- [1] S. K. Choi *et al.* (Belle Collaboration), Observation of a narrow charmonium-like state in exclusive $B^\pm \rightarrow K^\pm \pi^+ \pi^- J/\psi$ decays, *Phys. Rev. Lett.* **91**, 262001 (2003).
- [2] S. Navas *et al.* (Particle Data Group), Review of particle physics, *Phys. Rev. D* **110**, 030001 (2024).
- [3] N. Brambilla, S. Eidelman, C. Hanhart, A. Nefediev, C.-P. Shen, C. E. Thomas, A. Vairo, and C.-Z. Yuan, The XYZ states: Experimental and theoretical status and perspectives, *Phys. Rep.* **873**, 1 (2020).
- [4] R. Aaij *et al.* (LHCb Collaboration), Study of the doubly charmed tetraquark T_{cc}^+ , *Nat. Commun.* **13**, 3351 (2022).
- [5] R. Aaij *et al.* (LHCb Collaboration), Observation of an exotic narrow doubly charmed tetraquark, *Nat. Phys.* **18**, 751 (2022).
- [6] G. Barucca *et al.* (PANDA Collaboration), PANDA phase one, *Eur. Phys. J. A* **57**, 184 (2021).
- [7] V. D. Burkert *et al.*, Precision studies of QCD in the low energy domain of the EIC, *Prog. Part. Nucl. Phys.* **131**, 104032 (2023).
- [8] A. Abulencia *et al.* (CDF Collaboration), Analysis of the quantum numbers J^{PC} of the $X(3872)$, *Phys. Rev. Lett.* **98**, 132002 (2007).
- [9] S. Chatrchyan *et al.* (CMS Collaboration), Measurement of the $X(3872)$ production cross section via decays to $J/\psi \pi^+ \pi^-$ in pp collisions at $\sqrt{s} = 7$ TeV, *J. High Energy Phys.* **04** (2013) 154.
- [10] A. M. Sirunyan *et al.* (CMS Collaboration), Evidence for $X(3872)$ in Pb-Pb collisions and studies of its prompt production at $\sqrt{s_{NN}} = 5.02$ TeV, *Phys. Rev. Lett.* **128**, 032001 (2022).
- [11] M. Ablikim *et al.* (BESIII Collaboration), Observation of a new $X(3872)$ production process $e^+ e^- \rightarrow \omega X(3872)$, *Phys. Rev. Lett.* **130**, 151904 (2023).
- [12] M. Ablikim *et al.* (BESIII Collaboration), Coupled-channel analysis of the $\chi_{c1}(3872)$ line shape with BESIII data, *Phys. Rev. Lett.* **132**, 151903 (2024).
- [13] A. Esposito, L. Maiani, A. Pilloni, A. D. Polosa, and V. Riquer, From the line shape of the $X(3872)$ to its structure, *Phys. Rev. D* **105**, L031503 (2022).
- [14] M. Gell-Mann, A schematic model of baryons and mesons, *Phys. Lett.* **8**, 214 (1964).
- [15] G. Zweig, An $SU(3)$ model for strong interaction symmetry and its breaking. Version 1, 10.17181/CERN-TH-401 (1964).
- [16] F. Gross *et al.*, 50 years of quantum chromodynamics, *Eur. Phys. J. C* **83**, 1125 (2023).

- [17] H. Fritzsche, M. Gell-Mann, and H. Leutwyler, Advantages of the color octet gluon picture, *Phys. Lett.* **47B**, 365 (1973).
- [18] N. Brambilla *et al.*, Substructure of multiquark hadrons (Snowmass 2021 white paper), [arXiv:2203.16583](https://arxiv.org/abs/2203.16583).
- [19] N. Brambilla *et al.*, QCD and strongly coupled gauge theories: Challenges and perspectives, *Eur. Phys. J. C* **74**, 2981 (2014).
- [20] N. Brambilla *et al.*, Heavy quarkonium: Progress, puzzles, and opportunities, *Eur. Phys. J. C* **71**, 1534 (2011).
- [21] F.-K. Guo, C. Hanhart, Ulf-G. Meißner, Q. Wang, Q. Zhao, and B.-S. Zou, Hadronic molecules, *Rev. Mod. Phys.* **90**, 015004 (2018); **94**, 029901(E) (2022).
- [22] M. Karliner, J. L. Rosner, and T. Skwarnicki, Multiquark states, *Annu. Rev. Nucl. Part. Sci.* **68**, 17 (2018).
- [23] A. Ali, J. S. Lange, and S. Stone, Exotics: Heavy pentaquarks and tetraquarks, *Prog. Part. Nucl. Phys.* **97**, 123 (2017).
- [24] G. T. Bodwin, E. Braaten, E. Eichten, S. L. Olsen, T. K. Pedlar, and J. Russ, Quarkonium at the frontiers of high energy physics: A Snowmass white paper, in *Snowmass 2013: Snowmass on the Mississippi* (2013).
- [25] L. Maiani, J/ψ -pair resonance by LHCb: A new revolution?, *Sci. Bull.* **65**, 1949 (2020).
- [26] A. Ali, L. Maiani, and A. D. Polosa, *Multiquark Hadrons* (Cambridge University Press, Cambridge, England, 2019).
- [27] S. Godfrey and S. L. Olsen, The exotic XYZ charmonium-like mesons, *Annu. Rev. Nucl. Part. Sci.* **58**, 51 (2008).
- [28] R. F. Lebed, R. E. Mitchell, and E. S. Swanson, Heavy-quark QCD exotica, *Prog. Part. Nucl. Phys.* **93**, 143 (2017).
- [29] G. Mezzadri and S. Spataro, XYZ states: An experimental point-of-view, *Rev. Phys.* **8**, 100070 (2022).
- [30] S. L. Olsen, T. Skwarnicki, and D. Zieminska, Nonstandard heavy mesons and baryons: Experimental evidence, *Rev. Mod. Phys.* **90**, 015003 (2018).
- [31] N. A. Tornqvist, Isospin breaking of the narrow charmonium state of Belle at 3872-MeV as a deuson, *Phys. Lett. B* **590**, 209 (2004).
- [32] F. E. Close and P. R. Page, The $D^{*0}\bar{D}^0$ threshold resonance, *Phys. Lett. B* **578**, 119 (2004).
- [33] R. L. Jaffe, Multi-quark hadrons. 1. The phenomenology of $Q^2\bar{Q}^2$ mesons, *Phys. Rev. D* **15**, 267 (1977).
- [34] R. L. Jaffe and F. Wilczek, Diquarks and exotic spectroscopy, *Phys. Rev. Lett.* **91**, 232003 (2003).
- [35] A. Esposito, A. Pilloni, and A. D. Polosa, Multiquark resonances, *Phys. Rep.* **668**, 1 (2017).
- [36] L. Maiani, F. Piccinini, A. D. Polosa, and V. Riquer, Diquark-antidiquarks with hidden or open charm and the nature of X(3872), *Phys. Rev. D* **71**, 014028 (2005).
- [37] V. Baru, X.-K. Dong, M.-L. Du, A. Filin, F.-K. Guo, C. Hanhart, A. Nefediev, J. Nieves, and Q. Wang, Effective range expansion for narrow near-threshold resonances, *Phys. Lett. B* **833**, 137290 (2022).
- [38] L. Liu, G. Moir, M. Peardon, S. M. Ryan, C. E. Thomas, P. Vilaseca, J. J. Dudek, R. G. Edwards, B. Joo, and D. G. Richards (Hadron Spectrum Collaboration), Excited and exotic charmonium spectroscopy from lattice QCD, *J. High Energy Phys.* **07** (2012) 126.
- [39] G. K. C. Cheung, C. O'Hara, G. Moir, M. Peardon, S. M. Ryan, C. E. Thomas, and D. Tims (Hadron Spectrum Collaboration), Excited and exotic charmonium, D_s and D meson spectra for two light quark masses from lattice QCD, *J. High Energy Phys.* **12** (2016) 089.
- [40] S. M. Ryan and D. J. Wilson (Hadron Spectrum Collaboration), Excited and exotic bottomonium spectroscopy from lattice QCD, *J. High Energy Phys.* **02** (2021) 214.
- [41] Z. S. Brown and K. Orginos, Tetraquark bound states in the heavy-light heavy-light system, *Phys. Rev. D* **86**, 114506 (2012).
- [42] S. Prelovsek and L. Leskovec, Evidence for X(3872) from DD^* scattering on the lattice, *Phys. Rev. Lett.* **111**, 192001 (2013).
- [43] M. Padmanath, C. B. Lang, and S. Prelovsek, X(3872) and Y(4140) using diquark-antidiquark operators with lattice QCD, *Phys. Rev. D* **92**, 034501 (2015).
- [44] P. Bicudo and M. Wagner (European Twisted Mass Collaboration), Lattice QCD signal for a bottom-bottom tetraquark, *Phys. Rev. D* **87**, 114511 (2013).
- [45] P. Bicudo, K. Cichy, A. Peters, and M. Wagner, BB interactions with static bottom quarks from lattice QCD, *Phys. Rev. D* **93**, 034501 (2016).
- [46] P. Bicudo, K. Cichy, A. Peters, B. Wagenbach, and M. Wagner, Evidence for the existence of $ud\bar{b}\bar{b}$ and the non-existence of $ss\bar{b}\bar{b}$ and $cc\bar{b}\bar{b}$ tetraquarks from lattice QCD, *Phys. Rev. D* **92**, 014507 (2015).
- [47] P. Bicudo, J. Scheunert, and M. Wagner, Including heavy spin effects in the prediction of a $\bar{b}\bar{b}ud$ tetraquark with lattice QCD potentials, *Phys. Rev. D* **95**, 034502 (2017).
- [48] L. Leskovec, S. Meinel, M. Pflaumer, and M. Wagner, Lattice QCD investigation of a doubly-bottom $\bar{b}bud$ tetraquark with quantum numbers $I(J^P) = 0(1^+)$, *Phys. Rev. D* **100**, 014503 (2019).
- [49] S. Meinel, M. Pflaumer, and M. Wagner, Search for $\bar{b}\bar{b}us$ and $\bar{b}\bar{c}ud$ tetraquark bound states using lattice QCD, *Phys. Rev. D* **106**, 034507 (2022).
- [50] C. Alexandrou, J. Finkenrath, T. Leontiou, S. Meinel, M. Pflaumer, and M. Wagner, Shallow bound states and hints for broad resonances with quark content $\bar{b}\bar{c}ud$ in $B^* - \bar{D}$ and $B^* - \bar{D}$ scattering from lattice QCD, *Phys. Rev. Lett.* **132**, 151902 (2024).
- [51] C. Alexandrou, J. Finkenrath, T. Leontiou, S. Meinel, M. Pflaumer, and M. Wagner, $\bar{b}\bar{c}ud$ and $\bar{b}\bar{b}us$ tetraquarks from lattice QCD using symmetric correlation matrices with both local and scattering interpolating operators, *Phys. Rev. D* **110**, 054510 (2024).
- [52] T. Aoki, S. Aoki, and T. Inoue, Lattice study on a tetraquark state T_{bb} in the HAL QCD method, *Phys. Rev. D* **108**, 054502 (2023).
- [53] P. Mohanta and S. Basak, Construction of $bb\bar{u}\bar{d}$ tetraquark states on lattice with NRQCD bottom and HISQ up and down quarks, *Phys. Rev. D* **102**, 094516 (2020).
- [54] P. Junnarkar, N. Mathur, and M. Padmanath, Study of doubly heavy tetraquarks in lattice QCD, *Phys. Rev. D* **99**, 034507 (2019).
- [55] A. Francis, R. J. Hudspith, R. Lewis, and K. Maltman, Lattice prediction for deeply bound doubly heavy tetraquarks, *Phys. Rev. Lett.* **118**, 142001 (2017).

- [56] A. Francis, R. J. Hudspith, R. Lewis, and K. Maltman, Evidence for charm-bottom tetraquarks and the mass dependence of heavy-light tetraquark states from lattice QCD, *Phys. Rev. D* **99**, 054505 (2019).
- [57] R. J. Hudspith, B. Colquhoun, A. Francis, R. Lewis, and K. Maltman, A lattice investigation of exotic tetraquark channels, *Phys. Rev. D* **102**, 114506 (2020).
- [58] M. Padmanath and S. Prelovsek, Signature of a doubly charm tetraquark pole in DD^* scattering on the lattice, *Phys. Rev. Lett.* **129**, 032002 (2022).
- [59] R. J. Hudspith and D. Mohler, Exotic tetraquark states with two $\bar{b}$ quarks and $J^P = 0^+$ and 1^+ B_s states in a non-perturbatively tuned lattice NRQCD setup, *Phys. Rev. D* **107**, 114510 (2023).
- [60] B. Colquhoun, A. Francis, R. J. Hudspith, R. Lewis, K. Maltman, and W. G. Parrott, Improved analysis of strong-interaction-stable doubly-bottom tetraquarks on the lattice, *Phys. Rev. D* **110**, 094503 (2024).
- [61] M. Padmanath, A. Radhakrishnan, and N. Mathur, Bound isoscalar axial-vector $bc\bar{u}\bar{d}$ tetraquark T_{bc} from lattice QCD using two-meson and diquark-antidiquark variational basis, *Phys. Rev. Lett.* **132**, 201902 (2024).
- [62] A. Radhakrishnan, M. Padmanath, and N. Mathur, Study of the isoscalar scalar $bc\bar{u}\bar{d}$ tetraquark T_{bc} with lattice QCD, *Phys. Rev. D* **110**, 034506 (2024).
- [63] M. Luscher, Two particle states on a torus and their relation to the scattering matrix, *Nucl. Phys.* **B354**, 531 (1991).
- [64] R. A. Briceño, J. J. Dudek, and R. D. Young, Scattering processes and resonances from lattice QCD, *Rev. Mod. Phys.* **90**, 025001 (2018).
- [65] N. Brambilla, A. Pineda, J. Soto, and A. Vairo, Potential NRQCD: An effective theory for heavy quarkonium, *Nucl. Phys.* **B566**, 275 (2000).
- [66] N. Brambilla, A. Pineda, J. Soto, and A. Vairo, The QCD potential at $O(1/m)$, *Phys. Rev. D* **63**, 014023 (2000).
- [67] A. Pineda and A. Vairo, The QCD potential at $O(1/m^2)$: Complete spin dependent and spin independent result, *Phys. Rev. D* **63**, 054007 (2001); **64**, 039902(E) (2001).
- [68] N. Brambilla, D. Eiras, A. Pineda, J. Soto, and A. Vairo, Inclusive decays of heavy quarkonium to light particles, *Phys. Rev. D* **67**, 034018 (2003).
- [69] M. Berwein, N. Brambilla, J. Tarrús Castellà, and A. Vairo, Quarkonium hybrids with nonrelativistic effective field theories, *Phys. Rev. D* **92**, 114019 (2015).
- [70] N. Brambilla, G. a. Krein, J. Tarrús Castellà, and A. Vairo, Born-Oppenheimer approximation in an effective field theory language, *Phys. Rev. D* **97**, 016016 (2018).
- [71] R. Oncala and J. Soto, Heavy quarkonium hybrids: Spectrum, decay and mixing, *Phys. Rev. D* **96**, 014004 (2017).
- [72] J. Soto and J. T. Castellà, Nonrelativistic effective field theory for heavy exotic hadrons, *Phys. Rev. D* **102**, 014012 (2020).
- [73] E. Braaten and R. Bruschini, Exotic hidden-heavy hadrons and where to find them, *Phys. Lett. B* **863**, 139386 (2025).
- [74] M. Berwein, N. Brambilla, A. Mohapatra, and A. Vairo, Hybrids, tetraquarks, pentaquarks, doubly heavy baryons, and quarkonia in Born-Oppenheimer effective theory, *Phys. Rev. D* **110**, 094040 (2024).
- [75] K. J. Juge, J. Kuti, and C. J. Morningstar, *Ab initio* study of hybrid $\bar{b}gb$ mesons, *Phys. Rev. Lett.* **82**, 4400 (1999).
- [76] K. J. Juge, J. Kuti, and C. Morningstar, Fine structure of the QCD string spectrum, *Phys. Rev. Lett.* **90**, 161601 (2003).
- [77] G. S. Bali, B. Bolder, N. Eicker, T. Lippert, B. Orth, P. Ueberholz, K. Schilling, and T. Struckmann (TXL Collaboration), Static potentials and glueball masses from QCD simulations with Wilson sea quarks, *Phys. Rev. D* **62**, 054503 (2000).
- [78] G. S. Bali and A. Pineda, QCD phenomenology of static sources and gluonic excitations at short distances, *Phys. Rev. D* **69**, 094001 (2004).
- [79] S. Capitani, O. Philipsen, C. Reisinger, C. Riehl, and M. Wagner, Precision computation of hybrid static potentials in SU(3) lattice gauge theory, *Phys. Rev. D* **99**, 034502 (2019).
- [80] C. Schlosser and M. Wagner, Hybrid static potentials in SU(3) lattice gauge theory at small quark-antiquark separations, *Phys. Rev. D* **105**, 054503 (2022).
- [81] P. Bicudo, N. Cardoso, and A. Sharifian, Spectrum of very excited Σ_g^+ flux tubes in SU(3) gauge theory, *Phys. Rev. D* **104**, 054512 (2021).
- [82] A. Sharifian, N. Cardoso, and P. Bicudo, Eight very excited flux tube spectra and possible axions in SU(3) lattice gauge theory, *Phys. Rev. D* **107**, 114507 (2023).
- [83] R. Höllwieser, F. Knechtli, T. Korzec, M. Peardon, and J. A. Urrea-Niño, Hybrid static potentials from Laplacian eigenmodes, *Proc. Sci. LATTICE2023* (2024) 073 [arXiv:2401.09453].
- [84] S. Prelovsek, H. Bahtiyar, and J. Petkovic, Z_b tetraquark channel from lattice QCD and Born-Oppenheimer approximation, *Phys. Lett. B* **805**, 135467 (2020).
- [85] M. Sadl and S. Prelovsek, Tetraquark systems $\bar{b}b\bar{d}u$ in the static limit and lattice QCD, *Phys. Rev. D* **104**, 114503 (2021).
- [86] L. Mueller, P. Bicudo, M. Krstic Marinkovic, and M. Wagner, Antistatic-antistatic-light-light potentials from lattice QCD, *Proc. Sci. LATTICE2023* (2024) 064 [arXiv:2312.17060].
- [87] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/physrevd.107.014012> for the explicit expressions of the leading-order coupled Schrödinger equations for the $\chi_{c1}(3872)$ and $T_{cc}^+(3875)$ states along with the results of the tetraquark multiplets. It also presents an alternative method for computing the spin-splitting corrections, supplementing the approach described in the main text.
- [88] J. Bulava, F. Knechtli, V. Koch, C. Morningstar, and M. Peardon, The quark-mass dependence of the potential energy between static colour sources in the QCD vacuum with light and strange quarks, *Phys. Lett. B* **854**, 138754 (2024).
- [89] F. Alasiri, E. Braaten, and A. Mohapatra, Born-Oppenheimer potentials for SU(3) gauge theory, *Phys. Rev. D* **110**, 054029 (2024).
- [90] Y. Lyu, S. Aoki, T. Doi, T. Hatsuda, Y. Ikeda, and J. Meng, Doubly charmed tetraquark T_{cc}^+ from lattice QCD near physical point, *Phys. Rev. Lett.* **131**, 161901 (2023).

- [91] J. Tarrús Castellà, Heavy meson thresholds in Born-Oppenheimer effective field theory, *Phys. Rev. D* **106**, 094020 (2022).
- [92] M. Foster and C. Michael (UKQCD Collaboration), Hadrons with a heavy color adjoint particle, *Phys. Rev. D* **59**, 094509 (1999).
- [93] E. S. Swanson, Short range structure in the $X(3872)$, *Phys. Lett. B* **588**, 189 (2004).
- [94] E. S. Swanson, Diagnostic decays of the $X(3872)$, *Phys. Lett. B* **598**, 197 (2004).
- [95] Y. Dong, A. Faessler, T. Gutsche, and V. E. Lyubovitskij, $J/\psi\gamma$ and $\psi(2S)\gamma$ decay modes of the $X(3872)$, *J. Phys. G* **38**, 015001 (2011).
- [96] F.-K. Guo, C. Hanhart, Y. S. Kalashnikova, U.-G. Meißner, and A. V. Nefediev, What can radiative decays of the $X(3872)$ teach us about its nature?, *Phys. Lett. B* **742**, 394 (2015).
- [97] D. A. S. Molnar, R. F. Luiz, and R. Higa, Short-distance RG-analysis of $X(3872)$ radiative decays, *arXiv*: 1601.03366.
- [98] D. P. Rathaud and A. K. Rai, Dimesonic states with the heavy-light flavour mesons, *Eur. Phys. J. Plus* **132**, 370 (2017).
- [99] R. F. Lebed and S. R. Martinez, Diabatic representation of exotic hadrons in the dynamical diquark model, *Phys. Rev. D* **106**, 074007 (2022).
- [100] B. Grinstein, L. Maiani, and A. D. Polosa, Radiative decays of $X(3872)$ discriminate between the molecular and compact interpretations, *Phys. Rev. D* **109**, 074009 (2024).
- [101] T. Barnes and S. Godfrey, Charmonium options for the $X(3872)$, *Phys. Rev. D* **69**, 054008 (2004).
- [102] T. Barnes, S. Godfrey, and E. S. Swanson, Higher charmonia, *Phys. Rev. D* **72**, 054026 (2005).
- [103] F. De Fazio, Radiative transitions of heavy quarkonium states, *Phys. Rev. D* **79**, 054015 (2009); **83**, 099901(E) (2011).
- [104] B.-Q. Li and K.-T. Chao, Higher charmonia and X, Y, Z states with screened potential, *Phys. Rev. D* **79**, 094004 (2009).
- [105] A. M. Badalian, V. D. Orlovsky, Y. A. Simonov, and B. L. G. Bakker, The ratio of decay widths of $X(3872)$ to $\psi'\gamma$ and $J/\psi\gamma$ as a test of the $X(3872)$ dynamical structure, *Phys. Rev. D* **85**, 114002 (2012).
- [106] J. Ferretti, G. Galatà, and E. Santopinto, Quark structure of the $X(3872)$ and $\chi_b(3P)$ resonances, *Phys. Rev. D* **90**, 054010 (2014).
- [107] A. M. Badalian, Y. A. Simonov, and B. L. G. Bakker, $c\bar{c}$ interaction above threshold and the radiative decay $X(3872) \rightarrow J/\psi\gamma$, *Phys. Rev. D* **91**, 056001 (2015).
- [108] W.-J. Deng, H. Liu, L.-C. Gui, and X.-H. Zhong, Charmonium spectrum and their electromagnetic transitions with higher multipole contributions, *Phys. Rev. D* **95**, 034026 (2017).
- [109] F. Giacosa, M. Piotrowska, and S. Coito, $X(3872)$ as virtual companion pole of the charm-anticharm state $\chi_{c1}(2P)$, *Int. J. Mod. Phys. A* **34**, 1950173 (2019).
- [110] B. Aubert *et al.* (BABAR Collaboration), Evidence for $X(3872) \rightarrow \psi_{2S}\gamma$ in $B^\pm \rightarrow X_{3872}K^\pm$ decays, and a study of $B \rightarrow c\bar{c}\gamma K$, *Phys. Rev. Lett.* **102**, 132001 (2009).
- [111] V. Bhardwaj *et al.* (Belle Collaboration), Observation of $X(3872) \rightarrow J/\psi\gamma$ and search for $X(3872) \rightarrow \psi'\gamma$ in B decays, *Phys. Rev. Lett.* **107**, 091803 (2011).
- [112] R. Aaij *et al.* (LHCb Collaboration), Evidence for the decay $X(3872) \rightarrow \psi(2S)\gamma$, *Nucl. Phys. B* **886**, 665 (2014).
- [113] M. Ablikim *et al.* (BESIII Collaboration), Study of open-charm decays and radiative transitions of the $X(3872)$, *Phys. Rev. Lett.* **124**, 242001 (2020).
- [114] R. Aaij *et al.* (LHCb Collaboration), Probing the nature of the $\chi_{c1}(3872)$ state using radiative decays, *J. High Energy Phys.* **11** (2024) 121.
- [115] S.-Y. Pei, W. Li, T. Wang, and G.-L. Wang, Radiative transitions of $\chi_{cJ} \rightarrow \psi\gamma$ and $\chi_{bj} \rightarrow \Upsilon\gamma$, *J. High Energy Phys.* **08** (2024) 191.
- [116] C. Bignamini, B. Grinstein, F. Piccinini, A. D. Polosa, and C. Sabelli, Is the $X(3872)$ production cross section at Tevatron compatible with a hadron molecule interpretation?, *Phys. Rev. Lett.* **103**, 162001 (2009).
- [117] M. Takizawa and S. Takeuchi, $X(3872)$ as a hybrid state of charmonium and the hadronic molecule, *Prog. Theor. Exp. Phys.* **2013**, 093D01 (2013).
- [118] M. Aaboud *et al.* (ATLAS Collaboration), Measurements of $\psi(2S)$ and $X(3872) \rightarrow J/\psi\pi^+\pi^-$ production in pp collisions at $\sqrt{s} = 8$ TeV with the ATLAS detector, *J. High Energy Phys.* **01** (2017) 117.
- [119] S. Weinberg, Elementary particle theory of composite particles, *Phys. Rev.* **130**, 776 (1963).
- [120] S. Weinberg, Evidence that the deuteron is not an elementary particle, *Phys. Rev.* **137**, B672 (1965).
- [121] R. Aaij *et al.* (LHCb Collaboration), Study of the lineshape of the $\chi_{c1}(3872)$ state, *Phys. Rev. D* **102**, 092005 (2020).
- [122] N. Brambilla, W. K. Lai, J. Segovia, J. Tarrús Castellà, and A. Vairo, Spin structure of heavy-quark hybrids, *Phys. Rev. D* **99**, 014017 (2019); **101**, 099902(E) (2020).
- [123] N. Brambilla, W. K. Lai, J. Segovia, and J. Tarrús Castellà, QCD spin effects in the heavy hybrid potentials and spectra, *Phys. Rev. D* **101**, 054040 (2020).
- [124] J. Soto and S. Tomas Valls, Hyperfine splittings of heavy quarkonium hybrids, *Phys. Rev. D* **108**, 014025 (2023).
- [125] P. Pakhlov *et al.* (Belle Collaboration), Production of new charmoniumlike states in $e^+e^- \rightarrow J/\psi D^{(*)}\bar{D}^{(*)}$ at $\sqrt{s} \approx 10.6$ GeV, *Phys. Rev. Lett.* **100**, 202001 (2008).
- [126] P. Bicudo, M. Krstic Marinkovic, L. Müller, and M. Wagner, Antistatic-antistatic $\bar{Q}\bar{Q}qq$ potentials for u , d and s light quarks from lattice QCD, *Proc. Sci. LATTICE2024* (2025) 124 [arXiv:2409.10786].
- [127] A. Francis, P. de Forcrand, R. Lewis, and K. Maltman, Diquark properties from full QCD lattice simulations, *J. High Energy Phys.* **05** (2022) 062.
- [128] B. S. Tripathy, N. Mathur, and M. Padmanath, $bb\bar{u}\bar{d}$ and $bs\bar{u}\bar{d}$ tetraquarks from lattice QCD using two-meson and diquark-antidiquark variational basis, *Phys. Rev. D* **111**, 114504 (2025).