

Elastic Turbulence Hides in the Small Scales of Inertial Polymeric Turbulence

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Gaining a fundamental understanding of turbulent flows of dilute polymer solutions has been a challenging and outstanding problem for a long time. In this Letter, we examine homogeneous, isotropic polymeric turbulence at large Reynolds and Deborah numbers through direct numerical simulations. While at the largest scales at which the flow inertial turbulence exists, we find that the flow is fundamentally altered from Newtonian turbulence below the Kolmogorov scale. We demonstrate that “elastic turbulence” exists at the smallest scales of polymeric turbulence by quantifying multiple statistical properties of the flow—energy spectrum and flux in Fourier space as well as the spatial statistics of the velocity field—the structure functions and kurtosis, and energy dissipation. Our results show the coexistence of two fundamentally distinct types of turbulence in polymeric fluids and point to the ubiquity of elastic turbulence, which was hitherto only known to exist for negligible inertia.

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Turbulent flows of dilute polymer solutions have been of long-standing interest since the early work of Toms [1]. In addition to the Reynolds number (Re), which is a measure of fluid inertia, polymeric fluids are characterized by the Deborah number (De), a measure of the elasticity of the polymers [2,3]. The interaction between inertia, elasticity, and viscous dissipation means that polymeric fluids can exhibit a wide variety of turbulent flow behavior. The most well-known is that the addition of even a small concentration of polymers can dramatically modify wall-bounded turbulent flows, leading to reduced drag at large Re and De [4–7], as well as early transition to turbulence at moderate Re [8–10]. The nontrivial nature of polymeric turbulence is not limited to wall-bounded flows, as polymers affect both energy dissipation and associated flow structures in grid generated homogeneous, isotropic turbulence, as shown in several experiments [11–18] and simulations [19–25]. Recently, a novel “elasto-inertial range” scaling was discovered in polymeric turbulence for large Re and $De \sim O(1)$, where the kinetic energy spectrum scales as $E \sim k^{-2.3}$ instead of the usual Kolmogorov scaling ($\sim k^{-5/3}$) [26–28]. Strikingly, polymeric fluids also exhibit a distinct spatiotemporally chaotic state at negligible Re and large De , often termed as “elastic turbulence” [29–35]. Unlike inertial turbulence, elastic turbulence involves a

balance between the polymeric stresses and viscous dissipation, with negligible contribution from fluid inertia, and is characterized by a novel scaling of the fluid energy spectrum, $E(k) \sim k^{-4}$ [34–39]. It has been assumed that elastic and inertial turbulence occur in different parameter regimes, with elastic turbulence only existing at low global Re , so inertia is negligible everywhere.

Polymers modify turbulent flows through a wide range of mechanics and at multiple length scales, a comprehensive understanding of which is still lacking after years of study. Here, we examine the small scales of homogeneous, isotropic turbulence of polymer solutions using direct numerical simulations (DNSs). A few existing studies using DNSs [20,22,23,25] have suggested that polymers modify the fluid energy spectrum also in the dissipation range. One experiment, using polyethylene oxide solution in grid turbulence, also observed a modified energy spectrum, but only transiently [16]. Despite these results, the exact nature of the flow at small scales in polymeric turbulence is largely unknown.

In this Letter, we characterize the statistical properties of the flow at small scales in large Re – De polymeric turbulence, and show that they are the same as those of elastic turbulence occurring at negligible Re . Our results thus reveal that elastic turbulence need not be confined to low global Re , as has been presumed so far since its discovery [29,30,35]. We indeed demonstrate that inertial and elastic turbulence simultaneously coexist at the large and small scales in turbulent flows of polymers.

The dynamics of polymeric fluids are described by the Navier-Stokes and continuity equation for the fluid velocity field $\mathbf{u}$, coupled to a constitutive model for the evolution of the polymer conformation tensor $\mathbf{C}$. We use the Oldroyd-B model, which assumes that the polymer molecules deform

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affinely with the flow, and has already been shown to successfully capture the fundamental physics of polymeric fluids [2,3]. Thus, the governing equations are

$$\begin{aligned}\rho_f(\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u}) &= -\nabla p + \frac{\mu_p}{\tau_p} \nabla \cdot \mathbf{C} + \mu_f \nabla^2 \mathbf{u} + \mathbf{F}, \\ \nabla \cdot \mathbf{u} &= 0, \\ \partial_t \mathbf{C} + \mathbf{u} \cdot \nabla \mathbf{C} &= \mathbf{C} \cdot \nabla \mathbf{u} + \nabla \mathbf{u}^T \cdot \mathbf{C} - \frac{\mathbf{C} - \mathbf{I}}{\tau_p},\end{aligned}$$

where ρ_f is the fluid density, μ_f the fluid viscosity, μ_p the additional viscosity due to the polymers, τ_p the relaxation time of the polymer molecules, and the force $\mathbf{F}$ is used to sustain the turbulence. See Supplemental Material (SM) for additional details of the numerical simulation [40].

In order to focus on the large $Re-De$ regime, we choose the Taylor Reynolds number $Re_\lambda \approx 240$ and the Deborah number $De = \tau_p/\tau_L \approx 9$, where τ_L is the turnover time of the largest eddy, with $\beta = \mu_s/(\mu_s + \mu_p) = 0.9$. We term this parameter set, $Re_\lambda \approx 240, De \approx 9$ as “polymeric turbulence” (PT) in the following. We use a grid size of 2048^3 grid points for PT; this large resolution has not been previously achieved in DNS at large $Re-De$ and is required to sufficiently resolve the statistics at small scales. Furthermore, we consider elastic turbulence (ET) and Newtonian turbulence (NT) to compare, where we set $Re_\lambda \approx 40, De \approx 9, \beta = 0.9$, and $Re_\lambda \approx 460, De = 0$, respectively, simulated with 1024^3 grid points. To compare the different cases, we also define the Kolmogorov length scale, $\eta = \nu_f^3/\epsilon_f$, where $\nu_f = \mu_f/\rho_f$ and ϵ_f is the viscous energy dissipation by the solvent. Recall that in Newtonian turbulence, inertia and viscous dissipation dominate for $k\eta < 1$ and $k\eta > 1$, respectively [48,49].

First, we examine the fluid energy spectrum $E(k)$ in Fourier space versus $k\eta$, which shows two distinct scaling ranges in Fig. 1(a). For small wave numbers ($k\eta < 1$), the energy spectrum $E(k)$ in PT scales as $\sim k^{-5/3}$ like in classical NT. Here, since we have $De \gg 1$ with $\beta = 0.9$, following earlier work [27], we recover the Kolmogorov scaling in the inertial range. At larger wave numbers ($k\eta > 1$), we see a separate power-law scaling of $E \sim k^{-4}$, which is different from NT where the energy spectrum decays exponentially [50–53]. In PT we see no hint of an exponential scaling even at the largest wave numbers resolved, with instead the observed power-law scaling corresponding to the one of ET [29,33,35–37]. Remarkably, the Kolmogorov length scale η continues to demarcate the two different regimes. The SM [40] shows the kinetic energy spectrum in PT obtained using the FENE-P model for the polymers, where we once again find the k^{-4} regime. The results presented here are therefore independent of the constitutive model used.

To identify the dominant physical mechanism in each regime, we look at the variation of the energy flux across

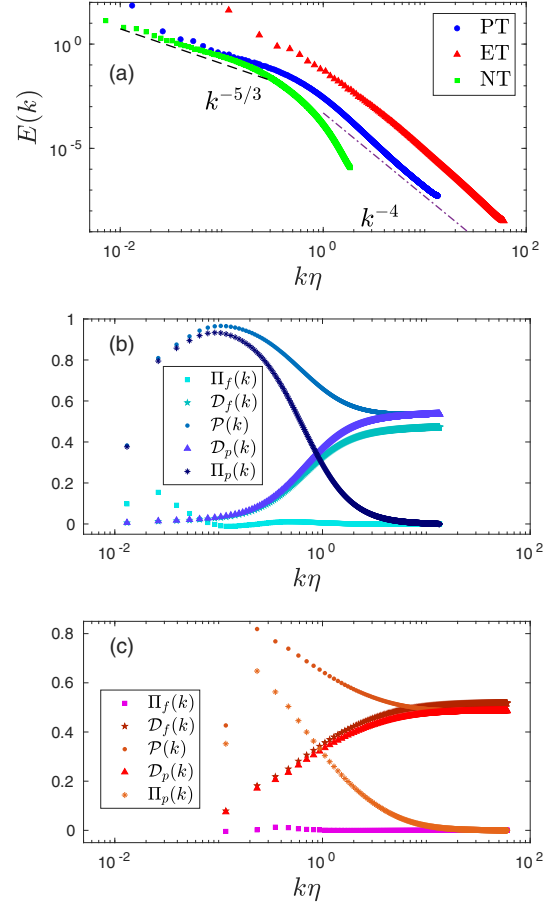


FIG. 1. (a) Energy spectrum $E(k)$ versus the nondimensional wave number $k\eta$, for polymeric (PT), elastic (ET), and Newtonian turbulence (NT). The various energy fluxes in Fourier space versus $k\eta$ for (b) polymeric and (c) elastic turbulence.

wave numbers, shown in Fig. 1(b). In polymeric turbulence, there are three distinct ways for energy redistribution: inertial Π_f , viscous $\mathcal{D}_f$, and polymeric $\mathcal{P}$ [24,27,39] (see SM for the derivation [40]). As expected, the inertial flux is only nonzero at the largest scales ($k\eta \ll 1$), while viscous dissipation is active at small scales ($k\eta \gg 1$). The polymeric flux has a nontrivial contribution across all wave numbers, showing that the elastic stresses continue to play an important role even for $k\eta \gg 1$ [55]. $\mathcal{P}$ can further be partitioned into a dissipative ($\mathcal{D}_p$) and nondissipative (Π_p) contribution, where $\mathcal{D}_p \rightarrow \mathcal{P}$ for $k\eta \gg 1$ [27]. With this decomposition, only the inertial (Π_f) and nondissipative component of the polymeric (Π_p) flux contribute to $k\eta \ll 1$, while both $\mathcal{D}_p$ and $\mathcal{D}_f$ dominate for $k\eta \gg 1$. Thus, we have a nontrivial balance between energy transfer due to polymers and viscous dissipation at large wave numbers, once again reminiscent of ET as shown in Fig. 1(c). The polymeric energy spectra for PT and ET also have the same scaling for $k\eta \gg 1$ (see SM [40]). Furthermore, the kinetic energy spectrum in the small scales of PT [$E(k) \sim k^{-4}$] agrees well with the existing

experimental results for ET at low Re , which find the exponent to range from 3.3 to 3.6 [29,33–35], as well as numerical simulations [36,37,55] and theoretical predictions for $E(k)$ [41]. However, the theory predicts different polymeric energy spectra $E_p(k)$ [40].

Thus, we have shown that the usual dissipation range is replaced by an “elastic-dissipation” range in polymeric turbulence. Both the energy spectrum and flux suggest that ET exists in this range, but this is not conclusive by itself. In the remainder of the Letter, we crucially demonstrate that these small scales also share the same velocity field and dissipation statistics as ET.

The spatial statistics of the velocity field in turbulence are revealed by the structure functions of two-point velocity differences $S_p(r) = \langle \delta u(r)^p \rangle$, with $\delta u(r) = (\mathbf{u}(\mathbf{x} + \mathbf{r}) - \mathbf{u}(\mathbf{x})) \cdot \hat{\mathbf{r}}$. Kolmogorov originally conjectured that the structure functions in NT would follow a similarity scaling, $S_p \sim r^{\zeta_p}$, predicting $\zeta_p = p/3$ in the inertial range based on dimensional analysis [48]. Figure 2(a) suggests that there are two distinct scaling regimes for S_p in PT: for $r/\eta > 1$,

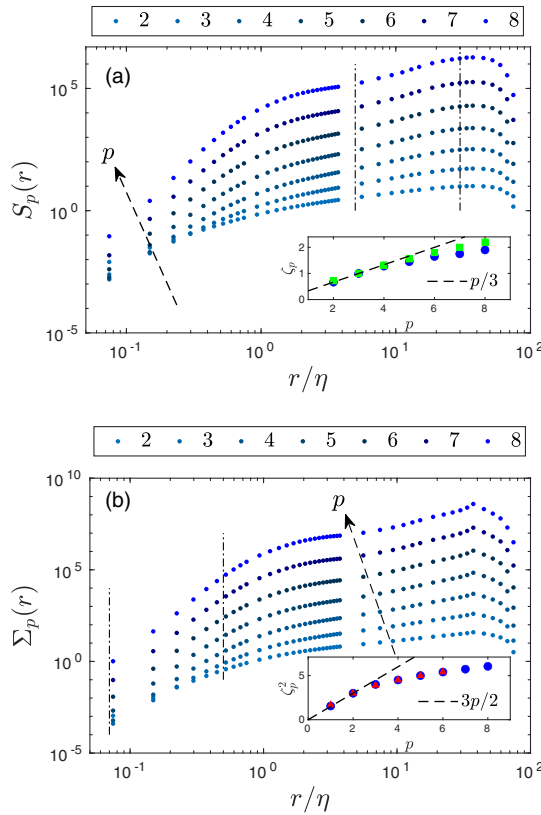


FIG. 2. The r variation of the structure functions (a) S_p and (b) Σ_p , for p from 2 to 8 for polymeric turbulence. The insets show the associated intermittency exponents in the inertial and elastic-dissipation ranges in (a) and (b), respectively (circles for PT), with the range where the scaling exponents are extracted marked in both figures. For comparison, we also show the results from [54] for inertial turbulence and [37] for elastic turbulence in the insets of (a) and (b) as squares and triangles, respectively.

we indeed see Kolmogorov’s prediction for $S_3 \sim r$, i.e., $\zeta_3 = 1$, while the scaling exponents ζ_p are known to differ from $p/3$ for larger p due to the intermittency [54,57–59]. The intermittency exponents ζ_p can be estimated from the slope of S_p versus r in the inertial range, and are plotted in the inset of Fig. 2(a), where ζ_p for PT are seen to be the same as in the inertial range of NT [27,54,57–59]. Thus, at large scales the inertial range intermittency statistics are recovered in polymeric turbulence. Instead, $S_p \sim r^p$ for $r/\eta < 1$, which follows from the trivial scaling of the velocity field $u \sim r$ for $r \ll 1$. Such analytic behavior of the velocity field is expected whenever the fluid energy spectrum decays faster than k^{-3} [48], which holds for both Newtonian and elastic turbulence. Hence, the small r behavior of $S_p(r)$ by itself is insufficiently revealing [37,60].

To differentiate the dynamics at small scales, we look at the structure functions of the symmetric three-point velocity difference, $\Sigma_p(r) = \langle \delta^2 u(r)^p \rangle$, with $\delta^2 u(r) = (\mathbf{u}(\mathbf{x} + \mathbf{r}) - 2\mathbf{u}(\mathbf{x}) + \mathbf{u}(\mathbf{x} - \mathbf{r})) \cdot \hat{\mathbf{r}}$, since it was recently shown that Σ_p reveals the intermittency in ET by removing the leading order analytic contribution [37]. Figure 2(b) shows that $\Sigma_p(r) \sim r^{\zeta_p^2}$ also has two distinct scaling regimes in PT, just like $S_p(r)$. In the inertial range, the velocity field is already rough at the leading order, so Σ_p shows the same scaling behavior as S_p , i.e., $\zeta_p^2 \approx \zeta_p$ for $r/\eta > 1$ (not shown). In contrast, for $r/\eta < 1$, $\Sigma_p(r)$ has a nontrivial scaling behavior, distinct from that of $S_p(r)$. Scaling analysis would suggest $\zeta_p^2 = 3/2$ for $r/\eta < 1$ [37], but the exponents reveal intermittency corrections and saturate below $3p/2$ for large p [see the inset of Fig. 2(b)]. More importantly, for $r/\eta < 1$, the exponents ζ_p^2 are the same as those discovered in ET [37], and thus, the statistics of the velocity fluctuations at the small scales in polymeric turbulence, including the intermittency behavior, are exactly the same as elastic turbulence. To further confirm this, SM [40] shows the comparison of Σ_2 for PT and NT for $r/\eta \ll 1$; for NT, $\Sigma_2(r) \sim r^4$, confirming the analyticity of the velocity field, while for PT $\Sigma_2(r) \sim r^3$.

We look at the kurtosis $\mathcal{K}(r) = \langle \delta u(r)^4 \rangle / \langle \delta u(r)^2 \rangle^2$, a measure of the deviation from a Gaussian distribution for which $\mathcal{K} = 3$; a larger kurtosis implies elongated tails with increased contribution from rare but extreme events, and $\mathcal{K}$ is used as another measure of the intermittency of the velocity field in NT [48,58,59]. Figure 3(a) shows two distinct scaling regimes of $\mathcal{K}$ for PT: for $r/\eta > 1$, $\mathcal{K}$ shows the same scaling behavior as the inertial range of NT ($\mathcal{K} \sim r^{-0.12}$) [54,58,59], while the scaling of the kurtosis is seen to be similar to ET for $r/\eta < 1$ ($\mathcal{K} \sim r^{-0.45}$). Further, since the kurtosis at small scales in PT increases faster than in the dissipation regime of NT, the velocity field in the elastic-dissipation range of polymeric turbulence at large $Re-De$ is thus more intermittent than the usual dissipation range in NT.

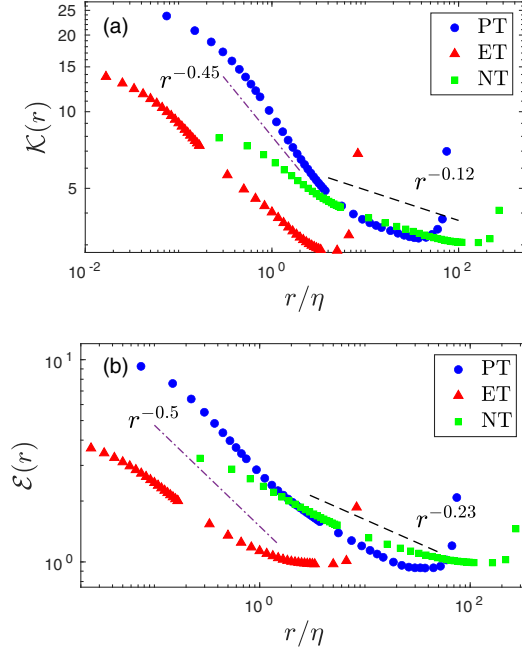


FIG. 3. (a) The kurtosis $\mathcal{K}(r) = \langle \delta u(r)^4 \rangle / \langle \delta u(r)^2 \rangle^2$ and (b) the covariance of the fluid dissipation $\mathcal{E}(r)$, versus the nondimensional distance r/η , for polymeric (PT), elastic (ET), and Newtonian (NT) turbulence.

Next, we examine the spatial statistics of the fluid dissipation $\epsilon_f = 2\nu_f \mathbf{S} : \mathbf{S}$, with $\mathbf{S} = 1/2(\nabla \mathbf{u} + (\nabla \mathbf{u})^T)$, using the spatial autocorrelation defined as $\mathcal{E}(r) = \langle \epsilon_f(x) \epsilon_f(x+r) \rangle / \langle \epsilon_f \rangle^2$. In the inertial range of NT, $\mathcal{E} \sim r^{-\mu}$, where the exponent μ is a measure of the intermittent and multifractal nature of energy dissipation [61–65]. In polymeric turbulence, $\mathcal{E}$ again has two distinct scaling ranges, as shown in Fig. 3(b): the inertial ($\mathcal{E} \sim r^{-0.23}$) and the elastic-dissipation ($\mathcal{E} \sim r^{-0.5}$) range, corresponding to Newtonian [61,63,65] and elastic turbulence, respectively. The increased scaling exponent at the smaller scales also implies that energy dissipation in polymeric turbulence is much more intermittent, in agreement with the results obtained using the kurtosis earlier. Finally, Fig. 4 compares the probability distribution function of the normalized, longitudinal velocity gradient in the three cases. The statistics of the velocity gradients independently characterize the small scales of turbulence and are important in determining the dynamics of sub-Kolmogorov scale particles as well as mixing [57–59,66]. Once again, the spatial structure of the flow at the small scales in polymeric turbulence well corresponds to that of elastic turbulence.

We have conclusively demonstrated that elastic turbulence exists for $r/\eta \ll 1$, while inertial turbulence occurs for $r/\eta \gg 1$ at large Re_λ and De . We now investigate *why* and *when* does elastic turbulence exist at the small scales of large Re_λ .

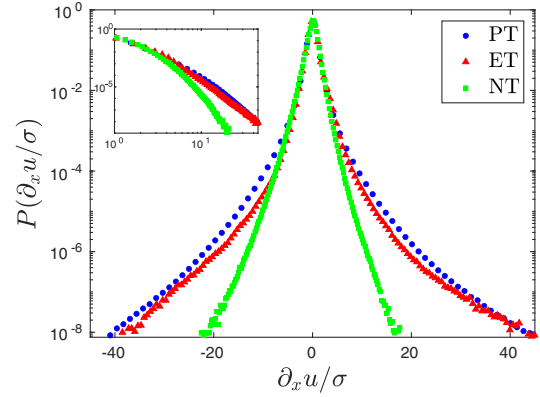


FIG. 4. The probability distribution function of the normalized, longitudinal velocity gradient $(\partial_x u / \sigma)$ for PT, ET, and NT, where σ is the standard deviation; the inset shows the tails of the probability distribution function on a log-log plot.

At a given length scale r , polymers experience the shear rate $\dot{\gamma}_r = (S_2)^{1/2}/r$, where $(S_2)^{1/2} = \delta u(r)$ is a scale-dependent measure of the fluid velocity. We can now define a local Deborah number $De_r = \tau_p (S_2)^{1/2}/r$, and, similarly, a scale-dependent Reynolds number $Re_r = S_2^{1/2} r / \nu$ [67,68]. Figure 5 shows the variation of Re_r and De_r with r/η . At large scales ($r/\eta > 1$), $Re_r \gg De_r$ and we see that inertia dominates. At small scales ($r/\eta < 1$), De_r asymptotes to a constant ($\sim r^0$ since $S_2^{1/2} \sim r$), while Re_r keeps decreasing with r ($\sim r^2$); thus, as the inertial effects decrease at smaller scales, eventually we have a transition length scale where Re_r becomes smaller than De_r . For ET to exist, the velocity fluctuations at small scales must be strong enough to sufficiently stretch the polymers; thus, we can expect ET to appear when $De_r \gg Re_r$ and $De_r \gg 1$ for $r \rightarrow 0$ with $Re_r \ll 1$, exactly the parameter range at which elastic turbulence is expected.

This suggests that elastic turbulence should exist if the local De_r number is large. To verify this hypothesis, we examine the energy spectrum for $Re_\lambda \approx 240$ with additional Deborah numbers of $1/9$, $1/3$, 1 , and 3 , shown in SM [40].

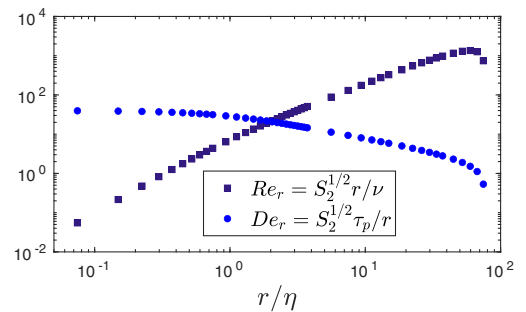


FIG. 5. The variation of the local Reynolds (Re_r) and Deborah (De_r) numbers with r/η for polymeric turbulence with $Re_\lambda \approx 240$ and $De \approx 9$.

For $De \approx 1/3$, 1, and 3, the local De_r for $r/\eta \ll 1$ is greater than 1, and indeed we observe the k^{-4} spectrum and hence ET. On the contrary, for $De \approx 1/9$, De_r for $r/\eta \ll 1$ is too small, and consequently ET does not exist and the energy spectrum recovers the usual exponential decay in the dissipation range.

In this Letter, we have shown that elastic turbulence is present in the small scales of turbulent flows of polymer solutions for a wide range of parameters—for any (even large) Re_λ whenever the local $De_r \geq 1$. We provide a crucial missing link in gaining a mechanistic understanding of polymeric turbulence by quantifying the flow at small scales, something that so far has lacked a quantitative understanding. Two competing theories have been proposed to explain drag reduction by polymer additives [69]: the viscous theory, where only the increased extensional viscosity due to polymers is important [70–72], and the elastic theory, where elastic effects instead dominate [41,67,68,73]. Our results suggest that elastic effects non-trivially modify the small scales in homogeneous turbulence and thus cannot be neglected. However, we do not find the “elastic-wave” range predicted by later extensions of the elastic theory [41,68,73], where inertia and elasticity balance. Instead, we find a balance between elasticity and viscous dissipation, similar to ET at low global Re .

Our results go beyond the expectations of existing theories by demonstrating that there is an exact correspondence between the statistics of PT at the small scales and ET. We thus expand the parameter range where ET is observed and show that it is quite ubiquitous in the flows of polymer solutions, a consistent theoretical description of which is still lacking [34–37,74]. ET has also been used to engineer devices with enhanced mixing and reaction kinetics recently [75,76]; therefore, our results also pose an intriguing question as to how ET at the small scale affects other global properties of the flow, such as mixing, in polymeric turbulence. Particularly an extension to wall-bounded flows could be relevant to polymeric drag reduction [5,69]. Finally, our results suggest future experimental measurements of structure functions and intermittency exponents for three-point velocity differences, extending recent work at large scales [17,18,26].

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Data availability—The data that support the findings of this Letter are not publicly available. The data are available from the authors upon reasonable request.

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