

Probing the Color-Octet Mechanism via Dihadron Fragmentation in χ_b Decays

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The color-octet (CO) mechanism is a cornerstone of nonrelativistic QCD, yet knowledge of its long-distance matrix elements remains limited, preventing stringent tests of the theory. We demonstrate that the Artru-Collins asymmetry in hadronic decays of the P -wave bottomonium state χ_{b2} provides a direct probe of CO dynamics. At leading order, the asymmetry arises exclusively from the CO decay channel, whereas the color-singlet (CS) contribution affects only the unpolarized rate, so that a nonzero signal constitutes unambiguous evidence of the CO mechanism. This observable provides a novel way to extract the ratio ρ_8 between CO and CS matrix elements. Focusing on $e^+e^- \rightarrow \Upsilon(2S) \rightarrow \gamma\chi_{b2}$ at Belle, we show that the asymmetric beam configuration preserves the asymmetry in the laboratory frame and avoids the strong suppression present in the center-of-mass frame. With the Belle II dataset, ρ_8 could be determined with sufficient precision to address the long-standing discrepancy between the lattice calculations and phenomenological determinations.

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Introduction—Five decades after the discovery of the J/ψ meson, heavy quarkonium production and decay processes remain a preeminent laboratory for probing the interplay between perturbative and nonperturbative regimes of quantum chromodynamics (QCD). The phenomenology of their spectrum, decay, and production facilitates the extraction of fundamental standard model parameters, such as the heavy quark mass m_Q and the strong coupling constant $\alpha_s(m_Q)$. Grounded in rigorous effective field theory [1], the nonrelativistic QCD (NRQCD) factorization [2] is now the most robust framework for describing heavy quarkonium inclusive decay, in which the decay width is factorized into short-distance coefficients (SDCs) to describe the annihilation of the heavy-quark pair ($Q\bar{Q}$) into light partons and nonperturbative long-distance matrix elements (LDMEs) to characterize the probability of that $Q\bar{Q}$ pair inside the meson. In contrast to the traditional color-singlet model, NRQCD factorization allows the intermediate $Q\bar{Q}$ pair to be in all possible spin and color

configurations, particularly in the color-octet (CO) state known as the CO mechanism (COM). The introduction of COM is not an *ad hoc* supplement, but rather an inevitable consequence of QCD factorization and NRQCD velocity power counting [1]. It is essential for ensuring infrared safety, restoring factorization to all orders of α_s in perturbative calculation. Consequently, a precise determination of the CO LDMEs provides definitive evidence of NRQCD factorization in the decay sector.

The P -wave quarkonium χ_{QJ} decay is viewed as the golden channel to probe COM because both color-singlet (CS) $^3P_J^{[1]}$ and CO $^3S_1^{[8]}$ states contribute at v^2 leading order (LO) in NRQCD factorization [2]. While the CS LDMEs $H_1^Q = \langle \chi_{QJ} | \mathcal{O}(^3P_J^{[1]}) | \chi_{QJ} \rangle$ can be related to the first derivative of the wave function at the origin within potential model calculations, the CO ones $H_8^Q(\mu_\Lambda) = \langle \chi_{QJ} | \mathcal{O}(^3S_1^{[8]}, \mu_\Lambda) | \chi_{QJ} \rangle$, where μ_Λ is the NRQCD factorization scale, have to be determined via simulations of lattice NRQCD or fits to experimental data [3,4]. In the charmonium case, the LDMEs predicted by lattice NRQCD [5] are in good agreement with the fits to χ_{cJ} decay [6,7]. However, for the bottomonium states $\chi_{bJ}(1P)$, the lattice prediction for the ratio $\rho_8(m_b) = H_8^b(m_b)m_b^2/H_1^b = 0.044 \pm 0.015$ [4,8] is significantly smaller than the value extracted from experimental fits to D^0 production in $\chi_{bJ}(1P)$ decays, $\rho_8(m_b) = 0.16_{-0.047}^{+0.071}$ [9]. Furthermore, $\rho_8(m_b)$ can also be related to the gluonic correlator $\mathcal{E}_3$ in potential-NRQCD (pNRQCD),

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which was fitted to χ_{cJ} decay data [10,11] or computed in the refined Gribov-Zwanziger theory [12]. An independent determination of $\rho_8(m_b)$ from χ_{bJ} decay is essential to examining the universal nature of $\mathcal{E}_3$ in strongly coupled pNRQCD.

In fits to decay rates, the CS and CO contributions are experimentally indistinguishable, precluding a precise determination of $\rho_8(m_b)$. In this Letter, we propose a novel approach utilizing the transverse spin correlations of quarks, described by the Artru-Collins asymmetry [13], as a direct probe of the COM in χ_{bJ} decays at lepton colliders. Because of J^{PC} conservation, at QCD LO, the CO $b\bar{b}({}^3S_1^{[8]})$ state decays into $q\bar{q}$ whereas CS $b\bar{b}({}^3P_J^{[1]})$ decays into gluons. Both the light quarks and gluons will subsequently fragment into dihadron pairs. Different from inclusive decay rates, the Artru-Collins asymmetry is sensitive to the spin structures of the fragmenting partons [13], thereby directly encoding the quantum numbers of the intermediate $q\bar{q}$ or gluons. We will demonstrate for the first time that CO and CS channels produce distinct dihadron azimuthal asymmetry patterns, enabling the isolation of the CO contribution through parton spin correlations in χ_{bJ} decays.

Kinematics and observable—At KEKB, χ_{bJ} states are produced predominantly via $E1$ transitions from the transversely polarized $\Upsilon(2S)$ in e^+e^- annihilation. In the heavy-quark limit, the χ_{bJ} polarization is completely determined by the $E1$ transition amplitude and directly reflects the parent $\Upsilon(2S)$ polarization [14]. For χ_{b0} , only the $b\bar{b}({}^3P_0^{[1]}) \rightarrow gg$ generates an Artru-Collins type asymmetry, while the CO $b\bar{b}({}^3S_1^{[8]}) \rightarrow q\bar{q}$ channel yields none due to the scalar nature of the χ_{b0} . For χ_{b1} , the two-gluon channel is forbidden by the Landau-Yang theorem, so the decay is CO dominated, consistent with the observed branching fraction of $\chi_{b1} \rightarrow D^0 X$ [15]. The resulting asymmetry is nevertheless insensitive to the LDMEs since they enter the polarized and unpolarized cross sections identically and cancel in the asymmetry ratio. By contrast, for χ_{b2} decay the CO channel generates nontrivial transverse $q\bar{q}$ spin correlations. Although individual χ_{b2} helicity states can generate a nonzero linear-polarization correlation between the two gluons in the CS gg channel, these contributions cancel exactly after summing over the helicity weighted by the $E1$ transition, which is a consequence of the spin-2 angular-momentum structure. Consequently, at LO, the azimuthal asymmetry is generated by the CO $q\bar{q}$ channel, while the CS channel contributes only to the unpolarized cross section and dilutes the signal. We thus focus on $\chi_{b2} \rightarrow q\bar{q}, gg \rightarrow (\pi^+\pi^-) + (\pi^+\pi^-) + X$, where the transverse $q\bar{q}$ spin correlations generate an asymmetry directly sensitive to the underlying CO dynamics.

Figure 1 illustrates the LO kinematics of this process. The χ_{b2} production is defined in the center-of-mass (c.m.) frame, while the subsequent partonic decay and its kinematics are described in the χ_{b2} rest frame. To characterize

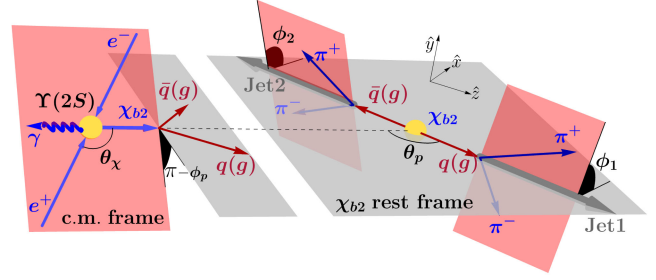


FIG. 1. Leading order kinematic configuration for the production of $\pi^+\pi^-$ -dihadron pairs in χ_{b2} decay at lepton colliders. The χ_{b2} production is defined in the c.m. frame, while the partonic subprocess is described in the χ_{b2} rest frame.

the dihadron kinematics, we construct a local coordinate system where the $\hat{z}$ axis is chosen along the thrust direction. The thrust axis, reconstructed from the final-state hadrons, approximates the parton direction. Each $\pi^+\pi^-$ pair is characterized by the total momentum $P_i = p_i^{\pi^+} + p_i^{\pi^-}$ and the relative momentum $R_i = (p_i^{\pi^+} - p_i^{\pi^-})/2$, with $i = 1, 2$. Experimentally, the two $\pi^+\pi^-$ pairs can be selected from opposite hemispheres in large-thrust events, thereby enhancing the back-to-back two-jet topology in the χ_{b2} rest frame and suppressing multijet contamination [16]. Within the collinear factorization framework [17,18] and in the limit $|P_i| \gg M_i$, the differential cross section for the $\pi^+\pi^-$ -dihadron pair final states can be expressed in terms of the invariant masses $M_i^2 \equiv P_i^2$, the azimuthal angles ϕ_i of the transverse component $\mathbf{R}_{i,T}$, and the kinematic variables $z_i = 2P_i \cdot p_X / m_X^2$, where p_X and m_X denote the momentum and mass of χ_{b2} , respectively.

Within this framework, the differential distribution of $\pi^+\pi^-$ pairs from χ_{b2} decay is obtained by combining the radiative production $\Upsilon(2S) \rightarrow \gamma\chi_{b2}$ in the c.m. frame with the partonic decay $\chi_{b2} \rightarrow q\bar{q}$ or gg in the χ_{b2} rest frame,

$$\begin{aligned} & \frac{d\sigma}{\sigma_0 dz_1 dz_2 dM_1 dM_2 d\phi_1 d\phi_2 d\cos\theta_\chi d\cos\theta_p d\phi_p} \\ &= H_8^b \sum_q \mathcal{C}_q [D_1^q(z_1, M_1) D_1^{\bar{q}}(z_2, M_2) \\ &+ \frac{1}{2} B H_1^{\leftarrow, q}(z_1, M_1) H_1^{\leftarrow, \bar{q}}(z_2, M_2) \cos(\phi_1 + \phi_2)] \\ &+ H_1^b \mathcal{C}_g D_1^g(z_1, M_1) D_1^g(z_2, M_2), \end{aligned} \quad (1)$$

where σ_0 denotes the production cross section of χ_{b2} . The angle θ_χ is the polar angle between the incoming electron and the produced χ_{b2} in the c.m. frame. The angles θ_p and ϕ_p are the polar and azimuthal angles of the partons in the χ_{b2} rest frame, defined relative to the χ_{b2} momentum direction in the c.m. frame (see Fig. 1). The hard coefficients $\mathcal{C}_{q/g}$ are determined by the unpolarized decay rates for $\chi_{b2} \rightarrow q\bar{q}/gg$, respectively, in which the LDMEs $H_{1,8}^b$

are factorized out and the NRQCD factorization scale is chosen to be m_b .

The hadronization process is described by the dihadron fragmentation functions (DiFFs), which encode the probability for a parton to fragment into a specific dihadron final state [13,19–34]. In Eq. (1), D_1^q and $H_1^{\leftarrow,q}$ denote the unpolarized and interference DiFFs for quarks, respectively, while D_1^g is the unpolarized gluon DiFF. We adopt the definitions and normalization convention used in the Jefferson Lab Angular Momentum (JAM) Collaboration analyses [28–30]. The functions $D_1^{q,g}$ determine the overall production rates, whereas the chiral-odd $H_1^{\leftarrow,q}$ probes transverse quark polarization. All DiFFs depend on the factorization scale μ , which we take as $\mu = m_\chi$, and the scale dependence is suppressed here for simplicity. Owing to this chiral-odd structure, DiFFs provide a clean probe of transverse spin dynamics of quarks and offer a sensitive avenue to search for potential new physics [25,35–40]. The coefficient $\mathcal{B} = C_{xx} - C_{yy}$ corresponds to the Bell variable, constructed from the quark pair spin-correlation matrix elements C_{xx} and C_{yy} [38].

Belle is an energy-asymmetric e^+e^- collider, so the produced $\Upsilon(2S)$ carries a longitudinal boost along the beam axis in the laboratory frame with velocity

$\beta = 0.3 \sim 0.4$, depending on the beam energies at Belle or Belle II [41,42]. The transformation of the χ_{b2} kinematics from the c.m. frame to the laboratory frame is therefore described by a Lorentz boost $\Lambda(\beta)$. Under this boost, the helicity state of the χ_{b2} undergoes a Wigner rotation associated with its little group, corresponding to a rotation of θ_χ by an angle $\theta_\beta \in [0, \pi]$, given by [43]

$$\cos \theta_\beta = \frac{v_\chi + \beta \cos \theta_\chi}{\sqrt{(1 + \beta v_\chi \cos \theta_\chi)^2 - (1 - \beta^2)(1 - v_\chi^2)}}, \quad (2)$$

where v_χ denotes the speed of χ_{b2} in the c.m. frame. Since the radiative transition $\Upsilon(2S) \rightarrow \gamma \chi_{b2}$ involves only a small recoil momentum, the χ_{b2} is nonrelativistic in the c.m. frame and satisfies $v_\chi \ll 1$. In this limit, the dependence of the θ_β on the velocity β cancels, and the Wigner rotation angle reduces to $\theta_\beta \simeq \theta_\chi$, so the Lorentz boost aligns the momentum of the χ_{b2} nearly collinearly with that of the parent $\Upsilon(2S)$ in the laboratory frame. As a consequence, the differential distribution in Eq. (1) becomes independent of θ_χ and ϕ_p in the laboratory frame. The Artru-Collins asymmetry can then be extracted experimentally from the average value of $\cos(\phi_1 + \phi_2)$,

$$A_{12} \equiv 2\langle \cos(\phi_1 + \phi_2) \rangle = \frac{1}{2} \frac{\rho_8(m_b) \mathcal{B} \sum_q C_q H_1^{\leftarrow,q}(z_1, M_1) H_1^{\leftarrow,\bar{q}}(z_2, M_2)}{\rho_8(m_b) \sum_q C_q D_1^q(z_1, M_1) D_1^{\bar{q}}(z_2, M_2) + m_b^2 C_g D_1^g(z_1, M_1) D_1^g(z_2, M_2)}. \quad (3)$$

At LO, the coefficients $\mathcal{B}$ and $C_{q/g}$ in the laboratory frame are given by

$$\begin{aligned} \mathcal{B} &= \frac{42 \sin^2 \theta_p}{21 \cos^2 \theta_p + 73}, \\ C_q &= \frac{\alpha_s^2}{480 m_\chi^2 \Gamma_\chi} (21 \cos^2 \theta_p + 73), \\ C_g &= \frac{8 \alpha_s^2}{15 m_\chi^4 \Gamma_\chi} (\cos^2 \theta_p + 1), \end{aligned} \quad (4)$$

where Γ_χ denotes the total width of the χ_{b2} . The $\sin \theta_p$ dependence of $\mathcal{B}$ maximizes the asymmetry in the central region, thereby enhancing the observable spin effect. By contrast, in the c.m. frame the explicit dependence of these coefficients on θ_χ and ϕ_p causes cancellations over phase space, suppressing the integrated signal. The asymmetric beam energies at Belle remove this angular dependence in the laboratory frame, allowing the asymmetry to survive integration and providing a clear advantage over energy-symmetric colliders.

Beyond the LO picture described above, higher-order QCD corrections modify the hard coefficients and induce

CS and CO mixing, thereby blurring the simple correspondence between the CS gg and CO $q\bar{q}$ channels. However, up to next-to-leading order (NLO) in QCD, because of the vanishing of the amplitude $b\bar{b}(^3S_1^{[8]}) \rightarrow gg$, the asymmetry still originates predominantly from the light-quark pair fragmentation in the large-thrust two-jet region, where the hard multijet radiation contribution is largely suppressed. By analyzing the Feynman diagrams at the one-loop level, it is straightforward to infer that the $q\bar{q}$ pair produced in the CS $b\bar{b}(^3P_2^{[1]}) \rightarrow q\bar{q}g$ channel has the same color-octet structure as that produced in the CO channel. We therefore expect the qualitative sensitivity of A_{12} to the CO contribution to remain stable. In such a case, the NRQCD factorization scale dependence of the SDCs for the CS $b\bar{b}(^3P_2^{[1]}) \rightarrow q\bar{q}g$ channel will be canceled by the CO LDME. Furthermore, the impact of NLO QCD corrections is expected to be reduced due to the partial cancellations in the normalized asymmetry, and a dedicated NLO calculation is needed for a precision extraction of $\rho_8(m_b)$.

Numerical results and discussion—With the method developed above, the Artru-Collins asymmetry in χ_{b2} decay, with the χ_{b2} produced in the laboratory frame, can be directly estimated. We evaluate this asymmetry

using the lattice central value $\rho_8(m_b) = 0.044$, defined at the scale $m_b \simeq 4.6$ GeV [8]. The statistical uncertainty of A_{12} in Eq. (3) can be estimated as

$$\delta A = \sqrt{\frac{2 - A_{12}^2}{N}} \simeq \sqrt{\frac{2}{N}}, \quad (5)$$

where N denotes the number of selected events after the kinematic cuts and the approximation holds for $A_{12} \ll 1$. To assess the statistical sensitivity to $\rho_8(m_b)$, the following projections include statistical uncertainties only, while the impact of the DiFF uncertainties is considered separately.

To quantify their impact on A_{12} , we use the JAM global-fit results [28–30]. The unpolarized quark DiFFs are typically constrained at the $\lesssim 5\%$ level, while the uncertainties of $H_1^{\text{q},q}$ are below approximately 20% over most of the relevant phase space. The dominant uncertainty arises from the unpolarized gluon DiFF D_1^q , whose current uncertainty is about 100%, mainly due to its indirect determination through QCD evolution [30,33]. Propagating the current DiFF uncertainties to A_{12} yields a relative uncertainty of about 50%–60%, whereas the quark DiFFs alone contribute 10%–20%, depending on the kinematics. An improved determination of D_1^q is therefore essential for a precision extraction of $\rho_8(m_b)$ and could benefit from dedicated gluon-sensitive dihadron measurements, such as χ_{b0} decays at Belle II [44]. With such improvements, the DiFF uncertainties are expected to become subleading relative to the projected statistical uncertainty.

The numerical results for A_{12} are shown in Fig. 2 as functions of z_1 and M_1 for two polar angle bins, $0.5 < |\cos \theta_p| < 1$ (blue) and $|\cos \theta_p| < 0.5$ (red), assuming an integrated luminosity of $\mathcal{L} = 1 \text{ ab}^{-1}$, with all other kinematical variables integrated over. This binning explicitly demonstrates the enhancement of the asymmetry in the central region. The dependence on z_1 and M_1 exhibits characteristic features consistent with existing observations of the Artru-Collins asymmetry in $\pi^+\pi^-$ -dihadron pair production at Belle [16,26,30]. The typical magnitude of the asymmetry reaches the percent level. Although this is smaller than that measured in the $\pi^+\pi^-$ -dihadron pair production at Belle [16] due to dilution from gluon channel, it remains well within the sensitivity of Belle. For comparison, the CELO extraction $\rho_8(m_b) = 0.160$ from measurements of $\chi_{bJ}(1P) \rightarrow D^0 X$ [9] enhances the relative CO $q\bar{q}$ contribution in Eq. (3), increasing the magnitude of A_{12} relative to the lattice benchmark while leaving the shapes of its z_1 and M_1 dependences essentially unchanged. In addition, the z_1 and M_1 dependences are not expected to exhibit the sharp endpoint enhancement discussed in Ref. [8] since the relevant partonic momentum fraction is integrated over in forming A_{12} . Any residual endpoint effect is part of the missing NLO corrections.

To quantify the sensitivity to $\rho_8(m_b)$, we perform a binned analysis in θ_p , exploiting the enhancement of the

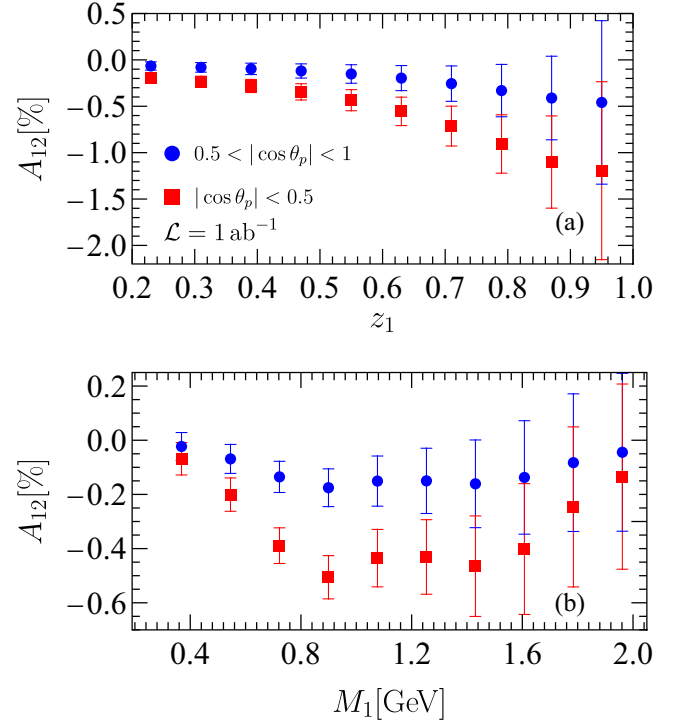


FIG. 2. Artru-Collins asymmetry A_{12} in χ_{b2} decay with the χ_{b2} produced in the laboratory frame at an integrated luminosity $\mathcal{L} = 1 \text{ ab}^{-1}$, shown as a function of z_1 (a) and M_1 (b) for two bins in $\cos \theta_p$, with all other kinematical variables integrated over.

asymmetry in the central angular region. Owing to the limited event statistics, the remaining kinematic variables are integrated over, with $z_i \in [0.19, 0.99]$ and $M_i \in [0.28, 2.05]$ GeV [30]. The projected constraints are obtained from a χ^2 analysis. The resulting 68% confidence level sensitivity to $\rho_8(m_b)$ in the laboratory frame as a function of integrated luminosity is shown by the blue band in Fig. 3, while the corresponding sensitivity in the c.m. frame is indicated by the orange band. In the c.m. frame, applying the same binning strategy as in the laboratory frame, namely retaining only θ_p and integrating over ϕ_p and θ_χ , causes the asymmetry to vanish due to complete phase space cancellations. We therefore bin instead in the production angle θ_χ for comparison. However, residual cancellations persist after integrating over θ_p and ϕ_p , substantially suppressing the asymmetry and yielding significantly poorer sensitivity than in the laboratory frame. For reference, the lattice result, $\rho_8(m_b) = 0.044 \pm 0.015$ [8], is shown as the brown band, while the CLEO extraction, $\rho_8(m_b) = 0.16^{+0.071}_{-0.047}$ [9], lies far above the lattice value and is therefore not displayed in the same figure. The projected precision in the laboratory frame surpasses the current lattice uncertainty with an integrated luminosity of $\mathcal{O}(0.1) \text{ ab}^{-1}$ and could reach the few-percent level at $\mathcal{L} \sim 10 \text{ ab}^{-1}$. This opens an opportunity to resolve the long-standing discrepancy in bottomonium between lattice

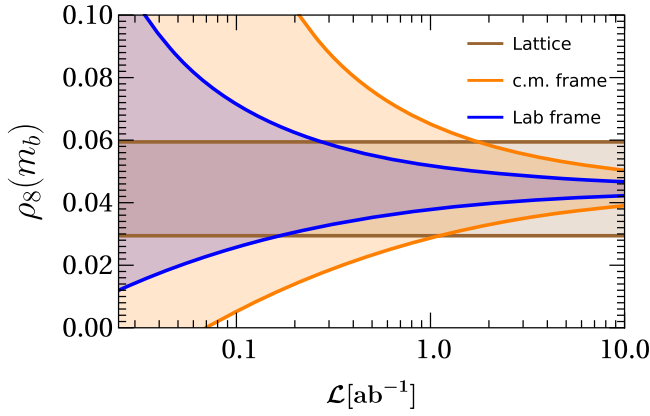


FIG. 3. Projected statistical sensitivity to $\rho_8(m_b)$ from the Artru-Collins asymmetry in χ_{b2} decay at Belle II as a function of the integrated luminosity, for χ_{b2} produced in the laboratory frame (blue band) and in the c.m. frame (orange band).

NRQCD calculations and phenomenological extractions of $\rho_8(m_b)$. A similar analysis can also be performed for the radiative transitions $\Upsilon(3S) \rightarrow \gamma\chi_{b2}(2P)$, which provides a nontrivial cross-check in view of the expected weak dependence of $\rho_8(m_b)$ on the radial quantum number [10] and will be investigated in future work.

Conclusion—In this Letter, we have proposed a novel and clean observable to directly probe the color-octet mechanism in NRQCD through the Artru-Collins asymmetry in hadronic decays of the P -wave bottomonium state χ_{b2} . From NRQCD power counting, P -wave quarkonia provide an ideal laboratory for this study since the color-singlet and color-octet contributions enter the decay rates at the same order, whereas, at LO, the spin-dependent asymmetry arises exclusively from the color-octet channel. This distinctive feature makes the asymmetry a direct and unambiguous signal of color-octet dynamics. A central result of our analysis is that the asymmetric beam energies at Belle preserve nontrivial transverse spin azimuthal correlations between the quark and antiquark in χ_{b2} decay in the laboratory frame, thereby avoiding the strong phase space cancellations that suppress the effect in the center-of-mass frame. By employing DiFFs from global fits, we have demonstrated that this observable enables a precise determination of the ratio of NRQCD long-distance matrix elements, $\rho_8(m_b)$. The projected statistical sensitivity surpasses current lattice QCD uncertainty with an integrated luminosity of $\mathcal{O}(0.1)$ ab^{-1} and could reach the few-percent level at Belle II, thereby providing an opportunity to resolve this long-standing discrepancy. Our results establish spin correlation in P -wave quarkonium decays as a powerful and complementary framework for tests of NRQCD.

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Data availability—The data that support the findings of this article are openly available [30].

- [1] G. P. Lepage, L. Magnea, C. Nakhleh, U. Magnea, and K. Hornbostel, *Phys. Rev. D* **46**, 4052 (1992).
- [2] G. T. Bodwin, E. Braaten, and G. P. Lepage, *Phys. Rev. D* **51**, 1125 (1995); **55**, 5853(E) (1997).
- [3] G. T. Bodwin, E. Braaten, and G. P. Lepage, *Phys. Rev. D* **46**, R1914 (1992).
- [4] G. T. Bodwin, D. K. Sinclair, and S. Kim, *Phys. Rev. D* **65**, 054504 (2002).
- [5] G. T. Bodwin, D. K. Sinclair, and S. Kim, *Phys. Rev. Lett.* **77**, 2376 (1996).
- [6] F. Maltoni, in *5th Workshop on QCD (QCD 2000)* (2000), pp. 301–314, arXiv:hep-ph/0007003.
- [7] G. T. Bodwin, *Int. J. Mod. Phys. A* **21**, 785 (2006).
- [8] G. T. Bodwin, E. Braaten, D. Kang, and J. Lee, *Phys. Rev. D* **76**, 054001 (2007).
- [9] R. A. Briere *et al.* (CLEO Collaboration), *Phys. Rev. D* **78**, 092007 (2008).
- [10] N. Brambilla, D. Eiras, A. Pineda, J. Soto, and A. Vairo, *Phys. Rev. Lett.* **88**, 012003 (2002).
- [11] N. Brambilla, H. S. Chung, D. Müller, and A. Vairo, *J. High Energy Phys.* **04** (2020) 095.
- [12] H. S. Chung, *Phys. Rev. D* **109**, 054001 (2024).
- [13] X. Artru and J. C. Collins, *Z. Phys. C* **69**, 277 (1996).
- [14] E. Eichten, S. Godfrey, H. Mahlke, and J. L. Rosner, *Rev. Mod. Phys.* **80**, 1161 (2008).
- [15] S. Navas *et al.* (Particle Data Group), *Phys. Rev. D* **110**, 030001 (2024).
- [16] A. Vossen *et al.* (Belle Collaboration), *Phys. Rev. Lett.* **107**, 072004 (2011).
- [17] J. C. Collins, S. F. Heppelmann, and G. A. Ladinsky, *Nucl. Phys. B* **420**, 565 (1994).
- [18] J. Collins, *Foundations of Perturbative QCD* (Cambridge University Press, Cambridge, England, 2011), Vol. 32.
- [19] R. L. Jaffe, X.-m. Jin, and J. Tang, *Phys. Rev. Lett.* **80**, 1166 (1998).
- [20] R. L. Jaffe, X.-m. Jin, and J.-a. Tang, *Phys. Rev. D* **57**, 5920 (1998).
- [21] A. Bianconi, S. Boffi, R. Jakob, and M. Radici, *Phys. Rev. D* **62**, 034008 (2000).
- [22] A. Bianconi, S. Boffi, R. Jakob, and M. Radici, *Phys. Rev. D* **62**, 034009 (2000).
- [23] A. Bacchetta and M. Radici, *Phys. Rev. D* **69**, 074026 (2004).
- [24] A. Bacchetta, F. A. Ceccopieri, A. Mukherjee, and M. Radici, *Phys. Rev. D* **79**, 034029 (2009).

- [25] J. Zhou and A. Metz, *Phys. Rev. Lett.* **106**, 172001 (2011).
- [26] A. Courtoy, A. Bacchetta, M. Radici, and A. Bianconi, *Phys. Rev. D* **85**, 114023 (2012).
- [27] M. Radici, A. Courtoy, A. Bacchetta, and M. Guagnelli, *J. High Energy Phys.* **05** (2015) 123.
- [28] D. Pitonyak, C. Cocuzza, A. Metz, A. Prokudin, and N. Sato, *Phys. Rev. Lett.* **132**, 011902 (2024).
- [29] C. Cocuzza, A. Metz, D. Pitonyak, A. Prokudin, N. Sato, and R. Seidl (JAM Collaboration), *Phys. Rev. Lett.* **132**, 091901 (2024).
- [30] C. Cocuzza, A. Metz, D. Pitonyak, A. Prokudin, N. Sato, and R. Seidl [Jefferson Lab Angular Momentum (JAM) Collaboration], *Phys. Rev. D* **109**, 034024 (2024).
- [31] T. C. Rogers, M. Radici, A. Courtoy, and T. Rainaldi, *Phys. Rev. D* **111**, 056001 (2025).
- [32] D. Pitonyak, C. Cocuzza, A. Metz, A. Prokudin, and N. Sato, *Phys. Rev. D* **113**, 038901 (2026).
- [33] V. Mahaut, L. Polano, A. Bacchetta, V. Bertone, M. Cerutti, M. Radici, and L. Rossi (MAP Collaboration), *J. High Energy Phys.* **02** (2026) 051.
- [34] T. C. Rogers, T. Rainaldi, M. Radici, and A. Courtoy, *Phys. Rev. D* **113**, 038902 (2026).
- [35] L. Yang, Y.-K. Song, and S.-Y. Wei, *Phys. Rev. D* **111**, 054035 (2025).
- [36] X.-K. Wen, B. Yan, Z. Yu, and C. P. Yuan, *Sci. China Phys. Mech. Astron.* **69**, 271011 (2026).
- [37] X.-K. Wen, B. Yan, Z. Yu, and C. P. Yuan, *Phys. Rev. D* **112**, 053004 (2025).
- [38] K. Cheng and B. Yan, *Phys. Rev. Lett.* **135**, 011902 (2025).
- [39] Q.-H. Cao, G. Li, X.-K. Wen, and B. Yan, *Rep. Prog. Phys.* **89**, 080501 (2026).
- [40] Q.-H. Cao, X.-K. Wen, B. Yan, and S.-T. Zhang, *Phys. Rev. Lett.* **136**, 231901 (2026).
- [41] T. Aushev *et al.*, [arXiv:1002.5012](https://arxiv.org/abs/1002.5012).
- [42] W. Altmannshofer *et al.* (Belle-II Collaboration), *Prog. Theor. Exp. Phys.* **2019**, 123C01 (2019); **2020**, 029201 (E) (2020).
- [43] Z. Yu and C. P. Yuan, *Phys. Rev. Lett.* **129**, 112001 (2022).
- [44] Z.-G. He, G. Li, Y. Liu, Y.-J. Tian, X.-K. Wen, and B. Yan, [arXiv:2608.10570](https://arxiv.org/abs/2608.10570).