

Holographic Origin of a Maximization and Higher-Derivative $\text{AdS}_5/\text{CFT}_4$

Kiril Hristov^{1,2}, Saurish Khandelwal³, Yi Pang^{4,5}, and Gabriele Tartaglino-Mazzucchelli⁶
¹Faculty of Physics, Sofia University “St. Kliment Ohridski,” James Bourchier Boulevard 5, 1164 Sofia, Bulgaria
²INRNE, Bulgarian Academy of Sciences, Tsarigradsko Chaussee 72, 1784 Sofia, Bulgaria
³Institut für Theoretische Physik Leibniz Universität Hannover, Appelstraße 2, 30167 Hannover, Germany
⁴Center for Joint Quantum Studies and Department of Physics,
School of Science, Tianjin University, 135 Yaguan Road, Tianjin 300350, China
⁵Peng Huanwu Center for Fundamental Theory, Hefei, Anhui 230026, China
⁶School of Mathematics and Physics, University of Queensland, St Lucia, Brisbane, Queensland 4072, Australia

 (Received 13 December 2025; revised 21 April 2026; accepted 29 June 2026; published 18 August 2026)

We develop a consistent partially off-shell framework for evaluating higher-derivative actions of five-dimensional $\mathcal{N} = 1$ gauged supergravity with Abelian vector multiplets on AdS_5 . Using the superconformal formalism, we show that the resulting holographic expression reproduces the trial a -anomaly coefficient of the dual conformal field theory, identifying the supergravity equations of motion with a maximization. We present an exact correspondence between Chern-Simons couplings and the anomaly polynomial of the boundary theory. We illustrate our proposal by applying it to all known two- and four-derivative actions, including the “off-diagonal” invariants never before considered in the gauged supergravity literature. Finally, we argue that all invariants beyond four derivative yield no genuinely new contributions to asymptotically AdS_5 BPS backgrounds, but instead reduce to linear combinations of the established two- and four-derivative actions.

DOI: [10.1103/nrlq-ktd1](https://doi.org/10.1103/nrlq-ktd1)

Introduction—The AdS/CFT correspondence is one of the major developments in modern theoretical physics, providing a nonperturbative definition of certain quantum gravity theories in terms of conformal field theories and yielding powerful insights into black-hole physics, strongly coupled gauge dynamics, and even condensed-matter systems. Most of our understanding of AdS/CFT relies on the leading large- N and two-derivative supergravity approximations, where genuine stringy and quantum-gravity effects are suppressed. Recent progress in exact $\text{AdS}_5/\text{CFT}_4$ [1–10] has highlighted the need for results beyond these approximations. On the CFT side, major advances have been made by using supersymmetric localization; see Refs. [11,12]. On the gravity side, by contrast, a systematic understanding of higher-derivative (HD) corrections is still lacking. This is essential for performing new precision tests in holography and for probing the structure of quantum gravity.

In this Letter, we initiate a systematic study of HD corrections in matter-coupled gauged five-dimensional supergravity relevant to holography. The superconformal (or off shell) formalism [13] provides a convenient

framework for constructing supersymmetry-invariant terms with various derivative orders, though distinct invariants typically involve different auxiliary fields and may yield linearly dependent on-shell actions. Building on advances in superconformal tensor calculus and superspace techniques—see, e.g., Refs. [7,14–34] for the 5D case as well as the reviews [35–39] and references therein—we organize all invariants according to their *on-shell* Chern-Simons (CS) terms. These CS couplings are dual to the ’t Hooft anomalies in the 4D superconformal field theory (SCFT) [40–43], with the structure of the CS terms holographically encoding the anomaly coefficients. This underpins the perturbative behavior of all supersymmetric partition functions of the dual theory.

Concretely, after integrating out the needed auxiliary fields [44], only two types of CS terms appear in the supergravity Lagrangian

$$\mathcal{L}_{\text{CS}} = \frac{1}{24}(k_{IJK}v^I \wedge dv^J \wedge dv^K - 3k_I v^I \wedge \mathcal{R}_{ab} \wedge \mathcal{R}^{ab}), \quad (1)$$

where v^I are the Abelian gauge fields in the vector multiplets, and $\mathcal{R}_{ab}$ is the Riemann curvature two-form. The coefficients k_{IJK} and k_I will be directly related to the cubic and linear ’t Hooft anomalies of the dual SCFT, respectively. These on-shell CS terms, involving only physical fields, can originate from different off-shell

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article’s title, journal citation, and DOI. Funded by SCOAP³.

invariants in 5D gauged supergravity, where we have integrated out only a small subset of all auxiliary fields and kept the rest of the action off shell. Thus our partially off-shell approach circumvents the complexity of solving all auxiliary equations of motion while retaining direct holographic relevance.

Guided by the CS-term classification, we consider the AdS_5 background with radius l , written in global coordinates with $\mathbb{R} \times S^3$ boundary. The conditions imposed by maximal supersymmetry uniquely determine all background fields except for the vector-multiplet scalars W^I , which remain unspecified constants. Evaluating the supergravity action on this background using the minimal holographic renormalization scheme—without imposing the remaining scalar equations of motion—yields

$$I_{\text{AdS}_5}^{\text{off shell}}(W) = \frac{\pi^2 \beta}{4} (l^2 k_{IJK} W^I W^J W^K - 18 k_I W^I), \quad (2)$$

where the (Wick-rotated) time direction is periodically identified with period β . The resulting expression depends only on the effective CS coefficients, not on the detailed couplings of the underlying off-shell invariants. The fully on-shell AdS_5 action follows from extremizing (2) under the supersymmetric constraint $l g_I W^I = \frac{3}{2}$,

$$\left. \frac{\partial I_{\text{AdS}_5}^{\text{off shell}}}{\partial W^I} \right|_{W=\bar{W}} = 0, \quad I_{\text{AdS}_5}^{\text{on shell}} := I_{\text{AdS}_5}^{\text{off shell}}(\bar{W}), \quad (3)$$

where $\bar{W}$ is the extremized value of W . This procedure is the gravitational analog of a maximization [45–47], which in field theory determines the exact superconformal R symmetry by maximizing the trial a anomaly over all possible R -symmetry mixings. This principle is crucial in distinguishing genuine SCFTs from generic supersymmetric field theories, where the R symmetry is not fixed dynamically. In this holographic context, we thus interpret the off-shell AdS_5 action as the trial a anomaly, while the scalar equations of motion play the role of the a -maximization conditions. Moreover, the constraint appears as the field equation of a certain auxiliary field. Implementing a maximization using off-shell supergravity allows us to go beyond the $N \rightarrow \infty$ limit, thus differing from the literature where discussions were mainly based on the on-shell two-derivative theory [48]. We believe that the present Letter can serve as a blueprint for evaluating arbitrary supersymmetric backgrounds in higher-derivative supergravity and, in this way, obtaining holographic predictions at finite N .

4D anomaly polynomial and (holographic) a maximization—To set the stage for our supergravity analysis, we briefly review key aspects of the holographically dual field theory, following the conventions of [9]. We consider a 4D $\mathcal{N} = 1$ SCFT with $(n+1)$ global $\text{U}(1)$ currents generated by Q_I . Without imposing any

normalization, their commutation relations with a supercharge $\mathcal{Q}$ are [53]

$$[Q_I, \mathcal{Q}] = -r_I \mathcal{Q}, \quad (4)$$

where r_I are rational constants. This is not the standard superconformal form, where one distinguishes the $\text{U}(1)_R$ generator R from the remaining flavor symmetries F_i ,

$$[R, \mathcal{Q}] = -\mathcal{Q}, \quad [F_i, \mathcal{Q}] = 0, \quad (5)$$

with $\mathcal{Q}$ normalized conventionally. The generators R and F_i are therefore specific linear combinations of the original Q_I . We parametrize the R symmetry through a *trial* combination

$$R^{\text{tr}}(s) := s^I Q_I, \quad r_I s^I = 1, \quad (6)$$

with the constraint on the s^I following from (5).

We define the cubic and linear 't Hooft anomaly coefficients for the global charges Q_I as

$$\mathcal{A}_{IJK} := \text{Tr}(Q_I Q_J Q_K), \quad \mathcal{A}_I := \text{Tr} Q_I, \quad (7)$$

which for many holographic theories of D3 branes and wrapped M5 branes are known *exactly* in the gauge group rank N , see e.g., Refs. [54,55]. From these, one constructs the anomaly polynomial

$$\begin{aligned} \mathcal{A}_{4\text{D}} &= \frac{\mathcal{A}_{IJK}}{6} c_1(\mathfrak{f}^I) \wedge c_1(\mathfrak{f}^J) \wedge c_1(\mathfrak{f}^K) \\ &\quad - \frac{\mathcal{A}_I}{24} c_1(\mathfrak{f}^I) \wedge p_1(T\mathcal{Y}_6), \end{aligned} \quad (8)$$

which encodes the variation of the partition function under background gauge transformations of the global symmetries, with field strengths $\mathfrak{f}^I := d\mathbf{a}^I$. Here, $\mathcal{A}_{4\text{D}}$ is a six form defined on an extension $\mathcal{Y}_6$ of the four-dimensional manifold $\mathcal{M}_4 = \partial\text{AdS}_5$. In local coordinates,

$$c_1(\mathfrak{f}^I) = \frac{\mathfrak{f}^I}{2\pi}, \quad p_1(T\mathcal{Y}_6) = \frac{1}{8\pi^2} \mathcal{R}_{ab} \wedge \mathcal{R}^{ab}, \quad (9)$$

where $\mathcal{R}_{ab}$ is the Riemann curvature two form on $\mathcal{Y}_6$.

When the R symmetry is not manifest, it can be uniquely determined by maximizing the *trial* a -anomaly coefficient,

$$a^{\text{tr}}(s) := \frac{3}{32} (3\mathcal{A}_{IJK} s^I s^J s^K - \mathcal{A}_I s^I), \quad (10)$$

whose extremization, subject to (6), yields

$$\left. \frac{\partial a^{\text{tr}}}{\partial s^I} \right|_{s=s} = 0, \quad a = a^{\text{tr}}(s), \quad R = R^{\text{tr}}(s). \quad (11)$$

This procedure, known as a maximization [45–47], is the exact field-theoretic counterpart of the gravitational

extremization principle (3). The correspondence follows from the holographic relation at the conformal boundary,

$$d\mathcal{L}_{\text{CS}}(v) = \mathcal{A}_{4\text{D}}(\mathbf{a}), \quad (12)$$

where we have implicitly extended the left-hand side on $\mathcal{Y}_6$. The holographic dictionary dictates proportionality between the bulk gauge fields and boundary global symmetries, $l^{-1}v^I = \mathbf{a}^I$, as well as $lg_I = r_I$, see Refs. [9,56,57], with the latter equality fixed by matching with the R -symmetry charge of the bulk Killing spinor (see next section). This yields the holographic identification between the CS and anomaly coefficients,

$$k_{IJK} = \frac{1}{2\pi^3 l^3} \mathcal{A}_{IJK}, \quad k_I = \frac{1}{48\pi^3 l} \mathcal{A}_I. \quad (13)$$

It was also shown in [9], to first order in the four-derivative expansion, that the on-shell action of global AdS₅ satisfies

$$I_{\text{AdS}_5}^{\text{on shell}} = \frac{3\beta}{2\pi l} a, \quad (14)$$

where l is the AdS radius and β the Euclidean time period, using minimal holographic renormalization. Our analysis extends this result to the exact *trial* a ,

$$I_{\text{AdS}_5}^{\text{off shell}}(W) = \frac{3\beta}{2\pi l} a^{\text{tr}}(s), \quad (15)$$

where the parameters s^I are proportional to the vacuum expectation values of the vector multiplet scalars W^I in the AdS background, $2W^I = 3s^I$, such that the gravitational constraint $lg_I W^I = \frac{3}{2}$ maps to (6).

Multiplets and invariants—We now introduce 5D $\mathcal{N} = 1$ superconformal invariants, focusing on a novel class of actions, termed “off diagonal” in [26]. To keep the discussion concise, we present only the primary fields of the composite multiplets that describe these invariants. The corresponding descendant components, along with their full superspace formulation, will appear in [58].

Fundamental multiplets: In five dimensional off shell supergravity, the standard Weyl multiplet [18] gauges the superconformal algebra $F^2(4)$ and contains 32 + 32 off-shell components. The independent gauge fields are the vielbein e_μ^a , gravitino $\psi_{\mu\alpha}^i$, $SU(2)_R$ gauge field ϕ_μ^{ij} , and dilatation gauge field b_μ . The spin connection ω_μ^{ab} , S -supersymmetry connection $\phi_{\mu\alpha}^i$, and special conformal connection $\mathfrak{f}_\mu^a$ are composite, determined by standard curvature constraints. The multiplet further includes the matter fields: a real antisymmetric tensor W_{ab} , a fermion χ_α^i , and a real auxiliary scalar D .

An Abelian vector multiplet, denoted henceforth by W , consists of a scalar field W [59], a gaugino λ_α^i , a gauge vector v_a with covariant field strength F_{ab} , and a $SU(2)$

TABLE I. The contribution to the Chern-Simons coefficients in (1) from different off-shell invariants. $C_{IJK}, g_I, \alpha_I, \beta_I, \gamma_I, \eta, a^I, d_{IJK}$ denote the various coupling constants of the corresponding supersymmetric actions, discussed in due course. The g_I 's play the role of the Fayet-Iliopoulos parameters that decide which linear combination of Abelian vectors is identified as $U(1)_R$ symmetry.

Invariant	k_{IJK}	k_I	References
2∂ theory	$3C_{IJK}$	0	[14–22,25,61]
$\mathcal{R}F$	$-12g_{(I}d_{JK)L}a^L$	0	[26,58]
Weyl ²	$8\alpha_{(I}g_Jg_K)$	α_I	[7,23]
log	$-4\beta_{(I}g_Jg_K)$	0	[7,28]
$\mathcal{R}^2$	$12\gamma_{(I}g_Jg_K)$	0	[24,28]
$\mathcal{R}F^3$	$-12\eta g_{(I}g_Jg_K)$	0	[26,58]
F^4	0	0	[26,58]

triplet of auxiliary fields X^{ij} . The linear multiplet, denoted by G , contains an $SU(2)$ triplet of scalars G^{ij} , a fermion φ_α^i , a vector H^a , Hodge dual of a covariant four-form field strength, and an auxiliary scalar F .

The BF action: In the superconformal tensor calculus, the BF action principle serves as a cornerstone for constructing general supergravity-matter couplings (see Refs. [16–20,22,25] for the 5D case). This relies on forming a suitable product between a linear multiplet (G) and an Abelian vector multiplet (W), $S_{\text{BF}} := \int d^5x e \mathcal{L}_{\text{BF}}$,

$$\mathcal{L}_{\text{BF}}(W, G) := -(WF + X_{ij}G^{ij} + v_a H^a), \quad (16)$$

where we neglected the fermions.

Given $(n+1)$ Abelian vector multiplets W^I , one can construct $(n+1)$ composite linear multiplets $\underline{G}_I$ [25] whose lowest components are

$$\underline{G}_I^{ij} = C_{IJK}(2W^J X^{iK} - i\lambda^{aJ}(\lambda_\alpha^i)^K), \quad (17)$$

with symmetric constants $C_{IJK} = C_{(IJK)}$. The remaining components of the multiplet, $\underline{\varphi}_I^{ai}$, $\underline{H}_I^a$, and $\underline{F}_I$, are obtained by taking successive Q -supersymmetry variations of $\underline{G}_I^{ij}$. Conversely, given a linear multiplet, one can construct a composite vector multiplet with the scalar field

$$\underline{W} = \frac{1}{4}FG^{-1} - \frac{i}{8}G_{ij}\varphi^{ia}\varphi_a^j G^{-3}, \quad (18)$$

with $G := \sqrt{\frac{1}{2}G^{ij}G_{ij}} \neq 0$. The remaining components of the multiplet, $\underline{\lambda}_\alpha^i$, $\underline{F}_{ab}$, and $\underline{X}^{ij}$, are obtained by taking successive Q -supersymmetry variations of $\underline{W}$. The explicit construction of the two-derivative supergravity theory, including the substitution of the composite multiplets into the BF action is presented in Sec. A of Supplemental

Material [60]. Here, we simply note that, as a result of substituting the composite multiplets (17) into the BF action (16), the coefficient of the Chern-Simons term is found to be proportional to C_{IJK} of Eq. (17).

All off-shell superconformal invariants considered in our analysis—including the two- and four-derivative terms—are listed in Table I together with their associated references. For the purposes of this Letter, we briefly outline the construction of the curvature-squared and off-diagonal invariants coupled to $(n+1)$ vector multiplets.

The three curvature-square invariants: Here, we present the primary fields corresponding to the composite multiplets that generate the three curvature-squared invariants—namely, the Weyl-squared, Ricci-squared, and scalar curvature-squared terms:

$$\begin{aligned} G_{\text{Weyl}^2}^{ij} &:= -\frac{i}{2} W^{\alpha\beta\gamma i} W_{\alpha\beta\gamma}^j - W^{ab} \phi_{ab}^{ij} + \frac{256i}{3} \chi^{ai} \chi_{\alpha}^j, \\ G_{\log}^{ij} &:= -\frac{3i}{40} \Delta^{ijkl} \nabla_{kl} \log(\sigma_I W^I), \\ G_{\mathcal{R}^2}^{ij} &:= \underline{G}^{ij}[\underline{W}] = 2\underline{W} \underline{X}^{ij} - i \underline{\lambda}^{(i} \underline{\lambda}_{\alpha}^{j)}, \end{aligned} \quad (19)$$

and corresponding actions $\mathcal{L}_{\text{BF}}(\alpha_I W^I, G_{\text{Weyl}^2})$, $\mathcal{L}_{\text{BF}}(\beta_I W^I, G_{\log})$, $\mathcal{L}_{\text{BF}}(\gamma_I W^I, G_{\mathcal{R}^2})$, respectively. Note that the definition of the matter-coupled log invariant, for which we refer the reader to [25,28] for its technical construction, allows for additional arbitrary coefficients σ_I in the action, which drop out of the final results. The explicit off-shell Chern-Simons terms corresponding to the four-derivative invariants are listed in Sec. B of Supplemental Material [60].

Off-diagonal invariants: We now turn to a wider class of *off-diagonal* invariants, obtained by coupling the composite vector multiplets $\underline{W}$ and $W_I^{F^4}$ (see below) to the composite linear multiplets $\underline{G}_I$, $G_{\text{Weyl}^2}^2$, $G_{\log}$, and $G_{\mathcal{R}^2}$. These interactions generate terms beyond the curvature-squared invariants associated with (19). While a subset was identified in [26], here we provide a classification of such invariants relevant to higher-derivative deformations in supergravity. The number of derivatives that these actions carry off shell is most directly seen by examining the BF action (16), together with Table II, which summarizes the composite-multiplet components.

A two-derivative [62] supersymmetric completion of the $\mathcal{R}F$ term arises from coupling the composite $\underline{W}$ to the linear multiplet $\underline{G}_I$ within the BF action (16),

$$\mathcal{L}_{\mathcal{R}F} := \mathcal{L}_{\text{BF}}(\underline{W}, a^I \underline{G}_I), \quad (20)$$

where, in (17), C_{IJK} are replaced by d_{IJK} (another set of arbitrary coefficients).

A four-derivative supersymmetric completion of the $\mathcal{R}F^3$ term arises by coupling the composite vector $\underline{W}$ and linear $G_{\mathcal{R}^2}$ multiplets within the BF action (16),

TABLE II. The derivative structure (over fundamental fields) of the bosonic components of the composite linear and vector multiplets used here. Combining each row on the left with a row on the right in a BF action produces a different off-diagonal invariant.

Linear	G^{ij}	H^a	F
$\underline{G}_I$	0∂	2∂	2∂
Weyl^2	2∂	4∂	4∂
$\log$	2∂	4∂	4∂
$\mathcal{R}^2$	2∂	4∂	4∂
Vector	W	v_a	X^{ij}
$\underline{W}$	0∂	0∂	2∂
F^4	2∂	2∂	4∂

$$\mathcal{L}_{\mathcal{R}F^3} := \eta \mathcal{L}_{\text{BF}}(\underline{W}, G_{\mathcal{R}^2}). \quad (21)$$

In analogy to these constructions, we are able to generate a couple of new superconformal invariants, replacing $G_{\mathcal{R}^2}$ above by $G_{\log}$ or G_{Weyl^2} , which will be discussed at length in [58]. Here, we finish with a supersymmetric completion of the F^4 term arising from a composite vector multiplet with the primary scalar field [26],

$$\begin{aligned} W_I^{F^4} &:= \frac{1}{4G} E_I - \frac{1}{8G^3} \left(G_{ij} \underline{G}_I^{ij} F + 2i G_{ij} \varphi^{ai} \underline{\varphi}_{\alpha}^j \right. \\ &\quad \left. + i \underline{G}_I^{ij} \varphi_i^{\alpha} \varphi_{\alpha j} \right) + \frac{3i}{16G^5} G_{ij} G_{kl} \underline{G}_I^{kl} \varphi^{ai} \varphi_{\alpha}^j, \end{aligned} \quad (22)$$

coupled to another set of composites $\underline{G}_I$,

$$\mathcal{L}_{F^4} := a^{IJ} \mathcal{L}_{\text{BF}}(W_I^{F^4}, \underline{G}_J), \quad (23)$$

resulting in a four-derivative invariant. Combining the F^4 multiplet with the other linear multiplets results in new invariants, also to be discussed in [58].

On-shell Chern-Simons terms: The explicit off-shell Chern-Simons terms for the four-derivative invariants are collected in Sec. B of Supplemental Material [60]. Unlike the two-derivative case, the Chern-Simons term at the four-derivative level often appears through the auxiliary field ϕ_a^{ij} . To extract the correct Chern-Simons coefficient, one must substitute ϕ_a^{ij} using the leading two-derivative equation of motion of ϕ_{ij}^a and H^a ,

$$\phi_a^{ij} = -g_I v_a^I \delta^{ij}, \quad H_a = 0. \quad (24)$$

The resulting coefficients of the Chern-Simons terms for all invariants are summarized in Table I. Importantly, Eq. (24) also gives the Killing spinor R -symmetry charge that we

used previously in the holographic dictionary, via the covariant derivative $\mathcal{D}_\mu \epsilon^i = \partial_\mu \epsilon^i + \phi_\mu^{ij} \epsilon_j + \dots$.

Partially off-shell AdS₅ action—We now describe the maximally supersymmetric AdS₅ background, which provides the setting for evaluating the invariants constructed above. The final results for each invariant, summarized in (2) using the table of Chern-Simons coefficients, have already been anticipated. We therefore omit intermediate details, which will be presented in [58] comprehensively. Here, we focus on the conditions satisfied by the relevant multiplet components under maximal supersymmetry, without imposing any equations of motion.

Since the $U(1)_R$ symmetry—gauged by a linear combination of the Abelian vectors—embeds into the original $SU(2)_R$ symmetry along an arbitrary direction, we introduce a unit tensor s^{ij} satisfying $s^2 = \frac{1}{2} s^{ij} s_{ij} = 1$. We will choose $s^{ij} = \delta^{ij}$. Maximal supersymmetry then constrains all $SU(2)_R$ tensors in the theory to align with this direction, i.e., X^{ij} and G^{ij} are proportional to δ^{ij} . For the metric, we take AdS₅ in the global coordinates,

$$ds_5^2 = -\left(1 + \frac{r^2}{l^2}\right) dt^2 + \frac{dr^2}{1 + \frac{r^2}{l^2}} + r^2 ds_{S^3}^2, \quad (25)$$

where $ds_{S^3}^2$ is the metric on S^3 with unit radius, such that the Ricci scalar is given by $\mathcal{R} = 20l^{-2}$ [63]. We further find that all other Weyl multiplet components vanish, notably including the auxiliary scalar field, $D = 0$. For the linear multiplet, maximal supersymmetry, together with the standard gauge fixing choice $G = 1$, sets

$$G^{ij} = \delta^{ij}, \quad \phi_\beta^i = 0, \quad H_a = 0, \quad F = \text{const.} \quad (26)$$

Similarly, for a vector multiplet we find

$$W^I = \text{const}, \quad \lambda_\alpha^I = 0, \quad F_{ab}^I = 0, \quad X^{ijI} = \frac{1}{6} F W^I \delta^{ij}. \quad (27)$$

All remaining multiplets, being composite, follow by an insertion of the identities above in their definition.

In order to truly evaluate the resulting actions, we need the additional input of the field equation of the auxiliary field F , which only gets a two-derivative contribution:

$$F = -\frac{6}{l} = -4g_I W^I, \quad (28)$$

giving an important constraint on the corresponding linear combination of vector multiplet scalars, since the AdS scale l needs to be taken into account in the holographic dictionary. This is the origin of the constrained extremization in (2), where the field F can be regarded as the holographic dual of the Lagrange multiplier enforcing (6).

With the above identifications, we have been able to explicitly evaluate the AdS₅ background on each of the Lagrangians we constructed, see more details in Supplemental Material [60]. In turn we need to discuss the holographic renormalization scheme that renders the corresponding actions finite. Importantly, by virtue of the supersymmetry preserved by the boundary geometry [64–66], in this case, $\mathbb{R} \times S^3$, all the possible finite counterterms or logarithmic divergences vanish, rendering our results unambiguous [58]. The holographically renormalized action can be simply computed in the following way,

$$I_M = \lim_{r \rightarrow \infty} S_{\text{sugra}}(M) + \sigma_{\text{ct}} S_{\text{GH}}(\partial M) + \sigma_{\text{ct}} S_{\text{ct}}(\partial M), \quad (29)$$

with $S_{\text{sugra}}(M)$ the original supergravity action with asymptotically AdS₅ background as in (25). The Gibbons-Hawking action is given by

$$S_{\text{GH}}(\partial M) = -\int_{\partial M} d^4 x \sqrt{-h} K, \quad (30)$$

where h_{ij} is the induced metric on the boundary, and $K = h^{ij} K_{ij}$ is the trace of the extrinsic curvature. The final counterterm is given by

$$S_{\text{ct}}(\partial M) = \int_{\partial M} d^4 x \sqrt{-h} \left(-\frac{3}{l} + \frac{l R(h)}{4} \right), \quad (31)$$

with $R(h)$ the scalar curvature of the induced metric h , which renders the full answer finite.

Note that we have allowed an arbitrary overall normalization of the original action. The constant σ_{ct} should then be chosen accordingly, such that the full answer is indeed finite. We consequently recover the partially off-shell results as advertised in the introduction.

Outlook—Building on the relation between the anomaly polynomial and the superconformal invariants established above, we conclude with an observation that merits further exploration. For holographic 4D SCFTs at large N , one expects that all supersymmetric partition functions (or large- N protected observables) are completely determined by the 't Hooft anomaly coefficients of the R -symmetry and flavor symmetries. Holographically, this suggests that the (partially) off-shell action for any supersymmetric asymptotically locally AdS₅ background is entirely determined by the Chern-Simons coefficients listed in Table I. Moreover, while higher-derivative invariants involving six or more derivatives can, in principle, be constructed within the present formalism (see Refs. [25,58]), these (up to integration by parts) cannot generate additional on-shell Chern-Simons terms beyond those in (1). We thus expect all such higher-order invariants, when evaluated on asymptotically AdS₅ BPS backgrounds, to reduce to linear combinations of the two-derivative and Weyl² actions, which already

exhaust the possible freedom in the coefficients k_{IJK} and k_I . This conclusion applies to all other invariants considered here, and will be extended to include a more complete set of invariants and building blocks in [58]. Here, we limited ourselves to consider systems of Abelian vector multiplets. However, the extension including off-shell non-Abelian vector multiplets is straightforward—see, e.g., Ref. [36]. By employing the formalism of [25,30], it would also naturally be very interesting to include in our analysis gauged hypermultiplets for a complete off-shell description.

To conclude, it is worth mentioning that an ambitious goal is being pursued within the framework of supergravity localization, initiated in [67] in the even-dimensional case. When applied to the present five-dimensional setting, see Refs. [68–70], the current implementations appear to require that equations of motion be imposed from the outset. This requirement prevents the direct evaluation of arbitrary higher-derivative actions of the type considered here. An important direction for future work would therefore be to better understand how these two approaches might be reconciled in the fully off-shell framework we have employed here.

Acknowledgments—We thank Robert Saskowski for discussions. K.H. is supported in part by the Bulgarian NSF Grant No. KP-06-N88/1. Y.P. is supported by the National Key R&D Program No. 2022YFE0134300 and the National Natural Science Foundation of China (NSFC) under Grants No. 12575076 and No. 12247103. G.T.-M. have been supported by the Australian Research Council (ARC) Future Fellowship FT180100353, ARC Discovery Project DP240101409, the Capacity Building Package at the University of Queensland, and a faculty start-up funding of UQ’s School of Mathematics and Physics. We are thankful for support during the MATRIX Program “New Deformations of Quantum Field and Gravity Theories” (Creswick, January 22–February 2, 2024), where this collaboration was initiated.

Data availability—There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

-
- [1] S. Cremonini, K. Hanaki, J. T. Liu, and P. Szepietowski, *J. High Energy Phys.* **12** (2009) 045.
 - [2] M. Baggio, N. Halmagyi, D. R. Mayerson, D. Robbins, and B. Wecht, *J. High Energy Phys.* **12** (2014) 042.
 - [3] N. Bobev, K. Hristov, and V. Reys, *J. High Energy Phys.* **04** (2021) 088.
 - [4] J. T. Liu and R. J. Saskowski, *J. High Energy Phys.* **05** (2022) 171.
 - [5] N. Bobev, V. Dimitrov, V. Reys, and A. Vekemans, *Phys. Rev. D* **106**, L121903 (2022).

- [6] D. Cassani, A. Ruipérez, and E. Turetta, *J. High Energy Phys.* **11** (2022) 059.
- [7] G. Gold, J. Hutomo, S. Khandelwal, M. Ozkan, Y. Pang, and G. Tartaglino-Mazzucchelli, *Phys. Rev. Lett.* **131**, 251603 (2023).
- [8] D. Cassani, A. Ruipérez, and E. Turetta, *J. High Energy Phys.* **06** (2023) 203.
- [9] D. Cassani, A. Ruipérez, and E. Turetta, *J. High Energy Phys.* **05** (2024) 276.
- [10] L. Ma, P.-J. Hu, Y. Pang, and H. Lu, *Phys. Rev. D* **110**, L021901 (2024).
- [11] V. Pestun, *Commun. Math. Phys.* **313**, 71 (2012).
- [12] V. Pestun *et al.*, *J. Phys. A* **50**, 440301 (2017).
- [13] By a mild abuse of terminology, we use the terms *superconformal* and *off shell* interchangeably when referring to the 5D supergravity formalism.
- [14] M. Zucker, *J. High Energy Phys.* **08** (2000) 016.
- [15] T. Kugo and K. Ohashi, *Prog. Theor. Phys.* **104**, 835 (2000).
- [16] T. Kugo and K. Ohashi, *Prog. Theor. Phys.* **105**, 323 (2001).
- [17] T. Fujita and K. Ohashi, *Prog. Theor. Phys.* **106**, 221 (2001).
- [18] E. Bergshoeff, T. de Wit, R. Halbersma, S. Cucu, M. Derix, and A. Van Proeyen, *J. High Energy Phys.* **06** (2001) 051.
- [19] T. Kugo and K. Ohashi, *Prog. Theor. Phys.* **108**, 1143 (2003).
- [20] E. Bergshoeff, S. Cucu, T. De Wit, J. Gheerardyn, R. Halbersma, S. Vandoren, and A. Van Proeyen, *J. High Energy Phys.* **10** (2002) 045.
- [21] M. Zucker, *Fortschr. Phys.* **51**, 899 (2003).
- [22] E. Bergshoeff, S. Cucu, T. de Wit, J. Gheerardyn, S. Vandoren, and A. Van Proeyen, *Classical Quantum Gravity* **21**, 3015 (2004).
- [23] K. Hanaki, K. Ohashi, and Y. Tachikawa, *Prog. Theor. Phys.* **117**, 533 (2007).
- [24] M. Ozkan and Y. Pang, *J. High Energy Phys.* **08** (2013) 042.
- [25] D. Butter, S. M. Kuzenko, J. Novak, and G. Tartaglino-Mazzucchelli, *J. High Energy Phys.* **02** (2014) 111.
- [26] M. Ozkan, *J. High Energy Phys.* **11** (2016) 157.
- [27] G. Gold, J. Hutomo, S. Khandelwal, and G. Tartaglino-Mazzucchelli, *Phys. Rev. D* **107**, 106013 (2023).
- [28] G. Gold, J. Hutomo, S. Khandelwal, and G. Tartaglino-Mazzucchelli, *J. High Energy Phys.* **07** (2023) 221.
- [29] S. M. Kuzenko and G. Tartaglino-Mazzucchelli, *Nucl. Phys. B* **785**, 34 (2007).
- [30] S. M. Kuzenko and G. Tartaglino-Mazzucchelli, *Phys. Lett. B* **661**, 42 (2008).
- [31] S. M. Kuzenko and G. Tartaglino-Mazzucchelli, *J. High Energy Phys.* **02** (2008) 004.
- [32] S. M. Kuzenko and G. Tartaglino-Mazzucchelli, *J. High Energy Phys.* **04** (2008) 032.
- [33] S. M. Kuzenko and G. Tartaglino-Mazzucchelli, *J. High Energy Phys.* **06** (2008) 097.
- [34] S. M. Kuzenko, J. Novak, and G. Tartaglino-Mazzucchelli, *J. High Energy Phys.* **10** (2014) 175.
- [35] D. Z. Freedman and A. Van Proeyen, *Supergravity* (Cambridge University Press, Cambridge, England, 2012).
- [36] E. Lauria and A. Van Proeyen, $\mathcal{N} = 2$ supergravity in $D = 4, 5, 6$ dimensions, in *Lecture Notes in Physics* (Springer, Cham, 2020), Vol. 966.

- [37] S. M. Kuzenko, E. S. N. Raptakis, and G. Tartaglino-Mazzucchelli, Superspace approaches to $\mathcal{N} = 1$ supergravity, in *Handbook of Quantum Gravity* (Springer, Singapore, 2023).
- [38] S. M. Kuzenko, E. S. N. Raptakis, and G. Tartaglino-Mazzucchelli, Covariant superspace approaches to $\mathcal{N} = 2$ supergravity, in *Handbook of Quantum Gravity* (Springer, Singapore, 2023).
- [39] M. Ozkan, Y. Pang, and E. Sezgin, *Phys. Rep.* **1086**, 1 (2024).
- [40] E. Witten, *Adv. Theor. Math. Phys.* **2**, 253 (1998).
- [41] M. Henningson and K. Skenderis, *J. High Energy Phys.* **07** (1998) 023.
- [42] Y. Tachikawa, *Nucl. Phys.* **B733**, 188 (2006).
- [43] O. Aharony and Y. Tachikawa, *J. High Energy Phys.* **01** (2008) 037.
- [44] Here, “auxiliary” refers to fields without kinetic terms in the two-derivative theory.
- [45] D. Anselmi, D. Z. Freedman, M. T. Grisaru, and A. A. Johansen, *Nucl. Phys.* **B526**, 543 (1998).
- [46] K. A. Intriligator and B. Wecht, *Nucl. Phys.* **B667**, 183 (2003).
- [47] D. Kutasov, A. Parnachev, and D. A. Sahakyan, *J. High Energy Phys.* **11** (2003) 013.
- [48] Our perspective is off shell and therefore intrinsically different from the approach of [42,49], which are based on on-shell models where the gauge coupling to the hypermultiplet effectively plays the role of a Lagrange multiplier. The on-shell nature of the hypermultiplets in those constructions makes it difficult to extend the analysis beyond the two-derivative level. One can rather view our results as complementary to the geometric perspective of a maximization in terms of the internal Sasakian volume, developed in [50–52].
- [49] P. Szepietowski, *J. High Energy Phys.* **12** (2012) 018.
- [50] D. Martelli, J. Sparks, and S.-T. Yau, *Commun. Math. Phys.* **268**, 39 (2006).
- [51] A. Butti and A. Zaffaroni, *J. High Energy Phys.* **11** (2005) 019.
- [52] D. Martelli, J. Sparks, and S.-T. Yau, *Commun. Math. Phys.* **280**, 611 (2008).
- [53] We leave the details of the spinorial representation of $\mathcal{Q}$ unspecified, except for its R charge, as they play no role here. See Ref. [9] and references therein for details.
- [54] S. Benvenuti, S. Franco, A. Hanany, D. Martelli, and J. Sparks, *J. High Energy Phys.* **06** (2005) 064.
- [55] I. Bah, C. Beem, N. Bobev, and B. Wecht, *J. High Energy Phys.* **06** (2012) 005.
- [56] C. Klare, A. Tomasiello, and A. Zaffaroni, *J. High Energy Phys.* **08** (2012) 061.
- [57] D. Cassani, C. Klare, D. Martelli, A. Tomasiello, and A. Zaffaroni, *Commun. Math. Phys.* **327**, 577 (2014).
- [58] K. Hristov, S. Khandelwal, Y. Pang, and G. Tartaglino-Mazzucchelli (to be published).
- [59] By a standard abuse of notation, we denote the complete multiplet in the same way as its lowest component. The distinction should be evident from the context in a given discussion or a formula.
- [60] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/nrlq-ktd1> for a review of the main ingredients of the two-derivative gauged supergravity theory and the terms in the four-derivative invariants which contribute to the computation of the trial a -central charges on the AdS5 background for each of the Lagrangians considered in our Letter.
- [61] M. Zucker, *Nucl. Phys.* **B570**, 267 (2000).
- [62] Note that classifying superconformal invariants by their derivative order is not unique. Because gauged supergravity contains a natural length scale l , one may instead classify invariants according to their scaling in l , which can be read off from Table I and Eq. (2). From this perspective, the $\mathcal{RF}$ invariant exhibits a scaling behavior distinct from that of the standard two-derivative action.
- [63] In the present convention for curvature, AdS₅ has a constant positive curvature.
- [64] M. de Roo, A. Wiedemann, and E. Zijlstra, *Classical Quantum Gravity* **7**, 1181 (1990).
- [65] J. T. Liu, L. A. Pando Zayas, and D. Reichmann, *J. High Energy Phys.* **10** (2012) 034.
- [66] B. Assel, D. Cassani, and D. Martelli, *J. High Energy Phys.* **11** (2014) 135.
- [67] P. Benetti Genolini, J. P. Gauntlett, and J. Sparks, *Phys. Rev. Lett.* **131**, 121602 (2023).
- [68] D. Cassani, A. Ruipérez, and E. Turetta, *J. High Energy Phys.* **12** (2024) 086.
- [69] E. Colombo, V. Dimitrov, D. Martelli, and A. Zaffaroni, [arXiv:2502.15624](https://arxiv.org/abs/2502.15624).
- [70] P. Benetti Genolini, J. P. Gauntlett, Y. Jiao, J. Park, and J. Sparks, *J. High Energy Phys.* **03** (2026) 080.