

Dissipative Phase Transition of Interacting Nonreciprocal Fermions

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(Received 31 May 2025; revised 16 April 2026; accepted 30 April 2026; published 26 June 2026)

While nonreciprocal couplings are ubiquitous in classical systems, their impact on quantum many-body criticality and entanglement remains largely unexplored. Using exact numerical simulations, we study an interacting fermionic chain subject to nonreciprocal gain and loss. We show that the interplay between dissipation and interactions drives a dissipative phase transition, marked by the opening of a many-body gap and a crossover from power law to exponential relaxation. The weakly interacting regime displays nonreciprocal signatures, including nonzero currents and directional charge accumulation reminiscent of the skin effect. Notably, despite this localization, quantum trajectories exhibit volume-law entanglement. Finally, reciprocity is dynamically restored above a critical interaction strength.

DOI: 10.1103/n8xv-kygx

Introduction—For classical and quantum systems in thermal equilibrium, the interactions between degrees of freedom are fundamentally symmetric or reciprocal. Far from equilibrium, however, asymmetric or nonreciprocal interactions are ubiquitous. Examples range from optics [1,2], to active [3–6] or living matter [7], ecology [8,9], robotic systems [10] and statistical physics [11–14]. Nonreciprocal systems display unusual properties, including unique critical behavior at phase transitions [15]. First investigated for classical systems, interest in nonreciprocity has branched out to the quantum world. Nonreciprocal phenomena can naturally arise in non-Hermitian (NH) quantum mechanics [16], the Hatano-Nelson model providing a well-known example of a system with nonreciprocal hopping [17,18], leading to rich physics such as enhanced sensitivity to boundary conditions, the so-called NH skin effect [19–21], NH topology [22,23], and entanglement phase transitions [24]. However, this approach neglects the contribution from quantum jumps. A fully consistent quantum-mechanical treatment of nonreciprocity is possible within the framework of cascaded master equations [25,26] and reservoir engineering [27,28].

The first steps toward applying these ideas to open quantum many-body systems [29] have recently been taken [30–41]. Outstanding questions are whether quantum many-body systems can exhibit robust nonreciprocal

steady states featuring rich entanglement structures, akin to those found in their NH counterparts, and whether quantum nonreciprocal phase transitions connecting such states can exist. In addition, one could ask whether these phase transitions can be classified within the framework of dissipative phase transitions or whether they instead represent a new universality class [42].

In this Letter, we consider an interacting fermionic chain featuring nonreciprocal hopping engineered via active and passive reservoirs (Fig. 1). The resulting gain and loss rates are characterized by nontrivial relative phases yielding an asymmetric coupling matrix—processes we refer to as nonreciprocal gain and loss. Crucially, these mechanisms can balance each other to produce gapless, critical nonreciprocal dynamics in the noninteracting limit. This

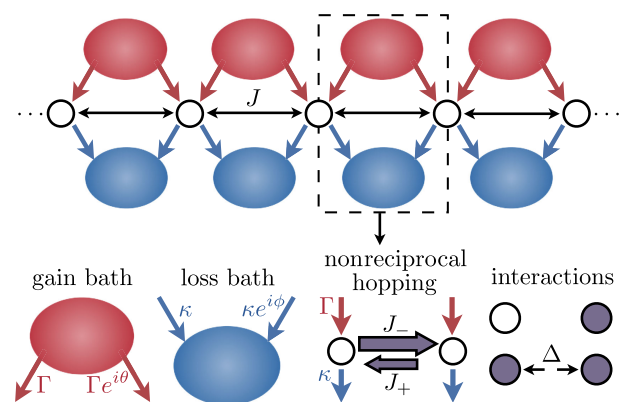


FIG. 1. Sketch of the setup. One-dimensional fermionic chain with nearest-neighbor hoppings, J , and density-density interaction, Δ . The gain (loss) bath injects (absorbs) particles to (from) neighboring sites, with relative phase θ (ϕ). Together with the hopping term, they realize nonreciprocal gain and loss processes.

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contrasts with purely local gain [36], which always exhibits a finite dissipative gap. Using large-scale diagonalization of the Lindblad master equation and quantum trajectories, we reveal an interaction induced transition marked by the opening of a finite dissipative gap and the onset of exponential relaxation. At this transition, reciprocity is dynamically restored, as evidenced by the suppression of boundary charge accumulation. Remarkably, unlike effective NH Hamiltonians featuring NH skin effect [24], the weakly interacting phase supports volume-law trajectory entanglement despite exhibiting identical boundary charge accumulation. Our Letter highlights the role of nonreciprocal gain and loss in giving rise to exotic dynamical and steady-state properties.

Model and setup—We consider an interacting fermionic chain of size L described by the Hamiltonian

$$\mathcal{H} = -\frac{J}{2} \sum_{j=1}^{L-1} \left[c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j \right] + \Delta \sum_{j=1}^{L-1} n_j n_{j+1}, \quad (1)$$

where $c_j^\dagger(c_j)$ is the fermionic creation(annihilation) operator that creates(annihilates) a particle at site j , $n_j = c_j^\dagger c_j$, J is the hopping strength, and Δ is the next-neighbor density-density interaction; see Fig. 1. We are interested in the dissipative dynamics generated by the Lindblad master equation for the system density matrix ρ [29]:

$$\partial_t \rho = -i[\mathcal{H}, \rho] + \sum_{j,\mu} L_{j,\mu} \rho L_{j,\mu}^\dagger - \frac{1}{2} \left\{ L_{j,\mu}^\dagger L_{j,\mu}, \rho \right\}, \quad (2)$$

where $L_{j,\mu}$ are a set of jump operators describing nonlocal gain and losses ($\mu = g, \ell$ respectively) of strength Γ, κ , that we take of the form

$$L_{j,\ell} = \sqrt{\kappa} \left(c_j + e^{i\phi} c_{j+1} \right), \quad L_{j,g} = \sqrt{\Gamma} \left(c_j^\dagger + e^{i\theta} c_{j+1}^\dagger \right). \quad (3)$$

A key feature of these jump operators is the relative phases ϕ, θ between the rates of loss and gain on neighboring sites. These phases can be seen to arise from the coupling to environments featuring synthetic gauge fields. Jump operators of this form can be realized within the framework of reservoir engineering [27,34,43–46], in various platforms including superconducting circuits and trapped ions [47–49]. For $\theta, \phi \neq 0$ the jump operators in Eq. (3) give rise to asymmetric gain and loss matrices, i.e., to nonreciprocal dissipative couplings. In the absence of coherent hopping ($J = 0$), one of the two phases can be removed from the Lindblad master equation by a local gauge transformation, as shown in the Supplemental Material [50]. On the other hand, for $J \neq 0$, the two phases give rise to a rich landscape of nonreciprocal behavior in dynamic and static properties, as we show below.

Noninteracting limit—We start by discussing the noninteracting limit $\Delta = 0$, which provides useful insight

(see also Ref. [32] where the case $\phi = -\theta = \pi/2$ has been studied for a Su-Schrieffer–Heeger model).

The dynamics of the correlation matrix $C_{ij} = \langle c_i^\dagger c_j \rangle$ takes the form

$$i\partial_t C_{ij} = \sum_l (H)_{jl} C_{il} - (H^\dagger)_{li} C_{lj} + iG_{ij}, \quad (4)$$

where $(H)_{jl} = -i(\kappa + \Gamma)\delta_{jl} - \frac{1}{2}J_+\delta_{j,l+1} - \frac{1}{2}J_-\delta_{j,l-1}$ encodes an effective NH Hamiltonian associated with the unconditional evolution, which in the presence of gain is different from the no-click limit [34], and noise matrix $G_{jl} = \Gamma(e^{-i\theta}\delta_{j,l+1} + e^{i\theta}\delta_{j,l-1} + 2\delta_{j,l})$. In our case H takes the form of the Hatano-Nelson model [17] with hopping amplitudes

$$J_\pm = J + i(\Gamma \cos \theta + \kappa \cos \phi) \pm (\Gamma \sin \theta - \kappa \sin \phi). \quad (5)$$

Unequal amplitudes $J_+ \neq J_-$ signify nonreciprocal hopping.

For periodic boundary conditions (PBC), we can compute exactly the dynamics of the occupation numbers [50] $\langle n_k \rangle = \langle c_k^\dagger c_k \rangle$, which read as

$$\langle n_k(t) \rangle = \langle n_k \rangle_{\text{SS}} + e^{-\lambda_k t} [\langle n_k(0) \rangle - \langle n_k \rangle_{\text{SS}}], \quad (6)$$

where λ_k represents the spectrum of decay rates

$$\lambda_k = 2(\Gamma + \kappa) + \text{Im}(J_+ + J_-) \cos k + (J_+ - J_-) \sin k, \quad (7)$$

while $\langle n_k \rangle_{\text{SS}}$ is the steady-state distribution

$$\langle n_k \rangle_{\text{SS}} = \frac{\Gamma(1 + \cos \theta \cos k + \sin \theta \sin k)}{2(\Gamma + \kappa) + \text{Im}(J_+ + J_-) \cos k + (J_+ - J_-) \sin k}. \quad (8)$$

We note that the phases of gain and losses θ, ϕ enter differently in the spectrum λ_k and in the steady-state occupation $\langle n_k \rangle_{\text{SS}}$. As a result, the two dissipative processes give rise to distinct nonreciprocal behavior in the dynamics or in the stationary state.

We start by discussing the spectrum and the dynamics. From Eq. (7), we see that the slowest decaying mode is at $k_* = \arctan[(\Gamma \sin \theta - \kappa \sin \phi)/(\Gamma \cos \theta + \kappa \cos \phi)]$ and around this value the spectrum reads as $\lambda_k = 2\delta + \alpha(k - k_*)^2$, with the dissipative gap

$$\delta = (\Gamma + \kappa) - \sqrt{\Gamma^2 + \kappa^2 + 2\Gamma\kappa \cos(\theta + \phi)}. \quad (9)$$

In the ϕ, θ plane the gap is finite everywhere, except along the line $\phi = -\theta$ where it closes as $\delta \propto |\phi - \phi_c|^2$. This implies that an excitation with wave vector k_* is not affected by dissipation and evolves due to the unitary part of evolution alone [50]. The existence of this gapless mode has direct consequences on the system's dynamics, making observables, such as the particle density, approach the steady state following a universal power-law decay [50]

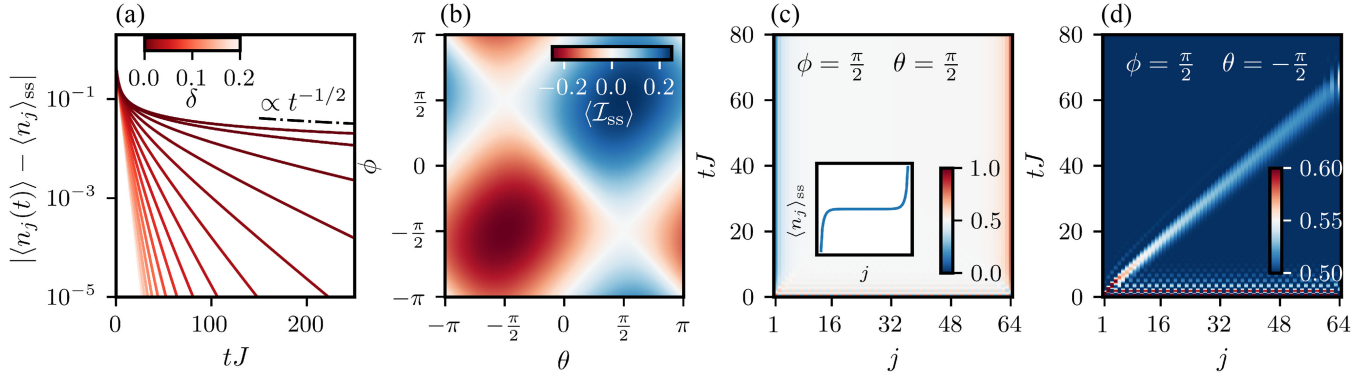


FIG. 2. (a) Time evolution of the particle density under PBC (rescaled by the steady-state value) for different values of the dissipative gap δ . (b) Steady-state particle current under PBC as a function of the angles. In (c) and (d), we depict the particle density dynamics with OBC for different values of the angles, with the initial state a CDW. The inset in (c) corresponds to the steady-state configuration. Other parameters: $\Gamma = \kappa = 0.1J$.

$\langle n(t) \rangle \sim \langle n \rangle_{ss} - \delta n / \sqrt{t}$. Moving away from this line the dissipative gap opens and the dynamics becomes exponential; see Fig. 2(a). This corresponds to a dissipative transition where the timescale to reach the steady state diverges. Importantly, the closure of the dissipative gap along the critical line $\phi = -\theta$ arises from the interference between nonreciprocal gain and loss. It therefore stands in contrast to the case of purely local gain [36], where the system is always gapped at finite dissipation, irrespective of the reciprocal vs nonreciprocal character of the dynamics (i.e., for any value of ϕ). In our system, in the presence of nonreciprocal losses, nonreciprocal gain is necessary for closing the gap. The dynamics along the line $\phi = -\theta$ shows both critical and nonreciprocal features, since the spectrum is asymmetric in momentum space, $\lambda_k - \lambda_{-k} = 2(J_+ - J_-) \sin k$, leading to a nonreciprocal transient occupation $\langle n_{-k}(t) \rangle \neq \langle n_k(t) \rangle$ and so to a nonzero transient current $\langle \mathcal{I} \rangle \sim \mathcal{I}_0 \sin \phi / \sqrt{t}$ that slowly vanishes at long times with a magnitude controlled by the phase of the gain and loss baths [50].

We now discuss the steady-state distribution, Eq. (8), which features two distinct sources of nonreciprocity, namely from the spectrum (denominator) and from the nonreciprocal gain (numerator). This leads to a nontrivial distribution across the ϕ, θ plane. Along the line $\phi = \theta$ and for $\Gamma = \kappa$, where the hopping is reciprocal ($J_+ = J_-$), the steady-state distribution in Eq. (8) is highly non-thermal [50]. Furthermore, the presence of nonreciprocal gain breaks inversion symmetry, as reflected in $\langle n_{-k} \rangle_{ss} \neq \langle n_k \rangle_{ss}$. This asymmetry results in a finite steady-state current, which reaches its maximum value at $\phi = \theta = \pi/2$ [50]. On the other hand, along the line $\phi = -\theta$, the two dissipative processes interfere destructively, and the distribution becomes flat in momentum space, $\langle n_k \rangle_{ss} = \Gamma / (\Gamma + \kappa)$, and as a consequence the steady-state current vanishes, just as in the case of local bulk gain and losses. When plotted in the ϕ, θ plane, the steady-state current shows a diamond structure reminiscent of Coulomb blockade physics [63], with lines of zero and

maximal currents that arise due to interference between the dissipative processes; see Fig. 2(b).

The steady-state correlation matrix decays exponentially across the phase diagram with a characteristic length that grows approaching the line $\phi = -\theta$ [50]. However, when the dissipative gap closes the momentum dependent gain also vanishes, preventing the correlation length from diverging [64,65]. Notably, along this line the system relaxes to a maximally mixed state with a particle density fixed by the ratio of Γ/κ [50].

We now consider the dynamics with open boundary conditions (OBC) [66]. In Figs. 2(c) and 2(d), we plot the dynamics of the particle density for different values of θ, ϕ and different initial states, at $\Gamma = \kappa$. In stark contrast to the PBC case, the relaxation toward the steady state is strictly exponential as the dissipative gap remains always finite, a manifestation of the Liouville skin effect [50]. For $\phi = \theta = \pi/2$ [see Fig. 2(c)], the initial charge-density wave (CDW) is quickly melted in the bulk after a few short-time revivals, in agreement with the previous discussion of exponential dynamics. However, at the edges the dynamics is nontrivial: charge is reshuffled across the system by the nonreciprocal gain and loss, and we observe the build up of a steady state featuring charge accumulation at the edges. Upon tuning the angle θ we see that the charge imbalance decreases and eventually disappears for $\theta = -\pi/2$, as the steady state corresponds to the maximally mixed state [50], a value at which, however, the dynamics displays clear directionality; see Fig. 2(d). We can understand these results by looking at the nonreciprocal hopping rates $J_{\pm}$ in Eq. (5). For $\Gamma = \kappa$ the maximum value of $J_+ - J_-$ is obtained precisely for $\phi = -\theta = \pi/2$, while for $\phi = \theta = \pi/2$ the hopping is perfectly reciprocal, and so is the dynamics. Overall, this behavior is reminiscent of the dynamics of the Hatano-Nelson model, yet it appears at the level of a fully Lindbladian evolution [Eq. (4)], which includes dissipation and noise and does not require post-selection [67].

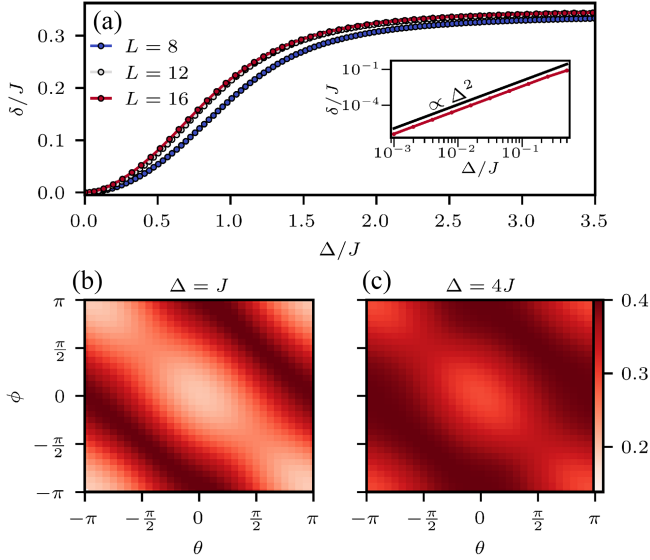


FIG. 3. Dissipative gap for PBC. (a) Gap as a function of the interaction strength ($\phi = -\theta = \pi/2$). (b),(c) Gap as a function of ϕ and θ for $L = 12$ with $\Delta = J$ (b) and $\Delta = 4J$ (c). Other parameters: $\Gamma = \kappa = 0.1J$.

Interaction-induced dissipative transition—Turning to the interacting case, we perform large-scale exact diagonalization of the many-body Lindbladian to investigate the dissipative gap δ [50]. We first focus on the noninteracting critical line $\phi = -\theta = \pi/2$, where the gap either vanishes exactly (if the system resolves the dark mode momentum k_*) or closes in the thermodynamic limit as $\delta \sim 1/L^2$ [50]. Upon adding interactions, Fig. 3(a) shows that δ scales quadratically for weak coupling before increasing rapidly near $\Delta \sim J$, which suggests a rapid crossover at weak interactions. Within computationally accessible system sizes, we find that the dissipative gap becomes independent of system size as Δ increases, consistent with bulk dissipation [68]. Tuning the angles [Fig. 3(b)] reveals that up to $\Delta \sim J$, the gap retains a structure similar to the noninteracting case, with a minimum along $\phi = -\theta$. This dependence persists at intermediate Δ but is suppressed as interactions increase, approaching a uniform gap [Fig. 3(c)].

In addition to the behavior of the dissipative gap, our numerically exact results give access to the symmetry-resolved many-body spectrum based on translation invariance and weak $U(1)$ symmetry, thus providing insight into the structure of the slowest mode [50]. In the noninteracting case, along the critical line, we found four eigenvalues with zero real part corresponding to two degenerate steady states with zero momentum and two states featuring single particle excitations at k_* . While a finite interaction partially lifts these degeneracies, a crossover takes place in the Lindbladian spectrum, separating the weak-interaction regime, where the slowest mode is almost degenerate between zero and at finite momentum, from the strongly interacting regime, where it belongs to the zero momentum

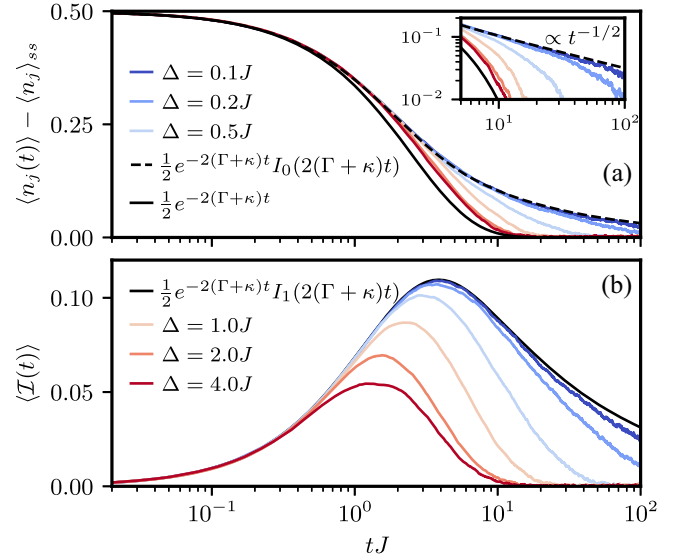


FIG. 4. Time evolution of (a) the particle density (rescaled by the steady-state value) and (b) the particle current for different values of the interaction strength Δ with PBC for $\phi = -\theta = \pi/2$; the system is initialized in the fully filled state. $I_\alpha(\cdot)$ correspond to the α order Bessel function of the second kind [50]. Other parameters: $L = 16$, $\Gamma = \kappa = 0.1J$.

sector and connects to the noninteracting steady state as $\Delta \rightarrow 0$ [50]. These results suggest that our system for $\phi = -\theta$ undergoes a dissipative phase transition driven by interactions via the opening of a many-body gap.

To complement the picture obtained by the dissipative spectrum, we focus on dynamics and solve the problem by unraveling Eq. (2) into quantum jump trajectories [29,51,69,70], where the NH evolution between subsequent quantum jumps is made with the recently developed Faber polynomial method [50,52]. We consider a finite chain with $\Gamma = \kappa = 0.1J$ and PBC, fixing $\phi = -\theta = \pi/2$ to test whether the power-law relaxation survives for finite interactions. In Fig. 4(a) we plot the dynamics of particle density for different values of Δ , while negligible at short times, interaction effects dominate the long-time decay toward the steady state.

Remarkably, for weak Δ , the dynamics remains compatible with a power law (the dashed line corresponds to the noninteracting case), possibly with an interaction-dependent exponent. However, for $\Delta \sim J$, the particle density exhibits a crossover to exponential decay governed by the total dissipation rate $2(\Gamma + \kappa)$. This points toward a genuine dissipative phase transition arising from the competition between interactions and nonreciprocity. Similar behavior is observed for the current [Fig. 5(b)] which vanishes asymptotically since the steady state remains maximally mixed for $\phi = -\theta$ independently of Δ [50]. At intermediate times, the peak in the time-dependent current is renormalized by interactions and pushed to shorter timescales. At long times, for $\Delta < J$, the decay

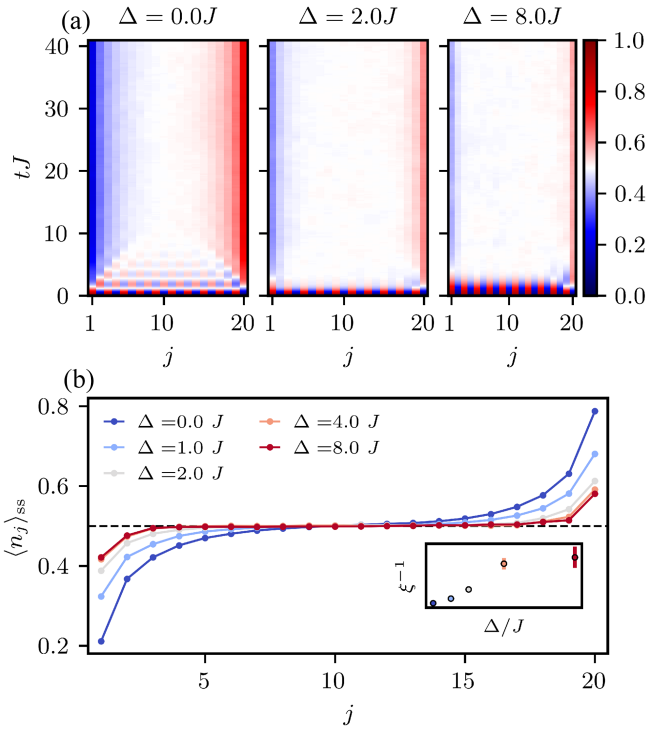


FIG. 5. (a) Time evolution of the particle density and (b) its value in the steady state for different values of Δ and with OBC and $\phi = \theta = \pi/2$. The system is initially in a CDW. Other parameters: $\Gamma = \kappa = 0.1J$.

is compatible with a power law, but is rapidly suppressed at larger Δ .

Dynamics and steady state under OBC—We now discuss the role of interactions in the OBC case. We consider the dynamics of the average particle density starting from a CDW state and for different values of Δ . We fix $\phi = \theta = \pi/2$, where in the noninteracting case there is charge accumulation at the boundaries; see Fig. 2(c). As we show in Fig. 5(a) increasing Δ has two major effects: (i) the tendency toward charge accumulation at the boundaries is reduced, although not suppressed completely even for $\Delta = 8J$, and (ii) the initial CDW state remains frozen for longer times. These two effects are indeed connected and result from an interaction-induced renormalization of nonreciprocal gain and loss.

To understand this, we derive an effective model in the regime $\Delta \gg J, \Gamma, \kappa$. Moving to a rotating frame yields modified jump operators (analogous for $L_{j,g}$) [50]

$$\tilde{L}_{j,\ell} = \sqrt{\kappa} \left(e^{i\Delta(n_{j-1}+n_{j+1})t} c_j + e^{i\phi+i\Delta(n_j+n_{j+2})t} c_{j+1} \right). \quad (10)$$

In this rotating frame, the Lindbladian is time periodic with terms rapidly oscillating at frequency $\omega \sim \Delta$, which can be averaged out as in the high-frequency expansion of Floquet systems [53,54,71]. From Eq. (10), we see that dissipative

processes involving occupied neighbors are highly off resonant and effectively blocked. This simple argument explains why in the strongly interacting regime the initial CDW state, containing no doublons, remains long lived and the nonreciprocal gain and loss become less efficient, reducing the charge accumulation at the edge. It is interesting to characterize in more detail the structure of the steady-state density profile, which we plot in Fig. 5(b) for different values of Δ with $\kappa = \Gamma = 0.1J$ and $\phi = \theta = \pi/2$. We see that the charge density in the bulk is pinned to half filling, because of equal gain and loss rates. The left (right) boundary displays charge depletion (accumulation), with a decay toward the bulk that is compatible with an exponential law $|\langle n_j \rangle_{ss} - \langle n_{L/2} \rangle_{ss}| = 1/2 + Ae^{-j/\xi}$ where ξ is the localization length and A a fitting constant. In the inset, we show that the ξ^{-1} increases with Δ and appears to saturate for $\Delta/J \gg 1$. Overall, these results suggest that interactions compete with the nonreciprocal gain and loss to reduce the tendency toward charge accumulation. Quite interestingly, in this regime, although the charge is localized at the edges and the skin effect would lead to area-law entanglement of NH free fermions [24], the unraveled steady-state for the interacting problem shows volume-law entanglement (see the End Matter for details). To conclude, our numerical results support the emergence of a robust nonreciprocal interacting many-body state at weak couplings.

Conclusions—We investigated the dynamics of an interacting fermionic chain subject to nonreciprocal gain and loss, in which nonreciprocity is controlled via two independent phases ϕ, θ . Using exact numerical simulations, we demonstrated the existence of a dissipative phase transition driven by the interplay between nonreciprocity and interactions. In the noninteracting limit, the single-particle gap closes along the critical line $\phi = -\theta$. Crucially, this shows that in order for criticality to persist at finite dissipation rates, nonreciprocal loss ($\phi \neq 0$) must be accompanied by nonreciprocal gain ($\theta \neq 0$). We then show that adding interactions opens a many-body gap along this critical line, inducing a crossover from power law to exponential relaxation. While the steady state on the critical line remains maximally mixed, deviations from it reveal nonreciprocal signatures, including currents due to broken inversion symmetry. Notably, despite a density profile reminiscent of the skin effect, unraveling the interacting steady state into quantum trajectories shows a volume-law entanglement entropy. Our Letter highlights the rich transport and entanglement properties emerging from the competition between nonreciprocal dynamics and many-body interactions.

Acknowledgments—We acknowledge A. Clerk, G. Lee, and A. Pocklington for inspiring discussions. M. B and M. S. acknowledge funding from the European Research Council (ERC) under the European Union’s Horizon 2020

research and innovation program (Grant Agreement No. 101002955—CONQUER). We acknowledge Collège de France IPH cluster where the numerical calculations were carried out.

Data availability—The data that support the findings of this article are openly available [72].

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End Matter

Impact of interactions on directional dynamics under OBC—We discuss the impact of interactions on the directional dynamics observed under OBC along the $\phi = -\theta$ line, corresponding to nonreciprocal hoppings $J_+ \neq J_-$ in the noninteracting model [see Fig. 2(d)]. To this end, we plot in Fig. 6 the dynamics of particle density for different values of the interaction Δ , starting from a CDW state. We see that the CDW pattern quickly melts after a few coherent revivals at short times. Furthermore, the directional dynamics observed in the noninteracting case, with a bump in the density profile traveling from the left edge of the system, remains well defined for moderate values of the interaction ($\Delta \lesssim J$). The effect of Δ is to increase the damping of this mode, which, already for $\Delta = 0.5J$, does not travel far enough to reach the boundary and dissipates in the bulk of the chain. For $\Delta > J$, on the

other hand, the directionality of the dynamics is barely visible, and the initial CDW state rapidly approaches a uniform steady state with no net particle current. As we analytically show in [50], the dynamics heats up the state to infinite temperature independently of Δ .

Entanglement entropy of quantum trajectories—Here we discuss the entanglement structure of our model of nonreciprocal driven-dissipative fermions. The density matrix $\rho(t)$ solution of the Lindblad master equation defined in the main text describes, in general, a mixed state. In order to characterize entanglement and quantum correlations, we use an unraveling of the system density matrix $\rho(t)$ into many-body quantum trajectories [29,51,69,70]. These describe the nonunitary, but purity-preserving, evolution of the system conditioned to a series of measurement outcomes associated to the set of jump operators $L_{j\mu}$. Given a quantum trajectory $|\psi(\xi_t, t)\rangle$ evolving under the quantum jumps dynamics, we introduce the conditional density matrix $\rho(\xi_t, t) = |\psi(\xi_t, t)\rangle\langle\psi(\xi_t, t)|$. Upon averaging this quantity over the measurement noise ξ_t , one recovers the mixed density matrix solution of the Lindblad master equation, i.e., $\rho(t) = \overline{|\psi(\xi_t, t)\rangle\langle\psi(\xi_t, t)|}$. As long as one is interested in standard average quantities $\langle O \rangle = \text{Tr}[\rho(t)O]$, the conditional and average (also called unconditional) state contain the same information. However, this is not the case for quantities that involve nonlinear functions of the state, for instance, probing higher moments of the conditional density matrix $\rho(\xi_t, t)$. A relevant example is provided by the von Neumann entanglement entropy, defined as [73]

$$S_A(t, \xi_t) = -\text{tr}_A[\rho_\xi^A(t) \ln \rho_\xi^A(t)], \quad (\text{B1})$$

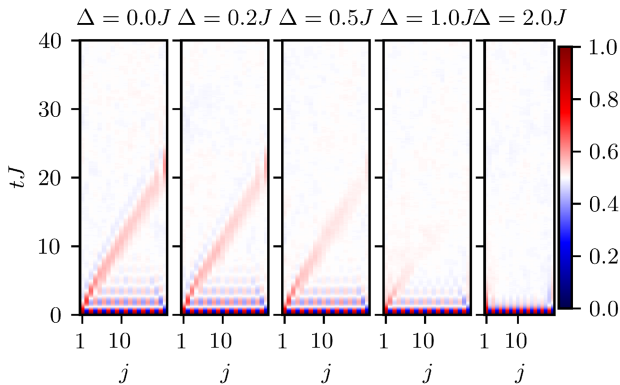


FIG. 6. Dynamics of particle density with OBC for different values of the interaction Δ and for $\phi = -\theta = \pi/2$, corresponding to nonreciprocal hoppings $J_+ \neq J_-$ in the single particle non-Hermitian Hamiltonian controlling the unconditional evolution. Other parameters: $L = 20$ and $\Gamma = \kappa = 0.1J$.

where we have introduced a bipartition $A \cup B$ of the system and defined the reduced density matrix $\rho^A(\xi_t, t) = \text{tr}_B |\psi(\xi_t, t)\rangle \langle \psi(\xi_t, t)|$. In the following, we are interested in the average entanglement entropy, given by $\bar{S}_A(t) = \int \mathcal{D}\xi P(\xi) S_A(\xi_t, t)$ where the average is taken over the Poisson measurement noise ξ_t . Clearly, the average entanglement entropy does not generally coincide with the entanglement entropy of the average state. The former is known to undergo a nontrivial evolution under continuous monitoring, leading to measurement-induced phase transitions.

In the present context, we focus on the effect of interaction and nonreciprocity on the dynamics of the entanglement entropy. In Fig. 7(a) we plot the time evolution of the average entanglement entropy for a bipartition of size $L/2$, different system sizes at fixed angles $\phi = \theta = \pi/2$, and for both $\Delta = 0$ (noninteracting fermions) and $\Delta = 2J$. Interestingly, we note that while in the noninteracting case the entanglement entropy has a very weak dependence on system size and saturates to a constant area-law value, for $\Delta \neq 0$ the entanglement dynamics clearly depends on the size of the subsystem. At long times, the entanglement saturates to a steady-state value that depends linearly on L (see inset), i.e., the scaling is compatible with a volume law. For interacting monitored quantum many-body systems, volume-law scaling of entanglement entropy is expected, at least for weak monitoring [74,75]. However, this is rather surprising in the present context of nonreciprocal fermions, given the fact that from the point of view of the steady state the charge is localized at the boundary as in the skin effect [24], as shown in Fig. 5. We contrast this behavior with the entanglement entropy scaling along the critical lines $\phi = -\theta = \pi/2$, shown in Fig. 7(b) which already in the noninteracting case displays a nontrivial system size dependence, likely reflecting the criticality of the modes along this line. Adding many-body interactions increases the entanglement entropy, which displays now a robust volume law scaling similar to what is shown in panel (a). Finally, we explore the dependence of the entanglement entropy dynamics from the nonreciprocity, by tuning the angles ϕ, θ for $\Delta = 0$ [Fig. 7(c)] and $\Delta = 2J$ [Fig. 7(d)]. By comparing the two panels, we first note that in the fully reciprocal case, corresponding to $\phi = \theta = 0$, the entanglement dynamics depends only weakly on the interaction Δ . Nonreciprocity, on the other hand, suppresses the

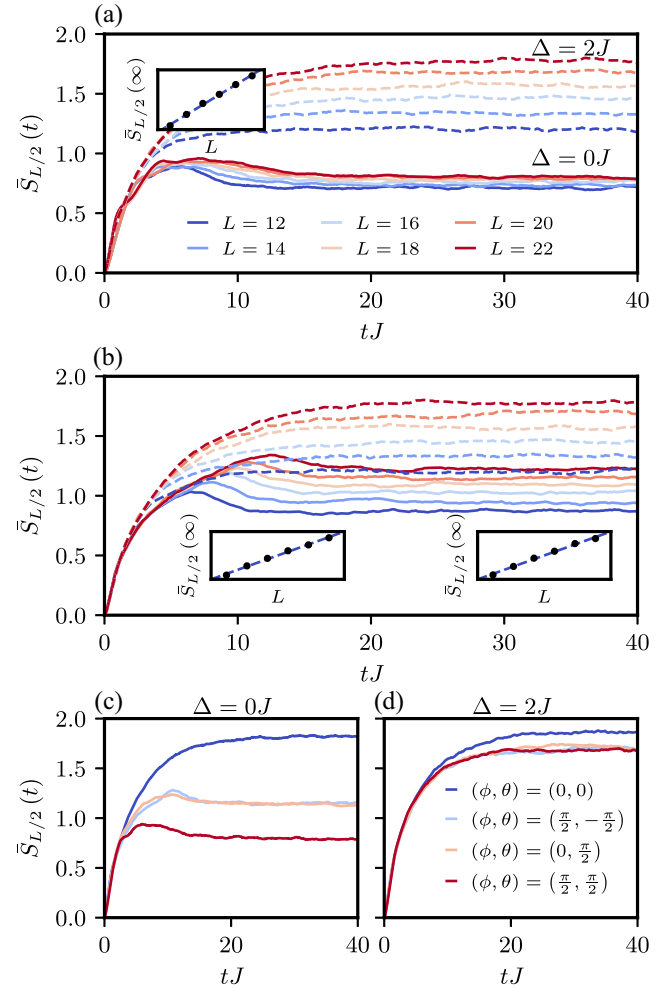


FIG. 7. (a),(b) Time evolution of the average entanglement entropy for different system sizes and interaction strength with $\phi = \theta = \pi/2$ (a) and $\phi = -\theta = \pi/2$ (b). The full lines correspond to the noninteracting case, while the dashed line corresponds to $\Delta = 2.0J$. (c),(d) Time evolution of the entanglement entropy for different values of the angles for $L = 20$. Panel (c) corresponds to the noninteracting case, while in (d) $\Delta = 2.0J$. The label is shared between panels (a), (b) and (c), (d). Other parameters: $\Gamma = \kappa = 0.1J$.

entanglement growth in the noninteracting case, an effect which is significantly less pronounced in the interacting nonreciprocal case, whose entanglement content appears to be more robust.