

Engineering a Multilevel Bath for Transmons with Three-Wave Mixing and Parametric Drives

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A quantum system with a tunable bath temperature provides an additional degree of freedom for quantum simulators. Such a system can be realized by parametrically modulating the coupling between the system and the bath. Here, by coupling a transmon qubit to a lossy superconducting nonlinear asymmetric inductive element (SNAIL) mode, we experimentally create a tunable bath for the qubit mode. The effective temperature of this bath can be precisely controlled, ranging from negative to positive values. We show that the qubit can be thermalized to equilibrium with different population distributions under different parametric pumping conditions. We further extend our method to the third level of the transmon, demonstrating its potential utility beyond the two-level case. Our results provide a useful tool that can be readily integrated with quantum simulators that would benefit from a nontrivial photon population distribution.

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Introduction—A quantum simulator [1] can address complex problems in various domains of physics by mapping problems that are otherwise difficult to probe or track in their original settings to an engineered quantum system over which experimenters have good control. Such simulators have provided insights into a wide range of fields, such as quantum materials simulation [2,3], quantum chemistry [4], quantum transport [5,6], many-body dynamics [7,8], and black holes [9]. Interactions between quantum systems and their uncontrolled environments are ubiquitous on different quantum simulation platforms, including superconducting microwave circuits [10], trapped ions [11], photonic quantum machines [12], and neutral atoms [13]. These interactions are usually viewed as detrimental, as they lead to the decoherence of the quantum systems. However, when engineered controllably, they tailor the properties of the environment seen by quantum systems, providing extra degrees of freedom for quantum simulations [14]. Temperature, as one of the fundamental properties of the environment, affects a quantum system by influencing its thermodynamics, state occupation, decoherence, and

transitions. Therefore, the ability to engineer an environment with a temperature that can be adjusted from negative to positive values is of great interest [15]. It enables the thermodynamical simulation of many-body systems [16], allows the preparation of the Gibbs state [17], and provides an important knob for quantum annealing [18]. So far, such systems have been realized in localized spin systems [19–21] and cold atoms in an optical lattice [22]. Recent efforts have also been made in the microwave domain using superconducting circuits, including the variational preparation of finite-temperature Gibbs state using unitary gates [23] and rapid generation of the Gibbs states with temperatures ranging from 110 to 500 mK within 100 ns using a quantum-circuit refrigerator [24]. In this letter, we address this problem from a driven-dissipative perspective, extending the accessible temperature tuning range to include negative values.

Superconducting circuits and circuit quantum electrodynamics (cQED) [10] provide a promising platform for this task. Bath engineering has become a powerful tool in cQED systems due to the strong coupling between superconducting circuits and their environment, as well as the controllable parametric interactions between different circuit elements [14]. In particular, it has been used to manipulate quantum systems, for example, to fast reset a qubit to the ground state with appropriate drives [25], to stabilize a qubit to an arbitrary state [26], and to stabilize a many-body system with a nontrivial state [27].

Here, we experimentally demonstrate a temperature-tunable bath for a transmon qubit. We start by parametrically coupling the qubit to a lossy mode. For the first experiments, the transmon is treated as a two-level system

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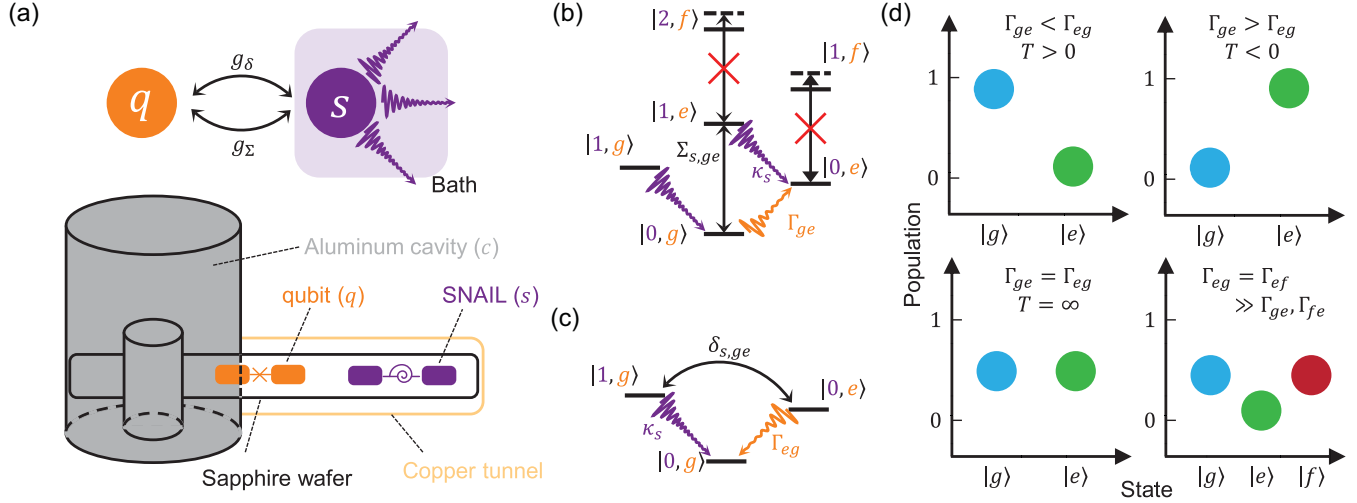


FIG. 1. Experiment concept and schematic. (a) Experiment setup. The qubit and SNAIL are fabricated on the same sapphire chip housed in a two-part enclosure comprising an aluminum cavity and a copper tunnel (see Supplemental Material [35] for the device picture). External parametric drives establish different interactions between the qubit and SNAIL at rates g_δ and g_Σ . (b), (c) Level diagrams of the system. (b) Heating: the Σ pump drives the system from $|0, g\rangle$ to $|1, e\rangle$, which rapidly decays to $|0, e\rangle$ due to the loss of the SNAIL, resulting in an effective “heating” process with rate Γ_{ge}^p . Because of the qubit anharmonicity and the frequency selectivity of the parametric process, this pump does not drive the transitions between other levels. (c) Cooling: the δ pump drives the system from $|0, e\rangle$ to $|1, g\rangle$, which rapidly decays to $|0, g\rangle$ due to the loss of the SNAIL, resulting in an effective “cooling” process with rate Γ_{eg}^p . (d) Artificial temperature control with parametric drives. Top left: positive temperature with $\Gamma_{ge} > \Gamma_{eg}$. Top right: negative temperature with $\Gamma_{eg} < \Gamma_{ge}$. Bottom left: infinite temperature with matched rates. Bottom right: nonthermal states where the transmon relaxes away from $|e\rangle$ without preference between $|g\rangle$ and $|f\rangle$.

whose dissipation rates are engineered by tuning the strengths of two parametric processes. We show that in the steady-state limit, the qubit reaches the Gibbs state: $\rho = e^{-\beta H} / \text{Tr}(e^{-\beta H})$, where $\beta = 1/k_B T$ is the inverse temperature set by the dissipation rates, and k_B is the Boltzmann constant. We verify this behavior by stabilizing the qubit to various steady states with different population distributions, including a population-inverted (negative-temperature) state, which cannot be achieved by coupling to a natural bath at positive temperatures.

Furthermore, the anharmonic, multilevel nature of the transmon allows us to go beyond the simple two-level model. We extend our parametric bath engineering technique to the third energy level of the transmon, creating a nontrivial steady state beyond the Gibbs state. Accessing the higher levels of the transmon extends its utility as a tunable bath when coupling to a larger quantum simulator, and also provides a potential tool to enter the regime of Gentile statistics [28–30], a generalization of Fermi-Dirac and Bose-Einstein statistics where the mode’s maximum occupation number (here, photons in a qubit) can be set between one (Fermions) and infinity (Bosons).

Model—Inspired by a proposal from Hafezi *et al.* [31], a temperature-tunable bath for a transmon qubit can be realized by parametrically coupling it to another lossy mode. Our model [Fig. 1(a)] consists of a transmon [32] capacitively coupled to a SNAIL mode [33], which acts as both the mixing element and the lossy mode. The qubit is

also coupled to a 3D cavity for dispersive measurement [34]. While realized in a cQED system, our model is platform-independent.

Central to our protocol are the following parametric interactions between the qubit and the SNAIL: two-mode squeezing and mode conversion, with Hamiltonians [50–52]

$$H_{\text{tms}} = g_\Sigma (s^\dagger q^\dagger + \text{H.c.}), \quad (1)$$

$$H_{\text{bs}} = g_\delta (s^\dagger q + \text{H.c.}), \quad (2)$$

where s and q are the annihilation operators of the SNAIL and qubit, g_Σ and g_δ are the interaction strengths of the corresponding processes. The subscripts Σ and δ reflect the sum and difference pump frequencies required to drive the respective processes, i.e., $\omega_p = \omega_s \pm \omega_q$ (here $\omega_{s(q)}$ are the frequency of the SNAIL(qubit)).

However, due to the transmon’s anharmonicity and the frequency selectivity of the parametric processes, each pump frequency addresses only a specific transition [Fig. 1(b)]. Thus, we rewrite Eqs. (1) and (2) for individual qubit levels. For two adjacent qubit levels $|i\rangle$ and $|i+1\rangle$, the respective interaction Hamiltonian terms are given by

$$H_{\Sigma, i, i+1} = g_{\Sigma, i, i+1} s^\dagger |i+1\rangle \langle i| + \text{H.c.}, \quad (3)$$

$$H_{\delta, i, i+1} = g_{\delta, i, i+1} s^\dagger |i\rangle \langle i+1| + \text{H.c.}, \quad (4)$$

where $g_{\Sigma_{s,i+1}}$ and $g_{\delta_{s,i+1}}$ are the effective coupling rates of the two individual processes (see Supplemental Material [35]), with the corresponding pump frequencies: $\omega_{\Sigma_{s,i+1}} = \omega_s + \omega_{i,i+1}$ and $\omega_{\delta_{s,i+1}} = \omega_s - \omega_{i,i+1}$ [$\omega_{i,i+1}$ is the frequency between the i th and $(i+1)$ th level of the qubit].

These parametric couplings, together with the lossy mode, enable the “heating” and “cooling” of the qubit. A pump at $\omega_p = \omega_{\Sigma_{s,ge}}$ coherently brings the system from $|0, g\rangle$ to $|1, e\rangle$, which will fall into $|0, e\rangle$ when the SNAIL is sufficiently lossy [i.e., the SNAIL bandwidth $\kappa_s \gg g_{\Sigma_{s,ge}}, 1/T_1^q$ (qubit energy relaxation rate)]. This effectively “heats” the qubit [Fig. 1(b)]. Similarly, a pump at $\omega_p = \omega_{\delta_{s,ge}}$ effectively “cools” the transmon to its ground state [Fig. 1(c)]. The tunability of this artificial temperature is realized by applying both pumps simultaneously. For a transmon treated as a two-level system, we show that by controlling the effective rates of the two processes, we can thermalize it to an arbitrary equilibrium state [Fig. 1(d)]. The density matrix of the equilibrium state can be described by a canonical ensemble,

$$\rho_q = e^{-\beta_q \hbar \omega_q \hat{\sigma}_q^z / 2} / \text{Tr}(e^{-\beta_q \hbar \omega_q \hat{\sigma}_q^z / 2}), \quad (5)$$

where $\hat{\sigma}_q^z$ is the Pauli for the qubit, and the inverse temperature β_q is given by

$$\beta_q = \frac{1}{\omega_q} \ln \left(\frac{g_{\Sigma}^2 + g_{\delta}^2 e^{\beta_B \omega_s}}{g_{\Sigma}^2 e^{\beta_B \omega_s} + g_{\delta}^2} \right). \quad (6)$$

This is a Gibbs state for a single qubit, whose temperature is a function of the controllable interaction strengths, g_{Σ} and g_{δ} . In principle, this bath engineering technique can be extended to prepare the Gibbs state at an arbitrary temperature for a more generalized system (see Supplemental Material [35]).

Before moving on to the experimental results, we note that since each decay event is heralded by a photon in the SNAIL mode, photon distinguishability must be considered when multiple processes occur simultaneously. In this Letter, we focus on the case of distinguishable photons; i.e., each “heating”/“cooling” event is uncorrelated with the others (see Supplemental Material Sec. III [35] for a detailed discussion).

Experiment results—Our experimental scheme is shown in the lower panel of Fig. 1(a), with circuit parameters listed in Supplemental Material [35]. A traveling wave parametric amplifier (TWPA) [36] is used for single-shot readout and transmon state differentiation. We perform a three-part division on the readout data and label all states from $|f\rangle$ and beyond as $|f^+\rangle$.

We begin with the implementation of the “heating” and “cooling” processes. In the “heating” experiment, the qubit is initialized in $|g\rangle$, then driven by the $\Sigma_{s,ge}$ pump with varying strength and duration before measurement [Fig. 2(a)]. The

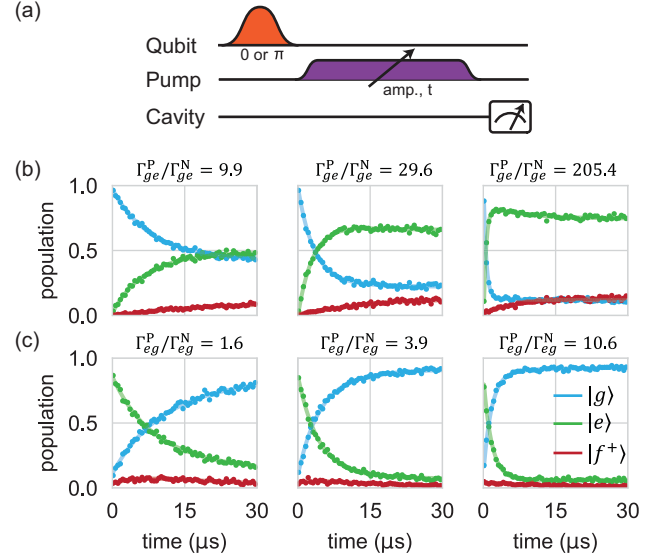


FIG. 2. Heating and cooling with a single pump. (a) Pulse sequence: the qubit is initialized in $|g\rangle$ (heating) and $|e\rangle$ (cooling), followed by pumps with varying strength and duration, and then measured. (b) Qubit state populations (blue $|g\rangle$, green $|e\rangle$ and red $|f^+\rangle$) as a function of pump duration in the heating process with increasing pump strength $g_{\Sigma_{s,ge}}$, from left to right. (c) Same as (b), but for cooling with increasing pump strength $g_{\delta_{s,ge}}$. Solid lines are fits to a semiclassical model used to extract transition rates (see Supplemental Material [35]).

qubit state population evolution is shown in Fig. 2(b), with increasing pump strength from left to right that results in an increase of the heating rate ($\Gamma^p_{ge}/\Gamma^N_{ge} = 9.9, 29.6, 205.4$ respectively; here p and N indicate the pumped and natural transition rates), as expected. The rates are extracted by fitting the data to the theoretical model (solid lines; see Supplemental Material [35]). The nontrivial $|f^+\rangle$ state population is attributed to the qubit’s natural thermal excitation from $|e\rangle$ to $|f\rangle$, and a sufficiently strong pump may cause extra parametric heating due to its finite spectral weight on the transmon’s higher transitions. We emphasize that for strong heating drives, the qubit population inverts, rather than spreading to the higher states—a scenario typically achieved by contacting it with a hot bath.

In the “cooling” experiment, the qubit is initialized in $|e\rangle$, then driven by the $\delta_{s,ge}$ pump with varying strength and duration before measurement [Fig. 2(a)]. As shown in Fig. 2(c), the qubit thermalizes to its equilibrium state faster as the pump strength increases from left to right ($\Gamma^p_{eg}/\Gamma^N_{eg} = 1.6, 3.9, 10.6$, respectively). In contrast to Ref. [25] where both the qubit and the cavity are driven to force the qubit fully into the $|g\rangle$ state, our method allows us to determine the $|g\rangle$ probability and only requires one drive on the SNAIL. There is no rise in the $|f^+\rangle$ state, as the qubit is actively pushed to its ground state, and no parametric processes are present to excite it to $|f^+\rangle$ near the $\delta_{s,ge}$ pump frequency.

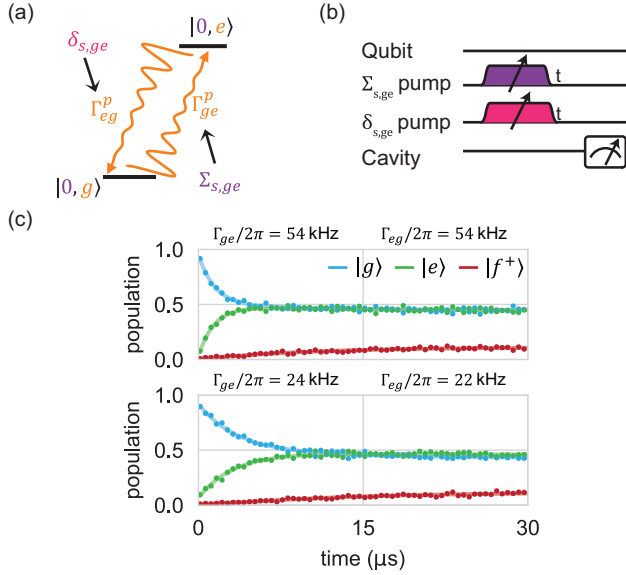


FIG. 3. Heating and cooling balance. (a) Schematic: simultaneous $\Sigma_{s,ge}$ and $\delta_{s,ge}$ pumps drive the system to a 50% $|g\rangle$ -50% $|e\rangle$ mixed state by balancing the transition rates. (b) Pulse sequence: the qubit is initialized in $|g\rangle$, followed by both pumps with varying durations, and then measured. (c) Qubit state populations as a function of pump duration for two different pump settings. The qubit thermalizes to the same nearly equal thermal mixture of $|g\rangle$ and $|e\rangle$ states, but with different rates.

It is worth noting that this “cooling” process is equivalent to the qubit reset problem, which has received sustained attention over the years (see, for example, Refs. [25,37,38,53,54]). In principle, our scheme can reset the qubit to an arbitrarily low temperature. However, in practice, the achievable temperature is limited by the coldness of the lossy mode and its frequency relative to the qubit (see Supplemental Material [35] for a detailed discussion).

The single pump experiments demonstrate our control over each parametric process by establishing equilibrium states with different effective temperatures. The steady state population distribution in the longtime limit is determined by the ratio of transition rates between different levels. We believe the effects of decay or leakage to and from a higher transmon level are minimal, as the data can be well described by our model without the need to introduce additional levels. However, we acknowledge that higher levels cannot be clearly distinguished in the current device. A conclusive answer to this question could be obtained in future experiments with optimized readout parameters. We also note that the difference in the final distribution for different single pump strengths does not conflict with Eq. (5) for the case with $g_\Sigma = 0$ or $g_\delta = 0$, as the qubit also weakly couples to the environment in practice.

Full bath control is achieved by applying both $\Sigma_{s,ge}$ and $\delta_{s,ge}$ drives simultaneously, enabling two competing

processes with rates set by their respective drive strengths [Fig. 3(a)]. To demonstrate this effect, we first initialize the qubit in $|g\rangle$, then apply pumps with matched “heating” and “cooling” rates ($\Gamma_{ge}^p/\Gamma_{eg}^p \sim 1$), before the final measurement of the qubit [Fig. 3(b)]. The qubit population as a function of the pump duration is shown in Fig. 3(c). The qubit thermalizes to the same mixed 50% $|g\rangle$ -50% $|e\rangle$ state (up to the residual population in the $|f^+\rangle$ state) at different rates. Moreover, by adjusting the rate ratio, we are able to set the transmon to a Gibbs state at an arbitrary temperature (see Supplemental Material [35]).

Finally, we extend our protocol beyond the two-level manifold to explore broader applications. For instance, a single “cooling” pump between $|f\rangle$ and $|e\rangle$ could potentially be used to reset the qubit to the computational subspace from leakage induced by measurement [55–58] and gate operations [59]. Similar to the two-level case, the Gibbs state of the three-level system with a tunable artificial temperature can be realized by properly driving the transitions between different transmon energy levels (see Supplemental Material [35]). Moreover, the flexibility of parametric control enables us to go beyond the Gibbs state. By applying external drives, we can engineer the diagonal elements of the steady-state density matrix, allowing the system to reach more exotic states with diverse population distributions. Furthermore, if the pump tones are engineered to be mutually coherent, the off-diagonal elements can also be manipulated, enabling applications such as qubit longitudinal readout [39] and synthetic squeezing [60].

As a demonstration of the exotic steady state, we stabilize the transmon in a mixture of 50% $|g\rangle$ and 50% $|f\rangle$ [Fig. 1(d), bottom right]. We simultaneously pump the $|e\rangle \rightarrow |g\rangle$ cooling process at $\omega_{\delta_{s,ge}}$ and the $|e\rangle \rightarrow |f\rangle$ heating process at $\omega_{\Sigma_{s,ef}}$ with matched transition rates away from $|e\rangle$ [Fig. 4(a)]. Together with the qubit’s natural excitation and decay, these drives establish the target mixed state [Fig. 4(c)]. These natural processes also account for the residual $\sim 15\%$ population in the $|e\rangle$ state. Although the simple semiclassical model can predict the steady-state population distribution, it does not fully capture the dynamics of the measured data (see Supplemental Material [35]); we attribute this to the residual coherence between the two pump tones and the finite transmon thermalization rate elevated by the pumps. By including these factors, we are able to numerically simulate the system’s behavior using QuTiP (see Supplemental Material [35]). Further suppression of the $|e\rangle$ population can be achieved by using a transmon with state-of-the-art coherence to reduce the natural decay and excitation rates, and by increasing its anharmonicity and optimizing the pulse shape to minimize undesired pump-induced transitions.

Discussion and conclusion—We present an engineered bath with a tunable temperature ranging from negative to positive values. A key application of this bath is the Gibbs

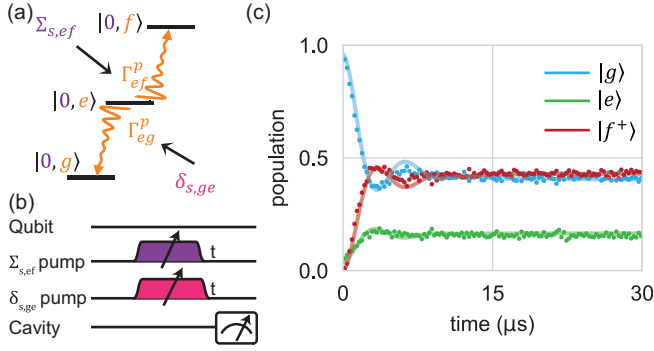


FIG. 4. Bath engineering for a three-level qubit. (a) Schematic: simultaneous $\delta_{s,ge}$ and $\Sigma_{s,ef}$ pumps drive the system to a 50% $|g\rangle$ -50% $|f\rangle$ mixed state by balancing the transition rates. (b) Pulse sequence: the qubit is initialized in $|g\rangle$, followed by both pumps with varying durations, and then measured. (c) Qubit state populations as a function of time under both pumps. The $|e\rangle$ population remains nonzero due to qubit decay from $|f^+\rangle$ and thermal excitations from $|g\rangle$.

state preparation, as we demonstrate with the transmon. Our approach can be extended to more general systems: given a bosonic system Hamiltonian H and a set of its eigenoperators A_i , we show that the steady state of the system will be the Gibbs state: $\rho = e^{-\beta H} / \text{Tr}(e^{-\beta H})$, if the system parametrically couples to lossy channels via its eigenoperators through the interactions similar to those in Eqs. (1) and (2) (see Supplemental Material [35]).

The ability to adjust bath temperature and the qubit's thermalization rate offers an opportunity for quantum simulation schemes across varied parameter regimes. For example, when simulating energy transport in a spin chain [61–63], our system can be attached to different spin sites with different effective temperatures, providing tunable boundary conditions for the simulator. In particular, with the temperature tuning range from negative to positive, this bath can be viewed as a photon source/sink. When combined with a large qubit array, it enables various simulations of quantum many-body systems, such as the investigation of Mott insulators [64,65], the study of entanglement and out-of-equilibrium steady states [66,67]. Moreover, access to the transmon's higher energy levels opens avenues for experimental studies of Gentile statistics [28–30] and the simulation of condensed matter systems, such as magnons and spin waves [68].

Going beyond the applications in quantum simulations, our system can be viewed as a single-cell open quantum battery from a quantum energy storage perspective [69]. The charging and discharging processes of the battery are enabled by engineered dissipation. Extending this setup to a multicell quantum battery by increasing the number of transmons is straightforward providing a platform to explore the charging power enhancement in collective charging schemes [70,71]. Additionally, the “heating” process forms a traditional lambda structure with a trapped

excited state, offering a pathway to generate highly coherent microwave signals [72].

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Data availability—The data that support the findings of this article are openly available [73].

- [1] R. P. Feynman, *Int. J. Theor. Phys.* **21**, 467 (1982).
- [2] M. Lewenstein, A. Sanpera, V. Ahufinger, B. Damski, A. Sen(De), and U. Sen, *Adv. Phys.* **56**, 243 (2007).
- [3] M. J. P. Gingras and P. A. McClarty, *Rep. Prog. Phys.* **77**, 056501 (2014).
- [4] M. Reiher, N. Wiebe, K. M. Svore, D. Wecker, and M. Troyer, *Proc. Natl. Acad. Sci. U.S.A.* **114**, 7555 (2017).
- [5] S. Krinner, T. Esslinger, and J.-P. Brantut, *J. Phys. Condens. Matter* **29**, 343003 (2017).
- [6] F. Giazotto, T. T. Heikkilä, A. Luukanen, A. M. Savin, and J. P. Pekola, *Rev. Mod. Phys.* **78**, 217 (2006).
- [7] D. A. Abanin, E. Altman, I. Bloch, and M. Serbyn, *Rev. Mod. Phys.* **91**, 021001 (2019).
- [8] F. Harper, R. Roy, M. S. Rudner, and S. Sondhi, *Annu. Rev. Condens. Matter Phys.* **11**, 345 (2020).
- [9] B. Swingle, *Nat. Phys.* **14**, 988 (2018).
- [10] A. Blais, A. L. Grimsmo, S. M. Girvin, and A. Wallraff, *Rev. Mod. Phys.* **93**, 025005 (2021).
- [11] C. Monroe, W. C. Campbell, L.-M. Duan, Z.-X. Gong, A. V. Gorshkov, P. W. Hess, R. Islam, K. Kim, N. M. Linke, G. Pagano, P. Richerme, C. Senko, and N. Y. Yao, *Rev. Mod. Phys.* **93**, 025001 (2021).
- [12] S. Slussarenko and G. J. Pryde, *Appl. Phys. Rev.* **6**, 041303 (2019).
- [13] L. Henriet, L. Beguin, A. Signoles, T. Lahaye, A. Browaeys, G.-O. Reymond, and C. Jurczak, *Quantum* **4**, 327 (2020).
- [14] P. M. Harrington, E. J. Mueller, and K. W. Murch, *Nat. Rev. Phys.* **4**, 660 (2022).
- [15] A. Shabani and H. Neven, *Phys. Rev. A* **94**, 052301 (2016).
- [16] I. M. Georgescu, S. Ashhab, and F. Nori, *Rev. Mod. Phys.* **86**, 153 (2014).
- [17] A. S. Trushechkin, M. Merkli, J. D. Cresser, and J. Anders, *AVS Quantum Sci.* **4**, 012301 (2022).

- [18] *Quantum Annealing and Other Optimization Methods*, edited by A. Das *et al.*, Lecture Notes in Physics Vol. 679 (Springer, Berlin, Heidelberg, 2005).
- [19] E. M. Purcell and R. V. Pound, *Phys. Rev.* **81**, 279 (1951).
- [20] A. S. Oja and O. V. Lounasmaa, *Rev. Mod. Phys.* **69**, 1 (1997).
- [21] P. Medley, D. M. Weld, H. Miyake, D. E. Pritchard, and W. Ketterle, *Phys. Rev. Lett.* **106**, 195301 (2011).
- [22] S. Braun, J. P. Ronzheimer, M. Schreiber, S. S. Hodgman, T. Rom, I. Bloch, and U. Schneider, *Science* **339**, 52 (2013).
- [23] R. Sagastizabal, S. P. Premaratne, B. A. Klaver, M. A. Rol, V. Negîrmeac, M. S. Moreira, X. Zou, S. Johri, N. Muthusubramanian, M. Beekman, C. Zachariadis, V. P. Ostroukh, N. Haider, A. Bruno, A. Y. Matsuura, and L. DiCarlo, *npj Quantum Inf.* **7**, 130 (2021).
- [24] T. F. Mörstedt, W. S. Teixeira, A. Viitanen, H. Kivijärvi, M. Tiiri, M. Rasola, A. M. Gunyho, S. Kundu, L. Lattier, V. Vadimov, G. Catelani, V. Sevriuk, J. Heinsoo, J. Rabinä, J. Ankerhold, and M. Möttönen, *Phys. Rev. Res.* **7**, L042010 (2025).
- [25] K. Geerlings, Z. Leghtas, I. M. Pop, S. Shankar, L. Frunzio, R. J. Schoelkopf, M. Mirrahimi, and M. H. Devoret, *Phys. Rev. Lett.* **110**, 120501 (2013).
- [26] Y. Lu, S. Chakram, N. Leung, N. Earnest, R. K. Naik, Z. Huang, P. Groszkowski, E. Kapit, J. Koch, and D. I. Schuster, *Phys. Rev. Lett.* **119**, 150502 (2017).
- [27] R. Ma, B. Saxberg, C. Owens, N. Leung, Y. Lu, J. Simon, and D. I. Schuster, *Nature (London)* **566**, 51 (2019).
- [28] G. Gentile J., *Il Nuovo Cimento* **17**, 493 (1940).
- [29] W.-S. Dai and M. Xie, *Ann. Phys. (Amsterdam)* **309**, 295 (2004).
- [30] C. S. Srivatsan, M. V. N. Murthy, and R. K. Bhaduri, *Pramana* **66**, 485 (2006).
- [31] M. Hafezi, P. Adhikari, and J. M. Taylor, *Phys. Rev. B* **92**, 174305 (2015).
- [32] J. Koch, T. M. Yu, J. Gambetta, A. A. Houck, D. I. Schuster, J. Majer, A. Blais, M. H. Devoret, S. M. Girvin, and R. J. Schoelkopf, *Phys. Rev. A* **76**, 042319 (2007).
- [33] N. E. Frattini, U. Vool, S. Shankar, A. Narla, K. M. Sliwa, and M. H. Devoret, *Appl. Phys. Lett.* **110**, 222603 (2017).
- [34] A. Blais, R.-S. Huang, A. Wallraff, S. M. Girvin, and R. J. Schoelkopf, *Phys. Rev. A* **69**, 062320 (2004).
- [35] See Supplement Material at <http://link.aps.org/supplemental/10.1103/mv4g-z64p> for additional information about relevant system parameters, a detailed description of the experimental setup, and a comprehensive theoretical model, which includes Refs. [10,15,34,36–49].
- [36] C. Macklin, K. O’Brien, D. Hover, M. E. Schwartz, V. Bolkhovskiy, X. Zhang, W. D. Oliver, and I. Siddiqi, *Science* **350**, 307 (2015).
- [37] Y. Zhou, Z. Zhang, Z. Yin, S. Huai, X. Gu, X. Xu, J. Allcock, F. Liu, G. Xi, Q. Yu, H. Zhang, M. Zhang, H. Li, X. Song, Z. Wang, D. Zheng, S. An, Y. Zheng, and S. Zhang, *Nat. Commun.* **12**, 5924 (2021).
- [38] Y. Sunada, S. Kono, J. Ilves, S. Tamate, T. Sugiyama, Y. Tabuchi, and Y. Nakamura, *Phys. Rev. Appl.* **17**, 044016 (2022).
- [39] C. Zhou, Superconducting quantum routers, modules, gates, and measurements based on charge-pumped parametric interactions, Ph.D. thesis, University of Pittsburgh, 2024.
- [40] V. Sivak, N. Frattini, V. Joshi, A. Lingenfelter, S. Shankar, and M. Devoret, *Phys. Rev. Appl.* **11**, 054060 (2019).
- [41] M. Reagor, H. Paik, G. Catelani, L. Sun, C. Axline, E. Holland, I. M. Pop, N. A. Masluk, T. Brecht, L. Frunzio, M. H. Devoret, L. Glazman, and R. J. Schoelkopf, *Appl. Phys. Lett.* **102**, 192604 (2013).
- [42] C. Axline, M. Reagor, R. Heeres, P. Reinhold, C. Wang, K. Shain, W. Pfaff, Y. Chu, L. Frunzio, and R. J. Schoelkopf, *Appl. Phys. Lett.* **109**, 042601 (2016).
- [43] J. M. Gertler, B. Baker, J. Li, S. Shirol, J. Koch, and C. Wang, *Nature (London)* **590**, 243 (2021).
- [44] L. Stefanazzi, K. Treptow, N. Wilcer, C. Stoughton, C. Bradford, S. Uemura, S. Zorzetti, S. Montella, G. Cencelo, S. Sussman, A. Houck, S. Saxena, H. Araldi, A. Agrawal, H. Zhang, C. Ding, and D. I. Schuster, *Rev. Sci. Instrum.* **93**, 044709 (2022).
- [45] A. A. Clerk, M. H. Devoret, S. M. Girvin, F. Marquardt, and R. J. Schoelkopf, *Rev. Mod. Phys.* **82**, 1155 (2010).
- [46] N. E. Frattini, V. V. Sivak, A. Lingenfelter, S. Shankar, and M. H. Devoret, *Phys. Rev. Appl.* **10**, 054020 (2018).
- [47] Z. Wang, S. Shankar, Z. Mineev, P. Campagne-Ibarcq, A. Narla, and M. Devoret, *Phys. Rev. Appl.* **11**, 014031 (2019).
- [48] T. Connolly, P. D. Kurilovich, V. D. Kurilovich, C. G. L. Böttcher, S. Hazra, W. Dai, A. Z. Ding, V. R. Joshi, H. Nho, S. Diamond, D. K. Weiss, V. Fatemi, L. Frunzio, L. I. Glazman, and M. H. Devoret, *arXiv:2506.05306*.
- [49] A. Frigerio, *Commun. Math. Phys.* **63**, 269 (1978).
- [50] A. Roy and M. Devoret, *C. R. Phys.* **17**, 740 (2016).
- [51] A. Kamal, A. Marblestone, and M. Devoret, *Phys. Rev. B* **79**, 184301 (2009).
- [52] N. Bergeal, F. Schackert, M. Metcalfe, R. Vijay, V. E. Manucharyan, L. Frunzio, D. E. Prober, R. J. Schoelkopf, S. M. Girvin, and M. H. Devoret, *Nature (London)* **465**, 64 (2010).
- [53] P. Magnard, P. Kurpiers, B. Royer, T. Walter, J.-C. Besse, S. Gasparinetti, M. Pechal, J. Heinsoo, S. Storz, A. Blais, and A. Wallraff, *Phys. Rev. Lett.* **121**, 060502 (2018).
- [54] M. A. Aamir, P. Jamet Suria, J. A. Marín Guzmán, C. Castillo-Moreno, J. M. Epstein, N. Yünger Halpern, and S. Gasparinetti, *Nat. Phys.* **21**, 318 (2025).
- [55] M. Khezri, A. Opremcak, Z. Chen, K. C. Miao, M. McEwen, A. Bengtsson, T. White, O. Naaman, D. Sank, A. N. Korotkov, Y. Chen, and V. Smelyanskiy, *Phys. Rev. Appl.* **20**, 054008 (2023).
- [56] M. F. Dumas, B. Groleau-Paré, A. McDonald, M. H. Muñoz-Arias, C. Lledó, B. D’Anjou, and A. Blais, *Phys. Rev. X* **14**, 041023 (2024).
- [57] W. Dai, S. Hazra, D. K. Weiss, P. D. Kurilovich, T. Connolly, H. K. Babla, S. Singh, V. R. Joshi, A. Z. Ding, P. D. Parakh, J. Venkatraman, X. Xiao, L. Frunzio, and M. H. Devoret, *arXiv:2506.24070*.
- [58] P. D. Kurilovich, T. Connolly, C. G. L. Böttcher, D. K. Weiss, S. Hazra, V. R. Joshi, A. Z. Ding, H. Nho, S. Diamond, V. D. Kurilovich, W. Dai, V. Fatemi, L. Frunzio, L. I. Glazman, and M. H. Devoret, *arXiv:2501.09161*.
- [59] T. Thorbeck, A. McDonald, O. Lanes, J. Blair, G. Keefe, A. A. Stabile, B. Royer, L. C. G. Govia, and A. Blais, *arXiv:2411.04101*.

- [60] L. C. G. Govia, A. Lingenfelter, and A. A. Clerk, *Phys. Rev. Res.* **4**, 023010 (2022).
- [61] C. Mejia-Monasterio and H. Wichterich, *Eur. Phys. J. Special Topics* **151**, 113 (2007).
- [62] C. X. Yu, L.-A. Wu, and D. Segal, *J. Chem. Phys.* **135**, 234508 (2011).
- [63] D. Roy, A. Dhar, H. Spohn, and M. Kulkarni, *Phys. Rev. E* **110**, 044110 (2024).
- [64] S.-l. Ma, X.-k. Li, X.-y. Liu, J.-k. Xie, and F.-l. Li, *Phys. Rev. A* **99**, 042336 (2019).
- [65] M. J. Mark, E. Haller, K. Lauber, J. G. Danzl, A. Janisch, H. P. Büchler, A. J. Daley, and H.-C. Nägerl, *Phys. Rev. Lett.* **108**, 215302 (2012).
- [66] S. Dutta and N. R. Cooper, *Phys. Rev. Res.* **3**, L012016 (2021).
- [67] B. Du, R. Suresh, S. López, J. Cadiente, and R. Ma, *Phys. Rev. Lett.* **133**, 060601 (2024).
- [68] Y. Shen, C.-C. Zhou, and Y.-Z. Chen, *Physica (Amsterdam)* **596A**, 127223 (2022).
- [69] F. Barra, *Phys. Rev. Lett.* **122**, 210601 (2019).
- [70] D. Ferraro, M. Campisi, G. M. Andolina, V. Pellegrini, and M. Polini, *Phys. Rev. Lett.* **120**, 117702 (2018).
- [71] F. Campaioli, S. Gherardini, J. Q. Quach, M. Polini, and G. M. Andolina, *Rev. Mod. Phys.* **96**, 031001 (2024).
- [72] C. Liu, M. Mucci, X. Cao, M. V. G. Dutt, M. Hatridge, and D. Pekker, *Nat. Commun.* **12**, 5620 (2021).
- [73] X. Cao, M. Mucci, G. Liu, D. Pekker, and M. Hatridge, Data and processing code for “Engineering a multilevel bath for transmons with three-wave mixing and parametric drives” (2025), [10.7910/DVN/DPMD3](https://doi.org/10.7910/DVN/DPMD3).