

Coherence-Induced Deep Thermalization Transition in Random Permutation Quantum Dynamics

Chang Liu¹, Matteo Ippoliti², and Wen Wei Ho^{1,3,*}

¹*Department of Physics, National University of Singapore, Singapore 117551*

²*Department of Physics, The University of Texas at Austin, Austin, Texas 78712, USA*

³*Centre for Quantum Technologies, National University of Singapore, Singapore 117543*



(Received 15 November 2025; accepted 18 February 2026; published 12 March 2026)

We report a phase transition in the projected ensemble—the collection of postmeasurement wave functions of a local subsystem obtained by measuring its complement. The transition emerges in systems undergoing random permutation dynamics, a type of quantum time evolution wherein computational basis states are shuffled without creating superpositions. It separates a phase exhibiting deep thermalization, where the projected ensemble is distributed over Hilbert space in a maximally entropic fashion (Haar random), from a phase where it is minimally entropic (“classical bit-string ensemble”). Crucially, this deep thermalization transition is invisible to the subsystem’s density matrix, which always exhibits thermalization to infinite temperature across the phase diagram. Through a combination of analytical arguments and numerical simulations, we show that the transition is tuned by the total amount of *coherence* injected by the input state and the measurement basis, and is exhibited robustly across different microscopic models. Our findings represent a novel form of ergodicity-breaking universality in quantum many-body dynamics, characterized not by a failure of regular thermalization, but rather by a failure of deep thermalization.

DOI: 10.1103/mjgs-y2z9

Introduction—Understanding universal behaviors of complex quantum systems out of equilibrium is a central goal of modern physics, with implications in statistical mechanics [1,2], condensed matter [3–5], high energy physics [6–8], and quantum information science [9,10]. Recently, a new universal feature was discovered in the dynamics of quantum many-body systems: the collection of conditional states of a local subsystem obtained by measuring its environment, known as the *projected ensemble* (PE) [11–13], was typically found to approach universal distributions at late times in dynamics, which satisfy generalized maximum-entropy principles [14–16]. For example, in spin systems, one obtains the Haar ensemble (the uniform distribution over the Hilbert space) [12,13,17,18] or the Scrooge ensemble (a deformation thereof) if conservation laws are present [12,14,19,20]; analogous maximally entropic distributions arise also for systems of Gaussian fermions and bosons [15,21,22]. The emergence of such universal ergodic ensembles has been dubbed *deep thermalization* [11–18,21–28], as it constitutes a stronger, more fine-grained notion of quantum

equilibration going beyond “regular” thermalization of the reduced density matrix to the Gibbs state. The inception of deep thermalization has led to a flurry of research activity, including studies on the timescales of convergence [17,18], generalizations to open quantum systems [29,30], connections to computational complexity and cryptography [28,31], as well as experimental realizations [11].

In this Letter, we uncover a striking exception to the aforementioned maximum-entropy paradigm within a class of quantum dynamics. Namely, we report a sharp transition in the PE separating a phase that exhibits deep thermalization from a phase that robustly fails to do so. The transition is completely invisible to the reduced density matrix: both phases always appear thermal at infinite temperature. Our findings thus represent a novel form of “deep” ergodicity breaking in quantum many-body systems, detectable not at the level of local expectation values, but rather in higher-moment observables.

Concretely, we study *random permutation dynamics* (RPD) [Fig. 1(a)], a type of quantum time evolution where computational basis states are randomly permuted [32–35]. The transition is tuned by the amount of *coherence* (the information-theoretic resource of superposition [36–38]) in the system, which is preserved under RPD and thus depends only on the choice of input state and final measurement basis. The high- and low-coherence phases yield, respectively, the Haar ensemble $\mathcal{E}_{\text{Haar}}$, wherein states are uniformly distributed in Hilbert space, and the “classical bit-string ensemble” $\mathcal{E}_{\text{CI}}$, wherein states are

*Contact author: wenweiho@nus.edu.sg

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International license](#). Further distribution of this work must maintain attribution to the author(s) and the published article’s title, journal citation, and DOI.

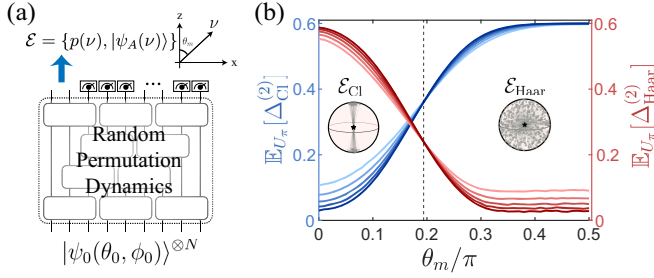


FIG. 1. (a) PE formed under random permutation dynamics. A single global random permutation unitary (dotted box) models the behavior of a deep quantum circuit made of local random permutation gates (brickwork circuit). For the tilted-basis model, input states and measurement basis are uniform product states, specified by Bloch angles (θ_0, ϕ_0) and (θ_m, ϕ_m) , respectively. (b) $k = 2$ trace distances of the PE (for $N_A = 2$) from the classical bit-string ensemble $\mathcal{E}_{\text{CI}}$ and Haar ensemble $\mathcal{E}_{\text{Haar}}$, generated from the tilted-basis model with $\theta_0 = \phi_0 = \pi/4$, $\phi_m = 0$, and variable θ_m . Different intensities indicate different system sizes $N = 16, 18, 20, 22, 24$ (lighter to darker). There is a common crossing at $\theta_m^* \approx 0.193\pi$ for both distances across all system sizes, signaling a singular change of the limiting PE.

distributed uniformly but only over the computational basis [Fig. 1(b)]; these are respectively the maximum- and minimum-entropy PEs compatible with an infinite-temperature density matrix. We establish the universality of this phenomenon across two different microscopic models theoretically and numerically; in particular, one of the models allows an analytical determination of the phase boundary. More generally, we argue the transition occurs generically for typical scrambling quantum dynamics, as long as it is coherence preserving. Lastly, we discuss generalizations of our analysis to transitions of other resources within the PE, like imaginarity [39,40], magic [41–43], or non-Gaussianity [44,45], suggesting the possibility of yet more novel universality classes in quantum many-body dynamics.

Projected ensemble from random permutation dynamics—Consider an N -qubit system bipartitioned into A and B (comprising N_A and N_B qubits, respectively) with initial state $|\Psi_0\rangle$. This state is acted upon by a randomly chosen global permutation unitary U_π , to yield a final state $|\Psi_\pi\rangle = U_\pi|\Psi_0\rangle$, which we denote as a *random permutation state* (RPS). Specifically, U_π is a unitary that permutes computational basis states $|z\rangle$ (with $z \in \{0, 1\}^N$) for some uniformly randomly chosen permutation π from the symmetric group S_{2^N} , i.e., $U_\pi|z\rangle = |\pi(z)\rangle$. The application of a single global permutation unitary can be understood as modeling the late-time behavior of quantum circuits composed of local permutation unitaries [see, e.g., Fig. 1(a)], and indeed in the End Matter, we show that such a simplification does not affect the physics to be discussed. RPD, long studied in the classical literature [46–48], has recently found applications in the quantum setting. This is because despite its “classical” action on bit-string states, it

can nevertheless reproduce general aspects of quantum dynamics like growth of state and operator entanglement [33,34,49–51], decay of out-of-time order correlators [35,52], and formation of state designs [53,54] when evolving from generic initial states.

Our aim is to characterize the limiting form of the PE of a fixed local subsystem A generated from a typical RPS given a fixed choice of initial state $|\Psi_0\rangle$, and measurements of the complementary region B in various bases. Here B is interpreted as the “bath,” and assumed much larger than A . We consider two models: (i) the “tilted-basis model,” with initial states taken as uniform product states $|\Psi_0\rangle = [\cos(\theta_0/2)|0\rangle + e^{i\phi_0}\sin(\theta_0/2)|1\rangle]^{\otimes N}$ and measurements also along a uniform local basis but with direction $\hat{n} = (\sin\theta_m\cos\phi_m, \sin\theta_m\sin\phi_m, \cos\theta_m)$. We henceforth set $\phi_m = 0$ for simplicity; θ_m thus continuously tunes the measurement basis from the z - to the x axis. (ii) The “mixed-basis model,” with initial states $|\Psi_0\rangle := |0\rangle^{\otimes(1-\alpha_0)N} \otimes |Y_+\rangle^{\otimes\alpha_0N}$ and $|Y_+\rangle = (1/\sqrt{2})(|0\rangle + i|1\rangle)$, and measurement schemes in which $(1-\alpha_m)N_B$ qubits ($\alpha_m N_B$ qubits) are measured in the z basis (x basis). Intensive parameters $\alpha_0 \in \{0, \dots, N-1\}/N$, $\alpha_m \in \{0, \dots, N_B-1\}/N_B$ tune the amount of superposition (over $|z\rangle$) of the initial state and measurement basis, respectively, which play analogous roles as θ_0, θ_m ; thus, the mixed-basis model can be thought of as a discrete analog of the tilted-basis model.

Upon measuring, one obtains a bit-string $\nu \in \{0, 1\}^{N_B}$ with Born probability $p(\nu)$, together with a postmeasurement pure state $|\psi_A(\nu)\rangle = (\mathbb{I}_A \otimes \langle\Phi_\nu|_B)|\Psi_\pi\rangle / \sqrt{p(\nu)}$ on subsystem A . Each 0(1) bit in ν denotes a measurement outcome aligned with (against) the particular local measurement basis, while $|\Phi_\nu\rangle_B$ is the product state on B associated with measurement outcome ν . The PE is defined as the ensemble of such projected states with probabilities

$$\mathcal{E}_{\text{PE}} := \{p(\nu), |\psi_A(\nu)\rangle\}. \quad (1)$$

Importantly, while the PE completely specifies the reduced density matrix (RDM) $\rho_A = \mathbb{E}_{\psi_A \sim \mathcal{E}_{\text{PE}}} [|\psi_A\rangle\langle\psi_A|]$, it contains strictly more information, as it describes a *distribution* over the Hilbert space $\mathcal{H}_A$ of A . Deep thermalization is the emergence of universal ergodic distributions describing the PE in the thermodynamic limit ($N_B \rightarrow \infty$, fixing N_A) consistent with generalized maximum entropy principles. A standard way to quantify the convergence of the PE $\mathcal{E}_{\text{PE}}$ to a target distribution $\mathcal{E}_*$ is to compare the closeness of their k th moments in the thermodynamic limit, such as through the trace distance $\Delta_*^{(k)} := \frac{1}{2} \|\rho_{\text{PE}}^{(k)} - \rho_*^{(k)}\|_1$, where $\rho_{(\cdot)}^{(k)} := \mathbb{E}_{\psi_A \sim \mathcal{E}_{(\cdot)}} [|\psi_A\rangle\langle\psi_A|^{\otimes k}]$ [12].

Deep thermalization and deep ergodicity breaking in the tilted-basis model—We begin by analyzing the first moment of the PE within the tilted-basis model. This is the RDM ρ_A , which is independent of the measurement

basis direction on B . We can rigorously show that local RDMs of almost all RPSs in the tilted-basis model are close to maximally mixed:

Theorem 1—Let $|\Psi_\pi\rangle$ be an N -qubit RPS in the tilted-basis model specified by an initial state $|\Psi_0\rangle$ with Bloch-sphere angles (θ_0, ϕ_0) not in $\{\theta_0 = 0, \theta_0 = \pi, (\theta_0 = (\pi/2), \phi_0 = 0)\}$. For any $\epsilon > 0$,

$$\mathbb{P}\left(\left\|\rho_A - \frac{\mathbb{I}_A}{d_A}\right\|_1 > \epsilon\right) < \frac{(d^{-\alpha_0} + d^{-2\beta_0})C + d^{-1}D}{\epsilon^2}, \quad (2)$$

where $d_A = 2^{N_A}$, $d = 2^N$ are the Hilbert-space dimensions of A and the full system, respectively, $0 < \alpha_0, \beta_0 < 1$ are factors determined by (θ_0, ϕ_0) , and C, D are $O(1)$ in system-size factors (see Supplemental Material [55] for details).

The proof [55] involves Weingarten calculus on the symmetric group [57] and a Markov inequality to bound fluctuations from the maximally mixed state.

Theorem 1 establishes that locally, the system almost always exhibits regular thermalization to the featureless infinite temperature state. We next turn to the higher moments ($k \geq 2$), which can now depend nontrivially on the choice of measurement basis. As a starting prediction, we employ the version of the maximum-entropy principle (MEP) put forth by [16]. It first prescribes that the exponentially many measurement outcomes ν entering the PE can be grouped into $O(N_B)$ equivalence classes $[\nu]$ defined by their Hamming weight (number of one-bits) [58]; each class yields a density matrix $\rho_{[\nu]}$ on average over RPSs. Then, it posits that the PE for a typical RPS is a statistical mixture across equivalence classes of the pure-state unraveling of $\rho_{[\nu]}$ with least *accessible information* [59] (the maximum amount of classical information extractable from the quantum-state ensemble); this yields the so-called “generalized Scrooge ensemble” [14,16]. In Supplemental Material [55], we present details of the analyses following the principle. We find that independent of measurement direction $\hat{n}$, the generalized Scrooge ensemble is the Haar ensemble

$$\mathcal{E}_{\text{Haar}} := \{d\psi_A, |\psi_A\rangle\}, \quad (3)$$

with $d\psi_A$ the Haar measure on $\mathcal{H}_A$. Thus, according to the MEP, we should expect that locally, the system is featureless not only at the level of the density matrix, but also of the PE as a whole. Indeed, when measurements are along the x basis ($\theta_m = \pi/2$), this prediction can be tested through an explicit computation of the expected PE: $\mathbb{E}_{U_\pi}[\mathcal{E}_{\text{PE}}] = \mathcal{E}_{\text{Haar}}$ (averaged over RPS for fixed $|\Psi_0\rangle$ with $\theta_0, \phi_0 \notin \{0, \pi\}$), using Weingarten calculus and combinatorics of set partitions; see Supplemental Material [55] for details. While this does not show convergence of $\mathcal{E}_{\text{PE}}$ to $\mathcal{E}_{\text{Haar}}$ for *individual* RPSs, it is a nontrivial necessary condition.

However, the MEP is only a guiding principle, and it is possible that the assumptions underlying its applicability fail, such that the limiting PE is in actuality not always the Haar ensemble. Interestingly, we find this is indeed the case for measurements along the z basis ($\theta_m = 0$): a projected state $|\psi_A(\nu)\rangle$ is typically some *computational basis state* $|z_A\rangle$ with $z_A \in \{0, 1\}^{N_A}$:

Theorem 2 (Informal)—Let $|\psi_A(\nu)\rangle = \sum_{z_A} c_{z_A}(\nu)|z_A\rangle$ be the projected state corresponding to some fixed bit-string outcome ν on RPS $|\Psi_\pi\rangle$ in the tilted-basis model with $\theta_0 \notin \{0, \pi/2, \pi\}$ and $\theta_m = 0$ (z -basis measurement). Then with unit probability over RPSs, only one coefficient $c_{z_A}(\nu)$ dominates in the thermodynamic limit.

See [55] for a precise formulation of the Theorem. The key idea is that the Born weight $|c_{z_A}(\nu)|^2$ depends on the Hamming weight h of the preimage bit-string $\pi^{-1}(z_A, \nu)$: $|c_{z_A}(\nu)|^2 \propto \cos^{2(N-h)}(\theta_0/2)\sin^{2h}(\theta_0/2)$. For RPSs, these Hamming weights are asymptotically distributed as independently and identically distributed binomial variables in limit $N \rightarrow \infty$. Fluctuations of independent binomials typically produce an exponentially large separation between the largest and the next-largest Born weight, so with unit probability a single coefficient dominates and the post-measurement state collapses to a single bit-string state $|\psi_A(\nu)\rangle \rightarrow |z_A(\nu)\rangle$. Strictly speaking, Theorem 2 tells us the *average* behavior of a projected state over RPSs, but we expect the same behavior for fixed *typical* RPSs. Furthermore, since the statement of Theorem 2 is agnostic as to which basis state is attained, it strongly suggests a uniform distribution over the computational basis on A . This defines the *classical bit-string ensemble*

$$\mathcal{E}_{\text{Cl}} := \{p(z_A) = 1/2^{N_A}, |z_A\rangle\}, \quad (4)$$

a discrete ensemble with the *least* entropy among the unravelings of $\rho_A = \mathbb{I}_A/d_A$, representing a maximal violation of deep thermalization [14–16,60].

Is this failure of deep thermalization a fine-tuned feature of computational basis measurements, or does it represent a robust phase? To probe this, we turn to numerics (see Supplemental Material [55] for details). We choose initial Bloch angles $(\theta_0, \phi_0) = (\pi/4, \pi/4)$ and generate PEs from RPSs, fixing $N_A = 2$, then compute the second-moment trace distances $\Delta_{\text{Cl}}^{(2)}$ and $\Delta_{\text{Haar}}^{(2)}$ from the classical bit-string and Haar ensembles, respectively, varying the measurement angle θ_m , see Fig. 1(b). Indeed, convergence to $\mathcal{E}_{\text{Cl}}$ and $\mathcal{E}_{\text{Haar}}$ (exponentially fast in system size) is seen for $\theta_m = 0, \pi$, respectively. However, strikingly, we also observe a critical value $\theta_m^* \approx 0.193\pi$ demarcated by crossings of both trace-distance curves over different system sizes (higher moments show a similar behavior with the same critical point [55]). Below (above) this value, the PE appears to converge to $\mathcal{E}_{\text{Cl}}$ ($\mathcal{E}_{\text{Haar}}$), strongly indicating that the absence of deep thermalization survives as a *robust*

phase away from the $\theta_m = 0$ point, separated from a deeply thermalized phase by a sharp phase transition at θ_m^* .

Coherence-induced phase transition—What is the mechanism behind the transition in the PE and the nature of the accompanying phases? A key observation is that the apparent limiting ensembles $\mathcal{E}_{\text{Cl}}$ and $\mathcal{E}_{\text{Haar}}$ are distinguished by the “amount” of superposition over the computational basis harbored by their constituent wave functions. This is captured by the resource-theoretic concept of *coherence* [36–38]. As a resource, coherence can be rigorously quantified by the relative entropy of coherence $C_r(\rho) := S(\rho_{\text{diag}}) - S(\rho)$ (with S the von Neumann entropy and ρ_{diag} the diagonal part of ρ , corresponding to full dephasing), which is a monotone under “free” incoherent operations (operations that map diagonal states to themselves). Among pure states, computational basis states achieve the minimum coherence, $C_r(|z_A\rangle) = 0$, while Haar random states are close to maximally coherent, $\mathbb{E}_{\psi_A \sim \text{Haar}(\mathcal{H}_A)}[C_r(|\psi_A\rangle)] = \sum_{k=2}^{d_A} 1/k$ [61,62]. Furthermore, RPD and z -basis measurements are incoherent operations while general measurements are not (they can increase coherence). This suggests that the transition in the PE should be viewed as one driven by coherence: coherence is injected globally by the initial state and choice of measurement basis before being scrambled nonlocally by RPD, which crucially does not create more superpositions; such coherence may then either proliferate and be detectable locally in the subsystem A or not. Further, the picture implies this physics should generically occur, i.e., be universal, as long as the dynamics is sufficiently scrambling and *coherence preserving*: indeed, the maximal set of such free incoherent unitary operations is precisely RPD, with the mild extension to include diagonal phase gates (see [55] for details). It also further suggests that the appropriate order parameter to consider is the ensemble-averaged coherence [63]

$$\overline{C}_r := \mathbb{E}_{|\psi_A\rangle \sim \mathcal{E}_{\text{PE}}}[C_r(|\psi_A\rangle)]. \quad (5)$$

To confirm this physical picture and illustrate it explicitly, we turn to the mixed-basis model. The key simplification of this model is that the distributions of computational-basis amplitudes of the initial state and measurement basis are all “flat,” allowing us to explicitly calculate coherence in the projected states through simple counting of the number of bit strings appearing in its decomposition (this is unlike the general case where the shape of the distribution also matters). As we will see, this yields an analytically predictable phase boundary.

To begin, consider the RPS associated with the mixed-basis model,

$$|\Psi_\pi\rangle = 2^{-\alpha_0 N/2} \sum_{z \in S} (-1)^{f_1(z)} i^{f_2(z)} |z\rangle. \quad (6)$$

Above, S is the image of the bit strings $0^{(1-\alpha_0)N} \times \{0, 1\}^{\alpha_0 N}$ under π ; this is a random subset of $\{0, 1\}^N$ of cardinality

$|S| = 2^{\alpha_0 N}$. $f_1(z)$, $f_2(z)$ are Boolean functions determined by π , but may be treated as effectively independent pseudorandom functions. We note Eq. (6) is a generalized version of so-called *random subset-phase states* [64], which have been recently studied in the context of pseudoentanglement [53,65–67].

Next, we analyze the form of a given projected state. Assume we obtain the measurement outcome $\nu = 00 \cdots 0$. The associated local postmeasurement state $|\psi_A(\nu)\rangle$ is proportional to

$$\sum_{z \in S} (-1)^{f_1(z)} i^{f_2(z)} \left(\prod_{j=N_A+1}^{N_A+(1-\alpha_m)N_B} \delta_{z_j,0} \right) |z_A(z)\rangle, \quad (7)$$

where $z_A(z) = (z_1, \dots, z_{N_A})$ is the restriction of the global bit string z to the subsystem A . The Kronecker deltas in Eq. (7) come from imposing compatibility with the z -basis measurement outcomes on $(1 - \alpha_m)N_B$ qubits; for the remaining $\alpha_m N_B$ qubits measured in the x basis, there is no such constraint. For other measurement outcomes ν , the projected states have similar forms upon suitably redefining the “pseudorandom” Boolean functions $f_{1,2}(z)$ and delta functions.

We now ask about the number l_A of nonzero terms that contribute to the sum in Eq. (7). This can be estimated as follows: the initial state provides $|S| = 2^{\alpha_0 N}$ bit strings; this number gets approximately halved by each Kronecker delta for a total of $(1 - \alpha_m)N_B$ times, resulting in $l_A \sim 2^{\alpha_0 N_A + (\alpha_0 + \alpha_m - 1)N_B}$. Therefore, $\alpha_0 + \alpha_m = 1$ demarcates the boundary between two distinct behaviors as $N_B \rightarrow \infty$: for $\alpha_0 + \alpha_m < 1$, the expected number of terms in the sum vanishes. Of course, the Born rule ensures only nonvanishing states are selected, so in practice $l_A \sim 1$. As long as $\alpha_0 N > N_A$, the PE covers all bit strings z_A uniformly, thus giving the classical bit-string ensemble $\mathcal{E}_{\text{Cl}}$. Conversely, for $\alpha_0 + \alpha_m > 1$, l_A diverges. Each bit-string z_A acquires many contributions with independent random coefficients $\pm 1, \pm i$. By the central limit theorem, this sum (upon normalization) converges in distribution to a complex standard Gaussian random variable, which describes a Haar random state. This collection of states is thus the Haar ensemble $\mathcal{E}_{\text{Haar}}$. (We note that a *complex* Haar random vector arose because of the initial state $|Y_+\rangle$ which provides imaginarity [39,40]; if instead it was real like $|X_\pm\rangle = (1/\sqrt{2})(|0\rangle \pm |1\rangle)$ we would obtain a *real* Haar random vector, i.e., $\mathcal{E}_{\text{Haar}}$ would be the state ensemble invariant under the real orthogonal group. See [55] for more details.)

To verify these predictions, we numerically evaluate the ensemble-averaged relative entropy of coherence $\overline{C}_r$ for the mixed-basis model with $\alpha_0 = 0.5$ (see [55] for simulations with other values of α_0). Figure 2(a) demonstrates an unambiguous transition at $\alpha_m = 0.5$, which becomes sharper with increasing N_B ; finite-size scaling shown in Supplemental Material [55] is consistent with a first-order

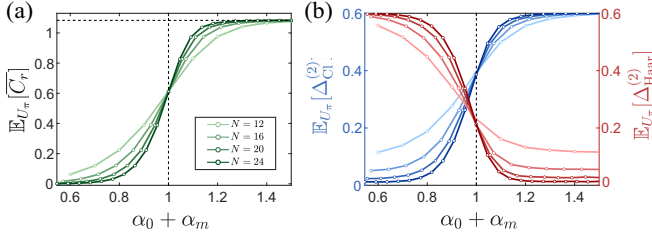


FIG. 2. (a) Ensemble-averaged coherence of the PE ($N_A = 2$) for the mixed-basis model with $\alpha_0 = 0.5$, showing a transition at $\alpha_m = 0.5$ (vertical dashed line) as predicted. Horizontal dashed line indicates the coherence of Haar random states. (b) Trace distances of the PE ($k = 2$) confirming convergence to the classical bit-string and Haar ensembles in the deeply nonergodic and ergodic regimes, respectively.

transition, which fits well with the analytical derivation presented above. Further, Fig. 2(b) shows convergences to the expected limiting ensembles $\mathcal{E}_{\text{Cl}}$ and $\mathcal{E}_{\text{Haar}}$ flanking the transition. This thereby provides strong evidence that both the tilted and mixed-basis models exhibit the same deep ergodicity-breaking physics, with coherence as the underlying driver behind the singular change in their PEs. This connection can be probed quantitatively: in the End Matter we explain how one may convert mixed-basis parameters (α_0, α_m) to tilted-basis parameters (θ_0, θ_m) of equal coherence (as measured by C_r); thus $\alpha_0 + \alpha_m = 1$ gives a prediction for the tilted-basis phase boundary. For the data in Fig. 1(b) this yields $\theta_m^* \approx 0.181\pi$, reasonably close to the observed $\theta_m^* \approx 0.193\pi$. However, we do not expect this connection to be exact, and indeed critical properties in the two models may be different—seemingly first order (second order) for the mixed (tilted) basis model; see [55]. Further study of the critical properties is an interesting direction for future work.

Lastly, to bolster our argument of universality of coherence-induced deep thermalization phases beyond the microscopic models studied here, we perform in Supplemental Material [55] an independent analytical calculation of the ensemble-averaged inverse participation ratio (IPR), a proxy for coherence, beginning from *generic* RPSs and under measurement schemes with varying IPRs. While based on an approximation replacing “quench averaging” over measurement outcomes with “annealed averaging,” it reveals distinct scalings in the IPR consistent again with the existence of deeply ergodic and nonergodic phases.

Discussion and outlook—Our work has identified a novel form of ergodicity-breaking transition in quantum many-body dynamics, defined not by a failure of conventional thermalization, but of deep thermalization. Importantly, this phenomenon is invisible in expectation values of standard observables, unlike known examples of ergodicity breaking such as localization [68–74] or many-body scarring [75–83]. Rather, it is similar to the phenomenon of measurement-induced phase transitions [10,84–88], where individual quantum trajectories exhibit

transitions not detectable within the average state. From a quantum information theoretic perspective, our transition can also be understood as a sharp change in the informational content of the PE: the accessible information of the limiting Haar (classical bit-string) ensemble is minimal (maximal), saturating the subentropy (Holevo) bounds [59,89,90].

We have further identified the transition as being driven by coherence, a quantum resource that is preserved by the dynamics, but is injected through the choice of initial states and measurement basis and transferred to the local unmeasured subsystem. This framework readily lends itself to generalizations based on other information-theoretic resources, such as imaginarity [39,40], nonstabilizerness [41–43,91] or non-Gaussianity [44,45]. Do resources injected by input states and measurements, and scrambled by “free” dynamics, generally give rise to PE transitions between a “resourceful” deeply thermalized phase and a “resourceless” ergodicity-breaking phase? How do such transitions, which require the constraint of dynamics being resource-preserving, interplay with conventional symmetry constraints like energy or charge conservation? Charting the landscape of resource-induced deep thermalization transitions, their implications for many-body dynamics, and possible applications in quantum information science represent exciting directions for future research.

Acknowledgments—We thank Andreas W. W. Ludwig and Tianci Zhou for helpful discussions, and Daniel Mark for useful comments. M.I. is supported by the U.S. Department of Energy, Office of Science, Office of Advanced Scientific Computing Research under Award No. DE-SC0025615. W. W. H. is supported by the Singapore National Research Foundation (NRF) Fellowship NRF-NRFF15-2023-0008 and through the National Quantum Office, hosted in A*STAR, under its Centre for Quantum Technologies Funding Initiative (S24Q2d0009). This research was supported in part by Grant No. NSF PHY-2309135 to the Kavli Institute for Theoretical Physics (KITP).

Data availability—The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

- [1] M. Rigol, V. Dunjko, and M. Olshanii, *Nature (London)* **452**, 854 (2008).
- [2] J. Eisert, M. Friesdorf, and C. Gogolin, *Nat. Phys.* **11**, 124 (2015).
- [3] R. Nandkishore and D. A. Huse, *Annu. Rev. Condens. Matter Phys.* **6**, 15 (2015).

- [4] D. A. Abanin, E. Altman, I. Bloch, and M. Serbyn, *Rev. Mod. Phys.* **91**, 021001 (2019).
- [5] L. M. Sieberer, M. Buchhold, J. Marino, and S. Diehl, *Rev. Mod. Phys.* **97**, 025004 (2025).
- [6] Y. Sekino and L. Susskind, *J. High Energy Phys.* 2008 (10), 065.
- [7] S. H. Shenker and D. Stanford, *J. High Energy Phys.* 2014 (03), 067.
- [8] J. Maldacena, S. H. Shenker, and D. Stanford, *J. High Energy Phys.* 2016 (08), 106.
- [9] P. Hayden and J. Preskill, *J. High Energy Phys.* 2007 (09), 120.
- [10] M. P. Fisher, V. Khemani, A. Nahum, and S. Vijay, *Annu. Rev. Condens. Matter Phys.* **14**, 335 (2023).
- [11] J. Choi, A. L. Shaw, I. S. Madjarov, X. Xie, R. Finkelstein, J. P. Covey, J. S. Cotler, D. K. Mark, H.-Y. Huang, A. Kale, H. Pichler, F. Brandao, S. Choi, and M. Endres, *Nature (London)* **613**, 468 (2023).
- [12] J. S. Cotler, D. K. Mark, H.-Y. Huang, F. Hernández, J. Choi, A. L. Shaw, M. Endres, and S. Choi, *PRX Quantum* **4**, 010311 (2023).
- [13] W. W. Ho and S. Choi, *Phys. Rev. Lett.* **128**, 060601 (2022).
- [14] D. K. Mark, F. Surace, A. Elben, A. L. Shaw, J. Choi, G. Refael, M. Endres, and S. Choi, *Phys. Rev. X* **14**, 041051 (2024).
- [15] C. Liu, Q. C. Huang, and W. W. Ho, *Phys. Rev. Lett.* **133**, 260401 (2024).
- [16] R.-A. Chang, H. Shrotriya, W. W. Ho, and M. Ippoliti, *PRX Quantum* **6**, 020343 (2025).
- [17] M. Ippoliti and W. W. Ho, *Quantum* **6**, 886 (2022).
- [18] M. Ippoliti and W. W. Ho, *PRX Quantum* **4**, 030322 (2023).
- [19] M. McGinley and T. Schuster, *arXiv:2511.17172*.
- [20] W.-K. Mok, T. Haug, W. W. Ho, and J. Preskill, *arXiv:2601.00266*.
- [21] M. Lucas, L. Piroli, J. De Nardis, and A. De Luca, *Phys. Rev. A* **107**, 032215 (2023).
- [22] M. Bejan, B. Béni, and M. McGinley, *Phys. Rev. Lett.* **135**, 020401 (2025).
- [23] P. W. Claeys and A. Lamacraft, *Quantum* **6**, 738 (2022).
- [24] T. Bhore, J.-Y. Desautels, and Z. Papić, *Phys. Rev. B* **108**, 104317 (2023).
- [25] A. Chan and A. De Luca, *J. Phys. A* **57**, 405001 (2024).
- [26] N. D. Varikuti and S. Bandyopadhyay, *Quantum* **8**, 1456 (2024).
- [27] H. Shrotriya and W. W. Ho, *SciPost Phys.* **18**, 107 (2025).
- [28] B. Zhang, P. Xu, X. Chen, and Q. Zhuang, *Nat. Commun.* **16**, 6341 (2025).
- [29] X.-H. Yu, W. W. Ho, and P. Kos, *Phys. Rev. Lett.* **135**, 260402 (2025).
- [30] A. Sherry and S. Roy, *arXiv:2507.14135*.
- [31] S. Chakraborty, S. Choi, S. Ghosh, and T. Giurgică-Tiron, *Phys. Rev. Lett.* **135**, 210603 (2025).
- [32] M. Aldana, S. Coppersmith, and L. P. Kadanoff, Boolean dynamics with random couplings, in *Perspectives and Problems in Nonlinear Science* (Springer, New York, 2003), pp. 23–89.
- [33] B. Bertini, K. Klobas, P. Kos, and D. Malz, *Phys. Rev. X* **15**, 011015 (2025).
- [34] D. Szász-Schagrin, M. Mazzoni, B. Bertini, K. Klobas, and L. Piroli, *arXiv:2505.06158*.
- [35] B. Bertini, K. Klobas, P. Kos, and D. Malz, *arXiv:2508.10890*.
- [36] T. Baumgratz, M. Cramer, and M. B. Plenio, *Phys. Rev. Lett.* **113**, 140401 (2014).
- [37] A. Winter and D. Yang, *Phys. Rev. Lett.* **116**, 120404 (2016).
- [38] A. Streltsov, G. Adesso, and M. B. Plenio, *Rev. Mod. Phys.* **89**, 041003 (2017).
- [39] A. Hickey and G. Gour, *J. Phys. A* **51**, 414009 (2018).
- [40] K.-D. Wu, T. V. Kondra, S. Rana, C. M. Scandolo, G.-Y. Xiang, C.-F. Li, G.-C. Guo, and A. Streltsov, *Phys. Rev. A* **103**, 032401 (2021).
- [41] S. Bravyi and J. Haah, *Phys. Rev. A* **86**, 052329 (2012).
- [42] M. Howard and E. Campbell, *Phys. Rev. Lett.* **118**, 090501 (2017).
- [43] Z.-W. Liu and A. Winter, *PRX Quantum* **3**, 020333 (2022).
- [44] Q. Zhuang, P. W. Shor, and J. H. Shapiro, *Phys. Rev. A* **97**, 052317 (2018).
- [45] L. Lami, B. Regula, X. Wang, R. Nichols, A. Winter, and G. Adesso, *Phys. Rev. A* **98**, 022335 (2018).
- [46] W. T. Gowers, *Comb. Probab. Comput.* **5**, 119 (1996).
- [47] S. Hoory, A. Magen, S. Myers, and C. Rackoff, *Theor. Comput. Sci.* **348**, 251 (2005).
- [48] A. Brodsky and S. Hoory, *Random Struct. Algorithms* **32**, 274 (2008).
- [49] S. Gopalakrishnan and B. Zakirov, *Quantum Sci. Technol.* **3**, 044004 (2018).
- [50] J. Iaconis, S. Vijay, and R. Nandkishore, *Phys. Rev. B* **100**, 214301 (2019).
- [51] J. Iaconis, A. Lucas, and X. Chen, *Phys. Rev. B* **102**, 224311 (2020).
- [52] J. Iaconis, *PRX Quantum* **2**, 010329 (2021).
- [53] X. Feng and M. Ippoliti, *J. High Energy Phys.* 2025 (02), 128.
- [54] W. Lee, H. Kwon, and G. Y. Cho, *arXiv:2411.03974*.
- [55] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/mjgs-y2z9> for details on statements, proofs of theorems, and numerics presented in the main text, which includes Ref. [56].
- [56] S. F. Edwards and P. W. Anderson, *J. Phys. F* **5**, 965 (1975).
- [57] B. Collins and M. Nagatsu, *arXiv:2503.18453*.
- [58] This is because the class of generator states (RPSs in this case) is invariant under qubit swaps in B .
- [59] R. Jozsa, D. Robb, and W. K. Wootters, *Phys. Rev. A* **49**, 668 (1994).
- [60] A. Touil, F. Anza, S. Deffner, and J. P. Crutchfield, *Quantum* **8**, 1494 (2024).
- [61] U. Singh, L. Zhang, and A. K. Pati, *Phys. Rev. A* **93**, 032125 (2016).
- [62] Asymptotically, this grows as $\ln d_A + \gamma - 1$ with corrections in $1/d_A$, where γ is the Euler-Mascheroni constant.
- [63] We emphasize that this is not the same as the coherence of the average state $C_r(|\mathbb{E}_{\psi_A \sim \mathcal{E}_{PE}} \psi_A\rangle\langle\psi_A|)$, since coherence is not a linear function of the state.
- [64] Generalized to include imaginary phases.
- [65] S. Aaronson, A. Bouland, B. Fefferman, S. Ghosh, U. Vazirani, C. Zhang, and Z. Zhou, in *15th Innovations in Theoretical Computer Science Conference (ITCS 2024)*, Leibniz International Proceedings in Informatics (LIPIcs). Vol. 287, edited by V. Guruswami (Schloss Dagstuhl—Leibniz-Zentrum für Informatik, Dagstuhl, Germany, 2024), pp. 2:1–2:21.

- [66] T. Giurgica-Tiron and A. Bouland, [arXiv:2312.09206](#).
- [67] F. G. Jeronimo, N. Magrafta, and P. Wu, [arXiv:2312.15285](#).
- [68] D. M. Basko, I. L. Aleiner, and B. L. Altshuler, *Ann. Phys. (Amsterdam)* **321**, 1126 (2006).
- [69] I. V. Gornyi, A. D. Mirlin, and D. G. Polyakov, *Phys. Rev. Lett.* **95**, 206603 (2005).
- [70] R. Nandkishore and D. A. Huse, *Annu. Rev. Condens. Matter Phys.* **6**, 15 (2015).
- [71] F. Alet and N. Laflorencie, *C. R. Phys.* **19**, 498 (2018).
- [72] D. A. Abanin, E. Altman, I. Bloch, and M. Serbyn, *Rev. Mod. Phys.* **91**, 021001 (2019).
- [73] V. Oganesyan and D. A. Huse, *Phys. Rev. B* **75**, 155111 (2007).
- [74] M. Žnidarič, T. Prosen, and P. Prelovšek, *Phys. Rev. B* **77**, 064426 (2008).
- [75] H. Bernien, S. Schwartz, A. Keesling, H. Levine, A. Omran, H. Pichler, S. Choi, A. S. Zibrov, M. Endres, M. Greiner *et al.*, *Nature (London)* **551**, 579 (2017).
- [76] S. Moudgalya, N. Regnault, and B. A. Bernevig, *Phys. Rev. B* **98**, 235156 (2018).
- [77] C. J. Turner, A. A. Michailidis, D. A. Abanin, M. Serbyn, and Z. Papić, *Nat. Phys.* **14**, 745 (2018).
- [78] S. Choi, C. J. Turner, H. Pichler, W. W. Ho, A. A. Michailidis, Z. Papić, M. Serbyn, M. D. Lukin, and D. A. Abanin, *Phys. Rev. Lett.* **122**, 220603 (2019).
- [79] W. W. Ho, S. Choi, H. Pichler, and M. D. Lukin, *Phys. Rev. Lett.* **122**, 040603 (2019).
- [80] C.-J. Lin and O. I. Motrunich, *Phys. Rev. Lett.* **122**, 173401 (2019).
- [81] S. Scherg, T. Kohlert, P. Sala, F. Pollmann, B. Hebbe Madhusudhana, I. Bloch, and M. Aidelburger, *Nat. Commun.* **12**, 4490 (2021).
- [82] G.-X. Su, H. Sun, A. Hudomal, J.-Y. Desaulles, Z.-Y. Zhou, B. Yang, J. C. Halimeh, Z.-S. Yuan, Z. Papić, and J.-W. Pan, *Phys. Rev. Res.* **5**, 023010 (2023).
- [83] D. Bhowmick, V. B. Bulchandani, and W. W. Ho, [arXiv:2505.05435](#).
- [84] Y. Li, X. Chen, and Matthew P. A. Fisher, *Phys. Rev. B* **98**, 205136 (2018).
- [85] Y. Li, X. Chen, and Matthew P. A. Fisher, *Phys. Rev. B* **100**, 134306 (2019).
- [86] B. Skinner, J. Ruhman, and A. Nahum, *Phys. Rev. X* **9**, 031009 (2019).
- [87] A. Chan, R. M. Nandkishore, M. Pretko, and G. Smith, *Phys. Rev. B* **99**, 224307 (2019).
- [88] Y. Bao, S. Choi, and E. Altman, *Phys. Rev. B* **101**, 104301 (2020).
- [89] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information: 10th Anniversary Edition* (Cambridge University Press, Cambridge, England, 2010).
- [90] A. S. Holevo, *Prob. Peredachi Inf.* **9**, 3 (1973), <https://www.mathnet.ru/eng/ppi903>.
- [91] H. Lóio, G. Lami, L. Leone, M. McGinley, X. Turkeshi, and J. D. Nardis, [arXiv:2510.13950](#).
- [92] V. Shende, A. Prasad, I. Markov, and J. Hayes, *IEEE Transact. Comput.-Aided Des. Integr. Circuits Syst.* **22**, 710 (2003).
- [93] C.-F. Chen, J. Haah, J. Haferkamp, Y. Liu, T. Metger, and X. Tan, [arXiv:2406.07478](#).
- [94] W. Gay, W. He, N. Kocurek, and R. O'Donnell, in *Annual International Cryptology Conference* (Springer, New York, 2025), pp. 651–678.

End Matter

Appendix A: Local RPD model—In the main text we analyzed initial states $|\Psi_0\rangle$ evolved under the application of a single *global* random permutation unitary U_π , where $\pi \in S_{2^N}$ (“global RPDs”). Here, we numerically confirm that the deep thermalization phase transition identified in the main text is indeed unaffected upon considering “local RPDs”—deep circuits made of random *local* permutation gates.

Concretely, we consider quantum circuits composed of local permutation unitaries, each of which acts on a fixed number of sites r and is stacked in a brickwork pattern up to some depth t , see, e.g., Fig. 1(a) for $r = 3$. Technically, such “ r -local” RPDs cannot reproduce exactly the distribution of global permutation unitaries for any $r < N$ even as $t \rightarrow \infty$, since they cannot generate *odd* permutations $\pi \in S_{2^N}$. However, for all $r \geq 3$, r -local RPDs generate the whole subgroup of even permutations (the *alternating group*) [92], which forms a k design over the global permutation group for all $k \leq 2^N - 2$, see [46,93,94]. (The cases $r = 1, 2$ are special as they only give Pauli and Clifford gates, respectively.) As the discrepancy between deep ($r \geq 3$)-local RPDs and global RPDs arises only at exponentially high moments, we expect it to be

unimportant toward the universal deep thermalization behavior.

To verify this, we numerically simulate three-local RPDs within the tilted-basis model, choosing initial Bloch angles $(\theta_0, \phi_0) = (\pi/4, \pi/4)$. We construct the PE for a subsystem of size $N_A = 2$ and track its ensemble-averaged coherence during circuit time evolution. Figure 3(a) shows that, for both z -basis and x -basis measurements, the ensemble-averaged coherence converges in time to that of the corresponding limiting ensembles, with errors vanishing in the thermodynamic limit. In Fig. 3(b) we plot the saturation value of the coherence, taken to be values in times $t \in [2N, 4N]$, versus θ_m , which reproduces very well the same deep thermalization phase transition behavior seen for global RPD in Fig. 1(b).

Appendix B: Estimation of critical point of the tilted-basis model from the mixed-basis model—Here, we estimate the critical point of the tilted-basis model by using our analytical result for the mixed-basis model, $\alpha_0 + \alpha_m = 1$, and matching the amount of coherence in the initial states and in the measurement bases of the two models.

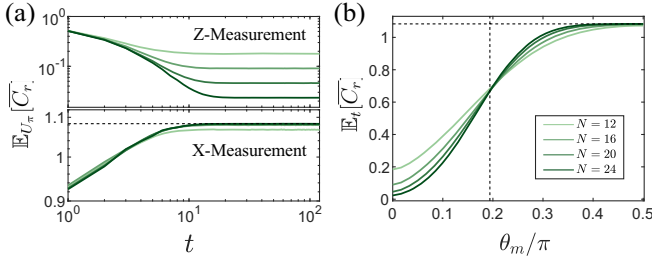


FIG. 3. (a) Time evolution under three-local RPDs of the ensemble-averaged coherence in the tilted-basis model, for z - (x -) basis measurements, shown in the top (bottom) panel. In both cases, the coherence saturates to that of the classical bit-string ensemble and Haar ensemble, respectively. (b) Saturation value as a function of the measurement angle θ_m . The critical angle $\theta_m^* \approx 0.193\pi$ of the tilted-basis model from Fig. 1(b) is shown as a vertical dashed line.

We convert between model parameters by using the relative entropy of coherence C_r . For any pure state $|\Psi\rangle$, the latter reduces to the Shannon entropy $S(\{p_z\}) := -\sum_z p_z \ln p_z$ of its population over the computational basis,

$$C_r(\Psi) = S(\{p_z\}), \quad (\text{B1})$$

with $p_z = |\langle z|\Psi\rangle|^2$. Below we denote the initial state by $|\Psi_0\rangle$ and the measurement-basis state by $|\Psi_m\rangle$.

In the tilted-basis model the bit-string distribution is a sum of independent binomials, so

$$\begin{aligned} C_r(\Psi_0) &= NH_2[\cos^2(\theta_0/2)], \\ C_r(\Psi_m) &= N_B H_2[\cos^2(\theta_m/2)], \end{aligned} \quad (\text{B2})$$

with H_2 the binary Shannon entropy $-p \ln(p) - (1-p) \ln(1-p)$, while for the mixed-basis model

$$C_r(\Psi_0) = \alpha_0 N \ln 2, \quad C_r(\Psi_m) = \alpha_m N_B \ln 2. \quad (\text{B3})$$

Equating the coherences of the input and measurement states between the two models yields α_0, α_m given θ_0, θ_m ; applying then the threshold condition $\alpha_0 + \alpha_m = 1$ gives

$$H_2[\cos^2(\theta_0/2)] + H_2[\cos^2(\theta_m/2)] = \ln 2. \quad (\text{B4})$$

For the parameters used in the main text, $(\theta_0, \phi_0) = (\pi/4, \pi/4)$, solving for θ_m yields a prediction $\theta_m^* \approx 0.181\pi$, which is close to the numerically extracted transition of $\theta_m^* \approx 0.193\pi$. A discrepancy is to be expected as the mixed-basis model can be thought of as a simplification of the tilted-basis amplitude distribution (effectively truncating amplitudes to 0 or 1). Nevertheless, the fact that these two values match well quantitatively is indication of the common physics underlying the two models.