

Dihadron Fragmentation and the Confinement Transition in Energy Correlators

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In this Letter, we relate the factorization for $e^+e^- \rightarrow h_1 h_2 X$ to the factorization for energy-energy correlators in the collinear limit. This enables us to give a nonperturbative proof of factorization for the energy correlators, relate the energy correlator jet function to transverse-momentum-sensitive dihadron fragmentation functions, and provide a rigorous description of the confinement transition region.

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Introduction—In high-energy collider experiments, the dynamics of quantum chromodynamics (QCD) are encoded in the energy distributions of hadrons recorded by detectors. A central puzzle is how the ultraviolet partons produced in hard scattering evolve into the observed infrared hadrons, and precisely how this confinement leaves its imprint on observables. Energy-weighted correlation functions of hadrons, known as energy correlators [1–4], have recently been identified as powerful probes of hadronization and confinement dynamics. For a recent review, see Ref. [5].

The simplest nontrivial correlator is between two hadrons, also known as energy-energy correlators (EEC), $\langle \Psi | \mathcal{E}(\vec{n}_1) \mathcal{E}(\vec{n}_2) | \Psi \rangle$, where $|\Psi\rangle = \mathcal{O}|0\rangle$ represents a state generated by a source operator $\mathcal{O}$, and the energy flow operator gives hadronic energies along the direction of the energy flow operator as $\mathcal{E}(\vec{n})|X\rangle = \sum_h E_h \delta^2(\Omega_{\vec{p}_h} - \Omega_{\vec{n}})|X\rangle$. For example, in $e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons}$, the electromagnetic current $J^\mu = e_f \bar{\psi}_f \gamma^\mu \psi_f$ can source a state with definite energy and momentum $q = l' - l$ with the lepton momenta l^μ and l'^μ as

$$\begin{aligned} & \langle \mathcal{E}(\vec{n}_1) \mathcal{E}(\vec{n}_2) \rangle_q \\ & \equiv \frac{1}{\sigma} \int d^4x \frac{e^{iq \cdot x}}{Q^2} L_{\mu\nu} \langle 0 | J^{\mu\dagger}(x) \mathcal{E}(\vec{n}_1) \mathcal{E}(\vec{n}_2) J^\nu(0) | 0 \rangle \\ & = \frac{1}{\sigma_{\text{tot}}} \sum_X \sum_{h_i, h_j} \int d\sigma_{e^+e^- \rightarrow h_i h_j X} \frac{E_{h_i} E_{h_j}}{Q^2} \\ & \quad \times \delta^2(\Omega_{\vec{p}_{h_i}} - \Omega_{\vec{n}_1}) \delta^2(\Omega_{\vec{p}_{h_j}} - \Omega_{\vec{n}_2}), \end{aligned} \quad (1)$$

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where $Q = \sqrt{q^2}$ is the center-of-mass energy of the e^+e^- , $L^{\mu\nu} = l^\mu l'^\nu + l'^\mu l^\nu - (l \cdot l') g^{\mu\nu}$ is the leptonic tensor, $\sigma = L_{\mu\nu} \times \int d^4x e^{iq \cdot x} \langle 0 | J^{\mu\dagger}(x) J^\nu(0) | 0 \rangle$, and $\sigma_{\text{tot}} = \sigma \times (4\pi\alpha_{\text{EM}})^2/2Q^6$ is the total cross section. Measuring only the angle between the two detectors via $z = (1 - \vec{n}_1 \cdot \vec{n}_2)/2$ gives the EEC,

$$\begin{aligned} \frac{1}{\sigma_{\text{tot}}} \frac{d\sigma_{\text{EEC}}}{dz} & \equiv \int d^2\Omega_{\vec{n}_1} d^2\Omega_{\vec{n}_2} \\ & \quad \times \delta\left(z - \frac{1 - \vec{n}_1 \cdot \vec{n}_2}{2}\right) \langle \mathcal{E}(\vec{n}_1) \mathcal{E}(\vec{n}_2) \rangle_q. \end{aligned} \quad (2)$$

As is clear from Eq. (1), the EEC of hadrons is directly related to dihadron production $e^+e^- \rightarrow h_i h_j X$ at various angles. This connection was, for example, leveraged to derive the EEC factorization in the back-to-back region ($1 - z \ll 1$) [6]. On the other hand, the connection between dihadron production and the EEC in the collinear region ($z \ll 1$) has not been harnessed. In this Letter, we explore this connection to provide a nonperturbative proof of the EEC collinear factorization and derive an explicit formula for the EEC jet function in terms of dihadron fragmentation functions, which enables us to make rigorous QCD-based predictions for the description of the confinement transition.

The EEC is an infrared safe observable that is perturbatively computable as long as its invariant-mass scale $Q\sqrt{z(1-z)} \gg \Lambda_{\text{QCD}}$. High precision fixed-order calculations [7,8], and the resummation of twist-2 and Sudakov double logarithms in the collinear and back-to-back regions [6,9–13], respectively, make the EEC one of the most precisely understood observables.

Near the $z \rightarrow 0, 1$ endpoints, the invariant mass scale becomes nonperturbative, $Q\sqrt{z(1-z)} \sim \Lambda_{\text{QCD}}$. For $z \rightarrow 1$, perturbative Sudakov logarithms and confinement both strongly suppress $d\sigma_{\text{EEC}}/dz$, making it hard to disentangle their individual effects. In contrast, in the small-angle regime, there is a clear signature of confinement with a

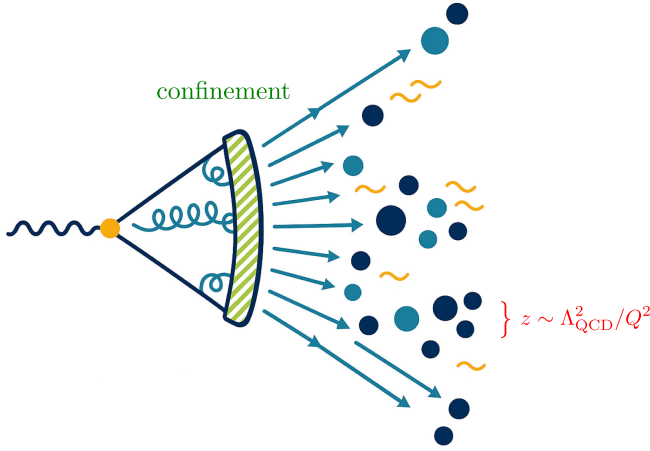


FIG. 1. Partons from a hard scattering undergo a cascade of partonic splittings at small angles $1 \gg z \gg \Lambda_{\text{QCD}}^2/Q^2$, and ultimately confine via hadronization into a dense cloud of uncorrelated hadrons separated by $z \sim \Lambda_{\text{QCD}}^2/Q^2$.

transition from a regime governed by the scaling of twist-two operators dressing a $\sim 1/z$ singularity to one of vanishing correlations among hadrons. At very small angles, nonperturbative confinement dynamics generate a dense cloud of hadrons as illustrated in Fig. 1, and wash out perturbative correlations. This parton-to-hadron transition has recently been observed experimentally [14–16] and sets confining theories such as QCD apart from conformal theories, where the light-ray operator product expansion (OPE) scaling [17–20] persists to arbitrarily small angles.

This striking feature of confinement has been studied in many recent publications [21–25], and one of our main goals is to provide a description of this transition from first principles.

One of the central organizing principles in high-energy hadronic processes is “QCD factorization,” which achieves predictive power by separating dynamics at disparate scales and isolating universal, nonperturbative functions that can be extracted in one kinematic regime and applied in another. Guided by this principle, it is then natural to ask: what is the universal nonperturbative structure underlying the small-angle confinement transition observed in the EEC, and how can we cleanly disentangle perturbative from genuine nonperturbative effects? In this Letter, we address these questions by connecting the EEC and dihadron production in the collinear limit. This allows us to derive the all-order factorization of the EEC in the full collinear region ($z \ll 1$) valid both in the OPE region ($\Lambda_{\text{QCD}}^2/Q^2 \ll z \ll 1$) and in the confinement transition region ($Q^2 z \sim \Lambda_{\text{QCD}}^2$), define universal nonperturbative jet functions that fully capture the confinement transition, and derive their perturbative evolution to enable predictive power.

Dihadron production—In collinear dihadron production, both hadrons emerge from the same fragmenting parton. Accordingly, the relevant nonperturbative functions describing such a fragmenting process are the transverse-momentum-sensitive dihadron fragmentation functions (FFs) [26–30]. For a quark, they are [26,27]

$$D_{q \rightarrow h_j h_k}(y_j, y_k, \vec{P}_{j\perp}, \vec{P}_{k\perp}) = \int \frac{dr^+ d^2 \vec{r}_\perp}{8N_c y_j y_k (2\pi)^6} e^{ik^- r^+} \sum_X \text{Tr}[\gamma^- \langle 0 | \psi_q(r^+, 0, r_\perp) | h_j, h_k; X \rangle \langle h_j, h_k; X | \bar{\psi}_q(0) | 0 \rangle], \quad (3)$$

where $\sum_X \equiv \sum_X \prod_{i \in X} \int \{ (d^3 \vec{p}_i) / [(2\pi)^3 2p_i^0] \}$, and similarly for a gluon. We use the convention $v^\pm \equiv (v^0 \pm v^3)/\sqrt{2}$ and suppress gauge links for convenience. The variable $y_a = p_{h_a}^-/k^-$ is the longitudinal momentum fraction of hadron h_a with respect to the fragmenting parton, and $\vec{P}_{a\perp}$ is the transverse momentum of h_a in the frame where this parton has no transverse momentum ($a = j, k$).

In the region $\{\Lambda_{\text{QCD}}^2, \vec{P}_{a\perp}^2\} \ll Q^2$, the two hadrons originate from a single fragmenting parton, and hence have the same $H \otimes D$ factorization [26–28] as the single-inclusive hadron production, with the same hard coefficients $H_i[x, (Q/\mu), \mu]$,

$$\begin{aligned} \frac{1}{\sigma_0} \frac{d\sigma^{h_j h_k X}}{d\xi_j d\xi_k d^2 \vec{P}_{j\perp} d^2 \vec{P}_{k\perp}} &= \sum_i \int dx dy_j dy_k H_i\left(x, \frac{Q}{\mu}, \mu\right) D_{i \rightarrow h_j h_k}\left(y_j, y_k, \vec{P}_{j\perp}, \vec{P}_{k\perp}, \mu\right) \delta(\xi_j - xy_j) \delta(\xi_k - xy_k) \theta(1 - y_j - y_k) \\ &= \sum_i \int_{\xi_j + \xi_k}^1 \frac{dx}{x^2} H_i\left(x, \frac{Q}{\mu}, \mu\right) D_{i \rightarrow h_j h_k}\left(\frac{\xi_j}{x}, \frac{\xi_k}{x}, \vec{P}_{j\perp}, \vec{P}_{k\perp}, \mu\right) \times \left[1 + \mathcal{O}(\vec{P}_{a\perp}^2/Q^2)\right], \end{aligned} \quad (4)$$

where σ_0 is the Born-level total cross section. Here H_i describe the production of parton i with momentum fraction $x = k^-/q^-$, and are now known to N³LO [31]. The observed momentum fraction of the hadrons is given by $\xi_a = p_{h_a}^-/q^-$. Further below we will exploit the fact that Eq. (4) remains valid when $\Lambda_{\text{QCD}}^2 \ll \vec{P}_{a\perp}^2$.

Equation (4) provides the input needed in Eq. (1) when $h_i \neq h_j$. The EEC also has a contribution with $h_i = h_j$, with the two detectors on the same hadron. This contribution comes from the single-hadron production given by

$$\begin{aligned} \frac{1}{\sigma_0} \frac{d\sigma^{hX}}{d\xi} &= \sum_i \int dx dy H_i \left(x, \frac{Q}{\mu}, \mu \right) D_{i \rightarrow h}(y, \mu) \\ &\quad \times \delta(\xi - xy) \theta(1 - y) \\ &= \sum_i \int_{\xi}^1 \frac{dx}{x} H_i \left(x, \frac{Q}{\mu}, \mu \right) D_{i \rightarrow h} \left(\frac{\xi}{x}, \mu \right), \end{aligned} \quad (5)$$

with single-hadron FFs

$$\begin{aligned} D_{q \rightarrow h}(y) &= \frac{1}{y} \frac{1}{2N_c} \int \frac{dr^+}{2(2\pi)} e^{ik^- r^+} \\ &\quad \times \sum_X \text{Tr}[\gamma^- \langle 0 | \psi_q(r^+, 0, 0_{\perp}) | h; X \rangle \langle h; X | \bar{\psi}_q(0) | 0 \rangle] \end{aligned} \quad (6)$$

for quark, and similarly for gluon. By RG consistency, the evolution equations of the single and dihadron FFs are described by the timelike Dokshitzer-Gribov-Lipatov-Altarelli-Parisi evolution kernel P^T , known up to three-loop accuracy [32–35], as

$$\frac{dD_{i \rightarrow h}(\xi, \mu)}{d \ln \mu^2} = \sum_j \int_{\xi}^1 \frac{dy}{y} D_{j \rightarrow h} \left(\frac{\xi}{y}, \mu \right) P_{ji}^T(y), \quad (7)$$

$$\begin{aligned} \frac{dD_{i \rightarrow h_k h_l}(\xi_k, \xi_l, \vec{P}_{k\perp}, \vec{P}_{l\perp}, \mu)}{d \ln \mu^2} \\ = \sum_j \int_{\xi_k + \xi_l}^1 \frac{dy}{y^2} D_{j \rightarrow h_k h_l} \left(\frac{\xi_k}{y}, \frac{\xi_l}{y}, \vec{P}_{k\perp}, \vec{P}_{l\perp}, \mu \right) P_{ji}^T(y). \end{aligned} \quad (8)$$

EEC in the collinear region—We now connect the results in Eqs. (4) and (5) with the z distribution of the EEC in Eq. (1). The transverse momentum difference between the two hadrons can be expressed in terms of z as

$$z = \frac{1 - \cos \theta_{jk}}{2} = \frac{(\xi_k \vec{P}_{j\perp} - \xi_j \vec{P}_{k\perp})^2}{(\xi_j \xi_k)^2 Q^2} \left(1 + \mathcal{O} \left(\frac{\vec{P}_{a\perp}^2, M_a^2}{Q^2} \right) \right). \quad (9)$$

This allows one to write

$$\begin{aligned} \left. \frac{d\sigma_{\text{EEC}}}{dz} \right|_{z \ll 1} &= \frac{1}{2^2} \frac{d}{dz} \left[\sum_{h_j \neq h_k} \int d\xi_j d^2 \vec{P}_{j\perp} d\xi_k d^2 \vec{P}_{k\perp} \xi_j \xi_k \Theta \left(z - \frac{(\xi_k \vec{P}_{j\perp} - \xi_j \vec{P}_{k\perp})^2}{(\xi_j \xi_k)^2 Q^2} \right) \frac{d\sigma^{h_j h_k X}}{d\xi_j d\xi_k d^2 \vec{P}_{j\perp} d^2 \vec{P}_{k\perp}} \right. \\ &\quad \left. + \sum_h \int d\xi \xi^2 \frac{d\sigma^{hX}}{d\xi} \Theta(z) \right] \\ &= \sigma_0 \frac{d}{dz} \left[\sum_i \int dx x^2 H_i \left(x, \frac{Q}{\mu}, \mu \right) J_i(x^2 Q^2 z, \mu) \right], \end{aligned} \quad (10)$$

where the EEC jet function is expressed in terms of fragmentation functions

$$\begin{aligned} J_i(x^2 Q^2 z, \mu) &= \frac{1}{2^2} \sum_{h_j \neq h_k} \int \frac{d^2 \vec{P}_{j\perp}}{(2\pi)^3} \frac{dy_j}{2y_j} \int \frac{d^2 \vec{P}_{k\perp}}{(2\pi)^3} \frac{dy_k}{2y_k} \Theta \left(x^2 Q^2 z - \frac{(y_k \vec{P}_{j\perp} - y_j \vec{P}_{k\perp})^2}{(y_j y_k)^2} \right) \\ &\quad \times y_j y_k (2\pi)^6 (2y_j 2y_k) D_{i \rightarrow h_j h_k}(y_j, y_k, \vec{P}_{j\perp}, \vec{P}_{k\perp}, \mu) + \frac{1}{2^2} \sum_h \int dy y^2 D_{i \rightarrow h}(y, \mu) \Theta(x^2 Q^2 z). \end{aligned} \quad (11)$$

The overall $1/2^2$ comes from normalizing by Q^2 in the denominator of Eq. (1). This formula for the EEC jet function applies even when $x^2 Q^2 z \sim \Lambda_{\text{QCD}}^2$ and makes explicit that the jet function only depends on the $x^2 Q^2 z$ combination.

Using the bare operator definitions of the single-hadron and dihadron FFs in Eqs. (6) and (3), we can obtain a more familiar expression for J . First, we shift the coordinate dependence of the quark field ψ_q and carry out the integration to produce the momentum-conserving delta

function. We then redefine X to include the explicit hadronic states in the FFs, identifying $h_j h_k X \rightarrow X$ in the dihadron case and $hX \rightarrow X$ in the single-hadron case. Finally, we note that the single-hadron sum $\sum_h$ coincides

with the $h_j = h_k$ sector of the double sum $\sum_{h_j, h_k}$. Altogether we then obtain after renormalization indicated by the subscript μ

$$J_q(x^2 Q^2 z, \mu) = \frac{1}{2N_c} \sum_X \sum_{h_j h_k \in X} \frac{E_j E_k}{x^2 Q^2} \Theta \left(x^2 Q^2 z - \frac{(y_k \vec{P}_{j\perp} - y_j \vec{P}_{k\perp})^2}{(y_j y_k)^2} \right) \times (2\pi)^3 \delta(xQ/\sqrt{2} - p_X^-) \delta^2(\vec{P}_{X,\perp}) \text{Tr}[\gamma^- \langle 0 | \psi_q(0) | X \rangle \langle X | \bar{\psi}_q(0) | 0 \rangle]_\mu, \quad (12)$$

and similarly for the gluon jet. What we have achieved is an extension of the factorization to the full collinear limit, including the region $Q^2 z \sim \Lambda_{\text{QCD}}^2$, where the jet function can be written as transverse-momentum-sensitive dihadron FFs as Eq. (11). Thus, we have identified the universal nonperturbative functions that are sensitive to the EEC's confinement transition!

Factorization in OPE region—The result in Eq. (12) reproduces the collinear jet function in the OPE region

$\Lambda_{\text{QCD}}^2/Q^2 \ll z \ll 1$ presented in [11], where the jet function can be perturbatively computed by taking $h_j h_k$ to be partons. It is interesting to see if this parton level result can be derived from the $\Lambda_{\text{QCD}}^2 \ll Q^2 z$ limit of Eq. (11). Based on standard collinear factorization the transverse-momentum-sensitive dihadron FFs in the limit $\Lambda_{\text{QCD}}^2 \ll \vec{P}_{a\perp}^2$ will refactorize as

$$D_{i \rightarrow h_j h_k}(y_j, y_k, \vec{P}_{h_j\perp}, \vec{P}_{h_k\perp}, \mu) = \sum_{j,k} \int_{y_j}^1 \frac{dw_j}{w_j} \int_{y_k}^1 \frac{dw_k}{w_k} C_{i \rightarrow jk}(w_j, w_k, \vec{P}_{j\perp}, \vec{P}_{k\perp}, \mu, \mu') D_{j \rightarrow h_j}\left(\frac{y_j}{w_j}, \mu'\right) D_{k \rightarrow h_k}\left(\frac{y_k}{w_k}, \mu'\right) + \sum_j \int_{y_j+y_k}^1 \frac{dw}{w^2} C_{i \rightarrow j}(w, \vec{P}_{j\perp}, \mu, \mu') D_{j \rightarrow h_j h_k}\left(\frac{y_j}{w}, \frac{y_k}{w}, \mu'\right) \delta^2\left(\vec{P}_{h_j\perp} - \frac{y_j}{y_k} \vec{P}_{h_k\perp}\right). \quad (13)$$

In the coefficient $C_{i \rightarrow jk}$, the transverse momentum argument, $\vec{P}_{j\perp} = (w_j/y_j) \vec{P}_{h_j\perp}$ denotes the transverse momentum of parton j in terms of the transverse momentum $\vec{P}_{h_j\perp}$ of the hadron into which it fragments. Here μ' is the refactorization scale between the partonic coefficients C and collinear FFs. Note that for $Q^2 z \gg \Lambda_{\text{QCD}}^2$ the contact terms in the last line of Eq. (13) can still be present. Since angular scales of size $Q^2 z \sim \Lambda_{\text{QCD}}^2$ are no longer tracked, the contact term is given by purely collinear dihadron FFs without explicit transverse momentum dependence. We believe this is the first presentation of the refactorization in Eq. (13), including its contact terms, although a similar expression for the first term can be found [36–38].

The contact term is important for infrared-collinear (IRC) safety of the cumulative energy correlators in the OPE region. After taking moments as in Eq. (12), by the momentum sum rule of the single-hadron FFs the first term in Eq. (13) becomes $\sum_{jk} \int dw_j dw_k w_j w_k C_{i \rightarrow jk}(w_j, w_k, \vec{P}_{j\perp}, \vec{P}_{k\perp}, \mu)$ (up to transverse momentum integrals we suppress for convenience), where $C_{i \rightarrow jk}$ is given by the same matrix element as Eq. (3) with hadronic states replaced by partonic states. Since the sum rule is independent of μ' , the μ' in C disappears in this moment. Taking the same moment of the last term in Eq. (13) and combining with the single-inclusive hadron contribution in Eq. (11), we find that the full contact term is

$$\sum_j \int_0^1 dw w^2 C_{i \rightarrow j}(w, \mu, \mu') \left[\sum_h \int dy y^2 D_{j \rightarrow h}(y, \mu') + \sum_{h_j \neq h_k} \int dy_j dy_k y_j y_k D_{j \rightarrow h_j h_k}(y_j, y_k, \mu') \right] \theta(x^2 Q^2 z) = \sum_j \int_0^1 dw w^2 C_{i \rightarrow j}(w, \mu) \theta(x^2 Q^2 z), \quad (14)$$

where we used the momentum sum rule of the collinear single-hadron and dihadron FFs. The coefficients $C_{i \rightarrow j}$ are given by the same matrix element as Eq. (6) with hadronic states replaced by partonic states. We thus obtain the jet function in Eq. (12) with h_j, h_k replaced with partonic states, as expected in the OPE region. This is consistent with the expectation that EEC is an IRC safe observable and that IR singularities cancel between the analogous two terms in partonic calculations. Exclusive measurements on hadronic states can introduce sensitivity to various collinear FFs organized in terms of moments of track functions as noted in [39–42], and this is apparent from our derivation here.

Predicting the Q dependence of $d\sigma_{\text{EEC}}/dz$ for the confinement transition—Although J is nonperturbative in the $Q^2 z \sim \Lambda_{\text{QCD}}^2$ region, the jet function given in terms of dihadron FFs has a perturbative RG evolution structure:

$$\frac{dJ_i(Q^2 z, \mu)}{d \ln \mu^2} = \sum_j \int_0^1 dy y^2 J_j(y^2 Q^2 z, \mu) P_{ji}^T(y), \quad (15)$$

which follows from the result in Eq. (11) and the RG evolution of the FFs in Eqs. (7) and (8). Equation (15) is also identical to the RG evolution of the perturbative collinear jet function in the OPE region [11].

We now follow the standard program of extracting the nonperturbative J_i at one scale and evolving them to others to test factorization. At some $\Lambda_{\text{trans}}^2 \gtrsim \Lambda_{\text{QCD}}^2$ we have the striking confinement transition. When $zQ^2 \gtrsim \Lambda_{\text{trans}}^2$, $d\sigma_{\text{EEC}}/dz \sim z^{-1-\delta}$ with anomalous dimensions $\delta \leq 0$ and power corrections [43–46] $\delta > 0$ both contributing to the scaling. On the other hand, when $zQ^2 \lesssim \Lambda_{\text{trans}}^2$, $d\sigma_{\text{EEC}}/dz \sim z^0$, i.e., flat due to confinement. This motivates one to use the following model to fit the jet function [47]:

$$J_i(Q^2 z, \mu_0) = \int_0^{Q^2 z} dv \frac{N_i}{1 + (\frac{v}{\Lambda_i})^{1+\delta_i}}, \quad (16)$$

where N_i is a normalization parameter, Λ_i denotes a transition scale, and $-1 - \delta_i$ is the scaling exponent when $Q^2 z > \Lambda_i$. Note that we did not exploit the (di)hadron FF formula in Eq. (11) to motivate Eq. (16). Instead, we apply Eq. (11) in reverse to provide deeper physical insight into the dihadron FFs through the confinement transition exhibited by the EEC. In the flat confining region this predicts that a weighted integral (“sum rule”) over the (di)hadron fragmentation function predicts a remarkable free streaming behavior from the cloud of hadrons produced via the confining fragmentation process. The analog in the OPE scaling region predicts that this weighted integral over the fragmentation functions has a perturbatively calculable dependence on zQ^2 , which eventually transitions into the free streaming hadrons as it becomes confined.

Using Eq. (16) we extract the quark and gluon jet functions at $\mu_0 = 10$ GeV using Pythia simulations of $e^+e^- \rightarrow \gamma^*/Z \rightarrow q\bar{q}$ and $H \rightarrow gg$ with $Q = m_H = 91.2$ GeV $= m_Z$, by evolving J_i to $\mu = m_Z$ and combining it with the leading order coefficients H_i . We chose this μ_0 since the transition begins at $\sqrt{z}Q \sim 10$ GeV. In Fig. 2 (upper panel) we show the results from leading logarithmic (LL) RG evolution (using Runge-Kutta), where the dashed $Q = m_Z$ curve is the starting input. We use J_i at $\mu = Q, Q/2, 2Q$ to obtain an estimate for the perturbative uncertainties, and observe that the evolution agrees with the general trend seen also in MC, though the uncertainties are fairly large. In Fig. 2 (lower panel), we repeat the exercise with next-to-leading order H_i , and carrying out next-to-leading logarithmic (NLL) RG evolution. We note a dramatic reduction in the perturbative uncertainty, and better agreement with Pythia at all Q values. This gives a useful cross check on the validity of our results.

Recent Letters in Refs. [21,22] have also explored the Q dependence of $d\sigma_{\text{EEC}}/dz$ in the $z \ll 1$ confinement transition region, but with transverse momentum dependent (TMD)-factorization inspired models rather than first-

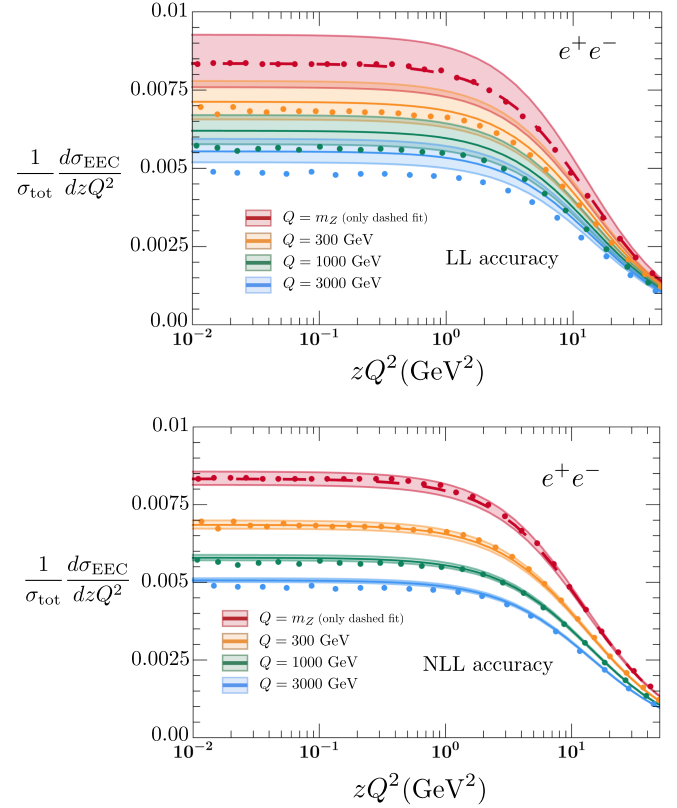


FIG. 2. LL (top) and NLL (bottom) predictions of EEC as a function of zQ^2 in the $zQ^2 \sim \Lambda_{\text{QCD}}^2$ region. Only the central curve (dashed) at $Q = m_Z$ is fit, while the rest of the curves are derived according to the perturbative evolution from Eq. (15).

principles QCD. In [21] $d\sigma_{\text{EEC}}/dz$ was modeled by the TMD distribution structure with a nonperturbative Sudakov exponent, and the Q dependence was predicted by Collins-Soper evolution within the Sudakov. In [22] they exploited collinear factorization, and then TMD modeling for J_i with a (different) nonperturbative Sudakov factor in the confinement transition region. They predicted the Q dependence by evolving their jet function at LL accuracy according to Eq. (15), and observed the same trend seen in our Fig. 2 (upper panel). Our Letter puts such tests of the Q dependence of the confinement transition on a firm theoretical footing.

Conclusions and outlook—In this Letter, we proved the collinear factorization of EEC in the hadron transition region $Q^2 z \sim \Lambda_{\text{QCD}}^2$ and determined its universal nonperturbative jet functions in terms of transverse-momentum-sensitive dihadron fragmentation functions. This enables predictions of the Q dependence through RG evolution, and a QCD based interpretation of the confinement transition. We also provide a factorization of dihadron FFs for $\Lambda_{\text{QCD}}^2 \ll \vec{P}_{a\perp}^2 \ll Q^2$ which is consistent with the collinear factorization of the EEC in the OPE region [11].

Our Letter provides exciting connections between the fields of energy correlators and multihadron production,

providing a powerful tool for phenomenological applications. It is straightforward to generalize our results to higher-point energy correlators [42,48–51], and when $\vec{P}_{a\perp}^2 \sim \Lambda_{\text{QCD}}^2$ we expect the nonperturbative dynamics to be described by higher multiplicity transverse-momentum-sensitive multihadron fragmentation functions. These can also be measured with different quantum numbers [39–41,52–54], such as spin, charge, and heavy-flavor number, and it would be interesting to track how this affects the nonperturbative confinement transition. It would also be interesting to calculate the EEC jet function with chiral perturbation theory [55–57]. Finally, applying this formalism to probe jet [58–65] and nuclear [66–68] substructures in the confinement transition region will be very interesting.

See also [69–72] for other work on this topic.

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Data availability—The data that support the findings of this article are openly available [73]; embargo periods may apply.

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