

Flocking Beyond One Species: Novel Phase Coexistence in a Generalized Two-Species Vicsek Model

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A hallmark in natural systems, self-organization often stems from very simple interaction rules between individual agents. While single-species self-propelled particle systems are well understood, the behavior of binary mixtures with general alignment interactions remains largely unexplored with a few scattered results hinting at the existence of a rich emergent phase behavior. Here, we investigate systematically a generalization of the two-species Vicsek model with reciprocal intra- and interspecies (anti)alignment couplings, uncovering a rich phenomenology of emergent states. Notably, we show that rather than destroying polar order, antialigning interactions can promote phase separation and the emergence of global polar order. In doing so, we uncover a novel mechanism for microphase separation. We further find these coexistence patterns can be generalized to multispecies systems with cyclic alignment interactions.

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For the past three decades, active matter has served as a playground for novel nonequilibrium emergent phenomena, where simple local interactions drive unexpected collective behaviors [1,2]. It also offers a quantitative approach to complex biological processes. Biological systems—ranging from animal herds to bacterial suspensions and cytoskeletal filaments—remarkably self-organize into coherent structures despite nature's inherent stochasticity [3–9]. A prototypical example of this is flocking, where local alignment interactions among self-propelled agents gives rise to macroscopic directed motion and true long-range order [4,10–23] at odds with the Mermin-Wagner-Hohenberg theorem [24,25].

Despite being under intense scrutiny since its inception three decades ago, Vicsek-like models still offer surprises [19,26–30], with even small modifications leading to novel mechanisms for long-range order [31,32]. Shown to be more robust but also more fragile than originally expected, flocks are unstable to small perturbations like the presence of obstacles [33–35], spatial anisotropy [36], or even dissenters [37,38], but equally arise without explicit alignment interactions [39–45]. Beyond the usual polar alignment rules, recent studies have explored the emergence of collective motion with nematic alignment [46], antipolar alignment [47,48], and mixed alignment rules [49,50], and in the presence of disorder [32,51–57].

Although foundational theoretical work in active matter has primarily focused on single-species systems, recent interest has turned to multispecies systems with more intricate interaction rules [31,58–60]. In particular, understanding the effect of nonreciprocal interactions in two-species systems has recently stolen the show [61–66]. Namely, nonreciprocity in the alignment—in which species A tends to align with species B, while species B antialigns with species A—leads to novel emergent behaviors such as (anti)parallel flocking [31,58,60] or chirality [61,67,68]. Interestingly, with the exception of Kürsten *et al.* [59], the vast majority of existing studies make the simplifying assumption of aligning (i.e., ferromagnetic-like) intraspecies interactions. As a consequence, a comprehensive study of a multispecies Vicsek-like model with generic (anti)alignment interactions is crucially lacking from the literature.

In this Letter, we explore a generalized two-species Vicsek model where self-propelled particles follow XY-like alignment rules with independent intra- and interspecies couplings. Mapping the phase space of reciprocal (anti) alignment interactions, we uncover a rich spectrum of flocking, antiparallel flocking, and coexistence states. Notably, we identify a novel microphase-separated state featuring stable periodic traveling bands, emerging from antiferromagnetic intraspecies and ferromagnetic interspecies interactions. We present a heuristic stability argument for this phase and demonstrate its persistence in multispecies systems with cyclic alignment coupling matrices.

Model—We study a continuous-time version of the Vicsek model (also called the flying XY model) composed of N self-propelled point-particles belonging to two species $\{A, B\}$ with distinct intra- and interspecies local alignment rules. Particles are placed in a two-dimensional periodic box of size $L \times L$ with a density of $\rho = N/L^2$. Here,

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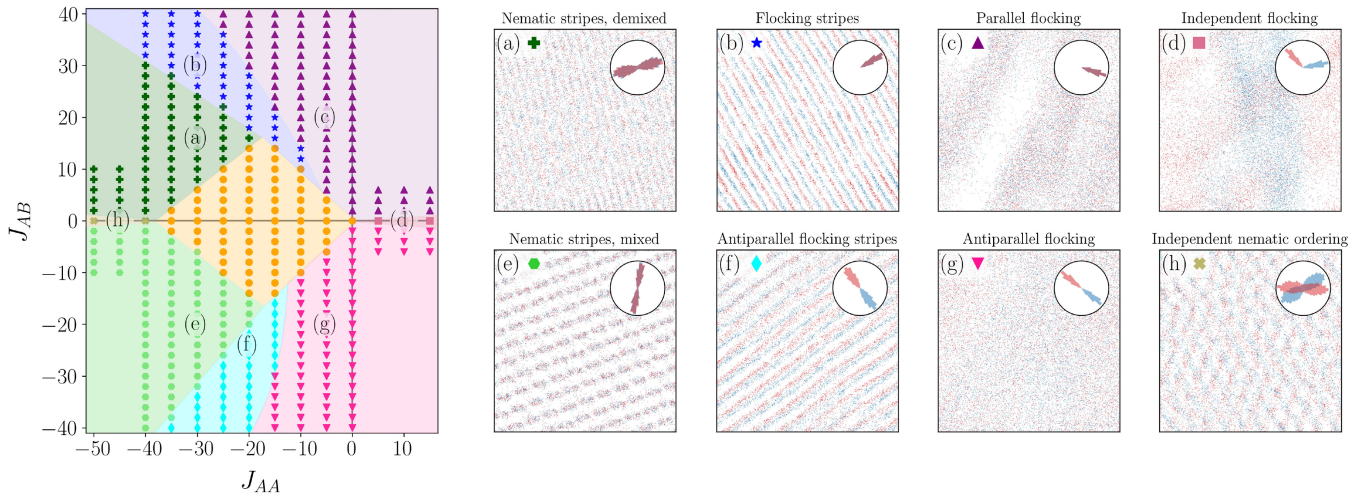


FIG. 1. Numerical phase diagram obtained from particle simulations. Here, we use the following parameters $N = 2 \times 10^4$, $\rho = 100$, and $D_r = 0.2$. Phases were classified numerically, with data averaged over 10 independent realizations for each data point. Here, we list the phases observed and the parameters used to obtain the associated example snapshots: (a) nematic stripes, demixed (dark-green plus), $J_{AA} = -30$, $J_{AB} = 16$; (b) flocking stripes (dark-blue star), $J_{AA} = -30$, $J_{AB} = 30$; (c) parallel flocking (purple upward triangle), $J_{AA} = -5$, $J_{AB} = 20$; (d) independent flocking (pink square), $J_{AA} = 10$, $J_{AB} = 0$; (e) nematic stripes, mixed (light-green hexagon), $J_{AA} = -30$, $J_{AB} = -20$; (f) antiparallel flocking stripes (light-blue diamond), $J_{AA} = -20$, $J_{AB} = -24$; (g) antiparallel flocking (pink downward triangle), $J_{AA} = -5$, $J_{AB} = -20$; (h) independent nematic ordering (khaki cross), $J_{AA} = -45$, $J_{AB} = 0$. Finally, we also observe a disordered hyperuniform phase (orange circles). Phase boundaries were added manually as a visual guide. See accompanying Videos S1–S9 and their descriptions in Supplemental Material [72].

particles move with constant speed v_0 along the self-propulsion direction $\hat{\mathbf{p}}_i = (\cos \theta_i, \sin \theta_i)$, which experiences rotational diffusion and a torque stemming from alignment interactions. The position $\mathbf{r}_i$ and orientation θ_i of particle i at time t are then governed by the following overdamped Langevin equations:

$$\dot{\mathbf{r}}_i = v_0 \hat{\mathbf{p}}_i, \quad (1a)$$

$$\dot{\theta}_i = \frac{1}{n_i} \sum_{j \in \mathcal{N}_i} J_{s(i),s(j)} \sin(\theta_j - \theta_i) + \sqrt{2D_r} \xi_i, \quad (1b)$$

where $\xi_i(t)$ is a zero-mean, unit-variance Gaussian white noise, and $\mathcal{N}_i = \{j: |\mathbf{r}_i - \mathbf{r}_j| \leq \sigma_I, i \neq j\}$ is the set of particles interacting with particle i , i.e., particles within the radius of interaction σ_I . The torque is normalized by the number of interaction neighbors $n_i = |\mathcal{N}_i|$, also known as a mean-sine model [69]. The coupling strength between two particles is $J_{s(i),s(j)}$, where $s(i)$ is the species of particle i . This coupling may be aligning ($J_{s(i),s(j)} > 0$) or antialigning ($J_{s(i),s(j)} < 0$). Populations A and B are taken to be in equal proportion such that there are $N/2$ particles of each species. For the purposes of this Letter, we will assume that the intraspecies couplings are identical across both species $J_{AA} = J_{BB}$, and the interspecies couplings are equal $J_{AB} = J_{BA}$ leading to pairwise reciprocal interactions [71]. We numerically solve Eq. (1) using an Euler-Maruyama method with time step $\Delta t = 0.01$. We set $v_0 = 1$ and $\sigma_I = 1$ unless stated otherwise, leaving the following free parameters: J_{AA} , J_{AB} , D_r , ρ , and N .

Rich emergent behavior and phase diagram—We explore the phase space spanned by (J_{AA}, J_{AB}) at fixed density and noise strength (see Fig. 1). Here, we first work at a large density to ensure system-spanning structures are observed. Note that the influence of density ρ and noise strength D_r are discussed below (see Fig. 2).

We observe a diverse range of emergent behaviors, which we classify according to a number of order parameters including polar order, nematic order, demixing, and spatial periodicity of observed structures. Further details of how

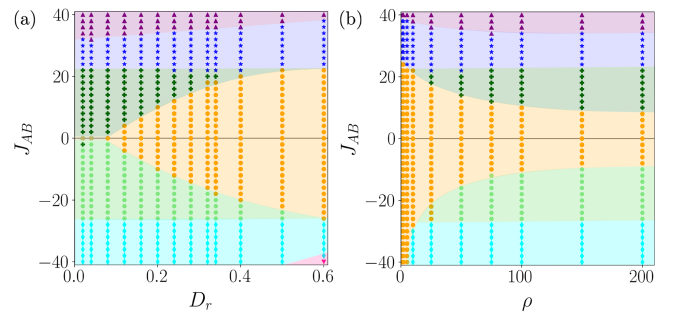


FIG. 2. Phase diagrams when varying noise strength and density. (a) Phase behavior in the (J_{AB}, D_r) plane at fixed density $\rho = 100$, intraspecies interaction strength $J_{AA} = -25$. (b) Phase behavior in the (J_{AB}, ρ) plane at fixed rotational diffusion $D_r = 0.2$ and intraspecies interaction strength $J_{AA} = -25$. Here again, the system size was taken to be $N = 2 \times 10^4$. Symbols used here are the same as those introduced in Fig. 1; phase boundaries were added manually as a visual guide.

these order parameters were calculated can be found in the End Matter and Supplemental Material [72]. First, we examine the phases observed for $J_{AA} > 0$. For $|J_{AB}| \ll J_{AA}$, the usual polar ordered flocking phase is observed with each species flocking independently [Fig. 1(d) and Video S4 in Supplemental Material [72]]. As J_{AB} increases, the interspecies alignment becomes strong enough to coordinate the flocking directions of the two species and we observe a globally polar ordered phase [Fig. 1(c) and Video S3 in Supplemental Material [72]]. Conversely, in systems with intraspecies alignment ($J_{AA} > 0$) but interspecies antialignment ($J_{AB} < 0$), the two populations display independently global polar order but are flocking in antiparallel directions [Fig. 1(g) and Video S7 in Supplemental Material [72]], leading to an absence of polar order at the system level. This phase was observed originally in the two-species Vicsek model [31]. Interestingly, we observe that the parallel and antiparallel flocking phases [see Figs. 1(c) and 1(g)] persist even in the case of intraspecies antialigning interactions ($J_{AA} < 0$) provided that the interspecies (anti)alignment J_{AB} is large enough. In this regime, we expect the large-scale phase behavior to be controlled by the interspecies interactions alone.

Strikingly, when $J_{AB} \approx -J_{AA}$, a novel phase emerges; this phase is characterized by traveling bands of alternating species exhibiting global polar order, and we refer to it as the *flocking stripes phase* [Fig. 1(b) and Video S2 in Supplemental Material [72]]. Interestingly, despite strong antialigning intraspecies coupling, particles self-organize to form a high-density system-spanning bands structure within their own species. We observe that these bands—which are reminiscent of the banding phase commonly observed in Vicsek models but stem here from a very different mechanism—are spatially periodic and bands of different species are nonoverlapping. The interspecies alignment, along with this interspecies demixing, facilitates flocking with strong global polar order. This novel phase behavior is analyzed in more detail below.

Upon reflection about the J_{AB} axis (i.e., in the regime where $J_{AB} \lesssim J_{AA}$), a similar flocking striped phase is found; however, in this case, the antialigning nature of the interspecies couplings leads to systems of bands for each species flocking in opposite directions [Fig. 1(f)]. Although reminiscent of the homogeneous density antiparallel flocking state [Fig. 1(g)], the system is now clustered; it displays a nontrivial global nematic order and is spatially periodic (in the direction of nematic order), with species-species demixing when averaged over time. We therefore call this the *antiparallel flocking stripes phase* (see Video S6 in Supplemental Material [72]).

For stronger intraspecies antialignment ($J_{AA} \ll -1$), we encounter three further periodically striped phases. In the regions $J_{AB} > 0$ and $J_{AB} < 0$, phases are shown to display nematic order for both species in the same direction [Figs. 1(a) and 1(e)]. In both cases, the structure exhibits

spatial periodicity, with regions of high-density stripes or lanes. However, for $J_{AB} > 0$, species remain demixed when averaged over time (see Video S1 in Supplemental Material [72]), whereas the particles form local mixed clusters for $J_{AB} < 0$ (see Video S5 in Supplemental Material [72]). We therefore call these phases *nematic stripes (demixed)* and *nematic stripes (mixed)*, respectively. When $J_{AB} \approx 0$ but $J_{AA} \ll -1$, the species behave independently as expected; nematic order is still observed within each population due to the very strong antialigning couplings (see Video S8 in Supplemental Material [72]). Similar nematic lanes and spatial structuring were also reported in previous models involving antialigning couplings [47–49].

Finally, for moderate value of intraspecies antialignment and interspecies interactions (either aligning or antialigning), we observe a homogeneous *disordered* phase. While no polar or nematic order is observed in the disordered phase, we have found strong spatial structuring; indeed, as shown in Supplemental Material [72], this is consistent with hyperuniform configurations. As this has been put into question by recent work on a related lattice model [74], confirming this result would require further analytical work, which is outside the scope of the current paper.

While the diverse phases in Fig. 1 naturally invite questions about transition orders and tricritical points, characterizing these remains a formidable challenge that took decades to resolve even for the simpler single-species model. We have added numerical order parameter plots across different phase boundaries to the End Matter to provide quantitative insight; however, a full characterization of these phase transitions, including the investigation of possible tricritical points, remains a major task for future research.

Robustness of the observed phases—As discussed above, we chose to study the phase diagram for high density in order to get system-spanning patterns. Here, we confirm that these emergent phases were observed over a wide range of noise strengths D_r and densities ρ [Fig. 2]. Notably, we observe that the disordered regime shrinks upon decreasing rotational diffusion D_r . Further, the parallel and antiparallel flocking stripes regions (blue stars and pink diamonds, respectively) persist across the wide range of noise strengths tested here. In fact, the flocking stripes region shows a weak expansion at higher J_{AB} values at larger noise strengths. In both cases, intraspecies nematic order is favored when $|J_{AB}|$ is decreased. We expect the disordered phase to take over at larger noises, eventually disrupting the polar and antiparallel polar stripes phases.

Examining the effect of density, the disordered region grows as ρ decreases. This is likely due to the fact that the relative sparseness of these systems allows the particles to alleviate their competing alignment frustration through spatial distancing, rather than being forced to alleviate this through collective directed motion. Just as in the case of varying D_r , the parallel and antiparallel flocking stripes phases persist for a remarkably wide range of densities.

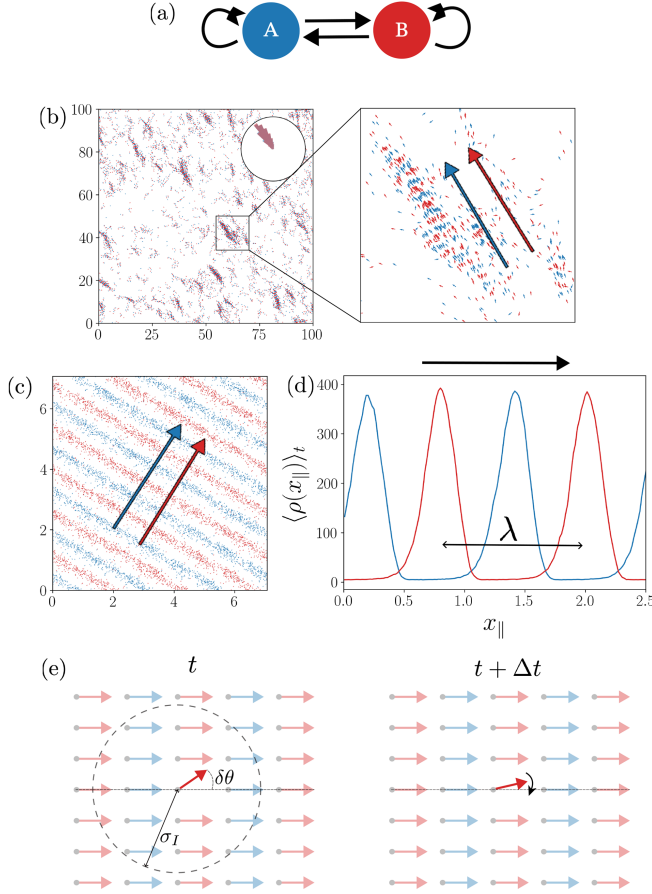


FIG. 3. (a) Sketch of interactions between species in the flocking stripes phase. The interspecies couplings are aligning, whereas the intraspecies coupling is antialigning. (b) Snapshot of a simulation at low density for $J_{AA} = -J_{AB} = -20$ (parallel flocking with phase separation) with enlarged snapshot showing the microphase separation. The inset shows a polar histogram of the particle angles. These simulations were performed with $N = 10^4$, $\rho = 1$, and $D_r = 0.02$. (c) Snapshot of a high-density simulation in the traveling band phase ($N = 10^4$, $\rho = 200$, $D_r = 0.2$, $J_{AA} = -J_{AB} = -20$) and (d) its time-averaged density profile, projected along the direction of travel $x_{\parallel}$. (e) Simplified schematic to explain the stability mechanism.

For lower densities, the systems displaying flocking stripes tend to form isolated clusters of striped patterns displaying polar order [see Fig. 3(b)]. Predicting the phase boundaries as well as the nature of all these transitions analytically remains a formidable challenge and is the focus of future studies.

Further, we tested the robustness of all observed phases to finite size effects by simulating a larger range of system sizes (see Supplemental Material [72]). We found the phases to persist up to the largest system sizes simulated ($N = 2 \times 10^5$). The flocking stripes phase is also robust to nonsymmetric parameter choices, such as unequal particle numbers or $J_{AA} \neq J_{AB}$ (see Supplemental Material [72]). Finally, we confirmed that our findings were robust to

changes in the exact microscopic interaction rules; in particular, upon rescaling of the interaction couplings, we find that an additive sine model displays qualitatively the same phase diagram and confirmed that the flocking stripes patterns are also found in models with soft particle repulsion (see Supplemental Material [72]). We note that interaction kernels that decay too fast tend to hinder flocking stripes (see Supplemental Material [72]).

Characterization of the flocking stripes phase—Finally, we focus for a moment on trying to characterize further the emergence and the stability of the flocking stripes phase, perhaps one of the most counterintuitive emergent behaviors presented here. As shown in Figs. 1 and 2, the flocking stripes phase appears over a wide range of parameters J_{AA}, J_{AB} . The two key ingredients giving rise to this phase are as follows: (1) one needs intraspecies antialignment ($J_{AA} < 0$) but interspecies alignment ($J_{AB} > 0$); (2) the intraspecies and interspecies interaction strengths should be similar in absolute value, $|J_{AA}| \approx |J_{AB}|$. For simplicity, we here focus on the case in which the couplings are equal in magnitude and of opposite signs. While particles align with the opposite species, they antialign within their own [Fig. 3(a)]; it thus seems quite counterintuitive at first that particles would cluster and form dense bands within their own species. Although these bands resemble those in the coexistence regime of the two-species Vicsek model [31], we argue that this novel phase separation stems from a distinct physical mechanism.

At low densities, particles self-organize into finite polar clusters exhibiting stripes of alternating species as seen in Fig. 3(b) (see also Video S10 in Supplemental Material [72]). Interestingly, this low-density clustered phase displays large global polar order. At high densities, the clusters become system-spanning traveling waves within the periodic box [Fig. 3(c)]. To characterize the stripes further, we project the particles onto the mean direction of travel and measure the density profile of the bands, averaged over time $\langle \rho(x_{\parallel}) \rangle_t$ [see Fig. 3(d)]. The stripes profiles are extremely regular, periodic in the direction of travel, and of equal amplitude forming an alternating smectic A pattern after a short transient. In the Vicsek model, traveling bands are found to eventually organize into a periodic pattern but only after very long transients [23]. Furthermore, in our case the local density between bands drops to almost zero, in contrast with the low-density gas state observed in the Vicsek model bands [26,36].

Measuring the wavelength between density peaks of the same species for various different interaction radii reveals that the wavelength is linearly dependent on the radius of interaction, with $\lambda \approx 1.23\sigma_I$, independently of the system density (see Supplemental Material [72]). Particles thus form evenly spaced bands of alternating species, with the peaks of each species just over σ_I apart. The same wavelength also appears in the other patterns, and even in the disordered hyperuniform state (see Supplemental

Material [72]). Notably, we also found that the correlations in the velocity perpendicular to the direction of motion $\mathbf{v}_\perp^{(i)}$ decay extremely fast with time, pointing at the stability of this polar ordered phase.

How are the flocking stripes so stable despite such strong antialignment between agents of the same species? To understand this, we devise the following mean-field argument: first assume without loss of generality that vertical stripes of alternating species have emerged, all with their polarization pointing in the x direction (i.e., $\theta = 0$) as in Fig. 3(e) [75]. As already argued, the distance from the center of one stripe to another one of the same species is approximately $1.2\sigma_I$; thus, the red particle highlighted in Fig. 3(e) only interacts with the particles in the same stripe (i.e., same species particles) and the two nearest neighbor stripes (i.e., opposite species particles) found directly in front and behind.

At time t , let us assume that red particle i experiences a fluctuation in its orientation $\delta\theta > 0$, due, for instance, to rotational diffusion. Because of the metric nature of the interactions, particle i interacts with more particles of the other species n_i^{blue} than with particles of its own species n_i^{red} . Denoting $\kappa = n_i^{\text{blue}} - n_i^{\text{red}} > 0$ and $J_{AB} = -J_{AA} \equiv J$, then the torque on particle i at time $t + \Delta t$ is given by $\mathcal{T}_i = -(J\kappa/n_i) \sin \delta\theta$, where $n_i = n_i^{\text{blue}} + n_i^{\text{red}}$. We conclude that $\delta\theta$ and $\mathcal{T}_i$ have opposite signs, which means that the torque is in the opposite direction to the direction of the original fluctuation and the particle's polarization is pulled back toward the mean direction of travel. The remaining particles experience the torque $\mathcal{T}_j = \pm(J/n_j) \sin \delta\theta$ (the sign depends on if it is a red or blue particle), which is much smaller in magnitude than $\mathcal{T}_i$. The timescale for particle i to correct its orientation is therefore much faster than the change in orientation experienced by the neighbors. We conclude that the system is robust to orientational fluctuations and is self-stabilizing.

Multiple species—We investigate whether alternating flocking stripes can form for any number of species. Extending our model to m species, we find that the flocking stripes phase persists under the following coupling rules:

$$J_{ab} = \begin{cases} -J, & a = b, \\ J, & b = (a + 1) \bmod m, \\ 0, & \text{otherwise,} \end{cases} \quad (2)$$

with $J > 0$ and species numbered $a, b \in \{0, 1, \dots, m-1\}$. As shown in Fig. 4, for $m > 2$, only odd m produce m distinct chasing stripes, while even m leads to overlap with every second species, effectively reducing the system to the $m = 2$ case. A similar parity dependence was previously observed in attraction-repulsion cyclic interactions [76]. The behavior of multispecies ($m > 2$) Vicsek models under general interactions presents intriguing open questions, which we will address in future studies.

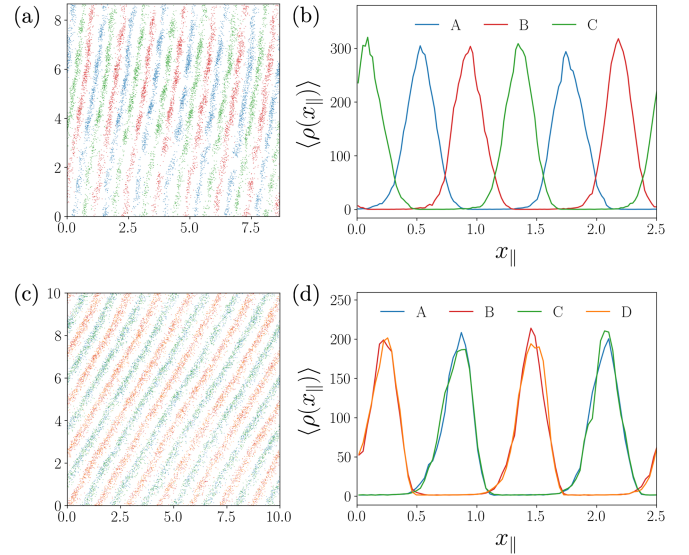


FIG. 4. Multispecies ($m > 2$) flocking. Snapshots of systems with alignment interactions governed by Eq. (2) with (a) $m = 3$ and (c) $m = 4$ species. (b)–(d) Time-averaged density profiles for the systems in (a)–(c), respectively. Simulation parameters were: $N = m \times 5 \times 10^4$, $\rho = 200$, $D_r = 0.2$, $J = 10(m + 1)$.

Conclusion and outlook—Despite being under intense scrutiny over the past three decades, Vicsek-like models still offer surprises. Recent studies have shown that multispecies systems of self-propelled particles can generate intricate patterns [76,77], including moving stripe and cluster formations under both reciprocal [31,58,60] and nonreciprocal [61–66,78–80] interactions. In this Letter, we have provided a comprehensive study of a generalized two-species Vicsek model with homogeneous and reciprocal couplings. Here, purely reciprocal alignment has led to a novel mechanism for interspecies phase separation and the emergence of collective motion. Distinct (anti)parallel flocking and nematic flocking phases were identified along the way. In particular, we highlighted a novel flocking stripes phase as a striking example of how alignment across species, yet antialignment within a species, can lead to a counterintuitive stable traveling bands state.

A theoretical approach to this problem is addressed in a companion paper [81], in which we derive a kinetic framework for the model and perform linear stability analysis. The theoretical results agree well with our particle-based simulation results. Strikingly, they also predict a finite wavelength instability with corresponding wavelength $\lambda = 1.23$ for the novel coexistence phases. The analytical results also explain the differences in parity for $m > 2$ species flocking stripes and their stripe ordering. The aforementioned extension of this problem to general interaction networks remains an open challenge that we will address in future work. We expect this to be fundamental to our understanding of the connection between the structure of the interaction couplings matrix and the

resulting emergent behavior, e.g., in drawing parallels with other cyclic interaction models [76,82–84]. By uncovering how global polar order emerges directly from species-level antialignment, we provide a new theoretical framework to interpret observed phase separation in coexisting biological populations. These robust “flocking stripe” and nematic phases offer an interesting set of rules that serve as a blueprint for controlling emergent behavior in synthetic systems, including programmable robotic swarms or active colloids with specifically designed interspecies interactions.

Note added—A preprint independently reporting the flocking stripes phase within a similar model [85] was released during the finalization of this manuscript.

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Data availability—The data that support the findings of this article are openly available [86].

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End Matter

Appendix—Here we provide a brief overview of the quantities used to classify the phases in Figs. 1 and 2. Further details on the calculation of the order parameters and classification of phases can be found in Supplemental Material [72].

For a set Ω of particles, the global *polar order* over the set Ω is given by

$$\Psi_{\Omega} = \left\langle \frac{1}{|\Omega|} \left| \sum_{i \in \Omega} \hat{\mathbf{p}}_i(t) \right| \right\rangle, \quad (\text{A1})$$

where $|\Omega|$ is the number of particles in the set Ω and the angular brackets denote an average over time, once in steady state. We measure the polar order within each species Ψ_A , Ψ_B , as well as the overall polar order Ψ . We also measure the difference in polar direction between the two species $\delta\theta_{\Psi}$.

Similarly, we can measure the *nematic order* parameters S_A , S_B , and S using

$$S_{\Omega} = \left\langle \frac{1}{|\Omega|} \left| \sum_{i \in \Omega} \hat{\mathbf{q}}_i(t) \right| \right\rangle, \quad (\text{A2})$$

where $\hat{\mathbf{q}}_i(t) = [\cos 2\theta_i(t), \sin 2\theta_i(t)]$ is the unit nematic orientation vector and $|\Omega|$ is the number of particles in the set Ω . We also measure the difference in nematic direction between the two species $\delta\theta_S$.

The degree of local species phase separation is measured via the following *demixing order parameter*:

$$\bar{d} = \left\langle \frac{1}{N} \sum_{i=1}^N \frac{1}{|\mathcal{D}_i|} \sum_{j \in \mathcal{D}_i} \delta_{s(i),s(j)} - \frac{1}{2} \right\rangle, \quad (\text{A3})$$

where $\mathcal{D}_i = \{j : |\mathbf{r}_i - \mathbf{r}_j| \leq r_d\}$ and $\delta_{s(i),s(j)}$ is a Kronecker delta. This quantity measures the average ratio of same species particles to total number of neighbors within a certain distance r_d of each particle. Note that we subtract a $1/2$ such that $\bar{d} > 0$ describes a demixed binary system. We are concerned with demixing that occurs on a length scale smaller than the interaction radius; thus, we choose $r_d = \sigma_I/2$ so that $\mathcal{D}_i$ is almost only comprised of particles of the same species in the striped phases. The value of r_d could be tuned further, but we found this to be sufficiently sensitive to changes in species demixing.

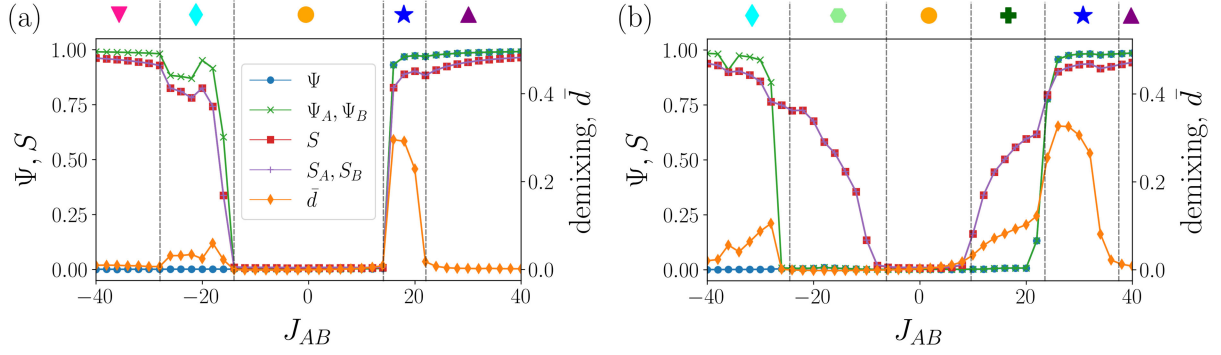


FIG. 5. Numerical order parameters (polar order, nematic order, and demixing) against J_{AB} for fixed (a) $J_{AA} = -15$ and (b) $J_{AA} = -25$. Vertical gray dashed lines show the phase boundaries between different phases, whose symbols plotted above correspond to those in Fig. 1.

Finally, for each system, we quantify the periodicity of spatial structures by checking for the appearance of a large peak around $k \approx 1/\lambda \approx 0.8$ in the power spectrum $|\delta\hat{\rho}(k)|^2$ of the density fluctuations parallel to the direction of order. This is a particularly helpful measure when the demixing order parameter is less sensitive to phase changes, for instance,

between antiparallel flocking (pink downward triangle) and antiparallel flocking stripes (light-blue diamond).

The numerical order parameters are plotted in Fig. 5 along the vertical slices at $J_{AA} = -15$ and $J_{AA} = -25$ from Fig. 1. A more refined characterization of the phase transitions is left as future work.