

Osinsky and Brilliantov Reply: Firstly, we note that the velocity distribution functions (VDFs) of dimers and trimers presented in the Comment [1] (simply combining two separate distributions) are not correct as the aggregation rates are velocity dependent, that is, one must solve instead a set of coupled Boltzmann equations [Eqs. (3)] of Ref. [2] for the respective VDFs.

Second, although the initially emerging distributions of dimers and trimers could be double-peaked, these and similar initial conditions are hardly relevant for the hydrodynamic theory of aggregating fluids. Indeed, such a theory addresses the hydrodynamic time scales when initial conditions are completely forgotten and the majority of aggregates are comprised of a large number of monomers. A cluster of a typical size $M \gg 1$ may be built up in many possible ways, from smaller clusters of different sizes and velocities. As a result, we obtain Gaussian (Maxwellian) VDF for such clusters, which is illustrated in Fig. 1 of the Supplemental Material (SM) of our article [2]. Figure 1 below also demonstrates that, as the aggregation process develops, VDF evolves toward a Maxwellian, even for relatively small clusters for $M = 5$.

Furthermore, the hydrodynamic stage of a process implies that *all* relevant characteristics of a system depend on space and time only through the five first moments of the VDF—density, flux velocity, and temperature (the velocity dispersion). Hence, these five moments are of primary importance, and if they are correct, the dynamics of a system may be described with good accuracy, even if the other moments deviate from the true ones. This is the reason that Euler’s hydrodynamics, which uses only these first moments (and a Maxwellian VDF), is so successful for many systems; it is also demonstrated in Figs. 1 and 2 of our paper [2]. Our Smoluchowski-Euler equations form a closed system for the five first moments of VDF, and the kinetic coefficients are expressed in terms of these moments only; see e.g., Eq. (7) in [2] for the coefficients C_{ij} .

A more general theory with a larger number of correct moments—the 14-moment Grad theory is developed in the SM of [2]. Here the deviations from a Gaussian distribution are expressed in terms of orthogonal polynomials expansion, Eq. (46) of the SM of [2]. For any VDF which has the same first five moments as for the Gaussian VDF, the kinetic coefficients are equal to the sum of the coefficients obtained for the Gaussian approximation, as C_{ij} in Eq. (7) of [2], and *corrections*, expressed in terms of the next-order moments, multiplied

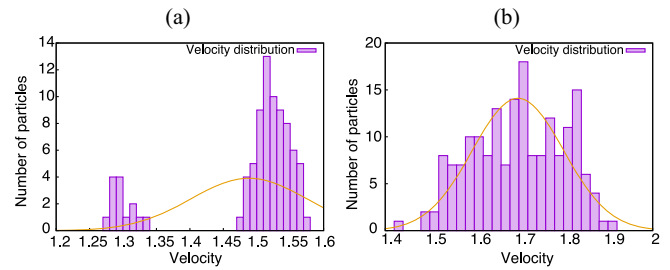


FIG. 1. VDF for clusters of size $M = 5$ for dust sedimentation with aggregation (see [2] for detail) for the height $z = -50m$ (a) and $z = -250m$ (b). As the process develops VDF tends to the Maxwellian.

by such coefficients as $C_{ij}^{(p,\alpha,\beta)}$ and $C_{ij}^{(q,\gamma)}$, in Eq. (65) of the SM of [2]. For Euler-Smoluchowski hydrodynamics these high-order coefficients are neglected, and the accuracy of this approach is confirmed by the numerical experiments shown in Figs. 1 and 2 of [2]. Hence, we demonstrate that for some important cases one can refrain from the usage of computationally heavy 14-moment Grad theory.

The statement of the author of the Comment that “...at the long-time stage the evolution is described by the standard Smoluchowski equation” is definitely not correct. Indeed, if by the “standard Smoluchowski equation” the author means phenomenological Smoluchowski Eqs. (2) of [2], with the steady velocities, predicted by the Stokes law and kinetic coefficients taken from the space-homogeneous Smoluchowski equations, Eq. (1) of [2], this is not correct as the velocities u_k in Eqs. (2) in [2] are very different from their steady (Stokes) values, without aggregation, as illustrated in Figs. 1(a) and 1(b) of our article [2]. Furthermore, an impact of aggregate temperatures, T_k , on the kinetic rates is not considered for “the standard Smoluchowski equations” (T_k assumed constant), although T_k change substantially during the aggregation processes. In the course of time, the velocities of clusters u_k of close sizes may become close [as in Figs. 1(a) and (b) of [2]], but they are very different from the steady values predicted by the Stokes law. Moreover, they evolve in time [the velocities in Figs. 1(a) and 1(b) of [2] are different]. The “standard Smoluchowski equations” cannot explain such a behavior, while our theory describes it quantitatively. Hence, the above statement in the Comment is not correct.

A. I. Osinsky^{1,2} and N. V. Brilliantov^{1,3}

¹Skolkovo Institute of Science and Technology
121205 Moscow, Russia

²Marchuk Institute of Numerical Mathematics of Russian Academy of Sciences
Moscow 119333, Russia

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International license](#). Further distribution of this work must maintain attribution to the author(s) and the published article’s title, journal citation, and DOI.

³Department of Mathematics, University of Leicester
Leicester LE1 7RH, United Kingdom

 Received 31 July 2025; accepted 10 October 2025;
published 13 November 2025
DOI: [10.1103/lnwl-t2rh](https://doi.org/10.1103/lnwl-t2rh)

- [1] A. Pikovsky, preceding Comment, *Phys. Rev. Lett.* **135**, [209701 \(2025\)](#).
- [2] A. I. Osinsky and N. V. Brilliantov, New hydrodynamic equations for space-inhomogeneous aggregating fluids, *Phys. Rev. Lett.* **133**, 217201 (2024).