

# Essay: Topological Phases and Exceptional Points in Non-Hermitian Systems

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(Received 18 February 2026; published 27 April 2026)

Non-Hermitian physics is an emerging field that intersects with condensed matter physics, quantum physics, and atomic, molecular, and optical physics. It provides a theoretical framework for open systems, which interact with their environment and undergo nonunitary evolution. This framework features mathematical structures, such as complex eigenvalues, exceptional points, and nonorthogonal eigenstates, fundamentally distinct from Hermitian theories. These structures not only facilitate new methods for precisely customizing non-Hermitian systems for various applications, but also give rise to topological phenomena absent in Hermitian systems, such as non-Hermitian skin effects, exceptional-point-enriched topological phases, and complex-energy band topology. As discoveries in non-Hermitian phenomena continue to grow, a key question arises: what critical impact will non-Hermitian physics have on fundamental physics? In this Essay, I will discuss my perspective on the important open problems and future applications associated with this research area, with an emphasis on novel topological phases and exceptional points in non-Hermitian systems that are relevant to fundamental physics and potential technological applications, aiming to inspire further ideas.

*Part of a series of Essays which concisely present author visions for the future of their field.*

DOI: [10.1103/ll76-j215](https://doi.org/10.1103/ll76-j215)

**Parity-time symmetry in non-Hermitian systems**—Non-Hermitian physics provides a novel framework for describing open quantum systems that interact with their environment [1–3]. Traditionally, physical theories assume isolated systems with no environmental interaction, which are described by Hermitian Hamiltonians. In reality, however, most systems are open and interact with their environments, which introduces dissipation and complicates their dynamics. Nowadays, various methods incorporate openness into quantum models. A rigorous quantum treatment of open systems is to replace the Hermitian Hamiltonian in the Schrödinger equation, responsible for the wave function’s time evolution, with a Liouvillian superoperator that governs the evolution of mixed states via the density matrix [4,5]. The resulting Lindblad master equation [6], rigorously captures the irreversible, dissipative evolution of open quantum systems and serves as the canonical model for Markovian quantum dynamics, combining coherent Hamiltonian evolution with environmental effects through Lindblad operators or jump operators. In this context, Markovian refers to memoryless dynamics, in

which the system’s evolution depends only on its current state and not on its history.

These methods show that when a system couples to an environment or bath, its effective dynamics become non-Hermitian and exhibit quantum jumps—discrete stochastic transitions between quantum states, induced by environmental interactions [7]. However, they are often technically demanding. This leads to the development of an effective non-Hermitian Hamiltonian formalism, which offers a more intuitive and computationally accessible alternative. It can be derived from the Lindblad master equation by neglecting the quantum jump term, and therefore, does not account for the associated jump processes (see Appendix D). A minimal physical picture is shown in Fig. 1(a), where a two-level system exchanges energy with an environment, leading to loss and decoherence that render the effective Hamiltonian non-Hermitian. This approach enables the exploration of phenomena that are challenging to investigate within traditional Hermitian frameworks [8–11], including unidirectional invisibility [12,13], enhanced sensitivity [14–19], topological energy transport [20–23], coherent perfect absorption [24–27], and single-mode lasing [28–30].

Non-Hermitian systems generally feature complex spectra. Remarkably, Bender and Boettcher [1] showed that certain non-Hermitian Hamiltonians can nonetheless exhibit entirely real spectra if they possess parity-time (*PT*) symmetry. While other antilinear symmetries can also yield real eigenvalues [31], *PT* symmetry stands out for its ease of physical implementation. *PT*-symmetric Hamiltonians effectively model systems with balanced

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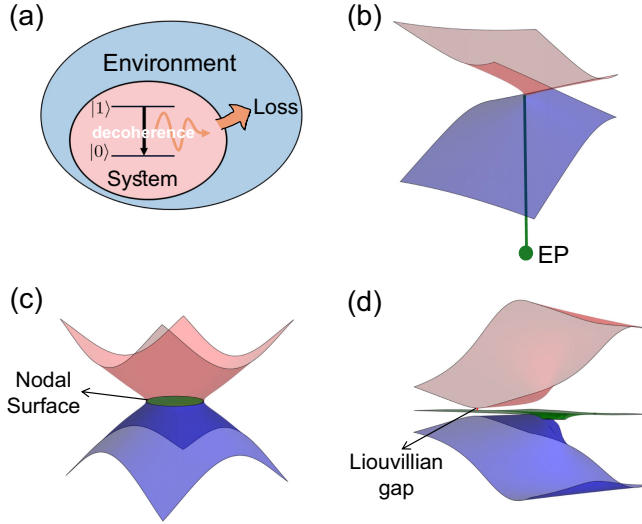


FIG. 1. (a) Schematic of a two-level open system interacting with its environment. (b) Eigenvalue spectrum of the non-Hermitian Hamiltonian of the two-level open system, featuring an exceptional point (EP). (c) Nodal structures composed of EPs. (d) Decoherence induces a finite gap on the Liouvillian-spectrum landscape. Pink (blue) denotes eigenenergies with larger (smaller) imaginary parts in (b) and (c), and larger (smaller) real parts in (d), corresponding to weaker (stronger) loss. Green highlights the EP in (b), the nodal surface in (c), and the Liouvillian eigenenergy in (d).

gain and loss [32,33]. Unlike Hermitian systems, their eigenfunctions are not orthogonal. When the system parameters, typically the balanced gain and loss, exceed a critical threshold,  $PT$  symmetry can spontaneously break, causing eigenvalues to become complex. This threshold marks the  $PT$ -symmetry-breaking transition, which is intimately connected to the concept of exceptional points (EPs) [9].

In this Essay, I discuss emerging non-Hermitian phenomena that challenge conventional paradigms in condensed matter and quantum physics. I also explore the interface between non-Hermitian physics and topology, with an emphasis on novel topological phases and EPs relevant to fundamental physics and applications, hoping to inspire the community to generate further groundbreaking ideas and next-generation applications.

**Exceptional points (EPs)**—A hallmark of non-Hermitian systems is the presence of EPs, branch-point singularities at which both the eigenenergies and the eigenvectors coalesce [Fig. 1(b)]. At these EPs, the eigenvectors become parallel, and the system exhibits enhanced sensitivity to perturbations [15,16] and the emergence of topologically nontrivial eigenspectrum structures in parameter space [34–42].

A minimal two-level model Hamiltonian illustrates these features [43]:

$$H_{PT} = s(\sigma_x + ia\sigma_z) = s \begin{pmatrix} ia & 1 \\ 1 & -ia \end{pmatrix},$$

where  $s \geq 0$  sets the energy scale and  $a \geq 0$  quantifies non-Hermiticity. The eigenenergies are  $E_{\pm} = \pm s\sqrt{1 - a^2}$ , with an EP at  $a = 1$ . Unlike Hermitian cases, the eigenstates satisfy  $H_{PT}|\psi_{R,\pm}\rangle = E_{\pm}|\psi_{R,\pm}\rangle$  and  $H_{PT}^{\dagger}|\psi_{L,\pm}\rangle = E_{\pm}^*|\psi_{L,\pm}\rangle$ , with  $\langle\psi_{R,+}|\psi_{L,-}\rangle \neq 0$ , indicating nonorthogonality. This Hamiltonian respects  $PT$  symmetry, yielding two distinct phases: a  $PT$ -unbroken phase with real eigenvalues and a  $PT$ -broken phase with complex eigenvalues. The EP marks a non-Hermitian phase transition, the  $PT$  transition, where quantities such as state distinguishability (a measure of the distinguishability between quantum states) and the recurrence time (the characteristic time over which the distinguishability returns to its initial value) follow power-law scaling [43], revealing critical behavior in the information flow. Once the symmetry is broken, however, this power-law behavior vanishes and information retrieval becomes suppressed. Consequently, EP-induced criticality arises only under specific symmetry conditions.

EPs are well established in gain-loss systems, especially in optics and photonics [44–46], where optical modes (analogous to eigenstates) experience loss via absorption or radiation. Such open systems are effectively modeled by non-Hermitian Hamiltonians. Beyond optics, EPs underpin novel phenomena in mechanics [47–49], electromagnetism [50–53], acoustic-phononic systems and materials [54,55], atomic and molecular physics [56–61], quantum phase transitions [62,63], and quantum chaos [64]. Notably, EPs have attracted broad interest for sensing applications [15,16]. Near an EP, small perturbations can induce characteristic spectral splittings that scale differently from those near Hermitian degeneracies [65–67]. At the same time, the coalescence of eigenstates may amplify noise, leaving open whether the sensitivity enhancement translates into an improved signal-to-noise ratio in realistic settings [68–73]. This pursuit naturally leads to broader questions: Beyond sensing, what are other practical applications of EPs? Moving beyond EPs, what unique phenomena does non-Hermiticity offer? Addressing these open questions is crucial for translating theoretical advancements into robust, functional technologies.

Beyond isolated EPs, non-Hermitian systems exhibit fundamentally different band structures from their Hermitian counterparts, enabling greater flexibility and unique topologies. This distinction has fueled growing interest in their rich topological characteristics, such as Berry phase [74,75] and vorticity [76], and can give rise to open Fermi arcs in bulk dispersions [77]. Most existing research has focused on isolated (0D) or linelike (1D) EPs. By contrast, higher-dimensional EP structures offer a richer avenue for exploring non-Hermitian topology [41,76,78–83], serving as natural extensions of nodal points to higher dimensions [64], such as nodal surfaces formed by EPs in parameter space [Fig. 1(c)]. These higher-dimensional EP structures are more robust and connect to open Fermi volumes, regions in momentum space where

the real part of the energy gap vanishes [84], thereby enriching electronic behavior. Consequently, non-Hermitian nodal semimetals transition into symmetry-protected metals whose EP surfaces bound Fermi volumes.

While nodal lines, chains, and knots have been extensively studied in Hermitian systems, non-Hermiticity introduces a regime where such structures emerge purely from topology, independent of symmetry [84,85]. Moreover, intrinsically knotted or linked exceptional lines (ELs) (see Appendix A) are topologically stable: their structure remains unchanged under deformation unless reconnection, crossing, or collapse to a point occurs. Thus, even under symmetry-breaking perturbations, the nodal topology persists over a finite range of perturbation strengths, rendering the knotted ELs robust against generic disturbances [84]. Beyond their conceptual novelty, knotted ELs offer practical advantages. Unlike isolated EPs, which are highly susceptible to fabrication errors and experimental imperfections, the extended structure of knotted ELs maintains robustness across a wide range of perturbations, thereby broadening the parameter space for sensing. Elucidating the interplay between robustness and sensitivity in higher-dimensional EP structures, and extending their implementation to photonic, microwave, and acoustic platforms, thus emerges as a compelling research direction.

*Topology in non-Hermitian systems*—Beyond EPs, the non-Hermitian skin effect (NHSE), another hallmark of non-Hermitian systems, has attracted growing attention [86–96]. In parallel with the development of topological insulators, which enable robust, dissipationless transport [97–99], recent studies have begun to uncover the interplay between topology and non-Hermiticity [100–102].

The bulk-boundary correspondence (BBC), central to topological phases in Hermitian systems [103,104], ensures that nontrivial bulk invariants predict edge states. In non-Hermitian systems, however, the BBC breaks down: under open boundary conditions (OBC), bulk eigenstates deviate drastically from their periodic-boundary condition (PBC) counterparts [105,106], leading to the NHSE, where bulk states localize at boundaries, in contrast to the extended modes characteristic of Hermitian systems [86]. Consequently, conventional Bloch-based invariants no longer capture the underlying topological physics.

To restore the BBC, non-Bloch band theory introduces generalized topological invariants that reflect the non-Bloch nature of bulk states [107,108]. The key idea is to deform the Bloch Hamiltonian by embedding an imaginary flux, thereby interpolating between PBC and OBC behavior. Central to this framework is the construction of the generalized Brillouin zone (GBZ), which captures localization effects absent in standard Bloch bands. This construction yields a non-Bloch Hamiltonian defined on the GBZ, enabling accurate computation of topological invariants and reliable prediction of edge states. Despite these advances, several theoretical challenges remain:

generalizing the GBZ framework to interacting, disordered, or higher-dimensional systems, and connecting it to quantum dynamics—such as Liouvillian spectra and stochastic quantum jumps.

Since physics is ultimately an experimental science, progress in non-Hermitian physics is closely tied to empirical observations and realizations of these phenomena. Recent advances in synthetic platforms, including photonics [45], phononics [109], electrical circuits [110], acoustic platforms [40,111–114], and mechanical systems [115], have enabled controlled realizations of non-Hermitian effects, spurring intense interest in non-Hermitian topological systems. Analogies to Hamiltonian quantum mechanics emerge naturally in many of these classical platforms. For instance, in configurations like optical waveguides [116–118], the propagation of light can be described by a wave equation resembling the time-dependent Schrödinger equation for non-Hermitian systems. Photonic and acoustic crystals rely on Bloch’s theorem, directly relating their periodic structures to quantum lattice models [119]. Mechanical metamaterials [115] possess dynamical matrices akin to Hamiltonians, where asymmetries induce wave-manipulating effects similar to those of non-Hermitian quantum states. Electrical circuits [110] encode non-Hermitian behavior in response functions such as impedance, mirroring the dynamics of quantum systems. These classical-quantum analogies provide a powerful route for translating non-Hermitian concepts into engineered materials and devices. However, several fundamental and practical questions remain open. How faithfully do these platforms capture genuine quantum features, such as entanglement dynamics and Liouvillian criticality [120]? How scalable and robust are they against disorder, decoherence, or fabrication imperfections?

Recent work goes beyond using non-Hermitian systems as simple analogies and has begun to realize non-Hermitian concepts in engineered platforms. Photonic lattices with tailored gain-loss profiles exhibit unidirectional transport and robust lasing modes [45], while topoelectrical circuits realize non-Bloch band topology for disorder-resilient signal control [121]. Acoustic metamaterials likewise show non-Hermitian wave localization, and topological transitions [109]. Together, these examples illustrate how non-Hermitian physics enables topological transport and mode-selective single-mode lasing—mechanisms with no Hermitian counterparts. However, it remains unclear which advantages survive beyond fine-tuned models, and how to harness them in realistic, noisy, and large-scale settings.

*Non-Hermitian effects in the quantum regime*—To date, simulations of non-Hermitian systems on classical platforms have largely relied on effective medium theories, which provide coarse-grained descriptions that replace a heterogeneous structure with an equivalent uniform medium [122]. While useful, these approaches often neglect essential quantum processes, such as stochastic

quantum jumps, that can critically influence system behavior. A promising direction is to investigate non-Hermitian effects in small-scale devices, where quantum effects are both relevant and potentially transformative. Such studies can be useful to reveal new aspects of the interplay between classical and quantum dynamics, uncovering phenomena that bridge these regimes and inspiring novel technologies.

In quantum-scale systems, coherence, entanglement, and stochasticity render traditional effective theories inadequate. Explicitly incorporating these ingredients could refine our understanding of non-Hermitian physics in realistic settings. This line of research also has the potential to significantly advance quantum sensing, information processing, and energy harvesting, particularly through phenomena such as EPs. Key questions arise. How do quantum processes reshape non-Hermitian dynamics? Which device configurations best harness these effects? How can we engineer systems that exploit them for technological benefit?

Quantum optics with indistinguishable photons provides a particularly compelling test bed at the intersection with non-Hermitian physics. Dissipation, through loss and dephasing, naturally introduces non-Hermitian effects, enabling phenomena such as EPs and mode switching. Recent work has demonstrated nonunitary evolution of single photons under  $PT$ -symmetric Hamiltonians [123]. Although based on first-quantized frameworks, the field is rapidly progressing toward a second quantization with multiple-photon platforms [124], heralding a new era of non-Hermitian quantum optics. Beyond the constraints of Hermiticity, one could ask, what novel correlated phases emerge in strongly interacting non-Hermitian systems [125]? More fundamentally, do non-Hermitian interactions reshape quantum criticality in many-body systems?

Within quantum optical systems, photonic quantum walks stand out as powerful tools for simulating non-Hermitian topological phases [89]. These walks—quantum analogs of classical random walks—are governed by Floquet operators that can be engineered to produce topologically nontrivial effective Hamiltonians. By tuning the coin and Floquet operators, one can access a rich spectrum of behaviors, from standard topological phases to exotic non-Hermitian dynamics. This versatility enables both fundamental studies and practical applications in open-system quantum information processing.

Nonunitary quantum walks have, for instance, realized the non-Hermitian Su-Schrieffer-Heeger (SSH) model [89,126]. This prototypical model combines the familiar topological features of the SSH chain with gain-loss asymmetry, providing a platform to probe non-Bloch topology and boundary states (see Appendix B). Experiments have identified boundary states at  $PT$ -symmetric domain walls and revealed topological invariants from photon displacement. These results underscore the promise of quantum walks as platforms for non-Hermitian topological physics and for emergent quantum technologies.

Another important example, quantum walks can be used in studying non-Bloch EPs [63], which emerge in the GBZ rather than the conventional Bloch framework, even in translationally invariant systems, and result directly from NHSE, highlighting its central role beyond standard band topology. Notably, non-Bloch EPs establish a direct connection between  $PT$ -symmetry breaking and bulk periodicity, revealing a general mechanism applicable across a broad class of non-Hermitian systems (see Appendix C). Under NHSE, Bloch spectra form loops in the complex plane [88,91] and cannot exhibit real energy bands or  $PT$  symmetry. Non-Bloch spectra from the GBZ, however, appear as open arcs or lines enclosing no area, enabling real spectra and thus  $PT$  symmetry. Consequently, non-Bloch  $PT$  symmetry is the only viable mechanism to restore  $PT$  symmetry in systems with NHSE. Non-Bloch topological invariants defined on the GBZ are introduced, capturing the deformation of bulk modes induced by NHSE. In quantum walks, these invariants are measurable via the biorthogonal chiral displacement [126]. When the quasienergy spectrum is real, as ensured by non-Bloch  $PT$  symmetry under OBC, the displacement provides an experimentally accessible measure of the non-Bloch topological invariants.

These concepts have been experimentally validated in photonic quantum walks [89], which exhibit both NHSE and non-Bloch band deformation. Given their generality, these phenomena are expected to arise in various quantum platforms, including solid-state spin systems [127], superconducting qubits [128], ultracold atoms [94], and trapped ions [129,130]. Their realization underscores non-Hermitian physics as a natural framework for realistic quantum systems, where dissipation, decoherence, and environmental coupling are intrinsic rather than negligible.

Non-Hermitian approaches thus enable systematic exploration of topology and criticality in open quantum systems, beyond idealized closed-system descriptions. Rather than implying intrinsic advantages, this framework captures experimentally accessible dynamics shaped by loss, noise, and spectral non-Hermiticity. A central challenge remains: assessing the validity of effective non-Hermitian descriptions in realistic settings, where noise and quantum jumps can qualitatively alter the dynamics and obscure which predictions remain robust to imperfections.

*Liouvillian EPs*—In classical treatments, non-Hermitian Hamiltonians are typically obtained by neglecting quantum jumps and focusing solely on dissipation [120,131,132]. A fully quantum framework explicitly incorporates these jump terms, which can significantly modify both dynamics and eigenspectrum (see Appendix D). Within this framework, degeneracies in the Liouvillian superoperator eigenspace define Liouvillian EPs [120,131]. Dissipative quantum systems thus exhibit two distinct kinds of EPs: one associated with the non-Hermitian Hamiltonian when jumps are neglected, and the other intrinsic to the Liouvillian structure. Crucially, these Liouvillian EPs differ

fundamentally from Hamiltonian EPs, as the Liouvillian and the non-Hermitian Hamiltonian generally possess different eigenspaces. Incorporating full quantum decoherence effects therefore enables Liouvillian EPs to serve as a powerful probe of non-Hermitian physics in open quantum systems [Fig. 1(d)]. These EPs underlie phenomena such as chiral state transfer [133,134], enabling directional control of quantum evolution and dissipation-assisted quantum manipulation. Understanding and exploiting these effects is essential for robust quantum control and sensing in open systems. However, major challenges remain: Identifying accessible Liouvillian EPs, clarifying the role of noise and measurement backaction, and extending control beyond few-level models.

*Non-Hermitian sensing*—The interplay between quantum noise and non-Hermitian sensing offers promising pathways to surpass classical limits in precision measurements.

Beyond the earlier discussion of EP-based sensing, a broader open question under active debate in the community is the following: can non-Hermiticity enhance sensing in realistic noisy environments? While many works suggest possible enhancement, recent studies [135] argue that EPs themselves do not provide genuine sensitivity gains once noise and quantum measurement theory are fully considered. This controversy highlights the need to explore alternative non-Hermitian mechanisms, such as nonorthogonal eigenstates [18,136], complex eigenvalues [137], and critical behavior near topological phase transitions [135,138,139], to assess whether true advantages can be realized in realistic settings.

Among these, criticality associated with non-Hermitian topological transitions can produce particularly sharp responses to perturbations [138], making it a natural sensing resource (see Appendix E). Because realistic probes inevitably experience decoherence, it is especially important to identify sensing protocols whose advantages persist in open-system settings. In this sense, non-Hermitian topology provides a promising route toward robust sensing in noisy environments, while also offering insight into the interplay among topology, decoherence, and dynamics in open quantum systems.

*Discussions and outlook*—Non-Hermitian systems challenge traditional notions of quantum mechanics, offering unique properties that can be leveraged in various applications, ranging from novel materials to advanced quantum devices. The exploration of non-Hermitian phenomena not only deepens our understanding of complex systems but also offers strategies for harnessing their behavior in ways previously deemed unattainable.

Looking ahead, non-Hermitian physics is poised to drive exciting developments, generating new ideas, concepts, and processes. Its implications extend far beyond theoretical interests: they carry substantial potential to contribute to the global competition in emerging technologies and products. By building on these insights, scientists and engineers can devise innovative solutions to pressing challenges in

energy, computing, nanotechnology, and related fields. As the research landscape in these fields evolves, sustained interdisciplinary collaboration will be key to unlocking the full potential of non-Hermitian physics. I conclude by highlighting several future directions:

*Topological quantum computing*: Exploiting the inherent robustness of non-Hermitian topology may pave the way for fault-tolerant quantum computation [140]. Encoding quantum information in topologically protected systems could mitigate decoherence, a primary obstacle to quantum technologies.

*Enhanced photonic devices*: The development of topological lasers opens the door to developing more efficient and stable photonic devices [141,142]. Harnessing non-Hermitian topological boundary states could yield lower-threshold lasers with enhanced performance in integrated circuits, potentially revolutionizing telecommunications and data processing.

*Novel materials discovery*: Ongoing research on non-Hermitian topological matter is likely to uncover new materials with unique electronic and optical properties [143]. Such materials may host exotic excitations or exhibit tunable topological features, expanding the material repertoire for future applications in electronics and optoelectronics.

*Applications in metrology and sensing*: The pronounced sensitivity of non-Hermitian systems to perturbations can be harnessed for advanced sensing technologies [18,137]. Leveraging the distinctive response of topological boundary states to external fields could enable more precise metrological tools for applications in quantum sensing and metrological tools.

*Non-Hermitian topology in correlated systems*: While non-Hermitian physics is well established in classical and single-particle settings, the conceptually complex scenario of open quantum many-body systems, effectively described by non-Hermitian Hamiltonians, remains a frontier [125]. This fascinating and challenging research direction may transform our understanding of quantum dynamics, offering fresh insights into strongly correlated phenomena [144,145], and quantum phase transitions [146].

The journey into non-Hermitian physics is still in its infancy, placing us at the threshold of a new era in materials science and quantum technologies. These uncharted territories hold substantial promise for advancing our understanding and enabling the next generation of technological breakthroughs.

*Acknowledgments*—This work has been supported by the National Key R&D Program of China (Grant No. 2023YFA1406701) and the National Natural Science Foundation of China (Grants No. 92265209 and No. 12025401).

*Data availability*—No data were created or analyzed in this study.



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## End Matter

*Appendix A: Exceptional lines*—As a brief illustrative example, consider a non-Hermitian two-band model with intrinsic knotted or linked ELs in momentum space,

$$H(\mathbf{k}) = \mathbf{d}_R(\mathbf{k}) \cdot \boldsymbol{\sigma} + i\mathbf{d}_I(\mathbf{k}) \cdot \boldsymbol{\sigma}, \quad (\text{A1})$$

where  $\boldsymbol{\sigma}$  are Pauli matrices,  $\mathbf{k}$  is the lattice momentum, and  $\mathbf{d} = \mathbf{d}_R + i\mathbf{d}_I$  is the complex Bloch vector with  $\mathbf{d}_R(\mathbf{k}) = [f_1(\mathbf{k}) - \epsilon, \epsilon, 0]$ ,  $\mathbf{d}_I(\mathbf{k}) = [0, f_2(\mathbf{k}), \sqrt{2}\epsilon]$ . Here  $f_{1,2}(\mathbf{k})$  are real, continuous scalar functions constrained by  $f_1(\mathbf{k}) + if_2(\mathbf{k}) = Z_0^p + Z_1^q$  with  $(Z_0, Z_1) \in \mathbb{C}^2$  satisfying  $|Z_0|^2 + |Z_1|^2 = 1$ , and  $\epsilon$  a sufficiently large constant [84,85].

If  $(p, q)$  are coprime (or both even), the resulting ELs exhibit  $(p, q)$  knot (or link) topology and bound topologically nontrivial open Fermi–Seifert surfaces.

*Appendix B: Effective Hamiltonian for a quantum walk*—As an example, let us consider a chiral symmetric nonunitary quantum walk on a one-dimensional lattice, which is governed by the Floquet operator [89]:

$$U = R(\theta_1)S_2R(\theta_2)MR(\theta_2)S_1R(\theta_1) := e^{-iH_{\text{eff}}}. \quad (\text{B1})$$

Here, the unitary coin operator  $R(\theta)$ , which determines the choice of the quantum walker, rotates the coin state by angle  $\theta$  about the  $y$  axis. The shift operator,  $S_1$  ( $S_2$ ), translates the walker one site to the right (left) conditioned on the coin state  $|1\rangle$  ( $|0\rangle$ ). Non-Hermiticity arises from the partial measurement,  $M = \mathbb{1}_w \otimes (e^\gamma|0\rangle\langle 0| + e^{-\gamma}|1\rangle\langle 1|)$ , where  $\mathbb{1}_w$  is the identity on the

lattice modes and  $\gamma$  is a tunable gain-loss parameter. The time-evolved state is  $|\Phi(t)\rangle = U^t|\Phi(0)\rangle$  for  $t = 0, 1, 2, \dots$ . This discrete-time quantum walk stroboscopically simulates the nonunitary evolution, governed by an effective non-Hermitian Hamiltonian,  $H_{\text{eff}}$ , which features NHSE and the breakdown of conventional BBC.

A key property of the Floquet operator, is its chiral symmetry,  $\sigma_x U \sigma_x = U^{-1}$  (with  $\sigma_x$  the Pauli matrix), equivalent to  $\sigma_x H_{\text{eff}} \sigma_x = -H_{\text{eff}}$ . This chiral symmetry corresponds to sublattice symmetry. Notably, when boundaries are present,  $U$  exhibits NHSE, wherein all eigenstates localize near the edges [89].

In momentum space, the Fourier component of  $U$  is  $U(k) = d_0\sigma_0 - id_1\sigma_x - id_2\sigma_y - id_3\sigma_z$ , with

$$\begin{aligned} d_0 &= -\cosh \gamma \sin \theta_1 \sin \theta_2 + \cosh \gamma \cos k \cos \theta_1 \cos \theta_2 \\ &\quad + i \sinh \gamma \sin k \cos \theta_1, \\ d_1 &= 0, \\ d_2 &= \cosh \gamma \cos \theta_1 \sin \theta_2 + \cosh \gamma \cos k \sin \theta_1 \cos \theta_2 \\ &\quad + i \sinh \gamma \sin k \sin \theta_1, \\ d_3 &= -\cosh \gamma \sin k \cos \theta_2 + i \sinh \gamma \cos k. \end{aligned} \quad (\text{B2})$$

Near  $k = 0$ ,  $d_3$  resembles  $\sin k + i\gamma/2$ , akin to the non-Hermitian SSH model, providing intuition for NHSE in  $U$ . Since  $U = e^{-iH_{\text{eff}}}$ , the eigenvectors of  $H_{\text{eff}}(k)$  follow from those of  $U(k)$ :  $U(k)|\psi_\pm\rangle = \lambda_\pm|\psi_\pm\rangle$  and

$[U^{-1}(k)]^\dagger |\chi_\pm\rangle = (\lambda_\pm^*)^{-1} |\chi_\pm\rangle$ . Here  $\lambda_\pm = d_0 \pm E_k$ , and  $E_k := i\sqrt{d_1^2 + d_2^2 + d_3^2}$ . A formal expression of  $H_{\text{eff}}(k)$  can be written as

$$H_{\text{eff}}(k) = i \left[ \ln(\lambda_+) \frac{|\psi_+\rangle\langle\chi_+|}{\langle\chi_+|\psi_+\rangle} + \ln(\lambda_-) \frac{|\psi_-\rangle\langle\chi_-|}{\langle\chi_-|\psi_-\rangle} \right], \quad (\text{B3})$$

which features NHSE and the breakdown of conventional BBC.

*Appendix C: OBC non-Bloch spectrum*—As a brief illustrative example, The  $PT$ -symmetry transition is characterized by the spectrum of the effective Hamiltonian  $H_{\text{eff}}$ : real eigenvalues correspond to the  $PT$ -unbroken phase, while complex conjugate pairs indicate symmetry breaking. EPs, where both eigenvalues and eigenvectors coalesce, signal the transition point and defy standard Bloch-band descriptions. Under OBC, a wave function  $\psi_{n,j}$  for a particle in unit cell  $n$  maps to  $\beta^n \psi_j$ , where  $\beta = e^{ik}$  and  $\psi_j$  are eigenvectors of the Bloch Hamiltonian. Solving for  $\beta$  defines the GBZ, which can be viewed as a complex deformation  $k \rightarrow k - i \ln r$ , with  $\ln r$  representing the inverse localization length of skin modes. Consistently, the Bloch-Floquet operator  $U(k)$ , defined over  $k \in [0, 2\pi]$ , fails to reproduce the correct spectrum due to NHSE, which causes exponential localization of eigenstates. To resolve this mismatch, the standard phase factor  $e^{ik}$  is replaced by a complex deformation  $\beta$ , with  $|\beta| \neq 1$  in general. The set of such  $\beta$  values forms the GBZ, a closed loop in the complex plane that deviates from the unit circle. The OBC spectrum is then obtained by analytically continuing  $U(k)$  to  $U(\beta)$  and taking the logarithm of its eigenvalues. Crucially,  $U(\beta)$  satisfies the  $\eta$ -pseudo-unitarity, i.e.,  $\eta U^{-1}(\beta) \eta^{-1} = U^\dagger(\beta)|_{\beta \in \text{GBZ}}$ , provided  $|\cos \theta_2^{L(R)}| > |\tanh \gamma|$ , where  $\eta = \sum_n |\chi_n\rangle\langle\chi_n|$  is constructed from the left eigenstates  $|\chi_n\rangle$  of  $U(\beta)$ . The condition  $|\cos \theta_2^{L(R)}| = |\tanh \gamma|$  thus marks the location of non-Bloch EPs in parameter space.

*Appendix D: Quantum jumps*—As a brief illustrative example, consider a two-level system that shows how quantum jump processes can be incorporated into a non-Hermitian framework via the Lindblad master equation, leading to the emergence of Liouvillian EPs [133]. The ground state  $|0\rangle$  is coherently coupled to the excited state  $|1\rangle$ , which undergoes population loss to the environment through an intermediate state  $|2\rangle$ . Additionally, spontaneous decay from  $|2\rangle$  back to  $|1\rangle$  induces dephasing in the  $\{|0\rangle, |1\rangle\}$  subspace. The system is governed by a hybrid Lindblad master equation:

$$\dot{\rho} := \mathcal{L}\rho = -i[H_L, \rho] + L\rho L^\dagger - \frac{1}{2}\{L^\dagger L, \rho\}, \quad (\text{D1})$$

where  $\mathcal{L}$  is the Liouvillian superoperator, and  $H_L = H_0 - i\Gamma|1\rangle\langle 1|$  is an effective non-Hermitian Hamiltonian with Hermitian part  $H_0$  and decay rate  $\Gamma$ . The quantum jump operator  $L = \sqrt{\gamma_\phi}|1\rangle\langle 1|$  represents dephasing with rate  $\gamma_\phi$ . While an EP exists in the spectrum of  $H_L$ , the Liouvillian spectrum  $\lambda_i$  ( $i = 1, \dots, 4$ ), obtained from  $\mathcal{L}\rho_i^R = \lambda_i \rho_i^R$ , can exhibit its own EP, generally at different parameter values. This distinction highlights the fundamentally different nature of Liouvillian and Hamiltonian EPs.

In the absence of dephasing, encircling a Hamiltonian EP in parameter space leads to chiral state transfer: depending on the direction of encirclement, the final state either switches or remains unchanged. This behavior arises from the interplay between the topology of the eigenspectrum and non-Hermitian amplification of nonadiabatic transitions. Specifically, the adiabatic trajectory leads to asymmetric evolution, yielding distinct outcomes for clockwise and counterclockwise encirclement. The inclusion of dephasing qualitatively alters this picture. As  $\gamma_\phi$  increases, the final states corresponding to opposite encircling directions gradually converge, and chirality diminishes. This can be understood via the eigenspectrum of the Liouvillian: dephasing induces a finite Liouvillian gap, causing the system to first relax toward, and then adiabatically follow, a quasi-steady state. In the long-time limit, this relaxation suppresses nonadiabatic effects, erasing the chiral behavior. Thus, quantum jumps enforce adiabaticity and eliminate the directional distinction inherent to the non-Hermitian Hamiltonian dynamics.

*Appendix E: Non-Hermitian enhanced sensor without EPs*—As a brief illustrative example, consider a generic non-Hermitian qubit sensor operating without EPs. The system is governed by the Hamiltonian  $H_S = (2\mathcal{E}/1 + a)[1(a/\delta)/\delta a]$ , where  $|a| \leq 1$ ,  $|\delta| \ll 1$ , and  $\mathcal{E}$  sets the energy scale. The system is perturbed by a weak external field  $V = \lambda \sigma_x$ , with  $\lambda$  the parameter to be estimated [18]. Crucially, the sensitivity enhancement does not rely on tuning to an EP.

The qubit is initialized in  $|0\rangle$ , near an eigenstate of  $H_S$ , and evolves into a state  $|\psi(t)\rangle \sim D|0\rangle + \delta|1\rangle$ , with  $D$  and  $\delta$  dependent on  $\lambda$ . When  $|D| \sim |\delta| \ll 1$ , the population in  $|0\rangle$  becomes highly sensitive to small variations in  $\lambda$ , providing a practical sensing mechanism. This sensitivity stems from two core features of non-Hermitian physics: nonorthogonal eigenstates, which allow broader exploration of the Hilbert space, and complex eigenvalues, which induce distinct dynamical responses such as amplification and decay. This sensor outperforms its Hermitian counterpart, particularly under technical noise that cannot be mitigated by repeated measurements. The approach is robust and broadly applicable, as non-Hermitian dynamics are routinely implemented in photonic, atomic, and superconducting platforms, and extend naturally to higher-dimensional systems.