

# Lindbladian versus Postselected non-Hermitian Topology

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(Received 23 July 2025; revised 16 October 2025; accepted 16 December 2025; published 6 January 2026)

The recent topological classification of non-Hermitian “Hamiltonians” is usually interpreted in terms of pure quantum states that decay or grow with time. However, many-body systems with loss and gain are typically better described by mixed-state open quantum dynamics, which only correspond to pure-state non-Hermitian dynamics upon a postselection of measurement outcomes. Since postselection becomes exponentially costly with particle number, we here investigate to what extent the most important example of non-Hermitian topology can survive without it: the non-Hermitian skin effect and its relationship to a bulk winding number in one spatial dimension. After defining the winding number of the Lindbladian superoperator for a quadratic fermion system, we systematically relate it to the winding number of the associated postselected non-Hermitian Hamiltonian. We prove that the two winding numbers are equal (opposite) in the absence of gain (loss), and provide a physical explanation for this relationship. When both loss and gain are present, the Lindbladian winding number typically remains quantized and non-zero, though it can change sign at a phase transition separating the loss and gain-dominated regimes. This transition, which leads to a reversal of the Lindbladian skin effect localization, is rendered invisible by postselection. We also identify a case where removing postselection induces a skin effect from otherwise topologically trivial non-Hermitian dynamics.

DOI: [10.1103/ljvt-w6hw](https://doi.org/10.1103/ljvt-w6hw)

**Introduction**—The topological classification of gapped condensed matter is currently in full swing across various corners of physical parameter space [1,2]. In the context of noninteracting electrons, the theory of topological insulators and (mean-field) superconductors can be considered particularly mature. What makes this theory so successful is that it relates readily available *single-particle* quantities, like a band structure’s Berry curvature, to quantized response functions such as the Hall conductivity, which are a property of the full *many-body* ground state. Moreover, the bulk-boundary correspondence guarantees that a topological band structure coincides with gapless edge excitations on top of the many-body ground state, e.g., the coveted Majorana zero modes. These examples demonstrate that we must really think of topological band structures with a many-body mindset to see their most striking physics, even though they describe essentially noninteracting materials.

Topological band theory has been fruitfully generalized to non-Hermitian (NH) Hamiltonians whose spectra lie in the complex plane [3–15]. Here there are different notions

of gaps, the most interesting one is the *point gap*, as it has no Hermitian counterpart—it denotes a region of complex energy space that is surrounded by, but does not contain, complex eigenvalues. We now have a full classification of the topological equivalence classes that emerge when a point gap is kept open in addition to various global symmetries [3]. At least at the single-particle level, a new NH bulk-boundary correspondence relates point-gap topology to robust boundary states without equilibrium counterpart: the most famous example is the NH skin effect in one dimension (1D), where a macroscopic number of single-particle edge states are induced by a bulk spectral winding around the point gap [16,17]. However, it is difficult to make sense of NH Hamiltonians within a many-body picture. E.g., for fermions, it is unclear how single-particle orbitals should be filled up to make a sensible many-body state. This problem is compounded by the fact that many-body NH Hamiltonians are inherently nonlocal [18–21].

Physically, NH Hamiltonians arise in open quantum dynamics under a postselection of measurement outcomes [22,23]. Starting with a state of  $N$  particles, if measurements of the particle number are performed to guarantee no particle is lost or gained from the environment, the resulting effective dynamics is described by a NH Hamiltonian. With  $N \gg 1$  however, such postselection becomes near impossible, as the chance that no quantum jump has occurred in any given time interval vanishes exponentially fast with the particle number. This means that pure states evolving under

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NH Hamiltonians—even those that are noninteracting—are often inadequate physical descriptions of open quantum condensed matter. To alleviate this shortcoming, we investigate to what extent postselected NH point-gap topology survives in a full Lindblad treatment of the open quantum dynamics of density matrices [22,24–27]. The analog of noninteracting NH Hamiltonians are quadratic Lindbladians with linear jump operators, which do not suffer from the aforementioned problems regarding their many-body interpretation. We first recall third quantization [28,29], which can be used to solve such quadratic systems, both bosonic and fermionic. While we focus on fermions here, the bosonic case can be treated analogously as shown in Supplemental Material (SM) [30].

*Lindbladian band structure*—Absent interactions, the Gorini-Kossakowski-Sudarshan-Lindblad (GKSL) equation takes as input a Hermitian Hamiltonian  $H$  determining the unitary part of the dynamics, and a family of jump operators  $\{J_m\}_m$  that capture the interaction with an environment. The time ( $\tau$ ) evolution of the density operator  $\rho$  describing the quantum system in question is then given by

$$i \frac{d\rho}{d\tau} = (H_{\text{post}}\rho - \rho H_{\text{post}}^\dagger) + i \sum_m J_m \rho J_m^\dagger. \quad (1)$$

Here we have defined the *postselected* NH Hamiltonian

$$H_{\text{post}} = H - \frac{i}{2} \sum_m J_m^\dagger J_m \quad (2)$$

that governs dynamics conditioned on the absence of quantum jump events [22]. To determine the role of postselection in stabilizing point gap topology, we consider a U(1) charge-conserving  $H_{\text{post}}$ ,

$$H_{\text{post}} = \sum_{ij} \mathcal{H}_{ij}^{\text{post}} c_i^\dagger c_j + \text{const.}, \quad (3)$$

where the single-particle NH hopping matrix  $\mathcal{H}_{ij}^{\text{post}}$  has translational symmetry and nontrivial point gap topology in periodic boundary conditions (PBC), and  $c_i^\dagger, c_i, i = 1 \dots N$ , are fermionic creation and annihilation operators satisfying the canonical anticommutation relations  $\{c_i, c_j\} = 0$ ,  $\{c_i, c_j^\dagger\} = \delta_{ij}$ .

To ensure  $H_{\text{post}}$  is of the form of Eq. (3), we restrict ourselves to a U(1) charge-conserving Hermitian Hamiltonian  $H$  and two distinct sets of jump operators: losses  $J_m = L_m$  and gains  $J_m = G_m$  [summed over in Eqs. (1) and (2)], where

$$H = \sum_{ij} \mathcal{H}_{ij} c_i^\dagger c_j, \quad \mathcal{H}_{ij} = \mathcal{H}_{ji}^*,$$

$$L_m = \sum_i \mathcal{L}_{mi} c_i, \quad G_m = \sum_i \mathcal{G}_{mi} c_i^\dagger, \quad (4)$$

so that, defining the Hermitian loss and gain matrices  $m^{(l)} = \mathcal{L}^\dagger \mathcal{L}$  and  $m^{(g)} = \mathcal{G}^\dagger \mathcal{G}$ ,

$$\mathcal{H}_{ij}^{\text{post}} = \mathcal{H}_{ij} - \frac{i}{2} m_{ij}^{(l)} + \frac{i}{2} m_{ji}^{(g)}. \quad (5)$$

This choice implies that the corresponding GKSL equation has *weak* U(1) charge conservation symmetry [31,32]: while the total particle number  $\langle n \rangle = \text{tr}(\rho \sum_i c_i^\dagger c_i)$  is *not* preserved due to the presence of the second term (the “jump term”) in Eq. (1),  $\rho$  does not develop quantum coherences between Hilbert space sectors of different particle number.

To define point gap topological invariants absent postselection, we employ third quantization [28]. This method transforms the GKSL equation into the vectorized form  $\text{id}|\rho\rangle\rangle/d\tau = \hat{\mathcal{L}}|\rho\rangle\rangle$ , where  $\hat{\mathcal{L}}$  is called the NH Lindbladian *superoperator*. Here we use the symbol  $|\rho\rangle\rangle$  to denote the vectorized  $4^N$ -dimensional state associated with the  $2^N \times 2^N$  density matrix  $\rho$ . Third quantization essentially amounts to flattening the density matrix in a way that naturally respects fermion statistics. For our purposes, the main result we need is that the superoperator  $\hat{\mathcal{L}}$  can be brought into the diagonal form

$$\hat{\mathcal{L}} = \sum_{\alpha=1}^{2N} \lambda_\alpha \hat{b}'_\alpha \hat{b}_\alpha, \quad \lambda_\alpha \in \mathbb{C}, \quad \text{Im } \lambda_\alpha < 0, \quad (6)$$

where the operators  $\hat{b}'_\alpha, \hat{b}_\alpha$  are called the normal master modes (NMMs) that *almost* satisfy canonical anticommutation relations

$$\{\hat{b}_\alpha, \hat{b}_\beta\} = 0, \quad \{\hat{b}_\alpha, \hat{b}'_\beta\} = \delta_{\alpha\beta}, \quad \{\hat{b}'_\alpha, \hat{b}'_\beta\} = 0. \quad (7)$$

Note that  $\hat{b}'_\alpha \neq \hat{b}_\alpha^\dagger$  because  $\hat{\mathcal{L}}$  is not Hermitian. The non-equilibrium steady state (NESS), corresponding to  $d\rho/d\tau = 0$ , is the NMM vacuum:  $\hat{b}_\alpha |\text{NESS}\rangle\rangle = 0$ . The “excitations” are the relaxation modes  $\hat{b}'_\alpha \hat{b}'_\beta |\text{NESS}\rangle\rangle$  that decay toward the steady state with a rate  $e^{-i(\lambda_\alpha + \lambda_\beta)\tau}$ . (Here we use two  $\hat{b}'$  operators instead of one, because physical density matrices have even fermion parity [30].) We show in SM [30] that both the “single-particle” eigenvalues  $\lambda_\alpha$  of  $\hat{\mathcal{L}}$ , as well as the NMM creation operators  $\hat{b}'_\alpha$  (but *not* the  $\hat{b}_\alpha$ ), are fully determined by the spectrum and eigenvectors of the effective NH Hamiltonian [33]

$$\mathcal{H}_{ij}^{\text{eff}} = \mathcal{H}_{ij} - \frac{i}{2} m_{ij}^{(l)} - \frac{i}{2} m_{ji}^{(g)}. \quad (8)$$

In particular, denote by  $\epsilon_\alpha$  the  $N$  eigenvalues of  $\mathcal{H}_{ij}^{\text{eff}}$ , then the  $\lambda_\alpha$  are given by

$$\begin{aligned}\lambda_\alpha &= \epsilon_\alpha & (\alpha = 1, \dots, N), \\ \lambda_\alpha &= -\epsilon_{\alpha-N}^* & (\alpha = N+1, \dots, 2N).\end{aligned}\quad (9)$$

The real space profile of the NMM creation operators  $\hat{b}'_\alpha$  is exactly the same as that of the eigenvectors of  $\mathcal{H}^{\text{eff}}$ . Another way to see that  $\mathcal{H}^{\text{eff}}$  governs the relaxation dynamics is the time evolution equation of the correlation matrix  $\Delta_{ij} = \langle c_i^\dagger c_j \rangle$  [17,34]:

$$\frac{d\Delta}{d\tau} = i(\mathcal{H}^{\text{eff}})^* \cdot \Delta - i\Delta \cdot (\mathcal{H}^{\text{eff}})^T + m^{(g)}. \quad (10)$$

Since all expectation values follow from  $\Delta$  via Wick's theorem, we focus on  $\mathcal{H}^{\text{eff}}$  rather than  $\hat{\mathcal{L}}$  in the following.

In presence of translational symmetry, the complex eigenvalues of  $\mathcal{H}^{\text{eff}}$  yield the “band structure” of the Lindbladian superoperator and we can study its NH point gap topological invariants. Crucially,  $\mathcal{H}^{\text{eff}}$  and  $\mathcal{H}^{\text{post}}$  are not the same: compared to  $\mathcal{H}^{\text{post}}$  in Eq. (5), the gain term  $m^{(g)}$  of  $\mathcal{H}^{\text{eff}}$  in Eq. (8) comes with the opposite sign. The comparison of quadratic Lindbladian dynamics and point gap topological invariants with and without postselection then becomes an exercise of comparing  $\mathcal{H}^{\text{post}}$  with  $\mathcal{H}^{\text{eff}}$ .

*NH skin effect with and without postselection*—Since  $\mathcal{H}^{\text{eff}}$  fully determines the NMMs but not the NESS, Lindbladian point-gap topology is a property of the evolution toward the NESS, rather than a topological characterization of the NESS itself. In the absence of gain  $m^{(g)} = 0$ , we can immediately deduce that the point gap topology of  $\mathcal{H}^{\text{post}}$  survives in the full Lindbladian dynamics—this means that all PBC topological invariants and associated open boundary conditions (OBC) edge states are mathematically the same, even though they might have a different physical interpretation in the language of open quantum systems. In presence of gain  $m^{(g)} \neq 0$ , the topological *classification* of  $\mathcal{H}^{\text{eff}}$  remains the same as that of  $\mathcal{H}^{\text{post}}$ , but topological invariants may take on different values.

We focus on the 1D winding number of NH symmetry class A (no symmetry) in the 38-fold way [3], which for  $\mathcal{H}^{\text{eff}}$  is defined by

$$\nu_{\text{eff}}(E) = \frac{1}{2\pi i} \int_0^{2\pi} dk \frac{\partial}{\partial k} \log \det[\mathcal{H}^{\text{eff}}(k) - E\mathbb{1}], \quad (11)$$

and similarly for  $\mathcal{H}^{\text{post}}$ . Here,  $E$  is a reference complex energy inside the point gap, and  $\mathcal{H}^{\text{eff}}(k)$  is the momentum space NH Bloch Hamiltonian that follows via a Fourier transform from Eq. (8). [Note that the winding number of the second set of eigenvalues in Eq. (9), measured around  $E$ , is *always* equal to  $\nu_{\text{eff}}(-E^*)$  [30], which is why we do not consider it separately here.] As mentioned already, in

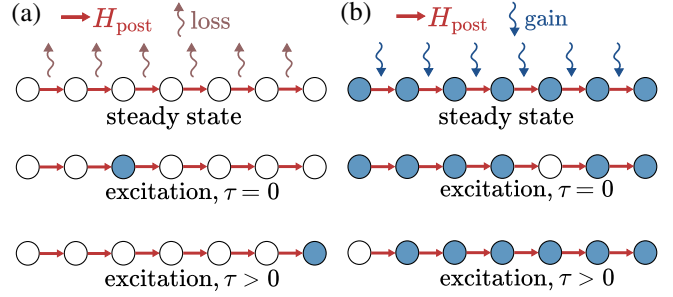


FIG. 1. Dynamics of the Lindbladian “Hatano-Nelson” model [Eq. (12)] under (a) loss only and (b) gain only in open boundary conditions. In both cases, the postselected NH Hamiltonian drives microscopic electrons towards the right edge (red arrows). For loss (a), the nonequilibrium steady state consists of all sites empty (white circles). If we add an electron into the system (blue circle), it moves to the right. For gain (b), the steady state has all sites occupied by electrons. The elementary excitations (relaxation modes) are now holes, which effectively move toward the left edge. Mathematically, this Lindbladian skin effect reversal is reflected in a Lindbladian bulk winding number that has a sign opposite to that of the postselected NH Hamiltonian.

presence of loss alone (no gain), we have  $\nu_{\text{eff}} = \nu_{\text{post}}$ . On the other hand, with gain alone (no loss), we have  $\mathcal{H}^{\text{eff}} = (\mathcal{H}^{\text{post}})^\dagger$ , from which it can be shown that  $\nu_{\text{eff}} = -\nu_{\text{post}}$ : the Lindbladian winding number and associated skin effect are *opposite* that of the postselected NH Hamiltonian! Fig. 1 illustrates this result physically for a specific  $\mathcal{H}^{\text{post}}$  that *always* has  $\nu_{\text{post}} > 0$  and therefore drives electrons to the right-hand side of the system, to result in a postselected NH skin effect on the right. In presence of gain alone, the Lindbladian steady state has all lattice sites fully occupied by electrons. The Lindbladian NMM relaxation modes are therefore *hole* excitations that effectively move to the left of the system, resulting in a NMM skin effect on the left and winding number  $\nu_{\text{eff}} < 0$ . In contrast, for loss alone, the NESS is the vacuum and the NMM excitations are electrons that move to the right, confirming  $\nu_{\text{eff}} = \nu_{\text{post}}$ . When gain and loss are both nonzero, the NESS is neither the vacuum nor the fully occupied state, and NMMs can have both electron and hole components. Surprisingly, even in this case—as long as the point gap remains open—*all* relaxation modes still move in one direction only, irrespective of whether they add or remove electronic density. We can see this by a thought experiment: e.g., starting with gain alone, introducing a little bit of loss will not immediately close the point gap. The associated winding number  $\nu_{\text{eff}}$  remains quantized and cannot change continuously, meaning that there is still a skin effect in OBC at only one end of the system. Numerical simulations using the model below confirm that in this case *all* perturbations on top of the steady state still move only toward the same end of the system [30,35]. Only when gain and loss are precisely balanced is there a point gap closing at which the winding

number and skin effect switches sides. This phase transition between different Lindbladian point-gap topological phases is invisible in the spectrum of  $\mathcal{H}^{\text{post}}$ , whose point gap remains open. Knowledge of the Lindbladian winding number therefore imposes a powerful constraint on the relaxation dynamics towards the steady state (but not on the steady state itself [36]), even when no interpretation as simple as that in Fig. 1 is available. This result is model independent and only relies on  $\nu_{\text{post}}$  remaining nonzero for all choices of gain and loss.

**Lindbladian “Hatano-Nelson” model**—We confirm our results in a simple Lindbladian model with a postselected skin effect that allows us to tune between gain and loss contributions. Consider an open quantum system with reciprocal hopping as well as gain and loss terms for each site, so that the index of the jump operators  $L_m$  and  $G_m$  becomes the same as position  $m = i, i = 1 \dots L$ :

$$H = t \sum_i (c_{i+1}^\dagger c_i + c_i^\dagger c_{i+1}),$$

$$L_i = \sqrt{\gamma_l}(c_i - i c_{i+1}), \quad G_i = \sqrt{\gamma_g}(c_i^\dagger - i c_{i+1}^\dagger), \quad (12)$$

where  $t, \gamma_l, \gamma_g \geq 0$  and PBC are implemented by setting  $c_{L+1} \equiv c_1$ . From these we obtain

$$\mathcal{H}^{\text{post}}(k) = 2t \cos k + i\gamma \sin k - i\delta,$$

$$\mathcal{H}^{\text{eff}}(k) = 2t \cos k + i\delta \sin k - i\gamma, \quad (13)$$

where  $\gamma = \gamma_l + \gamma_g, \delta = \gamma_l - \gamma_g$ . These expressions match that of the Hatano-Nelson model [7,37], up to an imaginary shift that in  $\mathcal{H}^{\text{eff}}(k)$  guarantees that all relaxation modes decay toward the steady state [34]. We plot the spectrum and OBC mode localization for both NH Hamiltonians in Fig. 2. The spectra show that the size of the point gap of  $\mathcal{H}^{\text{eff}}$  depends on the imbalance  $\delta = \gamma_l - \gamma_g$  between loss

and gain. Tuning to the critical point  $\gamma_l = \gamma_g$ , the spectrum collapses to a line [Fig. 2(b)]. The point gap in  $\mathcal{H}^{\text{eff}}$  closes when  $\gamma_l > \gamma_g$  changes to  $\gamma_l < \gamma_g$ , and the observed Lindbladian skin effect changes direction (concurrent with a sign change of the winding number). At the same time, the point gap in  $\mathcal{H}^{\text{post}}$  remains open, and its associated NH skin effect retains the same direction.

Interestingly, it is also possible to have a Lindbladian skin effect without a NH skin effect in the postselected Hamiltonian [17]. To see this, it suffices to replace the gain operators in Eq. (12) by  $G_i = \sqrt{\gamma_g}(c_i^\dagger + i c_{i+1}^\dagger)$ . This has the effect of flipping  $\gamma \leftrightarrow \delta$  in Eq. (13). Then, when  $\gamma_g = \gamma_l$ , the point gap of  $\mathcal{H}^{\text{post}}$  vanishes, while that of  $\mathcal{H}^{\text{eff}}$  remains open and exhibits a nontrivial winding number and an associated Lindbladian skin effect (see also Ref. [17]).

**Discussion**—Lindbladian superoperators with nontrivial point gap topology have previously been studied [36,38–57], and shown to exhibit Lindbladian skin effects, also dubbed Liouvillian skin effects (see also Refs. [42,58–66]). In addition, there have been initial explorations of the topological aspects of nonequilibrium steady states [67–70]. What sets our Letter apart is that we propose a systematic definition of the NH band structure associated with a quadratic Lindblad problem, and directly compare its NH topological invariants with those of the corresponding postselected NH Bloch Hamiltonian. We have shown that in the case of quadratic Lindbladians with a weak  $U(1)$  symmetry, while the 1D winding number remains a topological invariant without postselection, its value and physical interpretation in terms of a Lindbladian skin effect can change completely. A natural future direction is to generalize our approach to higher dimensions, other NH symmetry classes, and their respective topological invariants. Another promising idea is to determine the constraints that nontrivial Lindbladian point-gap

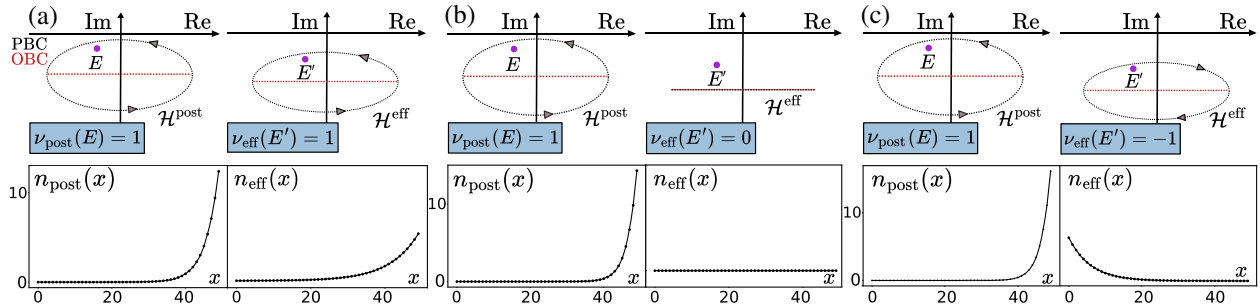


FIG. 2. Comparison of postselected NH Hamiltonian  $\mathcal{H}^{\text{post}}(k)$  with Lindbladian band structure calculated from  $\mathcal{H}^{\text{eff}}(k)$  for the Lindbladian “Hatano-Nelson” model [35] in Eq. (12). Top panels show the point gaps and their winding number [Eq. (11)] indicated by arrows for  $\mathcal{H}^{\text{post}}$  and  $\mathcal{H}^{\text{eff}}$ , side by side, for both periodic and open boundary conditions. The bottom panel shows the probability density  $n(x)$ , summed over all eigenstates, for  $\mathcal{H}^{\text{post}}$  [ $n_{\text{post}}(x)$ ] and  $\mathcal{H}^{\text{eff}}$  [ $n_{\text{eff}}(x)$ ], side by side. (a) When loss is greater than gain ( $\gamma_l > \gamma_g$ ), the winding number of  $\mathcal{H}^{\text{eff}}$  is the same as that of  $\mathcal{H}^{\text{post}}$ , and a Lindbladian skin effect occurs on the same edge as the postselected NH skin effect in open boundary conditions. (b) When loss equals gain ( $\gamma_l = \gamma_g$ ), the point gap of  $\mathcal{H}^{\text{eff}}$  collapses, while that of  $\mathcal{H}^{\text{post}}$  remains open, and there is no Lindbladian skin effect. (c) When gain is greater than loss ( $\gamma_l < \gamma_g$ ), the winding number of  $\mathcal{H}^{\text{eff}}$  is opposite that of  $\mathcal{H}^{\text{post}}$ , and the Lindbladian skin effect is reversed compared to the postselected NH skin effect.

topology imposes on the nonequilibrium steady state itself. Finally, as we have shown here, the topological gain-loss phase transition can become invisible after postselection, but it does not have to. One should compare this transition to the recent examples of measurement-induced phase transitions in the entanglement entropy that are *only* visible after postselection [71].

**Acknowledgments**—We thank Julia Hannukainen for a critical reading of the manuscript and insightful discussions. We also thank Peru d’Ornellas, Eva-Maria Graefe, Ryan Barnett, Nicolas Regnault, Titus Neupert, and Alessandro Romito for insightful discussions. A. C. acknowledges support from Imperial College London via a President’s PhD Scholarship. This work was supported by a UKRI Future Leaders Fellowship MR/Y017331/1.

**Data availability**—The data that support the findings of this article are openly available [35], embargo periods may apply.

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