

Field-Driven Band Asymmetry and Nonreciprocal Transport in a Helimagnet

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Helimagnets exhibit noncollinear spin arrangements characterized by a periodic helical modulation, giving rise to emergent chiral properties. These materials have attracted significant interest due to their potential applications in spintronics, particularly for robust information storage and the realization of topological spin textures such as skyrmions. In this work, we focus on Yoshimori-type helimagnets, where competing exchange interactions mediated by conduction electrons stabilize helical spin structures without requiring Dzyaloshinskii-Moriya interaction. We introduce a minimal model describing the electronic structure of a one-dimensional helimagnet in the presence of an external magnetic field and investigate its impact on nonreciprocal transport. We demonstrate how band asymmetry emerges in the conical phase induced by the external field, leading to a nonzero second-order electronic conductivity and injection photoconductivity. Our results provide insight into the interplay between the real space magnetic texture and electronic properties, paving the way for future studies on chirality-driven transport phenomena in centrosymmetric helimagnets.

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Chirality, the property of an object that cannot be superimposed on its mirror image, is a fundamental concept across physics, chemistry, and biology. It appears at various scales, from molecular structures to macroscopic crystals and biological forms [1]. In condensed matter physics, chirality underpins phenomena such as nonreciprocal transport, topological textures, and magnetoelectric effects [2]. It often originates from the crystal's symmetry, as encoded by its space group [3], with recent advances exploiting (magnetic-spin) space group data to classify topological structures [4–12].

Chirality can also emerge from magnetic ordering. Notably in helimagnets, it arises from spiral spin configurations, where magnetic moments rotate at fixed angles to form helical patterns [13]. These systems host exotic quasiparticles such as skyrmions [14], viewed as superpositions of multiple helices [15], and exhibit topological Hall effects [16]. Their two enantiomeric states enable

binary encoding, promising for robust information storage [17,18]. Understanding their chiral spin textures is therefore vital for spintronics and emergent magnetic phenomena.

Helimagnetism has historically been understood to arise from two distinct mechanisms. The first, proposed independently by Yoshimori, Kaplan, and Villain [13,19,20], is based on competing exchange interactions and was originally introduced to explain the helical spin structure observed in MnO_2 [21]. The second mechanism, proposed by Dzyaloshinskii [22], involves the stabilization of helimagnetism via spin-orbit coupling in noncentrosymmetric lattices. This interaction was later formalized as the Dzyaloshinskii-Moriya (DM) interaction [23,24], which is typically permitted only in chiral space groups [4,5,7–12]. In DM-type systems, the helical spin texture emerges from a competition between spin canting induced by spin-orbit coupling and ferromagnetic exchange interactions between magnetic sites. A notable example is $\text{Cr}_{1/3}\text{NbS}_2$, which hosts chiral magnetic solitons [25–28] and exhibits enhanced nonreciprocal transport in its conical phase [29]. A similar room-temperature response has been reported in $\text{Co}_8\text{Zn}_9\text{Mn}_3$, for which a theoretical model was recently proposed [30].

Nonetheless, in DM-type systems, the chirality is fixed at the Hamiltonian level by the crystal symmetry, making reversal by external fields challenging [31]. In contrast, field-controlled chirality has been demonstrated in

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centrosymmetric compounds such as MnP, MnAu₂, and α -EuP₃ [18,32,33]. In α -EuP₃, nonreciprocal transport has been linked to band asymmetry [33]. Since these materials lack inversion asymmetry, the DM interaction is absent. Motivated by our recent findings in α -EuP₃ [33], we develop and analyse a minimal model of a Yoshimori-type helimagnet under an external magnetic field. This model captures the essential physics behind the emergence of band asymmetry and nonreciprocal transport, offering a unified framework for understanding chirality-driven electronic responses in centrosymmetric systems.

Yoshimori helimagnetism—An essential feature of the exchange interaction in Yoshimori-type helimagnets is its extension beyond nearest neighbors. Denoting the interactions between neighboring and next-nearest spins as J_1 and J_2 , a helical ground state emerges when $|J_1| < |4J_2|$ and $J_2 < 0$ [34]. Intuitively, positive (negative) J_1 favors parallel (antiparallel) alignment of adjacent spins, while negative J_2 favors antiparallel alignment of next-nearest neighbors. The resulting compromise leads to spin canting and a helical arrangement. The structure is typically incommensurate, with the angle θ between adjacent spins determined solely by exchange couplings.

Figure 1(a) illustrates this scenario, where the competing interactions arise from Ruderman-Kittel-Kasuya-Yosida (RKKY) exchange via conduction electrons [35–38], common in systems with localized d - or f -electron moments. Being long ranged and oscillatory, RKKY interactions readily stabilize spiral states. Approximating conduction electrons as free, the exchange coupling's oscillatory behavior is shown in Fig. 1(b), with shaded regions marking parameters satisfying the helimagnetic condition—for instance, $2k_F a = 3\pi/2$ gives $J_1 > 0$, $J_2 < 0$, and $|4J_2| > |J_1|$; see the inset Fig. 1(b).

The spin modulation is characterized by a propagation vector $\mathbf{q}$, whose orientation relative to the rotation plane is influenced by magnetic anisotropy [34]. For fields applied perpendicular to the rotation plane, the system transitions from helimagnetic (HM) to conical, and finally to ferromagnetic (FM) phase above a critical field H_c , as depicted in Figs. 1(c) and 1(d).

Model Hamiltonian—To capture the evolution of the electronic structure and transport signatures across magnetic phases, we introduce a minimal model for a one-dimensional atomic chain with one orbital per site and local magnetic moments arranged helically. We assume a propagation vector $\mathbf{q} = (1/4, 0, 0)$ in reciprocal lattice units, corresponding to a 90° rotation between neighboring sites along the chain (x axis) [38], producing a magnetic supercell of four atoms, as illustrated in Fig. 2(a).

The Hamiltonian, adapted from the Kondo lattice model [39–41], reads

$$\hat{H} = \sum_{\langle i,j \rangle} t c_i^\dagger c_j + \text{H.c.} + \lambda \sum_i c_i^\dagger \mathbf{M}_i \cdot \boldsymbol{\sigma} c_i, \quad (1)$$

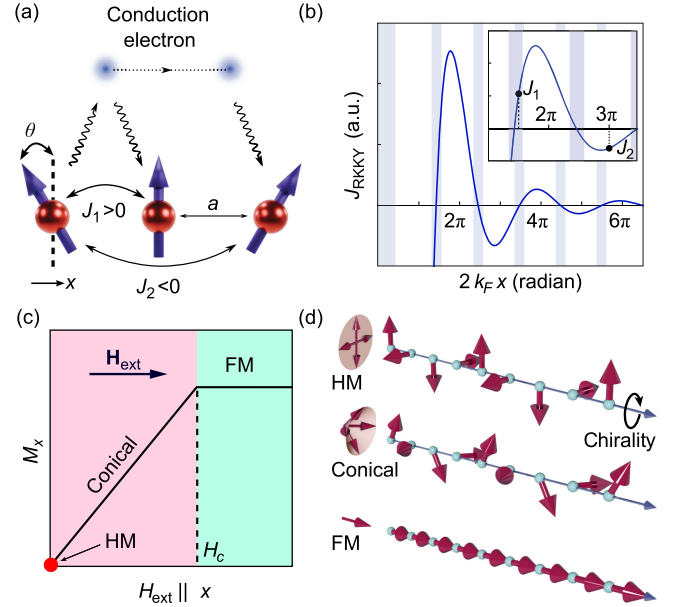


FIG. 1. RKKY interaction-induced helimagnetic ordering. (a) Coupling mechanism responsible for stabilizing helimagnetic order via competing exchange interactions mediated by conduction electrons between magnetic sites separated by the distance a . (b) RKKY coupling proportionality plotted as a function of $2k_F x$, where x is the interatomic distance and k_F the Fermi wave vector. Shaded regions highlight values of J_1 and J_2 satisfying the conditions for helimagnetic order, as illustrated in the inset. (c) Schematic evolution of magnetization along the chain (x direction) under an external magnetic field applied perpendicular to the helical plane. (d) Spin configurations under increasing field strength: helimagnetic phase at $H_{\text{ext}} = 0$, conical phase for $0 < H_{\text{ext}} < H_c$, and field-forced ferromagnetic phase for $H_{\text{ext}} \geq H_c$.

where $c_i^\dagger = (c_{i\uparrow}^\dagger, c_{i\downarrow}^\dagger)$ and $c_i = (c_{i\uparrow}, c_{i\downarrow})^T$ are the creation and annihilation operators at site i , t is the hopping amplitude, and λ is the exchange coupling to the local magnetization $\mathbf{M}_i$. The Pauli vector is $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$.

An external magnetic field $\mathbf{H}_{\text{ext}}$ applied along the chain and perpendicular to the spin-rotation plane tilts each magnetic moment by an angle α , giving

$$\mathbf{M}_i = M \left[\sin \alpha, \gamma \sin \left(\frac{i\pi}{2} \right) \cos \alpha, \cos \left(\frac{i\pi}{2} \right) \cos \alpha \right], \quad (2)$$

where $i = 0, 1, 2, 3$, M is the moment magnitude, and $\gamma = \pm 1$ denotes chirality [42]. Unless otherwise specified, we use $t = 1$ eV, $\lambda = 0.6$ eV, and $M = 1$.

Without an external field, the system forms a helical configuration with $\mathbf{q}$ normal to the rotation plane [Fig. 2(a)], and the resulting band structure [Fig. 2(b)] is symmetric and degenerate with respect to chirality. To highlight the effect of coupling to local magnetic moments, Fig. 2(c) shows the unfolded band structure projected onto the

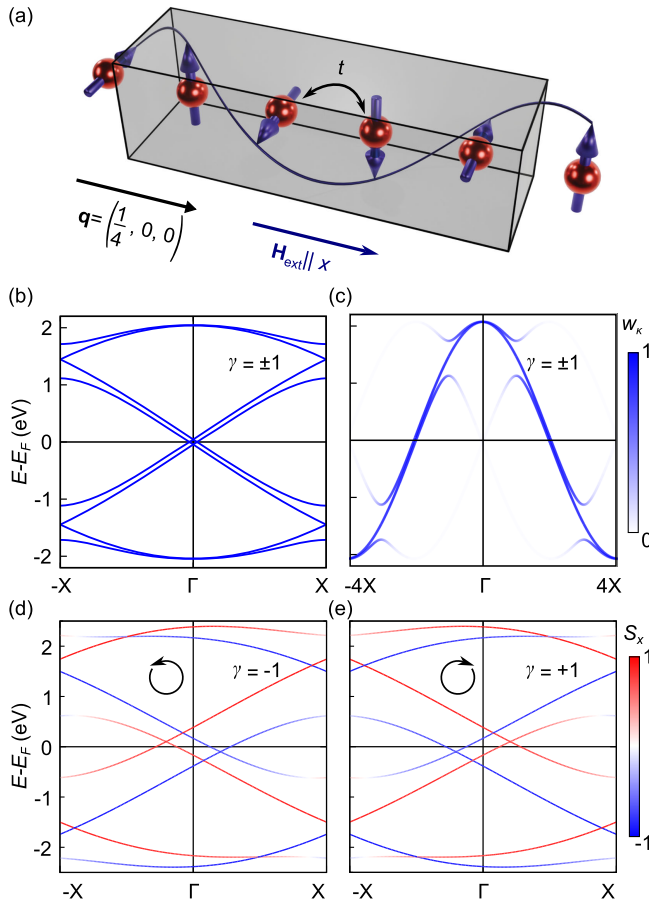


FIG. 2. Chirality dependence of electronic band structure in a one-dimensional helimagnet. (a) One-dimensional chain of atoms with helimagnetic order for which the q vector is along the chain. The electron hopping amplitude between nearest-neighbor sites is t . (b) Band structure for $\lambda = 0.3$ in the helical phase, which looks identical for both possible chiralities of the spiral. (c) Unfolded band structure to the primitive unit cell of the chain. The color indicates the spectral weight. (d), (e) Spin projected band structure for $\gamma = -1$ and $\gamma = +1$, respectively, in the conical phase with tilt angle $\alpha = 10^\circ$ and $\lambda = 0.8$ eV.

Brillouin zone (BZ) of the primitive unit cell. The spectral weight, computed following Ref. [44], reveals a $\cos k_x$ -like dispersion typical of a single-orbital chain, now punctuated by gaps arising from exchange interactions. Remarkably, when the system enters the conical phase, the band structure develops a chirality-dependent asymmetry about the Γ point, as shown in Figs. 2(d) and 2(e) for $\gamma = -1$ and $+1$, respectively.

Evolution under external field—To understand how band asymmetry influences transport, we compute the electronic conductivity up to second order using Boltzmann transport theory within the constant relaxation time approximation. For a one-dimensional system, the n th-order electronic conductivity at temperature T and chemical potential μ is given by

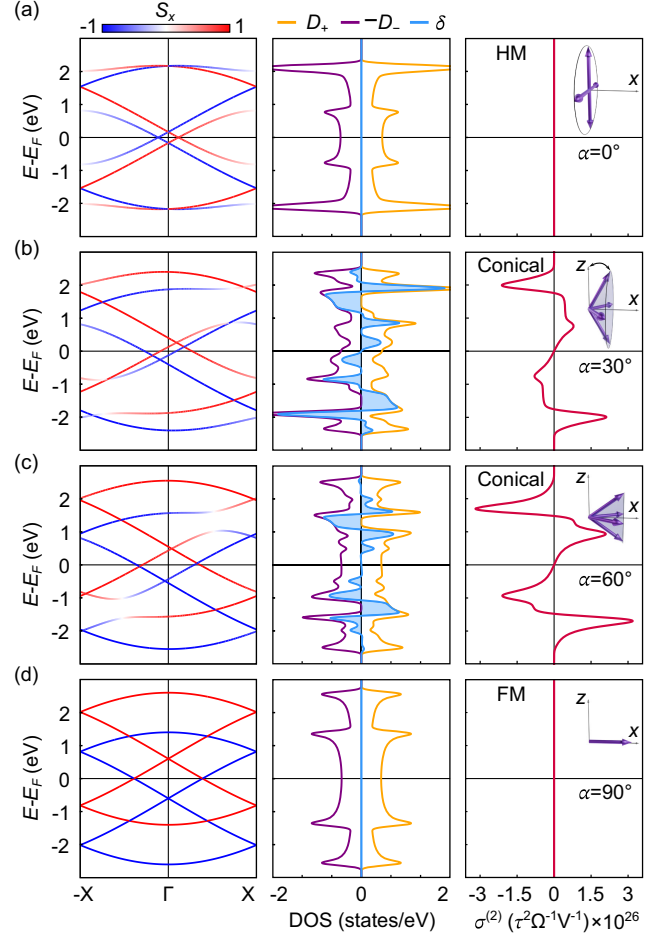


FIG. 3. Evolution of the electronic structure in an external magnetic field. (a) Helimagnetic phase, showing (from left to right) the spin-projected band structure, density of states for $k_x \leq 0$ (purple, with a negative sign) and for $k_x \geq 0$ (orange), and the difference between the two (light blue). The last panel on the right is the second-order conductivity, with the inset showing the arrangement of spins translated to the rotation plane. (b)–(d) Show the same information as (a) for the conical phase with $\alpha = 30^\circ$, $\alpha = 60^\circ$ and ferromagnetic, respectively, where α is the tilt angle of the spins with respect to the z axis.

$$\sigma^{(n)}(\mu, T) = -e \sum_m \int_{\text{BZ}} v_m(k) \left(\frac{e\tau}{\hbar} \frac{\partial f}{\partial k} \right)^n dk, \quad (3)$$

where m indexes the energy bands, e is the elementary charge, τ is the constant relaxation time, $v_m(k)$ is the group velocity of band m at wave vector k , and $f(E_m, \mu, T)$ is the Fermi-Dirac distribution.

Figure 3 shows the evolution of the band structure under an external magnetic field applied perpendicular to the spin-rotation plane. In the helimagnetic phase, the band structure is symmetric under $k_x \rightarrow -k_x$, and hence the density of states (DOS) is identical for $k_x \leq 0$ and $k_x \geq 0$. As a result, the second-order conductivity $\sigma^{(2)}(\mu, T)$ vanishes for any value of μ ; Fig. 3(a).

Upon applying an external field, this symmetry is broken, leading to band asymmetry and an imbalance in DOS between $k_x \leq 0$ and $k_x \geq 0$. Consequently, $\sigma^{(2)}(\mu, T)$ becomes finite in the conical phase. Figures 3(b) and 3(c) show this behavior for $\alpha = 30^\circ$ and 60° , respectively. When the system reaches its field-forced FM phase, the symmetry in DOS is restored, thereby again yielding vanishing $\sigma^{(2)}(\mu, T)$ as shown in Fig. 3(d).

A more detailed phase diagram of $\sigma^{(2)}$ across the conical phase is shown in Figs. 4(a)–4(c) for a range of carrier densities. As seen, the resulting transport behavior is highly tunable with carrier density, allowing for strictly positive, strictly negative, or sign-reversible second-order conductivity throughout the conical phase—consistent with previous experimental observations [45]. A detailed

explanation of how this model can be extended to reproduce first-principles calculations for α -EuP₃ is provided in Sec. 1 of Supplemental Material [46].

Optical responses—To further address how to observe such field-induced changes in a helimagnet, we consider dc (second-order) optical responses under linearly polarized light, including both shift and injection currents. These originate respectively from positional shifts of Wannier centers and asymmetric carrier injection due to band velocity differences between excited and ground states [47,48].

We adopt the quantum-geometric formalism introduced in Refs. [49,50], which expresses these nonlinear responses in terms of multiband Berry connections $\mathbf{A}_{nm} = i\langle u_{n,\mathbf{k}} | \nabla_{\mathbf{k}} | u_{m,\mathbf{k}} \rangle$, equivalent to interband dipole matrix elements for $n \neq m$ (see Sec. 3 of Supplemental Material for details [46]). The second-order shift and injection photoconductivities along direction a for linearly polarized light are then given by

$$\sigma_s^{a,aa}(\omega) = -\frac{2\pi e^3}{\hbar^2} \sum_{m,n} \int_{\text{BZ}} \frac{dk}{2\pi} \delta(\omega - \omega_{mn}) \times f_{nm,\mathbf{k}} |\mathbf{A}_{mn}^a|^2 R_{mn}^{a,a}, \quad (4)$$

$$\sigma_i^{a,aa}(\omega) = -\frac{\pi e^3 \tau_p}{\hbar^2} \sum_{m,n} \int_{\text{BZ}} \frac{dk}{2\pi} \delta(\omega - \omega_{mn}) \times f_{nm,\mathbf{k}} |\mathbf{A}_{mn}^a|^2 \partial_a \omega_{mn}, \quad (5)$$

respectively. Here, $f_{nm,\mathbf{k}} = f_{n,\mathbf{k}} - f_{m,\mathbf{k}}$ is the difference in Fermi-Dirac occupations, $\omega_{mn} = (E_m - E_n)/\hbar$ is the transition frequency, and τ_p is the relaxation time for the photoexcited carriers. The shift vector is defined as $\mathbf{R}_{mn}^a = \mathbf{A}_{mm} - \mathbf{A}_{nn} - \nabla_{\mathbf{k}} \text{Arg}(\mathbf{A}_{mn}^a)$, capturing the real-space displacement of photoexcited carriers [51].

We find that while the shift conductivity vanishes identically in all phases, the injection conductivity is nonzero only in the conical phase at finite chemical potential, similar to chiral magnets [63]; see Fig. 4. This provides a direct route for identifying the conical phase experimentally, as it is the only one exhibiting a finite photoconductive response. However, since the injection current is odd in μ , it vanishes at charge neutrality, necessitating alternative probes in that regime.

The behavior of the shift and injection photoconductivities can also be understood from the symmetries of the system. All three magnetic phases possess chiral symmetry, which can be decomposed into a particle-hole transformation followed by time reversal [64]. Under particle-hole symmetry, $\sigma_{s/i}^{a,aa} \rightarrow -\sigma_{s/i}^{a,aa}$, while under time reversal, $\sigma_s^{a,aa} \rightarrow \sigma_s^{a,aa}$ and $\sigma_i^{a,aa} \rightarrow -\sigma_i^{a,aa}$ [52]. Consequently, under their combination, $\sigma_s^{a,aa} \rightarrow -\sigma_s^{a,aa}$ and $\sigma_i^{a,aa} \rightarrow \sigma_i^{a,aa}$, implying that the shift must vanish identically, while the injection remains symmetry allowed. In contrast, the additional time-reversal or inversion symmetries in the helical and FM phases also forbid the injection [52], leaving only the conical

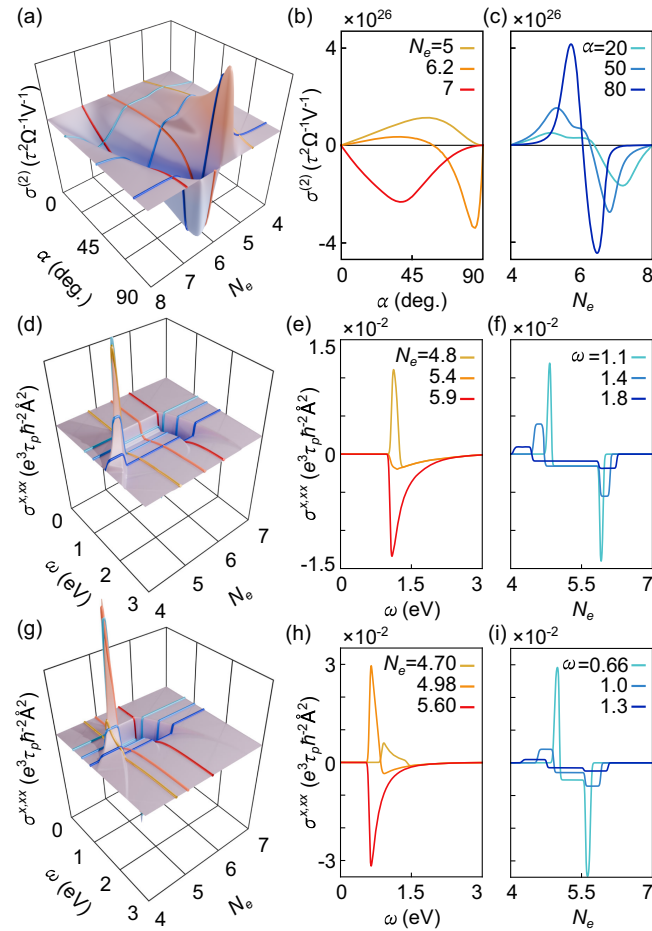


FIG. 4. Evolution of second-order conductivity and injection photoconductivity under an external magnetic field. (a) Second-order electronic conductivity $\sigma^{(2)}$ as a function of tilt angle α and carrier density N_e . Only positive N_e values are shown, as $\sigma^{(2)}(\mu, T) = -\sigma^{(2)}(-\mu, T)$. (b)–(c) Line cuts from (a) at fixed carrier densities and tilt angles within the conical phase, respectively. (d) Injection photoconductivity as a function of photon frequency and carrier density for $\alpha = 30^\circ$. (e), (f) Cuts from (d) at fixed carrier densities and photon frequencies, respectively. (g)–(i) Corresponding plots to (d), (e), and (f) for $\alpha = 60^\circ$.

phase with a finite response. This symmetry-based argument underscores the robustness of the injection photoconductivity as an experimental signature of the conical phase.

Conclusion—In conclusion, we examined the electronic structure and transport properties of Yoshimori-type helimagnets, focusing on the effects of an external magnetic field on their band structure, nonreciprocal conductivity, and photoconductivity. Unlike Dzyaloshinskii-Moriya-type systems, Yoshimori-type helimagnets were shown to rely on competing exchange interactions and to exhibit spontaneously broken chiral symmetry. When a magnetic field was applied perpendicular to the spin-rotation plane, this competition drove a phase transition from helimagnetic to conical and eventually to ferromagnetic order. The conical phase, in particular, exhibited band asymmetry, giving rise to finite second-order and injection conductivities. These findings highlight the potential of centrosymmetric helimagnets for chiral transport and spintronic applications.

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Data availability—The data that support the findings of this article are openly available [65].

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