

Weak-Correlation Universality and Macroscopic Fluctuations in Power-Law Recurrent Networks

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Balanced network theory describes the dynamic state of randomly connected systems with intrinsic negative feedback, predicting the suppression of correlations as connectivity increases. We show that this weak-correlation universality holds broadly but breaks down in power-law networks where the largest out-degrees scale with system size. In such networks, residual correlations cross over to a regime that retains macroscopic fluctuations in the thermodynamic limit. Unlike Erdős-Rényi networks, the degree heterogeneity of power-law topologies amplifies coherent population-level fluctuations. Our results provide a theoretical foundation for the strong correlations observed in real-world networks with sparse and heterogeneous connectivity.

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Collective dynamics in recurrent networks—whether in neural circuits, spin systems, or social interactions—are often studied through balanced network theory [1], where opposing signed interactions dynamically cancel to stabilize activity. A common approach to analyzing such systems is through mean-field methods, which approximate dynamics by averaging over large populations. Inspired by Glauber dynamics [2], these approaches typically assume Erdős-Rényi (ER) connectivity, with interaction degree K scaling linearly with system size N (i.e., $K \propto N$). Under these conditions, correlations and macroscopic fluctuations vanish as $\mathcal{O}(1/N)$, producing asynchronous and chaotic dynamic states in homogeneous networks [1,3]. However, real-world networks often exhibit heterogeneous, heavy-tailed connectivity, with sublinear dependence between K and N . This raises a pivotal question: Can structural heterogeneity sustain macroscopic fluctuations despite dynamic balance?

This Letter answers this question by developing a self-consistent mean-field theory for networks with arbitrary degree distributions. We show that, for all balanced networks with $K \propto N$, weak-correlation universality persists—even in nonexchangeable topologies like power-law (PL) networks. However, when the largest out-degree (i.e., hub size) scales with N , and K grows sublinearly, the classical $1/N$ scaling breaks down. Using a fluctuation expansion method within our Markov system theory, we uncover a

crossover to a nonperturbative regime in which finite- K systems sustain macroscopic correlations. Relaxing the Markovian assumption, we further demonstrate this effect in leaky integrate-and-fire (LIF) networks with power-law connectivity, where degree heterogeneity amplifies coherent fluctuations. Our framework offers a unifying explanation for the observed significant correlation in cortical activity, superspreader-driven outbreaks, and other emergent phenomena shaped by connectivity heterogeneity.

Consider a *network* comprising $\mathcal{Z}$ subpopulations, each containing N interacting binary units. Connectivity from unit j in subpopulation β to unit i in subpopulation α is encoded in the adjacency matrix $\mathbf{C}^{\alpha\beta}$, with $\alpha, \beta \in \{1, 2, \dots, \mathcal{Z}\}$. Each unit receives an expected K incoming connections (with possible multiple connections between units and no self-couplings). The in- and out-degrees per unit are defined as

$$\mathcal{K}_i^{\text{in}} := \sum_{j=1}^N C_{ij}^{\alpha\beta}, \quad \mathcal{K}_j^{\text{out}} := \sum_{i=1}^N C_{ij}^{\alpha\beta},$$

where $\mathcal{K}_i^{\text{in}}$ and $\mathcal{K}_j^{\text{out}}$ are independent across units and may have distinct distributions. The adjacency matrices $\mathbf{C}^{\alpha\beta}$ are generated via the configuration model with uncorrelated in- and out-degree distributions, as detailed in [4].

The state $\sigma_i^\alpha(t) \in \{0, 1\}$ of unit i in subpopulation α evolves via continuous-time Markov dynamics governed by the input field

$$h_i^\alpha(t) := \sum_{\beta=1}^{\mathcal{Z}} \frac{J_{\alpha\beta}}{K^{\gamma_0}} \sum_{j=1}^N C_{ij}^{\alpha\beta} \sigma_j^\beta(t) + K^{1-\gamma_0} I_\alpha, \quad (1)$$

where $J_{\alpha\beta} \sim \mathcal{O}(1)$ denotes the coupling strength from population β to α , I_α is a constant feedforward input,

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and $0 < \gamma_0 < 1$ controls the scaling of the input. We set $\gamma_0 = 0.5$ and consider only the invertible coupling matrix $\mathbf{J} := (J_{\alpha\beta})$ [3,4,10]. The transfer function $f(h_i^\alpha(t))$ governs the activation rate ($0 \rightarrow 1$), while $1 - f(h_i^\alpha(t))$ determines the deactivation rate ($1 \rightarrow 0$). The complete generator matrix specifying these Markovian dynamics is provided in [4].

Accordingly, the population-averaged activity in sub-population α is defined as

$$m_\alpha(t) := \frac{1}{N} \sum_{i=1}^N \sigma_i^\alpha(t). \quad (2)$$

Its time evolution is governed by

$$\frac{d}{dt} m_\alpha(t) = -m_\alpha(t) + F_\alpha(t), \quad (3)$$

with

$$F_\alpha(t) := \frac{1}{N} \sum_{i=1}^N f(h_i^\alpha(t)).$$

Expanding $F_\alpha(t)$ around the population mean input,

$$h_\alpha(t) = \sqrt{K} \left(\sum_{\beta=1}^Z J_{\alpha\beta} m_\beta(t) + I_\alpha \right), \quad (4)$$

one obtains the moment hierarchies associated with the input field in Eq. (1) [11]. Remarkably, with appropriate coupling, we have a self-consistent balancing solution for $m_\beta(t)$ as $K \rightarrow \infty$, since $(\sum_{\beta=1}^Z J_{\alpha\beta} m_\beta(t) + I_\alpha) \sim (1/\sqrt{K})$ and, thus, $h_\alpha(t) \sim \mathcal{O}(1)$ [3,4]. Hence, the corresponding $F_\alpha(t)$ series (with $\gamma_0 = 0.5$) converges to a Gaussian integral with mean and variance remaining $\mathcal{O}(1)$, when $\text{Var}(\mathcal{H}^{\text{in}}) \sim \mathcal{O}(K)$, in the regime $1 \ll K < N$ [3,4]. This requires weak correlations between units (i.e., $\sim \mathcal{O}(1/N)$) to truncate the moment hierarchies. In classical theory, the weak-correlation assumption implies that macroscopic fluctuations around the population mean input are small:

$$\langle [\delta h_\alpha(t)]^2 \rangle_t \ll \mathcal{O}(1),$$

where $\delta x(t) := x(t) - \langle x(t) \rangle_t$. More generally, as it detailed in [4], one obtains

$$\langle [\delta h_\alpha(t)]^2 \rangle_t = \frac{\kappa_{\text{out}}}{NK} \sum_{\beta=1}^Z J_{\alpha\beta}^2 A_\beta, \quad (5)$$

where $\mathcal{K}_{\text{out}} := \text{Var}(\mathcal{K}^{\text{out}})$ and

$$A_\beta := m_\beta - q_\beta, \quad q_\beta := \frac{1}{N} \sum_{j=1}^N \langle \sigma_j^\beta \rangle_t^2, \quad m_\beta := \langle m_\beta(t) \rangle_t.$$

Note that A_β is the variance of microscopic temporal fluctuations to units [3,4]. For ER networks, because $\kappa_{\text{out}} = K$, one finds

$$\langle [\delta h_\alpha(t)]^2 \rangle_t \sim \mathcal{O}\left(\frac{1}{N}\right), \quad (6)$$

which is consistent with the weak-correlation assumption. In contrast, networks with significant out-degree variance can exhibit arbitrarily large fluctuations, and, when $\lim_{K,N \rightarrow \infty} (\kappa_{\text{out}}/NK) \neq 0$, the dynamics become nonexchangeable across units [11]. In particular, for networks with PL out-degree distributions, an interesting case can emerge that

$$\langle [\delta h_\alpha(t)]^2 \rangle_t \sim \mathcal{O}(1), \quad (7)$$

indicating that population average mean input fluctuations remain finite and non-negligible [4]. Unlike ER networks, these fluctuations persist even in the large-network limit and must be explicitly accounted for in the fixed-point time-averaged mean-field equation $m_\alpha = \langle F_\alpha(t) \rangle_t$, as detailed in [4].

To assess the *consistency of the weak-correlation ansatz* in the presence of additional fluctuations, we use the Markovian generator matrix in [4] to derive the fixed-point equation for the population-averaged correlations between units in populations α and β . We define the symmetrized population-averaged correlation as

$$c_{(\alpha\beta)} := \frac{1}{2} (c_{\alpha\beta} + c_{\beta\alpha}), \quad (8)$$

where

$$c_{\alpha\beta} := \left\langle \left\langle \delta \sigma_i^\alpha(t) \delta f(h_j^\beta(t)) \right\rangle_t \right\rangle_{\mathbf{J}} \quad (9)$$

and the outer average $\langle \cdot \rangle_{\mathbf{J}}$ is taken over the disorder (i.e., over all pairs and connectivity realizations). Under the weak-correlation ansatz, expanding the transfer function $f(h_i^\alpha(t))$ about the time-average population mean input $\langle h_i^\alpha(t) \rangle_t$ yields

$$c_{\alpha\beta} = \sqrt{K} \bar{f}_0 + \frac{K}{N} \bar{f}_1 + \frac{\kappa_{\text{out}}}{N\sqrt{K}} \bar{f}_2 + \frac{\kappa_{\text{out}}}{NK} \bar{f}_3 + \dots, \quad (10)$$

where $\bar{f}_0, \bar{f}_1, \bar{f}_2$, etc., are coefficients that depend on the microscopic population fluctuation variance, A_β , and $c_{\alpha\beta}$ itself, which they have structure like $[(A_\beta/N) + c_{\alpha\beta}]$ with various prefactors, as detailed in [4]. The first term in Eq. (10), associated with $\bar{f}_0$, is the leading-order term for

balanced dynamics with ER topologies, for which

$$c_{(\alpha\beta)} \sim \mathcal{O}\left(\frac{1}{N}\right),$$

since the $\sqrt{K}$ amplification is exactly canceled by balancing of (A_β/N) and $c_{\alpha\beta}$ as the system size grows [4,10]. Equation (10), thus, suggests the *universality* of weak correlations for all networks satisfying $\kappa_{\text{out}} < KN$ with an identical structure, as all higher-order terms become sub-leading in large systems.

While the weak-correlation ansatz ($c_{(\alpha\beta)} \sim \mathcal{O}(1/N)$) holds under general conditions, deviations arise when out-degree heterogeneity introduces nontrivial structure in correlation scaling. We focus on a single-population network with negative recurrent couplings (i.e., $J = -1$) to explore this systematically. Specifically, we analyze a one-population system with Pareto-distributed out-degrees [4].

Regularized power-law networks—Using the configuration model as described in [4], we construct networks where the in-degree $\mathcal{K}_i^{\text{in}}$ for unit (i, α) is drawn from a Poisson distribution with mean K , while the out-degree $\mathcal{K}_i^{\text{out}}$ is drawn from a bounded Pareto distribution with parameters α_{PL} , L , and H , representing the shape parameter, the minimum, and the maximum number of possible outgoing connections, respectively [4]. In the regularized power-law (RPL) networks, we enforce

$$H = s_0 K \quad \text{with} \quad s_0 > 1$$

and adjust L so that the average number of outgoing connections remains K [4]. Under these conditions, the variance of the out-degrees scales as

$$\kappa_{\text{out}} \sim K^2,$$

for all values of α_{PL} , as detailed in [4].

For this single-population system, the expansion in Eq. (10) then becomes

$$c_{\text{RPL}} = \sqrt{K} \left(\bar{f}_0 + \frac{K}{N} \bar{f}_2 \right) + \mathcal{O}\left(\frac{K}{N}\right), \quad (11)$$

where $\bar{f}_0$ and $\bar{f}_2$ are the corresponding expansion coefficients, which depend on $[(A/N) + c_{\text{RPL}}]$ (see Ref. [4] for details). If the average connectivity K grows sublinearly with N , this expansion reduces to the classical result for ER networks [4]. In the case, $K \sim N$, both terms in Eq. (11) scale as $\sqrt{N}$, and the overall scaling of correlations remains

$$c_{\text{RPL}} \sim \mathcal{O}\left(\frac{1}{N}\right). \quad (12)$$

Nevertheless, the correlations' absolute magnitude becomes significantly enhanced and exhibits sensitivity

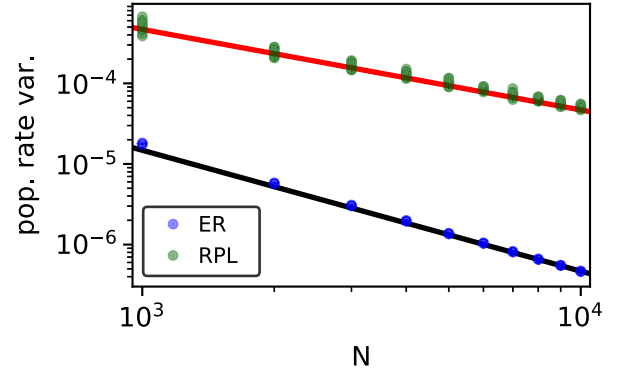


FIG. 1. Macroscopic fluctuations in RPL networks. Population-averaged rate variance is shown for individual trials in simulations of RPL (green dots) and ER (blue dots) networks with fixed $K/N = 0.05$. Each dot corresponds to one of 20 independent realizations of the network (dots largely overlap due to tight scaling). Theoretical predictions in solid lines (red, Var_{RPL} ; black, Var_{ER}) capture the fluctuations' scaling and numerical value. Simulations were performed using the Gillespie algorithm with a Heaviside transfer function [4,12]. Parameters: $J = -1$, $H = 20K$, $I = 0.1$, and $\alpha_{\text{PL}} = 0.5$.

to the network's topological parameters. Therefore, the structural difference between c_{RPL} and the correlations in ER networks (c_{ER}) has a remarkable impact on the variance of the population-averaged rate. As illustrated in Fig. 1, in ER networks we find

$$\text{Var}_{\text{ER}} \sim \frac{A}{N\sqrt{K}}, \quad (13)$$

indicating suppression of macroscopic fluctuations by additional $1/\sqrt{K}$ [4]. In contrast, in RPL networks, the population-averaged rate variance scales as

$$\text{Var}_{\text{RPL}} \sim \frac{A}{N}, \quad (14)$$

resulting in significantly larger macroscopic fluctuations. These findings show that, while the weak-correlation ansatz holds in both ER and RPL networks regarding scaling, the absence of the $1/\sqrt{K}$ suppression in RPL networks leads to an enhanced absolute magnitude of macroscopic fluctuations. These fluctuations linearly reflect the average pairwise correlation in the stationary regime [4]. In Supplemental Material [4], we provide further numerical confirmation of this scaling law, including direct estimates of pairwise correlations that quantitatively match the macroscopic rate variance results (see Fig. S1 in [4]).

Heterogeneous power-law networks—Analogous to the RPL case, we use the configuration model to construct networks with large hub structures in their connectivity. Here, we enforce

$$H = p_0 N \quad \text{with} \quad 0 < p_0 \leq 1,$$

so that the maximum number of outgoing connections scales extensively with the network size, and we adjust L to ensure that the average number of outgoing connections remains K [4]. Under these conditions, for $0 < \alpha_{\text{PL}} < 1$, the variance of the out-degrees scales as

$$\kappa_{\text{out}} \sim KN.$$

The heterogeneous power-law (HPL) topology also permits the average connectivity to grow as a power of the system size, i.e.,

$$K \sim N^{\beta_0} \quad \text{with} \quad 0 \leq \beta_0 \leq 1.$$

Substituting this scaling into the general expansion [Eq. (10)], we obtain

$$c_{\text{HPL}} = N^{\beta_0/2}(\bar{f}_0 + \bar{f}_2) + \bar{f}_3 + \mathcal{O}(N^{\beta_0-1}), \quad (15)$$

where the coefficients $\bar{f}_0$, $\bar{f}_2$, and $\bar{f}_3$ depend on the microscopic fluctuations (i.e., (A/N)) and on c_{HPL} itself (see Ref. [4] for details). Notably, when $\beta_0 = 1$, this series reduces to that of the expansion for RPL networks, i.e., $c_{\text{HPL}} \sim c_{\text{RPL}} \sim \mathcal{O}(1/N)$. In contrast, for $\beta_0 = 0$ (i.e., finite- K networks), the prefactor to the leading-order terms and the residual term all remain of the order of $\mathcal{O}(1)$; furthermore, the population rates stay away from their balancing solution [3], and, thus, these terms do not vanish. This indicates the emergence of a regime in which $c_{\text{HPL}} \sim \mathcal{O}(1)$. In comparison, ER networks maintain a residual term that is always suppressed as $\mathcal{O}(1/N)$, even when the average degree K is finite, and correlations remain $\mathcal{O}(1/N)$ in a self-consistent manner.

To determine the scaling behavior of c_{HPL} for intermediate values $0 < \beta_0 < 1$, we apply a fluctuation expansion approach. Specifically, we expand the microscopic fluctuation variance at the system's balance solution as

$$A = A_0 + \frac{A_1}{N^{\beta_0/2}} + \frac{A_2}{N^{\beta_0}} + \mathcal{O}\left(\frac{1}{N^{3\beta_0/2}}\right) \quad (16)$$

and posit a corresponding ansatz for the correlations:

$$c_{\text{HPL}} = \frac{c_0}{N^{\beta_0/2}} + \frac{c_1}{N^{\beta_0}} + \mathcal{O}\left(\frac{1}{N^{3\beta_0/2}}\right). \quad (17)$$

Substituting these series into Eq. (15), the dominant $\mathcal{O}(1/N^{\beta_0/2})$ terms allow canceling the leading divergence. At next-to-leading order, $\mathcal{O}(1/N^{\beta_0})$, the correlation scaling simplifies to

$$c_{\text{HPL}} \sim \frac{A}{N^{\beta_0}}, \quad (18)$$

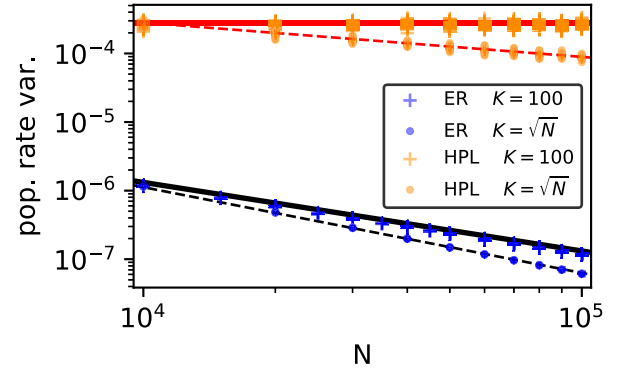


FIG. 2. Macroscopic fluctuations in HPL networks. Population-averaged rate variance is shown for individual trials in simulations of HPL and ER networks, with $K = 100$ (orange and blue plus signs) and $K = \sqrt{N}$ (orange and blue dots). Each symbol corresponds to one of 20 independent network realizations; individual points visually overlap due to tight scaling. Theoretical predictions for $K = 100$ are shown as solid lines (red, Var_{HPL} ; black, Var_{ER} , with $\beta_0 = 0$) and for $K = \sqrt{N}$ as dashed lines (red, Var_{HPL} ; black, Var_{ER} , with $\beta_0 = 0.5$), accurately capturing both the scaling and magnitude of fluctuations. Simulations use the Gillespie algorithm with a Heaviside transfer function [4,12]. Parameters: $J = -1$, $H = N$, $I = 0.1$, and $\alpha_{\text{PL}} = 0.5$.

revealing a continuous crossover in the correlation structure governed by the degree exponent $\beta_0 \in (0, 1)$ (see Ref. [4] for details). Accordingly, the variance of macroscopic fluctuations follows the same scaling:

$$\text{Var}_{\text{HPL}} \sim \frac{A}{N^{\beta_0}}. \quad (19)$$

As shown in Fig. 2, our numerical simulations of macroscopic variance agree well with this prediction. While this population-level measure is computationally efficient and experimentally relevant, we also provide direct confirmation using average pairwise correlations in Supplemental [4] Fig. S2, which further validates the theoretical scaling.

In addition to the single-population analysis, our framework naturally extends to systems composed of multiple interacting populations. A particularly well-studied example is the excitatory-inhibitory ($E-I$) network architecture, which captures fundamental antagonistic interactions in neural systems and beyond. These networks operate in a dynamically balanced regime that supports irregular spiking activity, as observed in cortical circuits [1]. In Supplemental Material [4], we provide a detailed derivation of the mean-field theory for pairwise correlations in such $E-I$ networks. Our results show that, consistent with the single-population case, PL topology leads to structurally different and significantly larger correlations, underscoring the impact of degree heterogeneity in multipopulation systems (see Ref. [4] for details).

Excessive correlations resulting from network topology can profoundly alter dynamics in complex systems, where

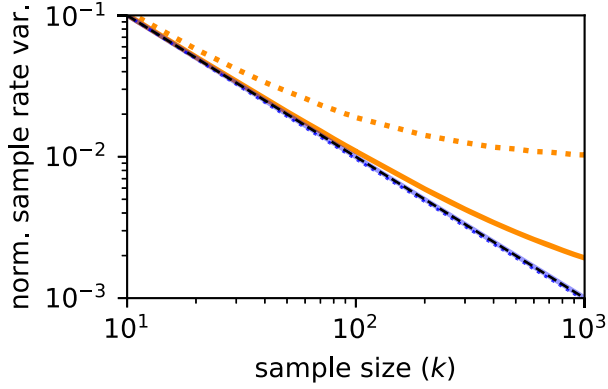


FIG. 3. Coherence in simulations of LIF spiking $E-I$ networks. The normalized temporal rate variance (coherence measure) of k units, averaged over 100 independent uniform samples, is plotted for ER and PL networks. The black dashed line indicates the theoretical prediction for an asynchronous state. The blue lines represent coherence in the ER network for two rate estimation windows: $\Delta = 1$ ms (solid) and $\Delta = 10$ ms (dotted), closely matching the asynchronous prediction. The orange lines show coherence in the PL network for the same windows, both deviating substantially from the asynchronous-state prediction. Parameters: $N = 10^5$, $K = 500$, $H = 10^4$, $\alpha_{PL} = 1.0$, and $T_{sim} = 100$ s. For further details, including neuron parameters and spiking statistics, see Ref. [4].

the internal dynamics of units impact signaling within the system, extending beyond *Markovian assumptions*. To demonstrate this, we consider a biologically inspired example of a network of LIF units. The LIF dynamics have diverse applications in the study of networked systems [13]. In short, each unit has internal dynamics governed by a membrane potential with continuous linear leakage and discrete spiking events [4]. The network is organized into recurrent excitatory (E) and inhibitory (I) populations, reminiscent of cortical microcircuits [4,14].

To quantify the impact of correlated fluctuations in this finite-size system, we measure coherence as proposed in [15]: We uniformly sample k units and compute the sample variance of their population-averaged spiking rates over a sliding observation window of width Δ . Figure 3 displays this coherence measure normalized by the system's total event rate.

The finite PL network coherence (orange solid line, $\Delta = 1$ ms) deviates substantially from the expected asynchronous regime (black dashed line), even for small sample sizes. This deviation becomes more pronounced at longer timescales (orange dotted line, $\Delta = 10$ ms), as the accumulated synchrony is integrated over windows comparable to the membrane time constant ($\tau_m = 10$ ms). In stark contrast, the coherence measure in the ER network (blue solid and dotted lines) aligns closely with the asynchronous-state prediction across all timescales for $k/N \ll 1$.

These results (Fig. 3) reveal a striking dynamic regime in the PL system: While individual units exhibit irregular

spiking (see Ref. [4]), their collective activity intermittently shows significant coherence. This highlights how network topology and out-degree heterogeneity can significantly influence the temporal coordination of finite-system spiking.

Discussion—We study the universality of weak correlation in a recurrent network that operates in a dynamically balanced state, where negative feedback stabilizes fluctuations in the population activity. This feedback is essential for maintaining weak correlations in the large- N limit [16]. By incorporating population fluctuation contributions into the mean-field theory, which is built upon previous work [16], we extend the analysis to networks with arbitrary in- and out-degree distributions. Our findings demonstrate that structural heterogeneity can fundamentally reshape the landscape of correlations in dynamically balanced networks. While classical mean-field theory predicts vanishing correlations as $1/N$ in diverse networks, this weak-correlation universality breaks down when a large degree of heterogeneity in outgoing connections is present in the network topology—in particular, in the networks where hub size is proportional to system size and interaction degree K scales sublinearly with system size. We showed a crossover phenomenon in these systems to a regime where macroscopic fluctuations can persist in the thermodynamic limit. This breakdown of weak-correlation universality is analytically captured by our generalized mean-field framework that considers degree distributions. Our analytical results are numerically confirmed in the stochastic simulation of networks with binary units.

Crucially, these findings extend beyond Markovian models. For example, in LIF neuronal networks with power-law connectivity, non-Markovian dynamics amplify coherent fluctuations, mirroring the high correlations observed in cortical circuits [17]. Similar mechanisms may explain superspreader-driven contagion—where hub nodes dominate outbreak coherence [18]—or animal swarms, where rare informed individuals synchronize group motion [19]. These examples highlight a universal principle: Degree heterogeneity can act as a latent dimension that amplifies macroscopic fluctuations, even in systems with intrinsic stabilizing feedback.

Our Letter broadens the theoretical foundation for fluctuation-driven phenomena in complex systems by bridging balanced network theory with topological heterogeneity. It demonstrates how degree variability can shape both chaotic and coherent dynamics across a range of scales, from microscopic to macroscopic. Future research may investigate how adaptive rewiring, spatial embedding, or structured motifs further modulate this interplay, potentially revealing new principles of fluctuation coordination in plastic network topologies.

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Data availability—The data that support the findings of this Letter are openly available [20].

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