# Muon Lifetime and Fermi Constant: An Update

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We present an updated prediction for the lifetime of the muon, including a detailed analysis of all relevant uncertainties. Our prediction includes QED corrections up to order $\alpha^3$ and state-of-the-art hadronic contributions based on dispersive methods. Radiative corrections and finite-electron-mass effects are parametrized by the correction factor $\Delta q$ in $\tau_\mu^{-1} = G_F^2 m_\mu^5 (1+\Delta q)/(192\pi^3)$, for which we obtain $\Delta q = (-4\,384\,700 \pm 34) \times 10^{-9}$. This reduces the uncertainty associated with $\Delta q$ by an order of magnitude compared to the previous prediction at order $\alpha^2$. We use our results to provide an updated value of the Fermi coupling constant, $G_F = 1.166\,378\,60\,(59) \times 10^{-5}$ GeV$^{-2}$ (consistent with the current Particle Data Group estimate). Further improvements will require better measurements of the muon lifetime and mass.

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*Introduction*—The Fermi coupling constant $G_F$ is a fundamental input parameter that enters the predictions of all quantities involving the electroweak sector of the standard model of particle physics. A numerical value of $G_F$ is obtained from its relation to the muon lifetime $\tau_\mu$ and mass $m_\mu$. (In certain applications $G_F$ is thus instead denoted $G_\mu$.) The current experimental precision is dominated by the final MuLan measurement, $\tau_{\mu^+} = 2196\,980.3(2.2)$ ps, which reached a relative uncertainty of 1.0 ppm and implies a determination of $G_F$ at the level of about 0.5 ppm [1,2].

The muon lifetime is conveniently expressed as

$$\frac{1}{\tau_\mu} = \Gamma(\mu^- \to e^- \nu_\mu \bar{\nu}_e) = \frac{G_F^2 m_\mu^5}{192\pi^3}\left[1 + \Delta q\right]. \tag{1}$$

Higher-order corrections and finite electron-mass effects are parametrized by the quantity $\Delta q$, which depends on $\rho = m_e/m_\mu$, the ratio between the electron mass, $m_e$, and the muon mass. By definition, $\Delta q$ contains only corrections to the decay process $\mu^- \to e^- \nu_\mu \bar{\nu}_e$ evaluated within the Fermi theory. Pure electroweak effects associated with the matching of the Fermi theory to the standard model are absorbed into the definition of $G_F$ [3,4], together with tree-level $W$-boson propagator effects [5,6]. A state-of-the-art evaluation of $\Delta q$ is therefore important for providing a robust baseline for the extraction of $G_F$ from the muon lifetime and for future high-precision global fits of the electroweak sector. In this Letter we present an updated result for $\Delta q$, including contributions through order $\alpha^3$, and update the corresponding prediction for the muon lifetime.

The one-loop QED corrections to $\Delta q$ were computed almost 70 years ago in Refs. [7–9]. The two-loop corrections have been known since the end of the last century from Refs. [10–12]. Results for the third-order corrections to $\Delta q$ were first obtained as a byproduct of the computation of the QCD corrections to the semileptonic $b \to c$ decay

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[13]. This calculation is based on an expansion in the mass difference $m_\mu - m_e$, which is extrapolated to $m_e \to 0$, inducing an uncertainty of about 15% for the coefficient of order $\alpha^3$ of $\Delta q$. An independent corroboration of parts of the calculation is provided by [14]. A dedicated calculation of the fermionic third-order QCD corrections to $b \to u$ in Ref. [15] can immediately be translated to muon decay. The combination with the nonfermionic parts from Ref. [13] leads to a reduced uncertainty as will be described below. In addition, as compared to Ref. [13], more expansion terms in $m_\mu - m_e$ are available. Results for the QED corrections are presented in the "QED contribution" section below.

At order $\alpha^2$, QCD corrections enter $\Delta q$ via the hadronic vacuum polarization (HVP) of the photon. Such contributions cannot be computed within the perturbative framework and nonperturbative methods have to be applied. The current prediction of the hadronic contribution is based on an analysis performed in 1998 [16]. It is therefore timely to update this contribution using modern experimental input for the $R$-ratio and contemporary analysis techniques. We include hadronic contributions based on the dispersive approach, accounting for the current tensions in different HVP evaluations (see the 2025 white paper of the Muon $g-2$ Theory Initiative [17–77] for more details). Details of the hadronic contributions can be found in the "Hadronic contribution" section.

Updated results for the prediction of the muon lifetime and the Fermi constant are shown below under "Muon lifetime and Fermi constant," followed by our conclusions.

*QED contribution*—We parametrize the quantity $\Delta q$ in Eq. (1) as

$$\Delta q = \sum_{i\geq 0} \Delta q^{(i)} = \sum_{i\geq 0} \left(\frac{\bar{\alpha}(m_\mu)}{\pi}\right)^i Y_{(i)}(\rho), \tag{2}$$

where the expansion parameter $\bar{\alpha}$ is the fine-structure constant in the $\overline{\text{MS}}$ scheme at the scale $\mu = m_\mu$. The current value for the fine-structure constant at zero momentum transfer is [78]

$$1/\alpha = 137.035\,999\,177\,(21). \tag{3}$$

For the lepton masses we use the current values from Ref. [79], which are given by

$$\begin{aligned} m_e &= 0.510\,998\,950\,00\,(15)\ \text{MeV}, \\ m_\mu &= 105.658\,3755\,(23)\ \text{MeV}, \\ \rho &= 483\,633\,169\,(11) \times 10^{-11}. \end{aligned} \tag{4}$$

Here, the uncertainty in $m_\mu$ is important for the determination of $G_F$, as will be discussed below. The uncertainty in $m_e$ can be neglected. Using the relation between $\alpha$ and $\bar{\alpha}(m_\mu)$ up to fourth order [80,81] we obtain

$$1/\bar{\alpha}(m_\mu) = 135.901\,240\,717\,(21). \tag{5}$$

Here, $\bar{\alpha}$ is the $\overline{\text{MS}}$ coupling with both electrons and muons as active flavors. Its conversion from the on-shell coupling $\alpha$ includes a finite muon-threshold matching term, which remains nonzero at $\mu = m_\mu$ and first contributes to the decay rate at order $\alpha^3$ (In contrast, Refs. [11,82] use a coupling with only the electron active in the running. At $\mu = m_\mu$, their conversion from the on-shell coupling differs from ours starting at order $\alpha^3$.). The theoretical uncertainty due to unknown $O(\alpha^5)$ corrections in the relation between $\alpha$ and $\bar{\alpha}$ can be estimated as $(\alpha/\pi)^5 \log^5(\rho)/243$ and is at the level of $10^{-11}$. The uncertainty in $\bar{\alpha}$ can be neglected.

In the following, we discuss the QED corrections to $\Delta q$ up to order $\alpha^3$. We separate the mass-independent and mass-dependent parts,

$$\Delta q^{(i)} = \Delta q_0^{(i)} + \Delta q_m^{(i)}. \tag{6}$$

At each perturbative order, the quantity $\Delta q^{(i)}$ is free from mass singularities, as guaranteed by the Kinoshita-Lee-Nauenberg [83,84] theorem. At leading order (with the neutrinos produced in the muon decay taken to be massless), the muon decay rate is described by the function

$$F(\rho) = 1 - 8\rho^2 - 24\rho^4 \log\rho + 8\rho^6 - \rho^8 \tag{7}$$

and the finite electron mass effect at leading order is given by

$$\Delta q_m^{(0)} = F(\rho) - 1 = -187\,050.828(8) \times 10^{-9}. \tag{8}$$

To first order in $\alpha$ one has

$$\begin{aligned} \Delta q^{(1)} &= \frac{\bar{\alpha}(m_\mu)}{\pi}\left[\frac{25}{8} - \frac{\pi^2}{2} - \rho^2[34 + 24\log(\rho)] + O(\rho^3)\right] \\ &= \frac{\bar{\alpha}(m_\mu)}{\pi}\left[-1.8098\ldots + 2.214\,217\,44\,(8) \times 10^{-3}|_{m_e}\right] \\ &= (-4238\,945.4 + 5\,186.2|_{m_e}) \times 10^{-9} \\ &= -4\,233\,759.2 \times 10^{-9}, \end{aligned} \tag{9}$$

where for the numerical evaluation of the finite electron mass effect at next to leading order we used the exact analytic expression from Ref. [85]. In the final value for $\Delta q^{(1)}$ we neglect the small uncertainty stemming from the ratio $\rho$. At order $\alpha^2$ the (QED) corrections are given by [11,82,86]

$$\Delta q^{(2)} = \left(\frac{\bar{\alpha}(m_\mu)}{\pi}\right)^2 \left[\frac{156\,815}{5184} - \frac{518}{81}\pi^2 - \frac{895}{36}\zeta_3 + \pi^2\frac{53\log(2)}{6} + \frac{67\pi^4}{720} - \frac{5\pi^2}{4}\rho + O(\rho^2)\right]$$
$$= \left(\frac{\bar{\alpha}(m_\mu)}{\pi}\right)^2 (6.74268\ldots - 0.079\,159\,704(2)|_{m_e})$$
$$= \left(36\,990.1 - 434.3|_{m_e}\right) \times 10^{-9}$$
$$= 36\,555.9 \times 10^{-9}. \quad (10)$$

Equation (10) includes the contribution from the decay channel $\mu^- \to e^-\nu_\mu\bar{\nu}_e e^+e^-$, and is therefore inclusive with respect to additional electron-positron pair emission. The small contributions arising at next to next to leading order (NNLO) from closed $\tau$ loops were calculated in Ref. [16],

$$\Delta q_\tau^{(2)} = -0.000\,58\left(\frac{\bar{\alpha}(m_\mu)}{\pi}\right)^2 = -3.2 \times 10^{-9}.$$

At third order we have

$$\Delta q_0^{(3)} = \left(\frac{\bar{\alpha}(m_\mu)}{\pi}\right)^3 (2.109\,799\,281 n_h - 0.018\,767\,889\,09 n_h^2 - 7.187\,551\,125 n_l - 6.919\,459\,635 n_l^2 - 0.012\,881\,141\,48 n_h n_l - 6.2 \pm 1.6)$$
$$= \left(\frac{\bar{\alpha}(m_\mu)}{\pi}\right)^3 (-18.3 \pm 1.6)$$
$$= (-235 \pm 20) \times 10^{-9}. \quad (11)$$

The fermionic corrections, marked by powers of $n_l = 1$ and $n_h = 1$ for closed electron and muon loops respectively, are available with high precision from Ref. [15]. The bosonic corrections [i.e., the terms independent of $n_l$ and $n_h$ in Eq. (11)] are taken from the $C_F^3$ color factor of the calculation performed in Ref. [13] for $b \to c l \bar{\nu}_l$. Expansion terms up to $\delta^{12}$ with $\delta = 1 - m_c/m_b$ have been computed in [13]. In Eq. (11), we use two additional expansion terms, up to $\delta^{14}$, which have been computed since the publication of [13]. The finite expansion depth in $\delta$ introduces an uncertainty when extrapolating the results to large values of $\delta$. Following [13], the uncertainty is estimated from the size of the last known term in the expansion ($\delta^{14}$) and by multiplying it by a factor of 5. From the behavior of the expansion terms in $\delta$ at $\delta = 1$, we obtain $-6.2 \pm 1.6$, as stated in Eq. (11). Our result of $-18.3$ can be compared with the estimate $Y_{(3)} \approx -20$ obtained in Ref. [4].

To understand the quality of the convergence of the expansion in $\delta$ we can compare furthermore, within QCD, the exact result for the third-order correction to $b \to u l \bar{\nu}_l$ from Refs. [87,88] with the extrapolation obtained from Ref. [13] using the results up to $\delta^{12}$ and setting $\delta = 1$ (i.e., $m_c = 0$). For the bosonic contribution at third order $X_{3,\mathrm{bos}}$ we have

$$X_{3,\mathrm{bos}}^{[87]} = -550.014, \qquad X_{3,\mathrm{bos}}^{[13]} = -526\,(38), \quad (12)$$

which differ by only 4.3% and are compatible within uncertainty. Note, however, that the absolute difference is dominated by the $C_A^2 C_F$ term, which is absent in QED. In Ref. [87] the QCD results are not presented for individual color factors. Thus, we cannot extract the coefficient of the color structure $C_F^3$, which is the relevant piece for the translation from QCD to QED.

At third order, the contribution proportional to $n_l$, taken from the exact calculation of Ref. [15], includes the corresponding $e^+e^-$ pair production at next to leading order, including part of the identical-electron interference. Electron-positron pair production in the remaining bosonic corrections is absent from the equal-mass expansion of Ref. [13] (see also Ref. [89]) but covered by our uncertainty estimate.

*Hadronic contribution*—Hadronic contributions to $\tau_\mu$ enter at order $\alpha^2$ and can be calculated using the dispersion integral

$$Y_{(2)}^{\mathrm{had}} = \int_{m_0^2}^{\infty} \frac{\mathrm{d}s}{3s} R(s) K_\Gamma(s), \quad (13)$$

a result for $Y_{(2)}^{\mathrm{had}}$ could alternatively be predicted from lattice QCD. Here, $m_0$ is the neutral pion mass and $R(s)$ is the hadronic $R$ ratio, defined as the total $e^+e^- \to \mathrm{hadrons}(+\gamma)$ cross section $\sigma_{\mathrm{had}}(s)$ normalized to the leading order massless muon cross section, i.e., $R(s) = \sigma_{\mathrm{had}}(s)[4\pi\alpha^2/(3s)]^{-1}$. The monotonic kernel $K_\Gamma(s)$ is given by Eq. (7) of [16]. As $s \to \infty$, $K_\Gamma(s) \sim 2m_\mu^2/(5s)\ln(m_\mu^2/s)$ so lower energy contributions are more heavily weighted.

The hadronic $R$ ratio used is that of Keshavarzi-Nomura-Teubner-Wright (KNTW) [90,91], based on the KNT19 data compilation [46,92]. This compilation consists of a sum of exclusive hadronic final states (channels) for $\sqrt{s} < 1.937$ GeV, inclusive hadronic measurements between 1.937 and 11.199 GeV, and perturbative QCD for $\sqrt{s} > 11.199$ GeV. These are supplemented by chiral perturbation theory estimates of the thresholds of leading low-energy channels and Breit-Wigner descriptions of the narrow $J/\psi$, $\psi'$ and $\Upsilon(1S-4S)$ resonances. Such additional contributions are not resolved in the experimental $e^+e^- \to$ hadrons data, but are wholly subdominant. The estimate of $Y_{(2)}^{\mathrm{had}}$ is dominated by the contributions from the $\pi^+\pi^-$ channel, which make up ~65% of the total. Uncertainties on $Y_{(2)}^{\mathrm{had}}$ are derived directly from the covariance matrices of the combination.

Despite the release of > 50 new high precision datasets since KNT19, an updated $R$-ratio compilation is not presently available. The KNTW analysis framework is

currently subject to software blinding and updated results cannot be obtained from it [90]. An important motivation for the introduction of this blinding was the persistent up to $\sim 5\sigma$ tensions between the recent CMD-3 measurement of the $\pi^+\pi^-$ cross section ($\sigma_{\pi\pi}$) [93,94] and earlier comparably precise $\sigma_{\pi\pi}$ datasets (e.g., [95–100]). The CMD-3 $\sigma_{\pi\pi}$ values ($\sigma_{\pi\pi}^{\mathrm{CMD-3}}$) exceed the older data by as much as $\sim 5\%$, consistently across the spectrum. Significant efforts to try to resolve this discrepancy [101–106] have not yet resulted in a satisfactory explanation, though studies (including new $\sigma_{\pi\pi}$ measurements) are ongoing [107–115].

In this Letter, both the KNT19 combination and the "KNT19/CMD-3" combination [47] are used to compute $Y_{(2)}^{\mathrm{had}}$. In the latter, the $\pi^+\pi^-$ data combination for the range 0.327–1.199 GeV (the range of the CMD-3 measurement) is replaced with $\sigma_{\pi\pi}^{\mathrm{CMD-3}}$. This provides a realistic upper limit on the values obtained for integrated observables such as $Y_{(2)}^{\mathrm{had}}$. These compilations are individually input into Eq. (13) to obtain

$$Y_{(2)}^{\mathrm{had}}[\mathrm{KNT19}] = -42.18(0.05)_{\mathrm{st}}(0.13)_{\mathrm{sy}}(0.08)_{\mathrm{rad}} \times (0.08)_{\mathrm{corr}}[0.18]_{\mathrm{tot}} \times 10^{-3}, \quad (14)$$

$$Y_{(2)}^{\mathrm{had}}[\mathrm{KNT19/CMD\text{-}3}] = -43.34(0.04)_{\mathrm{st}}(0.25)_{\mathrm{sy}}(0.07)_{\mathrm{rad}} \times (0.08)_{\mathrm{corr}}[0.27]_{\mathrm{tot}} \times 10^{-3}. \quad (15)$$

Here, "st" and "sy" refer to uncertainties propagated from experimental statistics and systematics, "rad" refers to uncertainties on the *a posteriori* radiative corrections necessary in the dispersive approach, "corr" refers to the new KNTW uncertainty accounting for the impact of imperfectly known systematic correlations [91] (note that this uncertainty is denoted $\rho$ in [91]), and "tot" denotes the total uncertainty obtained from a quadrature sum of the individual error sources (given in square brackets).

Accounting for correlation between Eqs. (14) and (15), this constitutes a $4.7\sigma$ tension. Since it will not be the leading source of uncertainty on $\tau_\mu$, it is reasonable here to take the unweighted average of these as a prediction for the central value and half of the spread as an additional uncertainty ($\pm 0.58 \times 10^{-3}$). This conservatively accounts for all plausible results including—as the $\pi^+\pi^-$ channel provides the dominant contribution to $Y_{(2)}^{\mathrm{had}}$—the potential impacts of other datasets missing since KNT19. Doing so, the result

$$\Delta q_{\mathrm{had}}^{(2)} = -(0.0428 \pm 0.0006)\left(\frac{\bar{\alpha}(m_\mu)}{\pi}\right)^2 = -(234.6 \pm 3.3) \times 10^{-9} \quad (16)$$

is obtained, the uncertainty on which is entirely dominated by the CMD-3 tension. This result is consistent with [16], with the uncertainty reduced by a factor of $\sim 3$. Equation (16) can be updated when the KNTW analysis is unblinded and a new combination properly inclusive of CMD-3 can be obtained.

To estimate the size of the hadronic correction at next to next to next to leading order, we first observe that the contribution from a closed muon loop at NNLO is $\Delta q_\mu^{(2)} = -200 \times 10^{-9}$, which is comparable in magnitude to the hadronic contribution in Eq. (16). Similar compatibility is seen for the muon loop and HVP contributions to the muon anomalous magnetic moment $a_\mu$ at orders $\alpha^2$ and $\alpha^3$ [46,116–123], with the NNLO hadronic contribution in Eq. (13) also dominated by the low-energy region. At $O(\alpha^3)$ the contribution from closed muon loops is $\Delta q_\mu^{(3)} = 27 \times 10^{-9}$ and, given the comparability of contributing Feynman diagrams, we expect $\Delta q_{\mathrm{had}}^{(3)}$ to be of the same sign and comparable size. We therefore base an estimate of $\Delta q_{\mathrm{had}}^{(3)}$ on $\Delta q_\mu^{(3)}$ and assign a conservative 100% relative uncertainty, i.e.,

$$\Delta q_{\mathrm{had}}^{(3)} = (27 \pm 27) \times 10^{-9}. \quad (17)$$

*Muon lifetime and Fermi constant*—The results discussed above are summarized order by order in Table I, which makes the good convergence of the perturbative expansion clear. Summing the uncertainties in quadrature, we obtain the final prediction,

$$\Delta q = (-4\,384\,699.5 \pm 3.3_{\mathrm{had}} \pm 20_{\mathrm{QED}} \pm 27_{\mathrm{h.o.had}}) \times 10^{-9} = (-4\,384\,700 \pm 34) \times 10^{-9}. \quad (18)$$

Here, the first uncertainty ("had") comes from the NNLO hadronic contribution in Eq. (16), the second ("QED") is induced by the uncertainty on $\Delta q_0^{(3)}$ in Eq. (11), and the third ("h.o. had") accounts for unknown hadronic corrections at $O(\alpha^3)$. It is common also to define the Fermi coupling constant by factorizing the leading-order phase-space function $F(\rho)$,

$$\frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3} F(\rho)[1 + \Delta q']. \quad (19)$$

TABLE I. QED and hadronic contributions to the muon lifetime, organized by perturbative order $i$. The $i$ th coefficient $Y_{(i)}$ multiplies $(\bar{\alpha}(m_\mu)/\pi)^i$ as indicated by Eq. (2).

| order ($i$) | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $Y_{(i)}^{\mathrm{QED}}$ | −0.000 187… | −1.808… | 6.663… | −18.3(1.6) |
| $Y_{(i)}^{\mathrm{had}}$ | | | −0.0428(6) | 2.1(2.1) |
| $Y_{(i)}^{\mathrm{total}}$ | −0.000 187… | −1.808… | 6.6202(6) | −16.2(2.6) |

With this alternative convention, we obtain

$$\Delta q' = (-4\,198\,434 \pm 34) \times 10^{-9}. \tag{20}$$

The size of additional higher-order contributions can be straightforwardly estimated. Finite-electron-mass effects have been ignored in the computation of three-loop QED corrections. These terms are expected to scale as $\Delta q_m^{(3)} \sim (\alpha/\pi)^3 \rho \log \rho$, which gives $\Delta q_m^{(3)} \sim 0.3 \times 10^{-9}$. This is about one order of magnitude smaller than the uncertainty associated with the NNLO hadronic corrections.

The $\overline{\text{MS}}$ coupling constant in Eq. (2) can in principle be evaluated at a scale $\mu$ different from our default choice $\mu = m_\mu$. To assess the residual scale dependence, we vary the $\mu$ between $m_e$ and $m_\mu$, consistently accounting for the scale dependence of both $\bar{\alpha}(\mu)$ and the perturbative coefficients $Y_{(i)}$. The resulting variation in $\Delta q$ is approximately $1 \times 10^{-9}$, well below the uncertainty in the third-order QED correction.

Using the average value for the muon lifetime [2,79],

$$\tau_\mu = 2.196\,981\,1\,(22) \times 10^{-6}\ \text{s}, \tag{21}$$

together with the muon mass [see Eq. (4)], we can then extract the Fermi coupling constant from Eq. (1),

$$\begin{aligned} G_F &= \sqrt{\frac{192\pi^3}{m_\mu^5 \tau_\mu (1+\Delta q)}} \\ &= 1.166\,378\,60\,(59) \times 10^{-5}\ \text{GeV}^{-2}. \end{aligned} \tag{22}$$

We obtain the following contributions to the relative uncertainty of $G_F$:

$$\begin{aligned} &\frac{1}{2}\frac{\sigma_{\Delta q}}{1+\Delta q} = 0.017\ \text{ppm}, \qquad \frac{5}{2}\frac{\sigma_{m_\mu}}{m_\mu} = 0.054\ \text{ppm}, \\ &\frac{1}{2}\frac{\sigma_{\tau_\mu}}{\tau_\mu} = 0.5\ \text{ppm}. \end{aligned} \tag{23}$$

This comparison clearly shows that the uncertainty in the muon lifetime is by far the dominant source of uncertainty in $G_F$. The muon-mass uncertainty provides the next-largest contribution, while the uncertainty induced by $\Delta q$ is significantly smaller and remains subleading even compared with the muon-mass uncertainty. The uncertainties associated with $\rho$ and $\bar{\alpha}(\mu)$ are entirely negligible.

For the total theoretical uncertainty, the main contribution comes from the uncertainty of the third-order correction and the unknown hadronic corrections at $O(\alpha^3)$, with the $O(\alpha^2)$ hadronic uncertainty now only making up around 1.0% of the theoretical error. Further improvements in the theoretical precision require an exact calculation of the bosonic contribution to $\Delta q^{(3)}$ and an explicit evaluation of the hadronic corrections $\Delta q_{\text{had}}^{(3)}$. Without the improvements presented in this Letter, the NNLO uncertainty induced by $\Delta q$ would be 0.15 ppm [2,11,86], which is of the same order as the uncertainty from the muon lifetime. Our result therefore reduces the theoretical uncertainty by an order of magnitude.

In order to meaningfully reduce the total uncertainty of the Fermi coupling constant, the experimental uncertainties in the muon lifetime and muon mass must be reduced further. In the near future, the latter is expected to be reduced, by a factor of ~20 by the upcoming MuMASS [124,125] and MuSEUM [126] experiments. This will be sufficient to make it the smallest of the three major error components considered in Eq. (23). However, we are unaware of any plans to make the improvements to the muon lifetime uncertainty that are necessary to substantially improve the precision of $G_F$.

*Conclusions*—We present an updated value of the Fermi constant $G_F$, derived from the muon lifetime. Our main result is given in Eqs. (18) and (22). It incorporates exact fermionic corrections up to order $\alpha^3$ [including $O(\alpha^3)$ QED corrections] and a resulting updated estimate of the remaining uncertainty. This leads to a reduction of the theory uncertainty by a factor of nearly 10. Furthermore, it features an updated dispersive hadronic contribution, whose uncertainty has been reduced by a factor of approximately 3. This contribution accounts for the significant tensions present in the input hadronic data, which are driven by the recent CMD-3 $\pi^+\pi^-$ cross section measurement.

Our result for $G_F$ in Eq. (22) coincides with the value reported in the 2026 edition of the Particle Data Group [127]. Following our improvements, the uncertainty in $G_F$ arising from higher-order corrections is a factor of 3 smaller than the uncertainty in the current muon mass and about a factor of 30 smaller than that of the muon lifetime; this underscores the need for improved measurements of $m_\mu$ and $\tau_\mu$. Since the uncertainty of $G_F$ is now dominated by the measurement of the muon lifetime, any improvement in this measurement leads directly to a more precise result for the Fermi constant.

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*Data availability*—There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.


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