

# High-Temperature Limit Penalizing High-Frequency Quantum Fluctuations

Graeme Pleasance<sup>1,2,\*</sup> Erik Aurell<sup>3,†</sup> and Francesco Petruccione<sup>1,2,4,‡</sup>

<sup>1</sup>Department of Physics, University of Stellenbosch, Stellenbosch, 7600, South Africa

<sup>2</sup>National Institute for Theoretical and Computational Sciences (NITheCS), Stellenbosch, South Africa

<sup>3</sup>KTH—Royal Institute of Technology, AlbaNova University Center, SE-106 91 Stockholm, Sweden

<sup>4</sup>School of Data Science and Computational Thinking, University of Stellenbosch, Stellenbosch 7600, South Africa



(Received 19 August 2025; revised 4 December 2025; accepted 26 May 2026; published 29 July 2026)

We revisit the Caldeira-Leggett model of quantum Brownian motion with Ohmic spectral density, and derive an additional contribution to the decoherence kernel in a new high-temperature limit at arbitrarily large cutoff frequency. This contribution reveals a novel mechanism for the classicalization of high-frequency quantum fluctuations. We further demonstrate that it leads to a Markovian master equation that is in guaranteed Lindblad form, and argue that this master equation describes the correct Markovian limit of quantum Brownian motion. Our approach considers in detail the behavior of the decoherence kernel at both the initial and final times of the process on the timescale of the bath memory.

DOI: 10.1103/kp9j-v4ss

**Introduction**—The time evolution of a quantum system interacting with an Ohmic bath of harmonic oscillators is one of the most widely studied models of open quantum systems theory [1]. It is applicable in numerous physical situations where the damping becomes frictionlike in the semiclassical limit [2], such as for photon and phonon baths [3], as well as superconducting qubits coupled to resistive elements in quantum circuits [4]. Caldeira and Leggett famously used this model to investigate the classical limit of quantum Brownian motion [5] and macroscopic tunneling [6,7]. It also played a central role in later investigations of the spin-boson problem [8] and decoherence [9–11], as well as recently being used to study gravitational noise effects in the arm lengths of the Laser Interferometer Gravitational Wave Observatory interferometer arising from interactions with a quantized bath of gravitons [12].

Under a path integral treatment [13], the influence of the bath splits into dissipation, where the memory is limited by the spectral cutoff  $\Omega$  of the interactions, and decoherence, which is also limited by the inverse thermal time  $k_B T/\hbar$ . For phenomenological models and interactions with phonon baths,  $\Omega$  is a real physical parameter while for interactions with photons it is a regularizer that can eventually be taken to infinity in renormalization [14,15]. In either case, dissipation is memoryless. In the Caldeira-Leggett (CL) master equation

[5] this dissipation accounts for a renormalization of the system Hamiltonian and two Lindblad terms, both of which are formed of ordered pairs of position and momentum operators  $(\hat{X}, \hat{P})$  and  $(\hat{P}, \hat{X})$ .

Decoherence is different, in that it has memory. Provided that the thermal time  $\hbar/k_B T$  is much longer than  $\Omega^{-1}$ , the cutoff  $\Omega$  plays a negligible role, and can be directly taken to infinity. As described in the pioneering paper by Feynman and Vernon [16], decoherence is indistinguishable from averaging over a classical Gaussian field  $\xi$  with the same force-force correlations as the memory kernel of the bath. The combined effects of dissipation and decoherence as outlined above are the ingredients of the stochastic Liouville simulations method of Stockburger and Mak [17] (see also Refs. [18,19]). The decoherence memory kernel  $k_r$  is therefore of great importance both in fundamental understanding of the influence of harmonic baths, and in practical simulation tools applied to quantum technologies. Similarly, its role in characterizing thermal fluctuations is particularly relevant to semiclassical descriptions of quantum Brownian motion at high temperatures.

In the standard high-temperature limit [5], the kernel  $k_r$  has no memory and produces Lindblad terms derived from ordered pairings of the position operator  $\hat{X}$  with itself, i.e.,  $(\hat{X}, \hat{X})$ . The classical random force  $\xi$  is then delta-correlated in time. In this Letter, we show that if correctly carried out, there is an additional  $(\hat{P}, \hat{P})$  term in the master equation that serves to both (i) damp high-frequency quantum fluctuations, and (ii) maintain the positivity of the reduced system density operator for parameters consistent with the Born-Markov approximations, similar to what has been identified in previous works [1,20–26]. However, our derivation considers a new high-temperature limit that applies to an arbitrarily large (ultraviolet) cutoff  $\Omega$  at fixed  $T$ , and in fact

\*Contact author: gpleasance1@gmail.com

†Contact author: eaurell@kth.se

‡Contact author: petruccione@sun.ac.za

Published by the American Physical Society under the terms of the Creative Commons Attribution 4.0 International license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI.

leads to a master equation that is completely independent of  $\Omega$ . Our result is therefore valid over a broader temperature range than for analogous master equations derived in [1,20,21], and offers new insights into the study of quantum Brownian motion at medium to low temperatures in the Markovian limit.

*Feynman-Vernon theory*—We focus on the CL model [5] describing a quantum particle of mass  $M$ , momentum  $\hat{P}$ , and position  $\hat{X}$ , moving in a potential  $V(\hat{X})$ . The particle interacts with a thermal bath of harmonic oscillators at (inverse) temperature  $T$  ( $\beta = 1/k_B T$ ). As such, the total Hamiltonian reads

$$\hat{H} = \hat{H}_S + \hat{H}_B + \hat{H}_I + \hat{H}_C, \quad (1)$$

where  $\hat{H}_S = \hat{P}^2/2M + V(\hat{X})$  and  $\hat{H}_B = \sum_k (\hat{p}_k^2/2m_k + \frac{1}{2}m_k\omega_k^2\hat{x}_k^2)$  are the system and bath Hamiltonians, respectively. The interaction part is taken to have a conventional bilinear form  $\hat{H}_I = -\hat{X} \otimes \sum_k c_k \hat{x}_k$ , with  $c_k$  denoting the system-bath coupling strength. The final part represents an additional counterterm  $\hat{H}_C = \hat{X}^2 \sum_k (c_k^2/2m_k\omega_k^2)$  that ensures the translational invariance of the Hamiltonian (1).

After integrating out the bath degrees of freedom, the time evolution of the reduced system density operator  $\langle X_f | \hat{\rho}_S(t) | X'_f \rangle$  is captured by the propagator  $\mathcal{J}(t_f, t_i)$  connecting the initial  $X_i$  and final  $X_f$  positions of the particle at times  $t_i$  and  $t_f$ . The propagator is expressed as a double path integral over the forward path  $X$  from  $X_i$  to  $X_f$  and backward path  $X'$  from  $X'_i$  to  $X'_f$  [2],

$$\mathcal{J}(t_f, t_i) = \int \mathcal{D}X \mathcal{D}X' e^{(i/\hbar)(S_S[X] - S_S[X'])} \mathcal{F}[X, X']. \quad (2)$$

Here,  $S_S[X] = \int_{t_i}^{t_f} dt [\frac{1}{2}M\dot{X}^2 - V(X)]$  is the free particle action along the path  $X(t)$ , and  $\mathcal{F}[X, X']$  is the Feynman-Vernon (FV) influence functional. Since the bath obeys Gaussian statistics, the influence functional can be written in the exponential form  $\mathcal{F}[X, X'] = e^{-(i/\hbar)S_i[X, X'] - (1/\hbar)S_r[X, X']}$ . The quantities  $S_r[X, X']$  and  $S_i[X, X']$ , which couple forward and backward paths, define the real (decoherence) and imaginary (dissipation) parts of the FV action. In terms of the quasiclassical path  $\bar{X}(t) = X(t) + X'(t)$  and fluctuation path  $\Delta X(t) = X(t) - X'(t)$  [2], they are written as

$$S_r[\Delta X] = \int_{t_i}^{t_f} dt \int_{t_i}^t ds k_r(t-s) \Delta X(t) \Delta X(s), \quad (3)$$

$$S_i[\bar{X}, \Delta X] = \int_{t_i}^{t_f} dt \int_{t_i}^t ds k_i(t-s) \Delta X(t) \bar{X}(s) + \mu \int_{t_i}^{t_f} dt \Delta X(t) \bar{X}(t), \quad (4)$$

with  $\mu = \sum_k c_k^2/(2m_k\omega_k^2)$  the bath reorganization energy ( $\mu \sim \Omega$ ). The kernels  $k_r$  and  $k_i$  denote the real and imaginary parts of the bath correlation function  $k(\tau) = \sum_k (c_k^2/2m_k\omega_k) [\coth(\hbar\omega_k/2k_B T) \cos(\omega_k\tau) - i \sin(\omega_k\tau)]$ . The real part—the FV kernel [16]—governs the decay of quantum fluctuations along the off-diagonal paths  $\Delta X(t)$ . In the FV influence functional, it plays the role of a force-force correlation function for a Gaussian stochastic force with memory time  $\tau_B$  [see Eq. (16) below]. For a bath with Ohmic spectral density  $J(\omega) = 2\eta\omega\theta(\Omega - \omega)$ , where  $J(\omega) = (\pi/2) \sum_k (c_k^2/m_k\omega_k) \delta(\omega - \omega_k)$ , the FV kernel reads

$$k_r(\tau) = \frac{2\eta}{\pi} \int_0^\Omega d\omega \omega \coth\left(\frac{\hbar\omega}{2k_B T}\right) \cos \omega\tau. \quad (5)$$

It is well known that the white-noise limit of  $k_r$  is realized at high temperature [5]. In particular, let  $\tau_B = \hbar/k_B T$ . By approximating  $\coth(\hbar\beta\omega/2) \approx 2/\hbar\beta\omega$  in the limit  $\tau_B \rightarrow 0$ , one obtains the singular kernel  $k_r^{(0)}(\tau) = (4\eta/\tau_B)\delta(\tau)$ , provided that the particle evolves on timescales much longer than  $\Omega^{-1}$ .

*Feynman-Vernon kernel and decoherence action*—We proceed to now derive a new high-temperature limit for which the FV kernel provides a nonsingular contribution to the decoherence action. As we will show, this leads to an additional diffusion-type term in the CL master equation that serves to damp high-frequency quantum fluctuations. Our approach throughout this Letter is to treat the FV kernel as a tempered distribution,  $k_r \in \mathcal{S}'(\mathbb{R})$  [27], where the corresponding test functions belong to  $\mathcal{S}(\mathbb{R})$  and are *a priori* assumed to exhibit negligible variation on timescales shorter than  $\Omega^{-1}$ . For this purpose, it is convenient to employ the following representation of the FV kernel [28]:

$$k_r(\tau) = \frac{2\eta}{\pi} \left[ -\frac{\pi^2}{\tau_B^2} \text{csch}^2(\pi\tau/\tau_B) + \Omega^2 O\left(\frac{1}{(\Omega\tau)^3}\right) \right]. \quad (6)$$

From this expression it is clear that  $k_r$  exhibits two regimes of behavior.

(i) Vacuum regime  $\Omega^{-1} \ll \tau \ll \tau_B$ : Using the asymptotic expansion  $\text{csch}^2 x = (1/x^2) - \frac{1}{3} + O(x^2)$ , Eq. (6) may be approximated as

$$k_r(\tau) \approx -\frac{2\eta}{\pi} \frac{1}{\tau^2}, \quad (7)$$

implying that the short-time behavior of  $k_r(\tau)$  is dominated by the vacuum contribution  $k_r^{\text{vac}}(\tau) = (2\eta/\pi) \int_0^\Omega d\omega \omega \cos \omega\tau$ . In particular,

$$k_r^{\text{vac}}(\tau) = \frac{2\eta}{\pi} \left[ -\frac{1}{\tau^2} + \Omega^2 O\left(\frac{1}{(\Omega\tau)^3}\right) \right]. \quad (8)$$

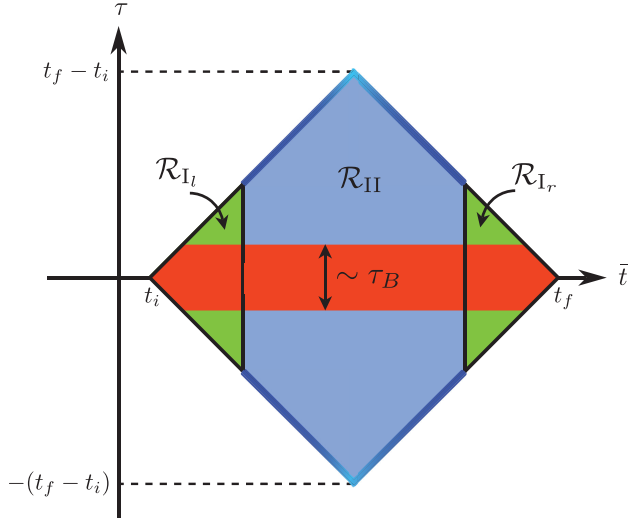


FIG. 1. Schematic depicting the full domain of integration and subdomains  $\mathcal{R}_I = \mathcal{R}_{I_l} \cup \mathcal{R}_{I_r}$  and  $\mathcal{R}_{II}$  of Eq. (10). Since the kernel is essentially zero outside the red-shaded area where  $\tau \gg \tau_B$ , the  $\tau$ -integration limits of the blue-shaded area may be extended to  $\pm\infty$  assuming  $|t_f - t_i| \gg \tau_B$ . The green-shaded areas indicate the support of the end contributions where the integration limits cannot be taken to infinity. A more detailed figure illustrating the boundaries of all three subdomains is provided in Ref. [28].

(ii) Exponential regime  $\tau \gg \tau_B \gg \Omega^{-1}$ : Using that  $\text{csch}^2 x \sim e^{-2x}$  for  $x \gg 1$ , we get

$$k_r(\tau) \approx -\frac{2\eta}{\pi} \frac{1}{\tau_B^2} e^{-2|\tau|/\tau_B}. \quad (9)$$

These relations indicate the kernel (5) to converge in the scaling limit  $\Omega \rightarrow \infty$  for  $\tau > 0$  and to vanish beyond times  $|\tau| \gtrsim \tau_B$  determined by the inverse bath temperature.

With the goal of evaluating the decoherence functional (3), we first choose to rewrite the double integral in terms of the new variables  $\bar{t} = \frac{1}{2}(t + s)$  and  $\tau = t - s$ ,

$$S_r[\Delta X] = \frac{1}{2} \iint_{\mathcal{R}} d\bar{t} d\tau k_r(\tau) \Delta X(\bar{t} + \tau/2) \Delta X(\bar{t} - \tau/2). \quad (10)$$

The integration region  $\mathcal{R}$  in the  $(\bar{t}, \tau)$  plane is depicted in Fig. 1. It comprises three subregions  $\mathcal{R}_{I_l}$  (left),  $\mathcal{R}_{I_r}$  (right), and  $\mathcal{R}_{II}$ , where the first two subregions represent the end term contributions to the decoherence action. To simplify the analysis, we choose here to evaluate  $S_r[\Delta X]$  without the end terms, which can be shown to play a negligible role at high temperature. A more detailed analysis that explicitly treats these end contributions is provided in Sec. III of the Supplemental Material [28]. Henceforth, the decoherence action shall be approximated by

$$S_r[x] \approx \frac{\eta}{\pi} \frac{1}{\tau_B} \int_{t_i}^{t_f} d\bar{t} \int_{-\infty}^{\infty} ds F_{\Omega\tau_B}(s) x_{\bar{t}}(s), \quad (11)$$

where  $x_{\bar{t}}(s) = \Delta X(\bar{t} + \tau_B s/2) \Delta X(\bar{t} - \tau_B s/2)$ , and  $F_{\Lambda}(s) = \int_0^{\Lambda} ds x \coth(x/2) \cos sx$ .

By repeatedly integrating Eq. (11) by parts, which is permissible since  $F_{\Lambda}$  is tempered, we are able to separate the inner integral above into two sets of terms: those depending on the leading contribution to the FV kernel given in Eq. (6), and boundary terms depending on the higher order contributions. In particular, the latter are shown to vanish in the scaling limit  $\Omega \rightarrow \infty$  (see Sec. II of Supplemental Material [28]). By then retaining only the leading terms, and defining the memory kernel  $k_r^{(m)}(\tau) = (\pi^2/\tau_B^2) \text{csch}^2(\pi\tau/\tau_B)$ , the decoherence action can be expressed as the sum of the following two functionals:

$$\begin{aligned} S_r[\Delta X] &:= S_r^{(0)}[\Delta X] + S_r^{(m)}[\Delta X] \\ &= \frac{2\eta}{\pi} \int_{t_i}^{t_f} dt \left[ \frac{\pi}{\tau_B} \Delta X^2(t) + \int_0^{\infty} d\tau k_r^{(m)}(\tau) (\Delta X^2(t) \right. \\ &\quad \left. - \Delta X(t + \tau/2) \Delta X(t - \tau/2)) \right]. \end{aligned} \quad (12)$$

The first functional  $S_r^{(0)}[\Delta X]$  is equivalent to the standard high-temperature result obtained for the decoherence action using the memoryless kernel  $k_r^{(0)}$ . More interestingly, the second functional  $S_r^{(m)}[\Delta X]$  depending on  $k_r^{(m)}$  is valid beyond the white-noise limit, and therefore represents a contribution to the decoherence action accounting for short-time memory effects. This is exemplified by the fact that  $S_r^{(m)}[\Delta X]$  vanishes when  $\tau_B \rightarrow 0$ , which is directly seen from expanding the path differences inside the inner brackets to lowest order in  $\tau$ . Hence, by including higher order terms within the expansion of the fluctuations paths, we are able to systematically derive corrections to the white-noise action  $S_r^{(0)}[\Delta X]$ .

*New high-temperature limit*—We proceed to expand  $x_i(s)$  to next leading order in  $\tau_B s$  (or  $\tau$ ),

$$x_i(s) = x_i^{(0)} + (\tau_B s)^2 x_i^{(1)} + \dots, \quad (13)$$

with  $x_i^{(0)} = \Delta X_i^2$ , such that Eq. (12) becomes

$$S_r[\Delta X] = S_r^{(0)}[\Delta X] + S_r^{(1)}[\Delta X] + \dots, \quad (14)$$

where

$$\begin{aligned} S_r^{(1)}[\Delta X] &:= \frac{\eta\tau_B}{6} \int_{t_i}^{t_f} dt \Delta X(t) \left( -\frac{\partial^2}{\partial t^2} \right) \Delta X(t) \\ &\quad + (\text{boundary terms}). \end{aligned} \quad (15)$$

Note that here we have used  $\int_0^{\infty} d\tau/\tau_B \tau^2 k_r^{(m)}(\tau) = \pi/6$ . Equation (15) is the first result of this Letter and provides the leading high-temperature correction to the white-noise action. We may elucidate its role within the functional (10)

by rewriting  $\exp(-S_r[\Delta X]/\hbar)$  as the average over an auxiliary Gaussian field  $\xi(t)$ , where  $\mathbb{E}[\xi(t)] = 0$  and  $\mathbb{E}[\xi(t)\xi(s)] = k_r^{\text{eff}}(t-s)$ . For this, let us introduce the differential operator  $K(t) = (1/\sigma^2)[-(\partial^2/\partial t^2) + \lambda^2]$  with  $\lambda = 2\sqrt{3}/\tau_B$  and  $\sigma = \sqrt{3/\eta\tau_B}$ , such that up to the leading order correction of Eq. (14) we have  $\exp(-S_r[\Delta X]/\hbar) = \exp[-(1/2\hbar) \int_{t_i}^{t_f} dt \Delta X(t) K[\Delta X](t)]$ . This exponential can then be recast using a Hubbard-Stratonovich transformation as [17,29,30]

$$\exp(-S_r[\Delta X]/\hbar) = (\text{trivial}) \int \mathcal{D}\xi P[\xi] \exp\left(-\frac{i}{\hbar} \int_{t_i}^{t_f} dt \xi(t) \Delta X(t)\right), \quad (16)$$

where  $P[\xi] = N^{-1} \exp[-(1/2\hbar) \int_{t_i}^{t_f} dt \xi(t) K^{-1}[\xi](t)]$  is a Gaussian probability functional. The trivial prefactor in this expression stems from the boundary terms of Eq. (15). Additionally,  $K^{-1}[\xi] = \int_{t_i}^{t_f} ds G(t-s)\xi(s)$ , where  $G(t-s)$  is the (symmetric) Green's function of the operator  $K(t)$ .

Since this kernel must be positive definite and satisfy  $(1/\sigma^2)[-(\partial^2/\partial t^2) + \lambda^2]G(t-s) = \delta(t-s)$ , it is straightforward to deduce that  $k_r^{\text{eff}}(t-s) = K(t-s)\delta(t-s)$ . Notably, this force-force correlation function is equivalent to the FV kernel expanded to its next leading term in the high-temperature limit, i.e.,  $k_r(\tau) \approx (4\eta/\tau_B)\delta(\tau) - (\eta\tau_B/3)\delta''(\tau)$ . This implies the correction term  $S_r^{(1)}[\Delta X]$  to be valid at the same level of approximation made by Diósi [20,21], where an analogous correction was derived by treating the FV kernel to  $O(\hbar\beta)$ . The correction there was shown to lead to an additional diffusion term in the CL master equation rendering it in Lindblad form [20,21]. In parallel, we demonstrate that our result also gives rise to a Lindblad form master equation but with Dekker coefficients [31,32] that are independent of the cutoff  $\Omega$ .

*Derivation of the Lindblad master equation from the Feynman-Vernon kernel*—For a Liouvillian  $\mathcal{L}$  that generates a completely positive and uniformly continuous dynamical semigroup, its most general representation is in Lindblad form [33,34],

$$\begin{aligned} \frac{\partial \hat{\rho}_S(t)}{\partial t} &= \mathcal{L}\hat{\rho}_S(t) \\ &= -\frac{i}{\hbar} [\hat{H}_{\text{eff}}, \hat{\rho}_S] + \sum_{i,j} a_{ij} \left( \hat{L}_i \hat{\rho}_S \hat{L}_j^\dagger - \frac{1}{2} \{ \hat{\rho}_S, \hat{L}_j^\dagger \hat{L}_i \} \right), \end{aligned} \quad (17)$$

where  $\hat{L}_i$  are Lindblad operators, and  $\hat{H}_{\text{eff}}$  is a Hamiltonian [35]. The positivity of the solution obtained from this equation is guaranteed given the matrix  $a$  is necessarily positive semidefinite,  $a \geq 0$ . In [1,5], the CL master equation derived under the Born-Markov approximations has the same form as Eq. (17) with Lindblad operators  $\hat{L}_1 = \hat{X}$ ,  $\hat{L}_2 = \hat{P}$ , and Kossakowski matrix

$$a_0 = \begin{pmatrix} 4\gamma M k_B T / \hbar^2 & -i\gamma/\hbar \\ i\gamma/\hbar & 0 \end{pmatrix}, \quad \gamma = \eta/M, \quad (18)$$

whose spectrum always possesses a negative eigenvalue ( $\det a_0 < 0$ ). The reason for this discrepancy is a missing  $(\hat{P}, \hat{P})$  term that emerges from the next leading contribution in the high-temperature expansion of the FV kernel [1,20,21,23–25]. In our case, rewriting the FV influence functional in superoperator representation [36] gives the Liouvillian

$$\begin{aligned} \mathcal{L}\hat{\rho}_S &= -\frac{i}{\hbar} [\hat{H}_S, \hat{\rho}_S] - i\frac{\gamma}{\hbar} [\hat{X}, \{\hat{P}, \hat{\rho}_S\}] - D_{PP}(\hat{X}, [\hat{X}, \hat{\rho}_S]) \\ &\quad - D_{XX}(\hat{P}, [\hat{P}, \hat{\rho}_S]) - 2D_{XP}(\hat{X}, [\hat{P}, \hat{\rho}_S]), \end{aligned} \quad (19)$$

where

$$D_{XX} = \frac{\gamma}{6Mk_B T}, \quad D_{PP} = \frac{2\gamma M k_B T}{\hbar^2}, \quad D_{XP} = 0, \quad (20)$$

are phenomenological Dekker coefficients [31,32]. We remark that (19) holds for any form of potential  $V(\hat{X})$  contained in  $\hat{H}_S$ , which solely influences the particle's free evolution and not the diffusion timescales (20) [37]. By setting  $\hat{L}_1 = \hat{X}$ ,  $\hat{L}_2 = \hat{P}$ , it is straightforward to show that the Kossakowski matrix obtained from the Liouvillian (19) satisfies the positivity constraint through  $\det a = 4D_{XX}D_{PP} - (\gamma^2/\hbar^2) \geq 0$ . As such, the position diffusion coefficient  $D_{XX}$  is sufficiently large to always render the master equation in Lindblad form. We further remark that since the additional  $(\hat{P}, \hat{P})$  term damps high-frequency quantum fluctuations, at sufficiently high frequency the quantum system interacting with the bath behaves classically—a new open quantum system avenue to classicalization [38].

Table I summarizes the key differences between the Markovian master equation derived in this Letter [Eqs. (19) and (20)] and those derived in Refs. [1,5,20,21].

*Connection to exact master equations*—Finally, we note that Eq. (17) with  $\hat{L}_1 = \hat{X}$ ,  $\hat{L}_2 = \hat{P}$ , and  $\hat{H}_{\text{eff}} = \hat{P}^2/2M + \frac{1}{2}M\omega_0^2\hat{X}^2$  has an inherently different structure to the exact Hu-Paz-Zhang (HPZ) master equation [40] describing non-Markovian quantum Brownian motion (see also Refs. [41–47] for similar exact results). This is due to the  $(\hat{P}, \hat{P})$  term that has no direct counterpart in the HPZ master equation. In Refs. [47,48], it was argued that this discrepancy stems from dissipation being an inherently non-Markovian feature of the CL model, and that dissipation in the Markovian limit can only be accounted for by introducing a momentum-mediated coupling to the bath [22,23,25,49]. In particular, a phenomenological approach including both a position and momentum coupling to an external noise was shown to unravel the time-local master equation [47],

TABLE I. Comparison between different Markovian master equations derived in the high-temperature limit with corresponding diffusion coefficients [39]. In [20,21], the CL master equation is rederived in Lindblad form for certain parameter limits. Note also that the cross diffusion term  $D_{PX}$  derived in [1] [Eq. (3.409) therein] is treated as negligible compared to  $D_{PP}$  when the system evolution is slow compared to  $\Omega^{-1}$ .

| Master equation       | Parameters                               | Lindblad form | $D_{XX}$           | $D_{PP}$                   | $D_{XP}$                         |
|-----------------------|------------------------------------------|---------------|--------------------|----------------------------|----------------------------------|
| <i>This Letter</i>    | $\hbar\gamma \ll k_B T \ll \hbar\Omega$  | Yes           | $(\gamma/6Mk_B T)$ | $(2\gamma Mk_B T/\hbar^2)$ | 0                                |
| Diósi [20,21]         | $\hbar\gamma \ll \hbar\Omega \leq k_B T$ | Yes           | $(\gamma/6Mk_B T)$ | $(2\gamma Mk_B T/\hbar^2)$ | $(\Omega\gamma/6\pi k_B T)$      |
| Breuer-Petrucione [1] | $\hbar\gamma \ll \hbar\Omega \leq k_B T$ | Yes           | $(\gamma/8Mk_B T)$ | $(2\gamma Mk_B T/\hbar^2)$ | $-(\gamma k_B T/\hbar^2 \Omega)$ |
| Caldeira-Leggett [5]  | $\hbar\gamma \ll \hbar\Omega \leq k_B T$ | No            | 0                  | $(2\gamma Mk_B T/\hbar^2)$ | 0                                |

$$\begin{aligned} \mathcal{L}_t \hat{\rho}_S = & -i[\hat{H}_{\text{eff}}(t), \hat{\rho}_S] + \Gamma(t)(\hat{X}, [\hat{X}, \hat{\rho}_S]) + \Theta(t)(\hat{X}, [\hat{P}, \hat{\rho}_S]) \\ & + \Xi(t)[\hat{X}^2, \hat{\rho}_S] + \Upsilon(t)[\hat{X}, \{\hat{P}, \hat{\rho}_S\}] + \gamma(t)(\hat{P}, [\hat{P}, \hat{\rho}_S]), \end{aligned} \quad (21)$$

where a  $(\hat{P}, \hat{P})$  term does appear. Since Eq. (21) reduces to (19) in the white-noise limit, it was concluded that a  $\hat{P}$  coupling is required to correctly describe quantum Brownian motion in the Markovian regime.

Alternatively, we argue that Markovian quantum Brownian motion is best described within the new high-temperature limit introduced in this Letter. This is based on the fact that for the other master equations listed in Table I, taking the Markovian limit  $\Omega \rightarrow \infty$ , also require  $T \rightarrow \infty$ , which amounts to retaining only the leading contribution to the FV kernel, i.e.,  $k_r \sim \delta(\tau)$ . This type of kernel leads to an unphysical description where the complete positivity of the dynamics is lost. In our approach, however, the next-leading contribution to  $k_r$  can be retained while also enforcing a truly memoryless bath  $\Omega \rightarrow \infty$ , suggesting that this provides the correct implementation of the Markovian limit. Ultimately, Eq. (19) with a harmonic potential  $V(\hat{X}) = \frac{1}{2}M\omega_0^2\hat{X}^2$  cannot be recovered as a limiting case of the HPZ master equation [1], thereby distinguishing our result from [40,44,45].

It is also worth noting that Eq. (19) admits an unraveling into the stochastic Schrödinger equation [45]

$$\dot{\psi}_t = \left( -\frac{i}{\hbar}\hat{H}_S + \sum_i \hat{L}_i \phi_i(t) - \frac{1}{2} \sum_{i,j} a_{ij} \hat{L}_j^\dagger \hat{L}_i \right) \psi_t, \quad (22)$$

with  $\phi_i(t)$  zero-mean Gaussian white noise, and  $\mathbb{E}[\phi_i(t)\phi_j^*(s)] = a_{ij}\delta(t-s)$ ,  $\mathbb{E}[\phi_i(t)\phi_j(s)] = 0$ . This equation contains a momentum-noise coupling term  $\hat{P}\phi(t)$  akin to that phenomenologically introduced in [47]. As such, the inclusion of the next-leading term of the FV kernel  $\sim \delta''(\tau)$  is sufficient to introduce the effective  $\hat{P}$  coupling required to maintain complete positivity in line with Refs. [22–26,49].

**Conclusions**—In this Letter, we have derived a new high-temperature limit of the CL model describing a quantum particle interacting with an Ohmic bath of harmonic

oscillators. The reduced system dynamics in this limit has shown to be captured by a Markovian master equation in Lindblad form, in contrast to the CL master equation used in the conventional white-noise treatment of quantum Brownian motion [5], among other similar works [20–26]. Furthermore, the master equation (19) applies to any form of potential  $V(\hat{X})$ , and does not require an additional momentum coupling to the bath to describe completely positive quantum Brownian motion. Since our result also applies to baths with an ultraviolet cutoff  $\Omega$ , it is likely to be applicable outside the temperature regime stipulated in Refs. [1,5,20] (cf. Table I), provided that the temperature is still large enough for the Born-Markov approximation  $\hbar\gamma \ll k_B T$  to be satisfied. A more formal comparison between these master equations could be explored for a harmonic oscillator system by benchmarking their predictions against the exact HPZ master equation [40]. Furthermore, our result may be systematically extended to lower temperature regimes by including higher order terms within the expansion of the fluctuation paths (13), which is a task left to future work.

**Data availability**—There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

- [1] H.-P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems* (Oxford University Press, New York, 2002).
- [2] U. Weiss, *Quantum Dissipative Systems* (World Scientific, Singapore, 2011).
- [3] E. Y. Wilner, H. Wang, M. Thoss, and E. Rabani, *Phys. Rev. B* **92**, 195143 (2015).
- [4] J. Leppäkangas, J. Braumüller, M. Hauck, J.-M. Reiner, I. Schwenk, S. Zanker, L. Fritz, A. V. Ustinov, M. Weides, and M. Marthaler, *Phys. Rev. A* **97**, 052321 (2018).
- [5] A. Caldeira and A. Leggett, *Physica (Amsterdam)* **121A**, 587 (1983).
- [6] A. O. Caldeira and A. J. Leggett, *Phys. Rev. Lett.* **46**, 211 (1981).
- [7] A. Caldeira and A. Leggett, *Ann. Phys. (N.Y.)* **149**, 374 (1983).

- [8] A. J. Leggett, S. Chakravarty, A. T. Dorsey, M. P. A. Fisher, A. Garg, and W. Zwerger, *Rev. Mod. Phys.* **59**, 1 (1987).
- [9] W. G. Unruh and W. H. Zurek, *Phys. Rev. D* **40**, 1071 (1989).
- [10] W. H. Zurek, S. Habib, and J. P. Paz, *Phys. Rev. Lett.* **70**, 1187 (1993).
- [11] J. P. Paz, S. Habib, and W. H. Zurek, *Phys. Rev. D* **47**, 488 (1993).
- [12] M. Parikh, F. Wilczek, and G. Zahariade, *Int. J. Mod. Phys. D* **29**, 2042001 (2020).
- [13] H. Kleinert, *Path Integrals in Quantum Mechanics, Statistics, Polymer Physics, and Financial Markets* (World Scientific, Singapore, 2009).
- [14] H.-P. Breuer and F. Petruccione, *Phys. Rev. A* **63**, 032102 (2001).
- [15] G. Baym and T. Ozawa, *Proc. Natl. Acad. Sci. U.S.A.* **106**, 3035 (2009).
- [16] R. Feynman and F. Vernon, *Ann. Phys. (N.Y.)* **24**, 118 (1963).
- [17] J. T. Stockburger and C. H. Mak, *J. Chem. Phys.* **110**, 4983 (1999).
- [18] J. T. Stockburger and C. H. Mak, *Phys. Rev. Lett.* **80**, 2657 (1998).
- [19] J. T. Stockburger and H. Grabert, *Phys. Rev. Lett.* **88**, 170407 (2002).
- [20] L. Diósi, *Physica (Amsterdam)* **199A**, 517 (1993).
- [21] L. Diósi, *Europhys. Lett.* **22**, 1 (1993).
- [22] J. Halliwell and A. Zoupas, *Phys. Rev. D* **52**, 7294 (1995).
- [23] S. Gao, *Phys. Rev. Lett.* **79**, 3101 (1997).
- [24] G. W. Ford and R. F. O'Connell, *Phys. Rev. Lett.* **82**, 3376 (1999).
- [25] B. Vacchini, *Phys. Rev. Lett.* **84**, 1374 (2000).
- [26] F. Petruccione and B. Vacchini, *Phys. Rev. E* **71**, 046134 (2005).
- [27] Here, we denote by  $\mathcal{S}'(\mathbb{R})$  the space of tempered distributions over  $\mathbb{R}$ —the continuous dual of the space of Schwartz functions  $\mathcal{S}(\mathbb{R})$ .
- [28] See Supplemental Material <http://link.aps.org/supplemental/10.1103/kp9j-v4ss> for further details on the derivation of Eq. (6), and evaluation of the end terms of the decoherence functional  $S_r[\Delta X]$ .
- [29] M. Xu, V. Vadimov, M. Krug, J. T. Stockburger, and J. Ankerhold, [arXiv:2307.16790](https://arxiv.org/abs/2307.16790).
- [30] M. Xu and J. Ankerhold, *Eur. Phys. J. Spec. Top.* (2025).
- [31] H. Dekker, *Phys. Rev. A* **16**, 2126 (1977).
- [32] H. Dekker, *Phys. Rep.* **80**, 1 (1981).
- [33] V. Gorini, A. Kossakowski, and E. C. G. Sudarshan, *J. Math. Phys. (N.Y.)* **17**, 821 (1976).
- [34] G. Lindblad, *Commun. Math. Phys.* **48**, 119 (1976).
- [35] The “eff” subscript on  $H_{\text{eff}}$  indicates that under a microscopic derivation of Eq. (17), the Hamiltonian contribution can contain additional Lamb shifts and/or frequency renormalization terms not in  $\hat{H}_S$ .
- [36] E. Aurell, R. Kawai, and K. Goyal, *J. Phys. A* **53**, 275303 (2020).
- [37] R. Alicki and K. Lendi, *Quantum Dynamical Semigroups and Applications* (Springer, Berlin, Heidelberg, 2007).
- [38] G. Dvali, G. F. Giudice, C. Gomez, and A. Kehagias, *J. High Energy Phys.* **08** (2011) 108.
- [39] G. Homa, J. Z. Bernád, and L. Lisztes, *Eur. Phys. J. D* **73**, 53 (2019).
- [40] B. L. Hu, J. P. Paz, and Y. Zhang, *Phys. Rev. D* **45**, 2843 (1992).
- [41] W. T. Strunz, *Phys. Lett. A* **224**, 25 (1996).
- [42] L. Diósi and W. T. Strunz, *Phys. Lett. A* **235**, 569 (1997).
- [43] L. Diósi, N. Gisin, and W. T. Strunz, *Phys. Rev. A* **58**, 1699 (1998).
- [44] T. Yu, *Phys. Rev. A* **69**, 062107 (2004).
- [45] W. T. Strunz and T. Yu, *Phys. Rev. A* **69**, 052115 (2004).
- [46] L. Diósi and L. Ferialdi, *Phys. Rev. Lett.* **113**, 200403 (2014).
- [47] L. Ferialdi, *Phys. Rev. Lett.* **116**, 120402 (2016).
- [48] L. Ferialdi, *Phys. Rev. A* **95**, 052109 (2017).
- [49] L. Diósi, *Europhys. Lett.* **30**, 63 (1995).